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## I. INTRODUCTION

The beam transfer lines TI 2 and TI 8 will be used to transport particles from SPS to LHC. Their main characteristics have been described in [1]. The geometrical layout is shown in figure 1. TI 2 branches off from the existing transfer line TT60 in the switchyard TCC 6 and ends near LHC point 2. TI 8 starts at LSS 4 and joins the LHC tunnel near LHC point 8. Each line is about 3 km long. The bending sections will use a FODO structure with 4 dipoles per half-cell and a half-cell length of 30.3 m. Figure 2 shows the half-cell layout, together with an enlarged view of a short straight section illustrating the sequence of quadrupole, beam position monitor and corrector magnet. Figure 3 gives a sketch of the present design of the beam position monitors which will re-use the button-type electrodes of the present LEP position monitors [2]. It has not yet been decided whether both planes will be equipped with electrodes (as shown, which improves the resolution in case the beam passes off-plane) or only one (either horizontal or vertical).

The beam parameters (energy 450 GeV, nominal intensity  $2.43 \cdot 10^{13}$  protons per SPS cycle per beam line, transverse emittance  $3.75 \mu\text{m} \cdot \text{rad}$  (r.m.s., normalized) [1]) require that the beams stay absolutely within the available aperture to avoid severe damage. The strongest aperture constraint comes from the main dipoles (MBI) [3] where most of the money and electrical power will go. Their nominal gap height of 25 mm results in a physical full aperture left for the beam of 20.4 mm as laid down in table 1.

From that the maximum allowed trajectory excursion  $E_{\text{max}}$  (after correction) is derived using the formula

$$A/2 = (6 \sigma + D \Delta p/p) k_{\beta} + E_{\text{max}} (\beta/\beta_{\text{max}})^{1/2}$$

where  $A$  is the full physical aperture,  $k_{\beta}$  the optical mismatch factor (1.1),  $\Delta p/p$  the momentum spread (0.1 % [4]), and  $\sigma = (\beta \cdot \varepsilon)^{1/2}$  (with the emittance  $\varepsilon = 0.0072955$ ) (the 6  $\sigma$  include (besides the beam size) kicker ripple, power supply ripple and SPS closed orbit imperfections). This leads to a numerical value of  $E_{\text{max}} = \pm 4.5$  mm in the vertical plane for the main dipoles close to defocusing quadrupoles (which constitutes the strongest constraint). TI 8 contains a special case where  $E_{\text{max}}$  is down to  $\pm 2.3$  mm and where additional beam observation and correction equipment needs to be installed.

Item	Size [mm]
Nominal gap height	25
Mechanical tolerances	$\pm 0.2$
Vacuum tube thickness (including tolerances and insulating foil)	$2 * 1.2$
Sagitta (hang-through)	$\pm 0.5$
Misalignment ( $\pm 2 \sigma$ , $\sigma = 0.2$ mm)	0.8
<b>Remaining physical full aperture A</b>	<b>20.4</b>

**Table 1: Physical vertical aperture in the main dipoles.**

The subject of the investigations described in this paper was to understand how many position monitors and correctors are needed, how they have to be placed and what their specifications have to be to fulfill the above requirement.

After a short description of the method used to calculate the trajectories, the elements contributing to trajectory errors will be reviewed before the results of the various investigations will be presented and discussed.

The analysis is based on the transfer line geometry and optics fitted to LHC version 5.0.

## II. METHOD

To calculate the trajectory assuming random values in all considered error parameters and to obtain the corrector strengths necessary to minimize the excursions the computer code "PATRAC" was used. This FORTRAN program was initially written some 15 years ago for **particle tracking** [5] but has recently been extended to allow also local trajectory correction in transfer lines. Some ad-hoc UNIX scripts and C utilities were added by one of the authors (VM) to facilitate command line input to PATRAC and to speed up result harvesting.

For each error parameter included in the calculation PATRAC defines its variation along the line, starting from one initial random seed per parameter. To obtain a representative sample of parameter permutations 1000 runs per investigated case were performed, each with a different set of the initial seeds. 1000 runs took approximately 11 minutes (CPU) on a HP 9000/735 workstation.

### III. ERRORS

The following errors were included in the calculation of the trajectory: quadrupole displacement, position monitor error and main dipole errors in field and tilt. These will be discussed in more detail below:

#### III.1. Quadrupole Displacement

All quadrupoles were assumed to be displaced in the horizontal and vertical plane, to account for the finite measurement resolution during the alignment, the settling during the pulsed operation and a normal (limited) deformation of the tunnel floor over time. The sum of these contributions was estimated to a  $\sigma$  of 0.2 mm one year after the last realignment [6]. A gaussian distribution was assumed, cut at  $\pm 3 \sigma$  which includes 99.7 % of all cases. Since each line contains not far from 100 quadrupoles the possibility to find a larger displacement can not be excluded, in particular in the case of rapid local floor movements, but it is assumed that these cases can be detected by carefully tracking and analysing the pattern of corrector settings. To be able to accommodate such movements within certain limits a safety margin has been included in the specification of the corrector magnets.

#### III.2. Monitor Error

Each beam position monitor has a limited electrical resolution (partly from the pickup electrode and partly from the electronics), a limited mechanical precision and a possible misalignment with respect to the nominal beam axis, both in the horizontal and vertical plane. All these contributions were lumped up to an error of  $\pm 0.5$  mm [2, 6] with a flat random distribution.

It should be mentioned that the precision of the position measurement in a given plane degrades with the distance of the beam from this plane if no reading in the opposite plane is available (i.e. if only one pair of adjacent pickup electrodes is used). However, since the correction of a trajectory excursion with feedback from a position reading is a converging process this effect can be compensated by iterative correction steps and is therefore disregarded.

#### III.3. Main Dipole Field Error

The main dipoles (MBI) form the largest number of magnets which are powered in series in each line. Field

errors in these magnets can therefore not be corrected by adjusting the individual magnet current. Due to their large number errors in these magnets will contribute perceivably to deviations from the nominal trajectory. All other groups of magnets contain far fewer elements and are therefore neglected.

The core of the MBI magnets consists of a stack of 1.5 mm steel sheets [3]. The specification requires that each magnet stays within  $\pm 5 * 10^{-4}$  of the average field. This will be reached - if the initial field after assembly is found outside the specification - by adding or removing the appropriate number of sheets so to come as close as possible to the nominal field. If the field happens to fall directly inside the tolerance no further effort to reach the nominal field is likely to be undertaken. For the sake of the present calculations the resulting distribution of the deflections is assumed to be gaussian, with a  $\sigma$  of 1.8  $\mu$ rad, cut at  $\pm 2 \sigma$ , which corresponds approximately to  $\pm 5 * 10^{-4}$  of the nominal deflection of 7.5 mrad.

If beneficial and realistic, a sorting of the MBI magnets prior to installation might reduce the contribution from this error type.

#### III.4. Main Dipole Tilt Error

With a width of 0.584 m and a height of 0.359 m, at a core length of 6.3 m, the MBI magnets are relatively prone to distortion around the beam axis, i.e. the direction of the field lines varies along the magnet length. This "tilt" results in additional vertical deflections proportional to the sine of the tilt angle whereas the normal horizontal deflections - proportional to the cosine - stay almost unchanged.

During alignment the mean tilt of a magnet will be calculated from a series of 5 measurements along the magnet, the resolution of each measurement being about 0.2 mrad [6]. The magnet supports will then be adjusted such that each local tilt coincides with the average tilt within the resolution. A possible variation in time has also to be taken into account. For the present calculations the distribution of the tilt error has been assumed to be gaussian with a  $\sigma$  of 1.5  $\mu$ rad ((with  $\sin(0.2 * 10^{-3} \text{ mrad}) \approx 0.2 * 10^{-3}$ ) =  $0.2 * 10^{-3} * 7.5 \text{ mrad}$  (the nominal horizontal deflection)), cut at  $\pm 4 \sigma$ .

#### III.5. Summary of Errors

To allow a better overview the assumed errors are listed again in the following table (they are also repeated on top of all tables in the annex).

Error Component	Value
Quadrupole Displacement	$\pm 3 \sigma$ ( $\sigma = 0.2 \text{ mm}$ )
Monitor Error	$\pm 0.5 \text{ mm}$ , flat distribution
Main Dipole Field Error	$\pm 2 \sigma$ ( $\sigma = 1.8 \mu\text{rad}$ )
Main Dipole Tilt	$\pm 4 \sigma$ ( $\sigma = 1.5 \mu\text{rad}$ )

**Table 2: Overview of error contributions.**

## IV. CORRECTION SCHEMES

The investigations were carried out assuming various correction “schemes”, i.e. different ways to arrange a number of correctors (and their respective beam position monitors) over the length of a beam line.

The following naming convention will be used in the remainder of the text: a “2-in-4” scheme for instance designates a configuration where - independently for each plane - two adjacent short straight sections out of every four are equipped with correctors (likewise “1-in-3”, “2-in-6”, etc.).

Inside the matching sections - which cover about 10 % of each line - a full correction scheme was chosen where all quadrupoles are followed by a corrector. This is particularly important for the final part of each line since the beams have to be delivered to the LHC with great precision.

For maximum sensitivity the position monitors giving the feedback information were placed at 90° phase advance. Horizontal (vertical) corrections were performed at QF (QD) quadrupoles.

## V. RESULTS

In the following the results obtained in the course of the analyses will be presented. As TI 2 and TI 8 show the same qualitative behaviour all detailed results are only given for TI 2.

### V.1. Individual Error Contributions

At first the individual influence of the different errors contributions was investigated. Basis for comparison was the “2-in-4” scheme which implies 62 corrector elements for TI 2 and 48 for TI 8.

The results of this comparison are shown in table 3 which gives the horizontal and vertical minima and maxima of the corrected trajectory excursions and the maximum required deflection over all correctors, over all 1000 runs per investigated case. The cut appearing in this table will be discussed in section V.3. It should be noted that the last two rows of this table, indicated by “All Errors”, give the result of the simulation if all errors contributions are included simultaneously, and are not the result of a quadratic summation of the preceding rows.

The error from the quadrupole misalignment comes out as by far the strongest effect, consuming alone around 75 % of the available aperture budget of  $\pm 4.5$  mm.

If only the monitor errors are included in the calculations the extrema of the excursions are close to  $\pm 0.5$  mm as expected, i.e. the beam is steered onto the misaligned and resolution-limited position monitors thus reproducing the error given as input. The fact that the found extrema are somewhat bigger than  $\pm 0.5$  mm is due

to the working method of PATRAC which uses also other beam observation points than only the position monitors to define the extrema of the excursions.

The contribution from the MBI tilt is of the same order of the one from the field error. As expected field errors of the main horizontal bends have - for TI 2 - only an effect in the horizontal plane. For TI 8 - which contains a number of tilted MBIs - there is some coupling into the vertical plane. However, the tilt error shows only up through additional deflections in the vertical plane. The effect from the small loss of deflection in the horizontal plane (cosine !) remains below the displayed resolution.

### V.2. Variation of Quadrupole Misalignment

The dominant effect of the quadrupoles was investigated further by varying the  $\sigma$  of the distribution of the horizontal and vertical misalignment between 0.2 mm and 0.3 mm, again for the “2-in-4” scheme, including quadrupole displacement as only error. As it can be seen from table 4 the dependence of the resulting trajectory extrema is linear, i.e. the extrema to be expected scale simply with the quality of the alignment. The same holds for the kicks necessary to correct the excursions. From this table it becomes clear that an alignment worse than about  $\sigma = 0.25$  mm can not be tolerated.

### V.3. $3\sigma$ Cut

To get an impression of the significance of the obtained corrected trajectory extrema over 1000 runs their distribution was plotted in figure 4, both for the horizontal and vertical plane, again for the “2-in-4” scheme, including all errors contributions. The distributions are almost gaussian but show some tails towards lower minima and higher maxima, probably due to the mean values which are relatively small compared to the width of the distributions.

There is of course a small but non-vanishing probability to encounter these extreme excursions in the real transfer lines. However, in order not to over-design the correction scheme it may be asked whether these cases could not be detected and appropriate counter-measures be taken to avoid them.

To this end the extreme cases were looked at in greater detail. As example figure 5 shows the corrected vertical trajectory along TI 2 together with the vertical displacements of QF and QD quadrupoles and the kicks of the vertical correctors, for the run (among the 1000 runs) which resulted in the greatest maximum vertical excursion (among all observation points along a line) after correction. For completeness the dashed line gives the trajectory before correction<sup>1</sup>. As can be seen the maximum corrected

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<sup>1</sup> The maximum of the uncorrected trajectories (over 1000 runs) was found at  $\pm 35$  mm.

excursion which PATRAC found at 3.898 mm<sup>2</sup> and some of the other large excursions coincide with large settings of the following correctors (marked with vertical bars). These excursions are caused by large displacements of the immediately upstream quadrupoles, perhaps together with unfavourable contributions from the other error components.

Although apparently neither each large corrector setting indicates necessarily a large upstream quadrupole displacement nor each large quadrupole displacement is necessarily followed by a large excursion, this tendency is taken as sufficient security to assume that these cases can be detected and corrected by quadrupole realignment and therefore as justification to eliminate these cases in the final result. As qualitative criterion the  $\sigma$  of the distribution of the minima and maxima was taken and a cut at 3  $\sigma$  was performed as indicated in figure 4.

The effect of this cut is represented in all tables in the shaded rows immediately following the rows giving the values before the cut. Usually this cut removes only a few percent of all cases. Exceptionally high fractions of cuts occur in circumstances where the distribution of the extrema from the runs is very narrow and their outmost values are anyway far below the critical limit. In these cases the cut is in principle useless but the cut values are nevertheless given for completeness.

To verify the reasonable choice of the numerical cut level table 5 gives the percentage of removed seeds in dependence of the number of  $\sigma$  of the cut. Whereas a cut at 4  $\sigma$  removes practically none of the seeds, a cut at 2  $\sigma$  eliminates with 11 % too many of the obtained cases. As most reasonable cut level 3  $\sigma$  was chosen which removes 1 % of all cases. This value can be interpreted as the probability to find TI 2 after initial alignment in a situation requiring a realignment of a few quadrupoles in order to be able to operate the line correctly.

#### ***V.4. Comparison of Different Correction Schemes***

After these preparatory investigations table 6 gives the synoptic results for several different correction schemes. All errors are included. Figure 6 shows a graphical representation of the horizontal (lower full line) and the vertical (dashed line) maximum excursions after the 3  $\sigma$  cut, in dependence of the number of corrector elements, with the dotted line at 4.5 mm indicating the available aperture budget. For completeness the full line on top gives the corresponding maximum deflection necessary.

As can be seen the “2-in-4” scheme fulfills the aperture requirements with the lowest number of correctors. Despite a considerably larger number of correctors the “2-in-3” scheme does not perform significantly better which is due to the phase advance in the transfer lines of 90°. The additional correctors of a “2-in-3” scheme fall in positions

where they can not be useful. The number of effective correctors is therefore not greater than in the “2-in-4” scheme.

Hence the “2-in-4” scheme is the scheme which fulfills (for both lines) the requirements in the most cost-effective way and thus becomes the preferred scheme.

Table 7 gives the synoptic result for both transfer lines using this “2-in-4” scheme. The overall extreme excursions after the cut are in the horizontal plane 3.598 mm and in the vertical plane 3.517 mm. The overall maximum corrector deflection needed is 62.2  $\mu$ rad. A total of 110 elements is used for correction. The difference in the number of correctors between TI 2 and TI 8 is founded in the different length of the lines and in the fact that TI 2 uses a number of correctors already installed in TT60.

#### ***V.5. Effect of Unusable Position Monitors***

To get a feeling of the tolerance of the transfer lines against malfunctioning of the position monitoring system table 8 gives the trajectory excursions in dependence of the number of missing monitors, for the preferred “2-in-4” scheme. For each number of missing monitors 10 cases of 100 runs each were considered and their extrema calculated. It should be noted that the position of the missing monitors was entirely chosen at random which means that for 2 and more missing monitors the case could occur where two consecutive monitors (even in the same plane) were assumed as missing. This case is however quite unlikely.

It can be seen that the extrema of the excursions degrade sensibly with the increase in the number of missing monitors. Five missing monitors - which correspond for TI 2 to about 8 % of all monitors - move the extrema of the excursions above the tolerable limit. If - as it seems technically desirable [2] - monitors are grouped into sub-stations comprising a number of successive monitors the effect will be even more drastic if such a group fails.

It is however noteworthy that such failures can probably be ignored for a while once the lines have been set up correctly and no re-steering is necessary. Repair can in this case be postponed until the next convenient access. Immediate repair or re-adjustment is only mandatory during the setting up or whenever a modification to the corrector setting has to be made.

#### ***V.6. Effect of Rapid Floor Movements***

To study the effect of a local bump as a result of e.g. an unexpected rapid movement of the tunnel floor a separate investigation was done in which one of the defocusing quadrupoles was systematically displaced in the vertical plane by 1 mm. Table 9 shows that already this relatively small and very localized bump increases sensibly the trajectory excursions.

It is nevertheless hoped that such cases can be detected in the real lines by carefully tracking and analysing the

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<sup>2</sup> This extreme excursion occurs around an area not equipped with a position monitor in this scheme.

pattern of corrector settings. To be able to accommodate floor movements to a certain extent without extraordinary re-alignment some safety margin was incorporated in the specification of the maximum corrector strength.

## VI. SPECIFICATION OF THE CORRECTORS

Table 10 gives the main parameters of the correction dipoles. Two different types (MCIA and MCIB) are specified which will have different gap heights in order to optimize their performance. The specified bending power of 0.08 Tm includes, as already mentioned, some safety margin to be able to continue the operation of a line within certain limits if the tunnel floor starts to move more rapidly than expected.

Whereas TI 2 and TI 8 will together use 110 corrector elements there are currently less than 100 “new” correctors required. The difference is made up by the correction elements already forming part of TT60 and a few beam line dipoles which can also serve as correctors. All “new” correctors will be recuperated from LEP together with their power supplies. Their enormous gap will be reduced by pole pieces with sufficient width to provide a good field.

Further details will be contained in a separate report [7].

## VII. CONCLUSIONS

The expected trajectory excursions of the LHC injection transfer lines TI 2 and TI 8 have been calculated taking into account quadrupole displacements, errors in the beam position monitors, and errors in the field and tilt of the main bending magnets (MBI). The maximum uncorrected excursions found are  $\pm 35$  mm and there is little chance for the beam to reach LHC without correction. In fact TI 2 and TI 8 need a large number of correctors, in view of the beam parameters, their length and their relatively tight aperture.

To keep the corrected trajectory excursions within the limit of  $\pm 4.5$  mm defined by the vertical aperture in the main dipoles a correction scheme is proposed in which two adjacent short straight sections out of every four are equipped with correctors. This scheme requires a total of 110 corrector elements for both lines. Apart from the correctors which form already part of TT60 and a few cases where ordinary bending magnets can also be used as correctors, the bulk of correctors and power supplies will be recuperated from LEP and adapted to its new function. The largest kick found during the simulation (pathological cases discarded) stays below  $65 \mu\text{rad}$ .

The beam position monitors will use button-type electrodes which can also be recuperated from LEP.

The largest contribution to the trajectory errors comes from the displacement of quadrupoles. There is not much tolerance in this parameter which might mean that the quadrupoles will need to be frequently realigned.

It is hoped that rapid movements of the tunnel floor can be detected through a careful tracking of the pattern of

corrector settings. To accommodate such effects up to a certain limit the specified bending power of the correctors (0.08 Tm) contains some safety margin.

It is also found that only a low number (about 5) of missing beam position monitors can be tolerated, at least when a re-steering of the line has to be carried out.

## ACKNOWLEDGEMENTS

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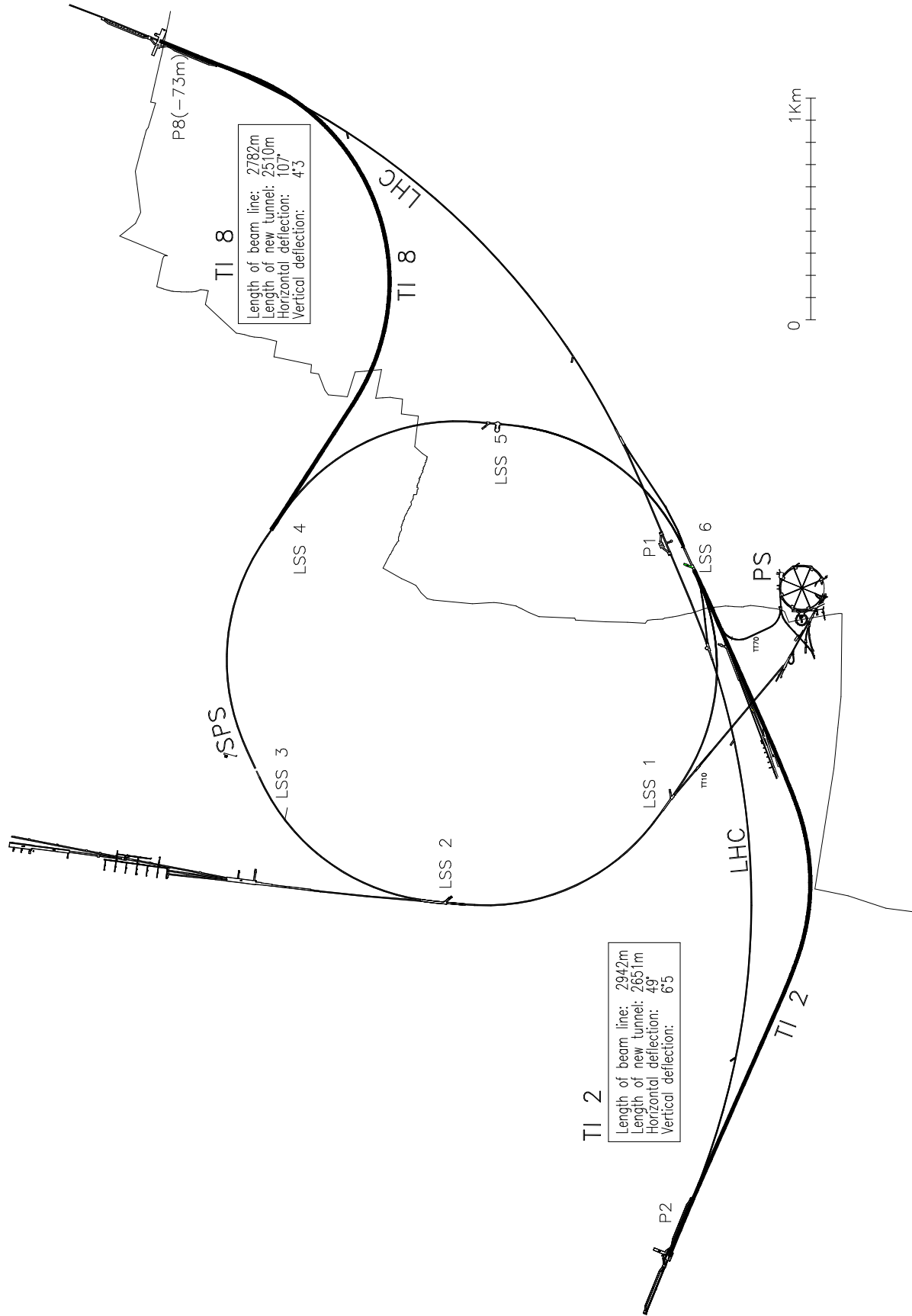
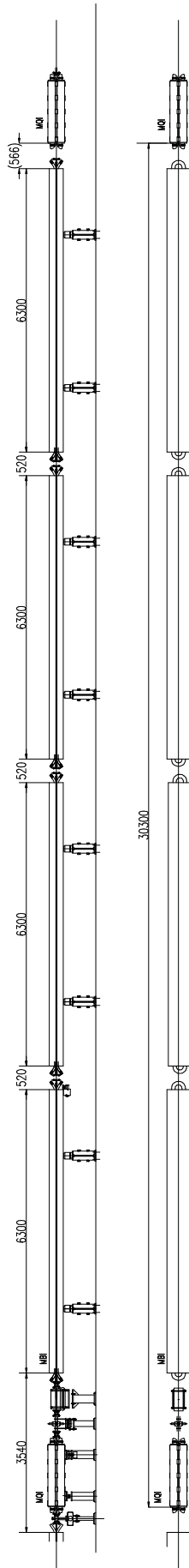


Figure 1: Geometrical layout of the LHC injection transfer lines.

T12 + T18 HALF CELL



T12 + T18 SHORT STRAIGHT SECTION

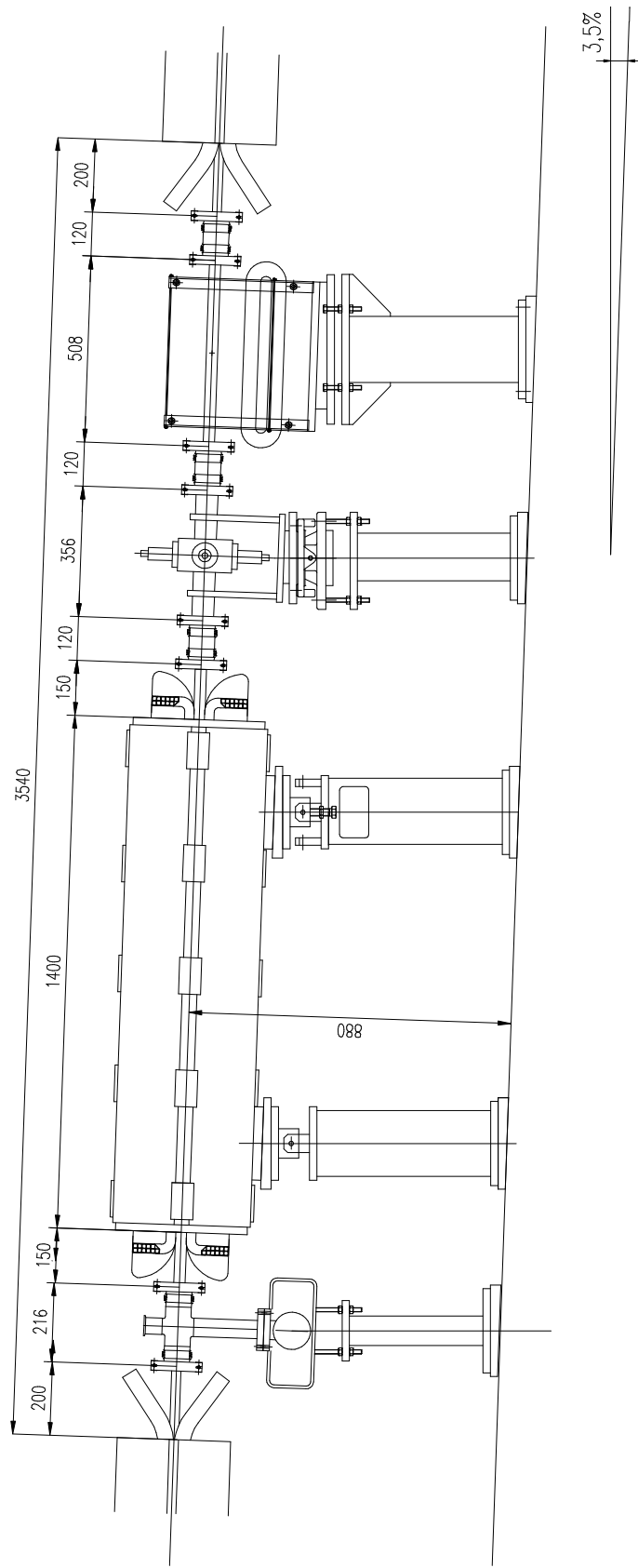


Figure 2: Layout of half cells (above) and short straight sections (below).



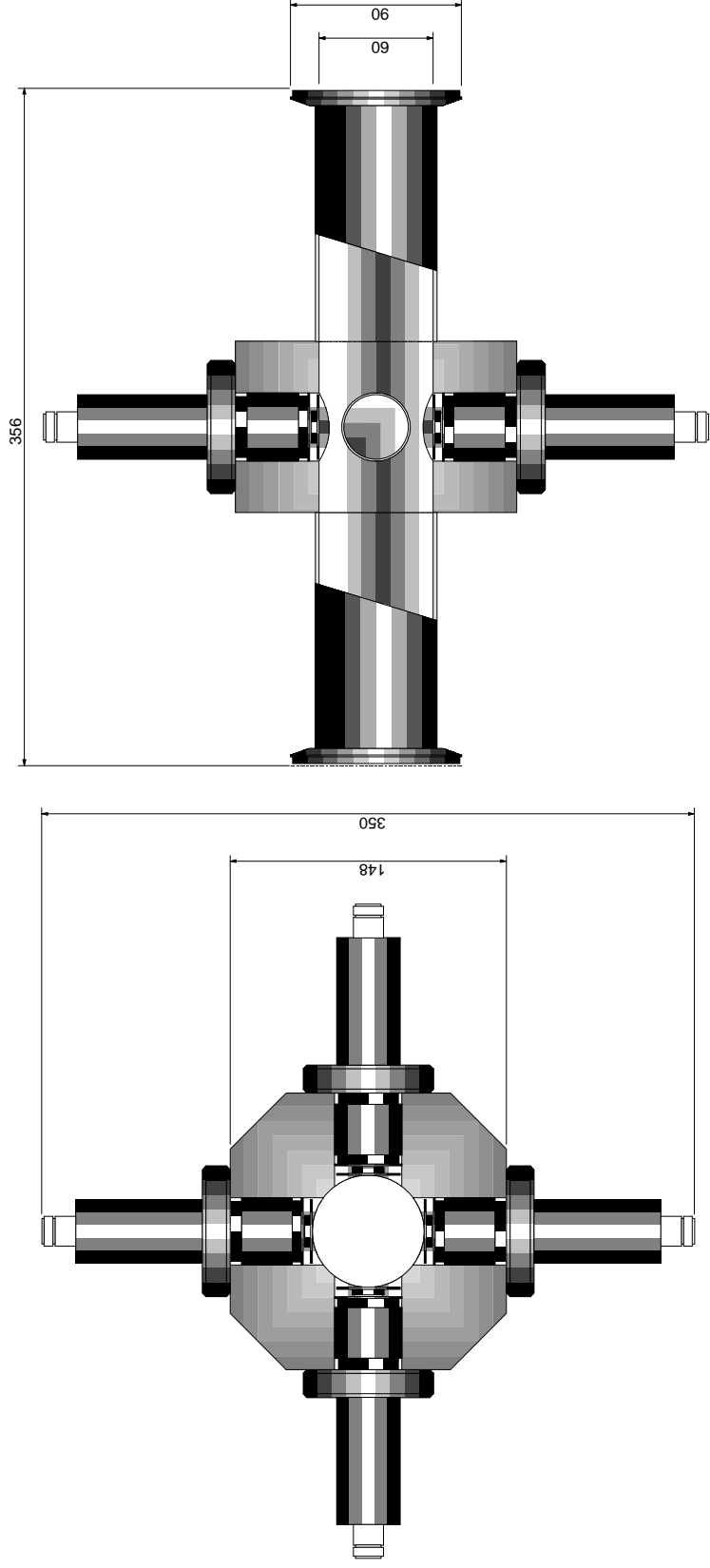


Figure 3: Transverse (left) and longitudinal cut (right) of the beam position monitors.

## TI 2 Trajectory Correction - Individual Error Contributions

Assumed Errors:	
Quadrupole Displacement:	$\pm 3 \sigma$ ( $\sigma = 0.2$ [mm])
Monitor Error (Displacement + Resolution):	$\pm 0.5$ [mm], randomly distributed
MBI Field Error:	$\pm 2 \sigma$ ( $\sigma = 1.8$ [ $\mu$ rad])
MBI Tilt Error:	$\pm 4 \sigma$ ( $\sigma = 1.5$ [ $\mu$ rad])

Cut Criterium: remove seed if Max > Average(Max) + 3  $\sigma$  \* (Maximum(Max) - Average(Max)) (analogously for Min)

Correction Scheme	Error	Cut	Removed Seeds [%]	Number of Correctors			Trajectory Excursions (corrected)				Kicks	
				H	V	$\Sigma$	Horizontal		Vertical		Min	Max
				[ ]	[ ]	[ ]	Min [mm]	Max [mm]	Min [mm]	Max [mm]	Min [ $\mu$ rad]	Max [ $\mu$ rad]
"2-in-4"	Quadrupoles	no	-	31	31	62	-3.379	3.720	-3.478	3.368	-	59.4
		yes	1.0				-3.116	3.282	-3.314	3.310	-	59.4
	Monitors	no	-				-0.586	0.591	-0.635	0.617	-	32.9
		yes	4.1				-0.553	0.549	-0.562	0.557	-	32.9
	MBI Field	no	-				-1.149	1.063	0.000	0.000	-	16.3
		yes	0.6				-0.869	0.830	0.000	0.000	-	16.3
	MBI Tilt	no	-				0.000	0.000	-1.050	1.036	-	16.3
		yes	1.3				0.000	0.000	-0.877	0.913	-	16.3
	All Errors	no	-				-3.677	3.831	-4.072	3.898	-	59.3
		yes	1.0				-3.549	3.423	-3.401	3.517	-	59.3

**Note:** 1000 seeds per case.

**Note:** The rows marked with "All Errors" are obtained if all error contributions are included simultaneously in the simulation (and are not a result of a quadratic summation of the previous rows)

Table 3: Individual error contributions (TI 2 / 2-in-4 scheme) (note that the rows marked with "All Errors" are obtained if all individual error contributions are included simultaneously in the calculation; they are not the result of a quadratic summation of the previous rows).

## TI 2 Trajectory Correction - Dependence of Trajectory Excursions and Corrector Kicks on Quadrupole Displacement

Assumed Error:	
Quadrupole Displacement:	$\pm 3 \sigma$

Cut Criterium: remove seed if  $\text{Max} > \text{Average}(\text{Max}) + 3 \sigma$  \* (Maximum(Max) - Average(Max)) (analogously for Min)

Correction Scheme	$\sigma^4$ [mm]	Cut	Removed Seeds [%]	Number of Correctors			Trajectory Excursions (corrected)		Kicks		
				H	V	$\Sigma$	Horizontal Min [mm]	Horizontal Max [mm]	Vertical Min [mm]	Vertical Max [mm]	Max [ $\mu\text{rad}$ ]
"2-in-4"	<b>0.20</b>	no	-	31	31	62	-3.379	3.720	-3.478	3.368	59.4
	<b>0.25</b>	yes	1.0				-3.116	3.282	-3.314	3.310	59.4
	<b>0.30</b>	no	-				-4.224	4.650	-4.348	4.210	74.3
		yes	1.0				-3.894	4.103	-4.143	4.138	74.3
		no	-				-5.068	5.580	-5.217	5.052	89.2
		yes	1.0				-4.673	4.923	-4.971	4.965	89.2

**Note:** 1000 seeds per case.

Table 4: Dependence of extrema of trajectory excursions and corrector kicks on quadrupole displacement (TI 2 / 2-in-4 scheme).

### TI 2 - Distribution of Minima and Maxima of Corrected Trajectory Excursions (2-in-4 Scheme)

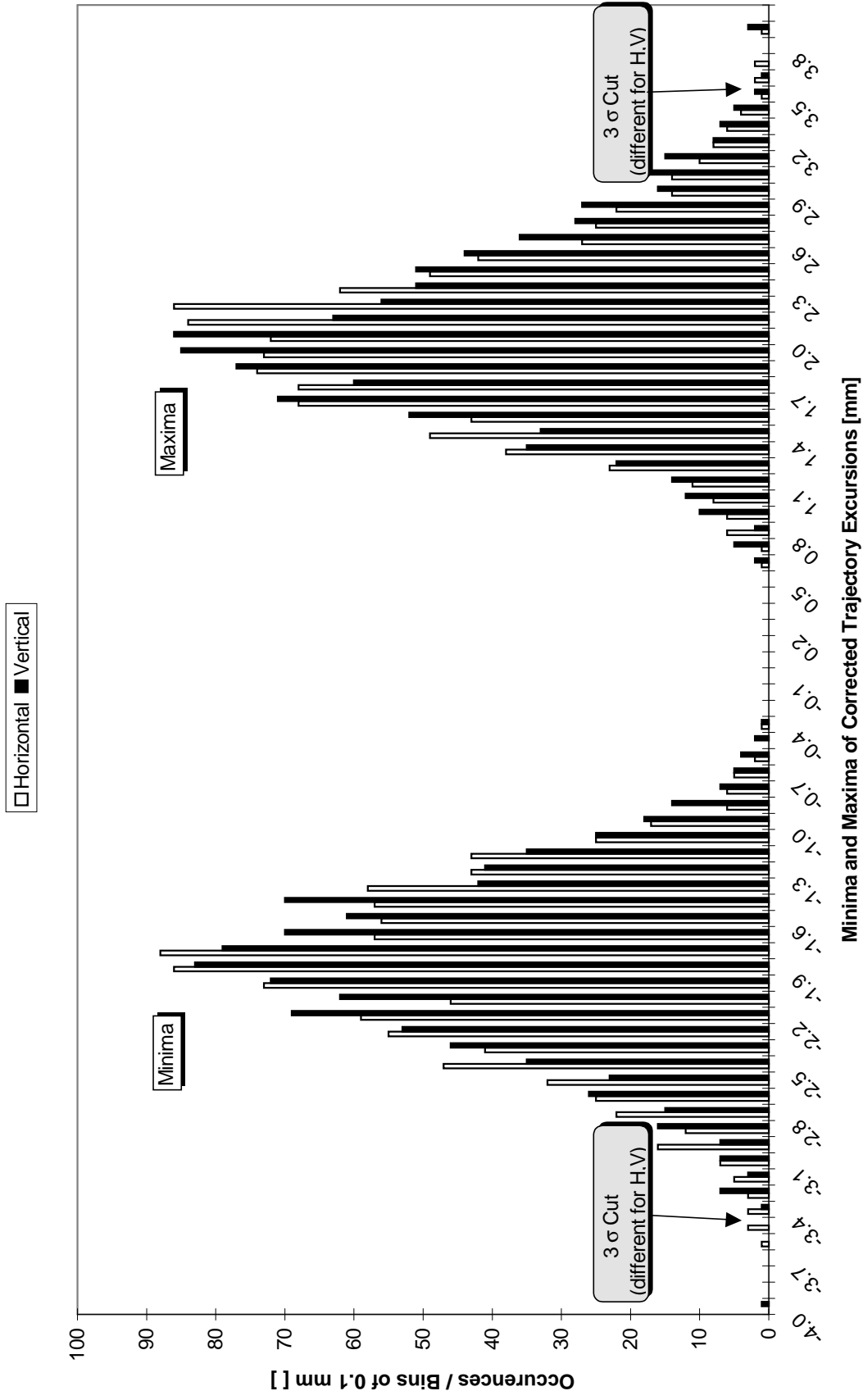


Figure 4: Distribution of minima and maxima of corrected trajectory excursions (TI 2 / 2-in-4 scheme).

TI 2 - Uncorrected and Corrected Trajectories, Quadrupole Displacements and Corrector Kicks for Seed with Maximum Vertical Corrected Excursion (2-in-4 scheme - vertical plane only)

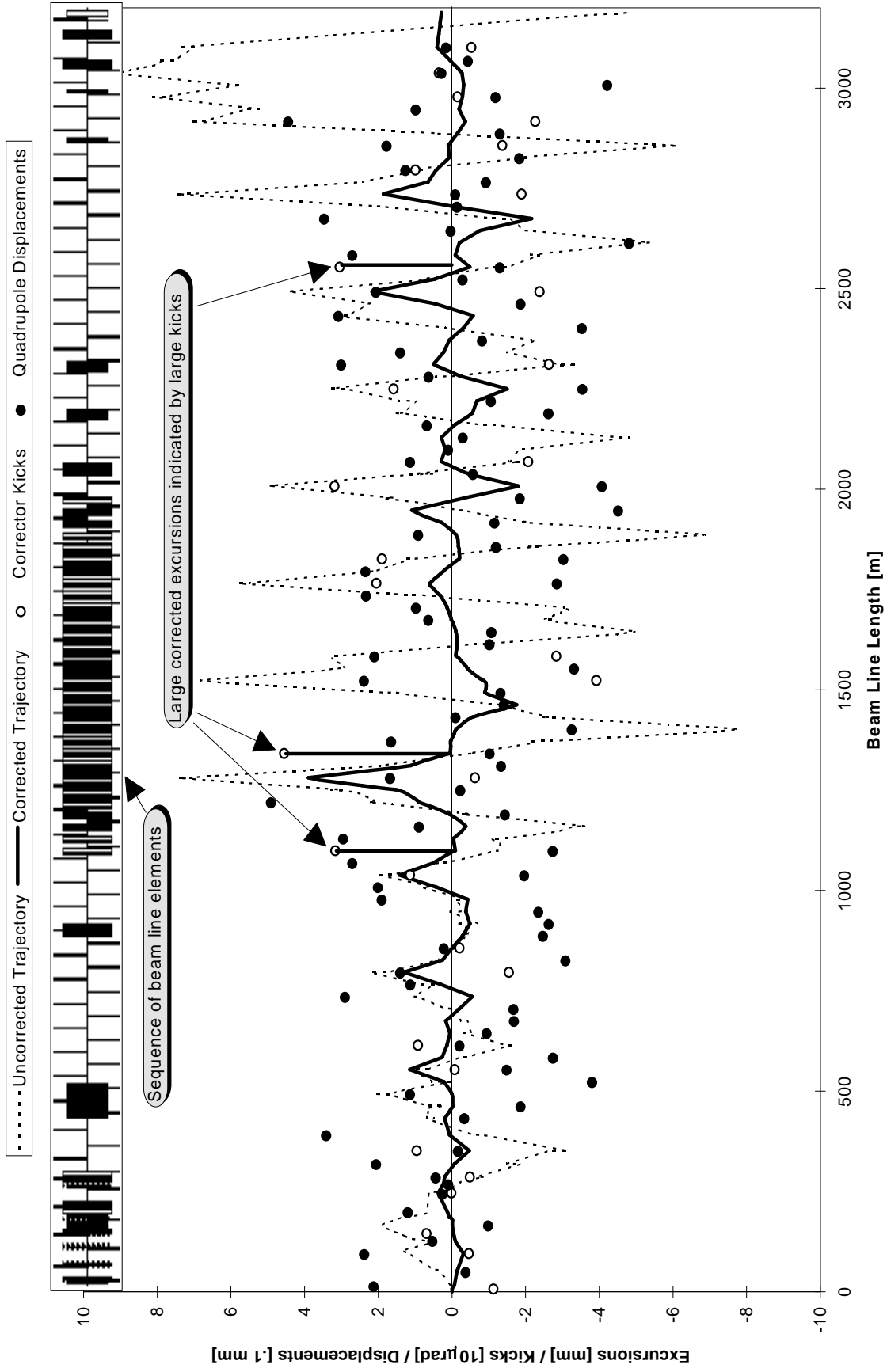


Figure 5: Uncorrected and corrected trajectories, quadrupole displacements and corrector kicks for seed with maximum vertical corrected excursion (TI 2).

## TI 2 Trajectory Correction - Dependence of Trajectory Excursions and Corrector Kicks on $\sigma$ Cut

Assumed Errors:	
Quadrupole Displacement:	$\pm 3 \sigma$ ( $\sigma = 0.2$ [mm])
Monitor Error (Displacement + Resolution):	$\pm 0.5$ [mm], randomly distributed
MBI Field Error:	$\pm 2 \sigma$ ( $\sigma = 1.8$ [ $\mu$ rad])
MBI Tilt Error:	$\pm 4 \sigma$ ( $\sigma = 1.5$ [ $\mu$ rad])

Cut Criterion: remove seed if  $\text{Max} > \text{Average}(\text{Max}) + x \sigma^*$  (Maximum(Max) - Average(Max)) (analogously for Min)

Correction Scheme	Cut	$x$ [ ]	Removed Seeds [%]	Number of Correctors			Trajectory Excursions (corrected)		Kicks Max [ $\mu$ rad]		
				H [ ]	V [ ]	$\Sigma$ [ ]	Horizontal Min [mm]	Horizontal Max [mm]		Vertical Min [mm]	Vertical Max [mm]
"2-in-4"	no	-	-	31	31	62	-3.677	3.831	-4.072	3.898	59.3
	yes	4.0	0.1				-3.677	3.831	-3.401	3.898	59.3
	yes	3.5	0.3				-3.677	3.638	-3.401	3.825	59.3
	yes	3.0	1.0				-3.549	3.423	-3.401	3.517	59.3
	yes	2.5	3.7				-3.295	3.222	-3.256	3.320	59.3
	yes	2.0	11.0				-3.049	2.977	-2.998	3.056	59.3

**Note:** 1000 seeds per run.

Table 5: Dependence of extrema of trajectory excursions and corrector kicks on  $\sigma$  cut (TI 2 / 2-in-4 scheme).

## TI 2 Trajectory Correction - Dependence of Trajectory Excursions and Corrector Kicks on Correction Scheme

Assumed Errors:	
Quadrupole Displacement:	$\pm 3 \sigma$ ( $\sigma = 0.2$ [mm])
Monitor Error (Displacement + Resolution):	$\pm 0.5$ [mm], randomly distributed
MBI Field Error:	$\pm 2 \sigma$ ( $\sigma = 1.8$ [ $\mu$ rad])
MBI Tilt Error:	$\pm 4 \sigma$ ( $\sigma = 1.5$ [ $\mu$ rad])

Cut Criterium: remove seed if  $\text{Max} > \text{Average}(\text{Max}) + 3 \sigma$  (Maximum(Max) - Average(Max)) (analogously for Min)

Correction Scheme	Cut	Removed Seeds [%]	Number of Correctors			Trajectory Excursions (corrected)		Kicks		
			H [ ]	V [ ]	$\Sigma$ [ ]	Horizontal Min [mm]	Horizontal Max [mm]	Vertical Min [mm]	Vertical Max [mm]	Max [ $\mu$ rad]
"2-in-6"	no	-	23	23	46	-5.197	6.014	-5.598	5.304	84.1
"1-in-3"	yes	3.3	24	24	48	-4.532	4.811	-4.804	4.733	84.1
"2-in-4"	no	-	31	31	62	-5.248	5.075	-5.431	5.303	68.9
"2-in-3"	yes	2.9	38	38	76	-4.491	4.672	-4.784	4.755	68.5
H: "2-in-4" / V: full	no	-	31	31	62	-3.677	3.831	-4.072	3.898	59.3
full	yes	1.0	51	51	102	-3.549	3.423	-3.401	3.517	59.3
	no	-	38	38	76	-3.797	3.620	-3.665	3.887	59.3
	yes	1.3	31	31	62	-3.313	3.355	-3.439	3.299	59.3
	no	-	31	31	62	-3.677	3.831	-0.659	0.650	57.0
	yes	4.5	51	51	102	-3.549	3.423	-0.571	0.563	57.0
	no	-	51	51	102	-0.790	0.800	-0.659	0.650	46.2
	yes	9.1	51	51	102	-0.591	0.613	-0.571	0.563	46.2

**Note:** 1000 seeds per scheme.

Table 6: Dependence of extrema of trajectory excursions and corrector kicks on correction scheme (TI 2).

TI 2 - Maximum Trajectory Excursions and Corrector Kicks vs. Number of Correctors (with 3  $\sigma$  cuts)

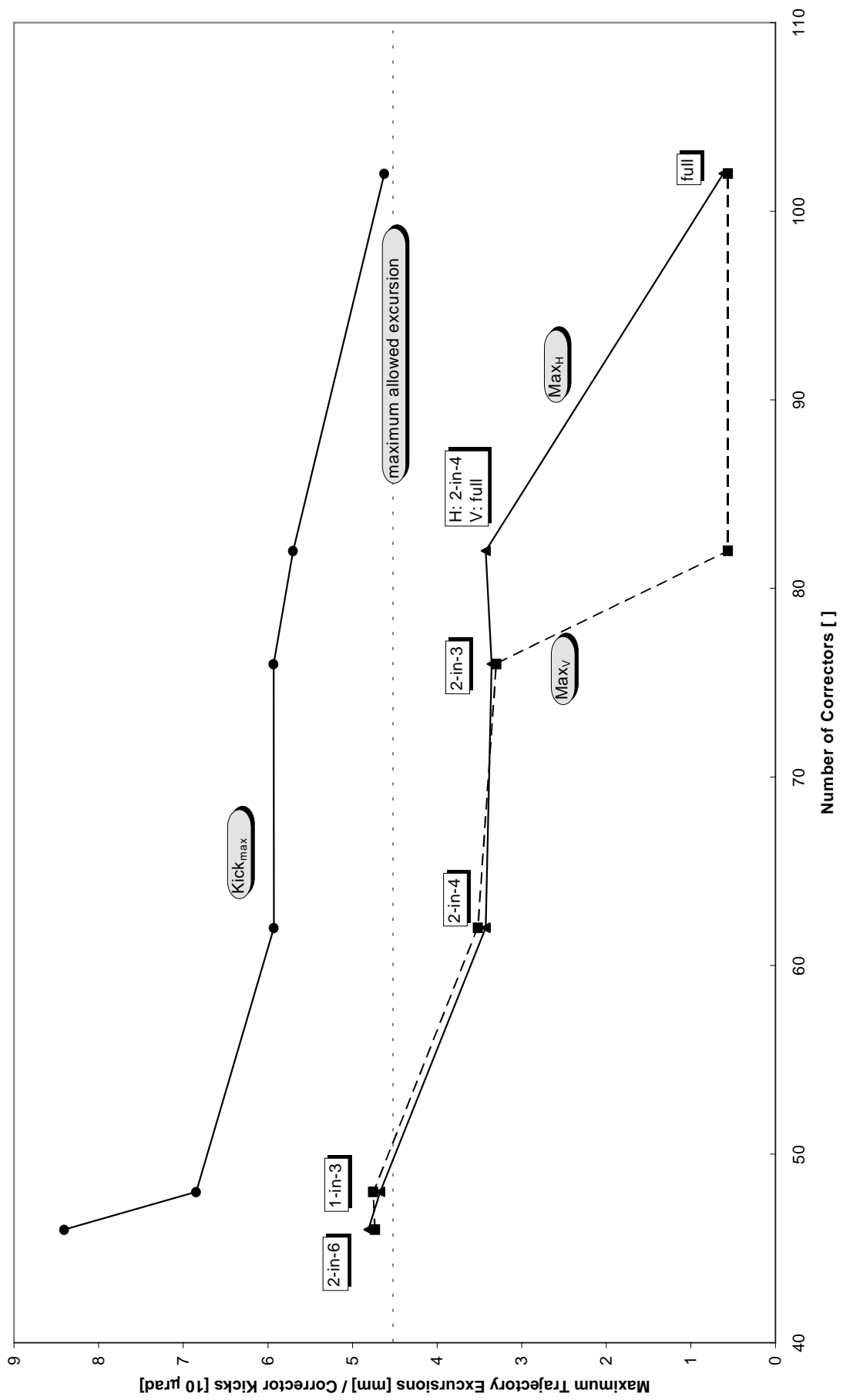


Figure 6: Dependence of maximum trajectory excursions and corrector kicks after cuts on number of correctors for various correction schemes (TI 2).



## TI 2 and TI 8 Trajectory Correction - Overview

Assumed Errors:	
Quadrupole Displacement:	$\pm 3 \sigma$ ( $\sigma = 0.2$ [mm])
Monitor Error (Displacement + Resolution):	$\pm 0.5$ [mm], randomly distributed
MBI Field Error:	$\pm 2 \sigma$ ( $\sigma = 1.8$ [ $\mu$ rad])
MBI Tilt Error:	$\pm 4 \sigma$ ( $\sigma = 1.5$ [ $\mu$ rad])

Cut Criterion: remove seed if  $\text{Max} > \text{Average}(\text{Max}) + 3 \sigma$  \* (Maximum(Max) - Average(Max)) (analogously for Min)

Correction Scheme	Cut	Removed Seeds [%]	Number of Correctors			Trajectory Excursions (corrected)				Kicks Max [ $\mu$ rad]
			H	V	$\Sigma$	Horizontal [mm]		Vertical [mm]		
<b>TI 2</b>										
"2-in-4"	no	-	31	31	62	-3.677	3.831	-4.072	3.898	59.3
	yes	1.0				-3.549	3.423	-3.401	3.517	59.3
<b>TI 8</b>										
"2-in-4"	no	-	23	25	48	-4.053	4.392	-3.892	4.097	66.5
	yes	1.5				-3.598	3.497	-3.488	3.408	62.2
<b>Overall</b>										
			110			-3.598		3.497		62.2

**Note:** 1000 seeds per transfer line.

Table 7: Number of correctors and extrema of trajectory excursions (TI 2 and TI 8 / 2-in-4 scheme).

## TI 2 Trajectory Correction - Dependence of Trajectory Excursions and Corrector Kicks on Number of Missing BPMs

Assumed Errors:	
Quadrupole Displacement:	$\pm 3 \sigma$ ( $\sigma = 0.2$ [mm])
Monitor Error (Displacement + Resolution):	$\pm 0.5$ [mm], randomly distributed
MBI Field Error:	$\pm 2 \sigma$ ( $\sigma = 1.8$ [ $\mu$ rad])
MBI Tilt Error:	$\pm 4 \sigma$ ( $\sigma = 1.5$ [ $\mu$ rad])

Cut Criterium: remove seed if  $\text{Max} > \text{Average}(\text{Max}) + 3 \sigma$  \* (Maximum(Max) - Average(Max)) (analogously for Min)

Correction Scheme	Missing BPMs [ ]	Cut	Removed Seeds [%]	Number of Correctors			Trajectory Excursions (corrected)		Kicks Max [ $\mu$ rad]		
				H [ ]	V [ ]	$\Sigma$ [ ]	Horizontal Min [mm]	Max [mm]		Vertical Min [mm]	Max [mm]
"2-in-4"	0	no	-	31	31	62	-3.677	3.831	-4.072	3.898	59.3
	1	yes	1.0				-3.549	3.423	-3.401	3.517	59.3
	2	no	-			61	-5.006	4.283	-4.498	5.674	73.8
	3	yes	2.3			60	-3.658	3.445	-3.704	3.692	63.4
	4	no	-			59	-5.724	5.260	-4.795	5.153	75.3
	5	yes	3.5			58	-3.994	3.925	-3.880	3.718	60.3
		no	-			57	-6.466	5.462	-6.219	5.011	91.9
		yes	3.9			57	-4.122	4.028	-4.061	4.006	91.9
		no	-			58	-5.674	5.203	-5.879	7.222	70.1
		yes	4.3			57	-3.986	3.977	-4.458	4.306	65.5
		no	-			57	-6.456	5.417	-7.257	6.753	86.5
		yes	4.9			57	-4.053	4.084	-4.551	4.484	86.5

**Note 1:** 10 schemes of missing BPMs \* 100 seeds (= 1000 runs) per case.

**Note 2:** Position of missing BPMs independently chosen at random (i.e. consecutive missing BPMs are possible but unlikely).

Table 8: Dependence of trajectory excursions and corrector kicks on number of missing BPMs (TI 2 / 2-in-4 scheme).

## TI 2 Trajectory Correction - Effect of Local Bump

Assumed Errors:	
Quadrupole Displacement:	$\pm 3 \sigma$ ( $\sigma = 0.2$ [mm])
Monitor Error (Displacement + Resolution):	$\pm 0.5$ [mm], randomly distributed
MBI Field Error:	$\pm 2 \sigma$ ( $\sigma = 1.8$ [ $\mu$ rad])
MBI Tilt Error:	$\pm 4 \sigma$ ( $\sigma = 1.5$ [ $\mu$ rad])

Cut Criterium: remove seed if  $\text{Max} > \text{Average}(\text{Max}) + 3 \sigma$  \* (Maximum(Max) - Average(Max)) (analogously for Min)

Correction Scheme	Cut	Removed Seeds [%]	Number of Correctors			Trajectory Excursions (corrected)				Kicks Max [ $\mu$ rad]
			H	V	$\Sigma$	Horizontal [mm]		Vertical [mm]		
"2-in-4"	no	-	31	31	62	-3.677	3.831	-4.072	3.898	59.3
	yes	1.0				-3.549	3.423	-3.401	3.517	59.3
<b>No Bump</b>										
<b>One QD Displaced Vertically by 1 mm (in area not equipped with Beam Position Monitor)</b>										
"2-in-4"	no	-	31	31	62	-3.677	3.831	-4.323	3.898	62.0
	yes	1.3				-3.549	3.423	-4.072	3.517	62.0

**Note:** 1000 seeds per case.

Table 9: Effect of local bump of 1 mm on trajectory excursions and corrector kicks (TI 2 / 2-in-4 scheme).

Type	Number <sup>1)</sup>	Bending power [Tm]	Gap height [mm]	Good field region <sup>2)</sup> [mm]
MCIA	58	0.08	26	± 20
MCIB	50	0.08	32	± 25

1) comprising 10 % reserve

2) region in which field differs from nominal by not more than 1 %

**Table 10: Number of correctors to be recuperated from LEP (TI 2 and TI 8 / 2-in-4 scheme).**