

EVIDENCE OF NO k-SELECTION IN GAIN SPECTRA OF QUANTUM WELL
AlGaAs LASER DIODES

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PACS: 42.55 Px

Abstract

It is suggested, contrary to present views, that the processes giving rise to radiation in undoped or lightly doped quantum well laser diodes are not subject to a k-selection rule. The reason is contained in the good fit of experimental TE gain spectra which we obtain on the basis of this assumption. This does not rule out the possibility that spectra can in principle be obtained in the future which are subject to the k-selection rule.

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The gain curves as a function of wavelength in single or multiple quantum well lasers can in principle be worked out theoretically by evaluating the emission rate per unit volume per unit energy range at emission energy E . Normally one takes this quantity to be an average over the directions of the photon wavevector k and the two directions $\lambda = 1, 2$ of polarisation. For a review with special reference to this average and the question of k -selection versus no k -selection in the emission process see [1].

In the case of k -selection the net material gain in the active region is

$$g(\eta) = A(\eta) [f_c(\eta + \eta) - f_v(\eta)] - B \quad (1)$$

where $\eta = E/k_B T$, $k_B = 8.617 \times 10^{-5} \text{ eV K}^{-1}$ is the Boltzmann constant and T is the operating temperature. $A(\eta)$ incorporates the matrix element and the density of states factors. The term B allows for losses. We neglect here any η -dependence of B . The functions f_c , f_v are Fermi-Dirac probabilities for electrons in the conduction and valence bands respectively. η is the normalised (i.e. divided by $k_B T$) energy of a hole at k and $\eta + \eta$ the normalised energy of an electron at the same k -value. It is at once clear that f_c decreases and f_v increases with η so that $f_c - f_v$ has a negative slope. In three dimensions the density of states factor in A pulls $g(\eta)$ down to zero at $\eta = \eta$ where $\eta =$

$E_g/k_B T$ is the normalised energy gap and hence $g(\eta)$ has a maximum, the rise at low η being due to the density of states and the drop at larger η being due to the probability term in $g(\eta)$. In two dimensions this changes, since A is now approximately constant, because the density of states are constants [2] and consequently $g(\eta)$ has a negative slope [3,4]. We have in this case taken A to be independent of the emission energy η , and we find broad agreement with Figure 1 of [4], except that downward sloping weakly bowed lines replace the straight lines of this figure. Note that the minimum energy of transition, η , from the conduction band to the heavy hole band is slightly larger than the energy gap, η , of bulk GaAs. This is due to the addition to η of the quantum well ground state energies in each band [5, equation (3)]. This effect is superimposed on the reduction of η by band gap shrinkage in the presence of high carrier concentrations [6] which probably is at least of a similar magnitude in quantum well structures as in the bulk material.

If k -selection has been relaxed because the impurities can take up significant parts of the electronic momentum, or for some other reason, one finds

$$g(\eta) = \int_0^{\eta-\eta} A(\eta) [f_c(\eta + \eta) - f_v(\eta)] d\eta - B = A(\eta) I(\eta) - B \quad (2)$$

In three dimensions the density of states term leads to a rather slower rise in $g(\eta)$, but one has again a maximum which lies nearer the high energy end [1, Figures 2.4 and 2.5]. It has been shown [7] that the maximum lies at

$$\eta_{\max} = \frac{1}{3} [\eta_c + 2(\gamma_e - \gamma_h)]$$

provided that $\gamma_e - \gamma_h - \eta_c \leq 2$, where $\gamma_e = F_e/k_B T$, $\gamma_h = F_h/k_B T$, and F_e , F_h are the quasi-Fermi levels. In two dimensions we take $A(\eta)$ as approximately independent of η . The integral $I(\eta)$ gives exactly [8]

$$I(\eta) = \ln \frac{[1 + \exp(\gamma_e - \eta_c)][1 + \exp(\gamma_h - \eta_v)]}{[1 + \exp(\gamma_e - \eta_v - \eta)][1 + \exp(\gamma_h + \eta - \eta_c)]} \quad (3)$$

where η_c and η_v represent the band edges. $I(\eta)$ goes to zero at $\eta = \eta_c - \eta_v = \eta$. It vanishes again at $\eta = \gamma_e - \gamma_h$, and it has a maximum half-way between these values. The energy positions of the electron and hole quasi-Fermi levels are calculated on the basis of equal numbers of electrons and holes in the quantum well region.

Using this background information we have inspected recently determined quantum well laser gain curves. The symmetry of the three gain curves measured by Dutta et al. for single-quantum-well lasers [9, Figure 1] persuaded us that the results should be representable by (2) using (3), i.e., by two-dimensional emission without k-selection. As

shown in Figure 1, the fit is excellent using the parameters given in Table 1 and $m_c = 0.067m_0$, $m_v = 0.082m_0$, $m_v = 0.45m_0$ [10], where m_0 is the free-electron mass. The superscripts l, h here and in Table 1 label parameters corresponding to light and heavy holes, respectively. The energy levels $E_{n, \alpha}$, $\alpha = c, v$, $n = 1, 2, \dots$ in the conduction and valence bands are obtained from the numerical solution of a basic quantum-mechanical problem of a particle in a symmetrical rectangular potential well [11]. The electron concentration N is calculated as a sum over all subbands corresponding to allowed quantum-well energy levels. The ratio of the barrier heights in the conduction and valence bands is taken as 0.85/0.15 [10, 12], the As content in the cladding layers $x=0.36$ [13], and the thickness of the quantum-well layer $L_z = 210\text{\AA}$ [9]. Only the transitions conforming to the $\Delta n = 0$ selection rule are considered. Contributions from these transitions, each given by the integral (3) with various η_c , η_v determined by E_{cn} , E_{vn} , are summed together to give the material gain. In the wavelength range of Figure 1 only the two lowest levels in each band contribute to the calculated gain spectra. The momentum matrix element is assumed energy-independent with $A^l/A^h = m_v/m_c$. The net optical gain $G(\eta)/\Gamma$ plotted in Figure 1 is given by

$$G(\eta)/\Gamma = AI(\eta) - B' = AI(\eta) - B - [(1-\Gamma)\alpha_c + \alpha_{ext}]/\Gamma,$$

(4)

where r is the optical confinement factor, α_c is the loss in the cladding layer, and α_{ext} is the end mirror loss. The adjustable parameters are A^h , B' (independent of current), effective bandgap E_g and $F_e - F_h$.

The broken line resembling a saw-tooth in Figure 1 is an example of the net optical gain spectrum calculated assuming k -selection (cf Eq. (1)). It is clear that the assumption of k -selection is incompatible with the experimental data.

Another pair of gain curves (TE polarisation) at lower emission energies but also at room temperature was given by Kobayashi et al. in [14, Figure 1]. The no k -selection fit using the data of Table 2 is again good, as shown by curves (a), (b) in Figure 2. The energy levels given in Table 2 correspond to the two types of quantum well present in the laser structure used in Ref. 14. For an inner well, a symmetrical well is assumed with the As-content of barrier layers $x=0.17$. In the case of an outer well, one of the barriers corresponds to a cladding layer with $x=0.26$, hence an asymmetrical potential well has to be considered. The thickness of the well layer is the same in both cases ($L_w = 104\text{\AA}$). Since the laser structure of Ref. 14 consisted of eight well layers, the gain originating from outer and inner wells is weighted as 1:3. In the spectral range of Figure 2, only the lowest levels in each band are involved in the gain calculations.

The broken curve in Figure 2 illustrates again the divergence of the k-selection model and the experimental data. The four maxima of the k-selection gain spectrum correspond to the energy separations of the first conduction band level and the first heavy (light) hole level in the symmetrical well - HS (LS) and in the asymmetrical well - HA (LA).

Our fit (curves (c), (d) in Figure 2) also shows that the measured TM gain spectra given in [14] agree reasonably well with the no-k-selection theory when a polarisation-dependent selection rule is taken into account. According to this rule [14, 15] only the transitions between the light-hole band and the conduction band can contribute to the emission of TM-polarised light from two-dimensional structures, whereas both the light and heavy holes contribute to the TE polarisation. The theoretical curves in Figure 2 confirm that this model can account for the shift of the TM gain peak relative to the TE gain peak. It is worth noting that in order to obtain a good agreement with the measured spectra the constant A^1 has to be taken 3.6 times larger for TM than for TE polarisation. This indicates that the probability of the electron-light hole recombination resulting in the emission of TM-polarised light is 3-4 times larger than for TE-polarised light. Finally, we note that the shift of the TM gain peak compared to the TE gain peak amounts to $\sim 4\text{meV}$,

that is to a half of the separation of the lowest heavy- and light-hole levels in the inner well, and not to the full separation as suggested by Kobayashi et al. [14].

In the above analysis of the multi-quantum-well laser we have neglected the splitting of the levels caused by an interaction of bound states from adjacent wells separated by a very thin barrier [12]. This effect should manifest itself in some broadening of the gain spectrum, hence one can expect even better agreement of the no-k-selection theory with the measured spectra of Figure 2 if a more detailed examination was undertaken.

It should be emphasised that the experimental gain spectra of Refs. 9, 14 have been measured for lasers with lightly p-doped [13] or undoped active layers [14]. The common presumption until now was that the k-selection rule could be relaxed only in heavily doped quantum well structures (see e.g. Refs. 3,4) and it has been argued recently that even under heavy-doping conditions the k-selection rule should be obeyed [16]. A possible explanation of our observation may inhere in a strong carrier-carrier interaction, enhanced compared to conventional lasers by the higher carrier concentrations required at threshold in quantum-well lasers (c.f. Tables 1c, 2c). This interpretation seems to be supported by the fact that the emission spectra of lasers used in Refs. 9, 14 as well as in other quantum-well lasers [17] are shifted towards longer wavelengths than those predicted without any bandgap

shrinkage. We conclude that current ideas that the processes leading to the emission of radiation in undoped or lightly doped quantum wells involve k-selection are probably not always right, and special steps would have to be taken to obtain curves which represent such processes.

We are indebted to Drs. M.J. Adams and M.G. Burt, British Telecom Research Laboratory, for comments and discussions.

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Table 1 Parameters used for the fit of the gain spectra of Dutta et al. [9].

a. Energy levels in meV

n	E_{cn}	E_{vn}	E_{vn}
1	12.7	1.9	10.4
2	50.9	7.6	41.8
3	114.6	17.1	94.0

b. Current-independent parameters

	$A^h(\text{cm}^{-1})$	$A^l(\text{cm}^{-1})$	$B'(\text{cm}^{-1})$
no k-selection fit	74×10^3	13.5×10^3	2.1×10^3
k-selection fit	5×10^3	0.9×10^3	0.2×10^3

c. Current-dependent parameters

	I (mA)	E_g (eV)	$F_e - F_h$ (eV)	$N(\text{cm}^{-3})$
no k-selection fit	67	1.4157	1.5960	5.00×10^{18}
	63	1.4159	1.5954	4.98×10^{18}
	60	1.4159	1.5949	4.97×10^{18}
k-selection fit	67	1.3897	1.5960	5.00×10^{18}

Table 2 Parameters used for the fit of the gain spectra of Kobayashi et al [14]

a. Energy levels in meV

Inner well (symmetric)				Outer well (asymmetric)		
n	E _{cn}	E _{vn}	E _{vn}	E _{cn}	E _{vn}	E _{vn}
1	28.6	4.4	13.2	30.1	4.6	14.4
2	107.9	16.9	-	114.9	17.9	-
3	-	31.7	-	-	-	-

b. Current-independent parameters

Polarisation		A ^h (cm ⁻¹)	A ^l (cm ⁻¹)	B(cm ⁻¹)
TE	no k-selection fit	108 x 10 ³	20 x 10 ³	10
	k-selection fit	50 x 10 ³	9.1 x 10 ³	10
TM	no k-selection fit	0	73 x 10 ³	10

c. Current-dependent parameters

	I (mA)	E _g (eV)	F _e -F _h (eV)	N(cm ⁻³)
no k-selection fit	101	1.3890	1.4645	2.40x10 ¹⁸
	91	1.3903	1.4579	2.20x10 ¹⁸
k-selection fit	101	1.3963	1.4645	2.40x10 ¹⁸

Figure Captions

Figure 1 Experimental points from Ref. 9, a fit of net optical gain spectra calculated from the no k-selection theory (solid lines), and a fit of the spectrum at 67 mA obtained from the strict k-selection theory (broken line).

Figure 2 Experimental points from Ref. 14, a fit of net material gain spectra calculated from the no k-selection theory (solid lines), and a fit of the TE-polarisation gain at 101 mA obtained from the strict k-selection theory (broken line). Curves a,c correspond to $I = 101$ mA, and curves b,d to $I = 91$ mA.