R-Parity Violating SUSY Signals in Lepton-Pair Production at the Tevatron

J. Kalinowski\textsuperscript{1,2}, R. Rückl\textsuperscript{3,\ast}, H. Spiesberger\textsuperscript{4,\ast}, and P.M. Zerwas\textsuperscript{1}

\textsuperscript{1} Deutsches Elektronen-Synchrotron DESY, D-22607 Hamburg
\textsuperscript{2} Institute of Theoretical Physics, Warsaw University, PL-00681 Warsaw
\textsuperscript{3} Institut für Theoretische Physik, Universität Würzburg, D-97074 Würzburg
\textsuperscript{4} Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld

ABSTRACT

In supersymmetric theories with $R$-parity breaking, sleptons can be produced in quark-antiquark annihilation at the Tevatron through interactions in which two quark fields are coupled to a slepton field. If at the same time trilinear slepton-lepton-lepton couplings are present, the sleptons can be searched for as resonances in $p\bar{p} \rightarrow \tilde{\nu} \rightarrow l^+l^-$ and $\tilde{\tau} \rightarrow l\nu$ final states. Existing Tevatron data can be exploited to derive bounds on the Yukawa couplings of sleptons to quark and lepton pairs. Similar bounds can also be obtained from $e^+e^-$ annihilation to hadrons at LEP2.

\* Supported by Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, Bonn, Germany, Contracts 05 7BI92P (9) and 05 7WZ91P (0).
1. Supersymmetric theories may include interactions with $R$-parity breaking [1, 2]. In one of the possible scenarios, the superpotential involves terms in which three lepton superfields and lepton-quark-quark superfields are coupled:

$$W_R = \lambda_{ijk} L_i L_j E^c_{k} + \lambda'_{ijk} L_i Q_j D^c_{k}$$

(1)

The indices $ijk$ denote the generations; the couplings $\lambda_{ijk}$ are non-vanishing only for $i < j$. The left-handed doublets of the leptons are denoted by $L$, the right-handed singlets by $E$, correspondingly the quark doublets by $Q$ and the quark singlets by $D$. Both terms violate the lepton number, yet conserve the baryon number. In four-component Dirac notation the Yukawa interactions have the following form

$$\mathcal{L}_R = \lambda_{ijk} \left[ \bar{\nu}_L^i \tilde{e}_R^k \ell_L^i + \bar{e}_R^k (\tilde{\nu}_L^i)^c \nu_L^i + \bar{\ell}_L^i \tilde{e}_R^k \nu_L^i - \bar{\nu}_L^i \tilde{e}_R^k \ell_L^i - \bar{\nu}_L^i (\tilde{\nu}_L^i)^c \nu_L^i - \bar{\ell}_L^i \tilde{e}_R^k \nu_L^i \right] + h.c. \quad (2)$$

In the second line, the up (s)quark fields in the first parentheses and/or the down (s)quark fields in the second may be Cabibbo rotated in the mass-eigenstate basis. However, as we will discuss mainly sneutrino induced processes, we will assume the basis in which only the up sector is mixed (i.e., $NDD^c$ is “diagonal”). We will add comments on the other choice where relevant for the discussion.

This scenario can be explored in various processes. The coupling $\lambda LLE^c$ could give rise to sneutrino production in $e^+e^-$ annihilation at LEP2 [3-7], the coupling $\lambda' LQD^c$ to squark production in positron-quark collisions at HERA [8]. Moreover, this interaction could lead to the formation of sneutrinos and charged sleptons in quark-antiquark annihilation at the Tevatron [3, 9]. Even though sneutrinos and charged sleptons are expected to have small widths, it is difficult to extract the signal in the hadronic environment from jet decays of these particles since the production rate is small and the background of QCD jets is large. However, if both interaction terms are present in the superpotential (1), the sneutrinos and charged sleptons may decay also to pairs of leptons:

$$pp \rightarrow \tilde{\nu} \rightarrow \ell^+ \ell^- \quad (3)$$
$$pp \rightarrow \tilde{\ell} \rightarrow \ell \nu \quad (4)$$

If the $R$-parity violating couplings $\lambda, \lambda'$ are taken consistent with current upper limits from low-energy experiments, such effects could be observed for sufficiently low masses of the sleptons. In any case, the existing data for lepton pair production can be used to derive new interesting bounds on the two types of Yukawa couplings.

The phenomenological analysis of the processes (3) and (4) is the main subject of this letter. We will also include a discussion of the mirror process $e^+e^- \rightarrow q\bar{q}$, which can be studied in hadron production at LEP2\(^1\).

\(^1\)Note that the existence or non-existence of squarks in the HERA range [10] is not linked directly to the production of sleptons in the Tevatron and LEP2 range; sleptons are generally expected to be lighter than squarks.
2. The coexistence of the lepton-number violating couplings \( \lambda \) and \( \lambda' \) is not forbidden by the non-observation of proton decay. However, since they may induce FCNC processes (meson mixing and decays that are suppressed or forbidden in the SM), strong bounds on these couplings have been established from low-energy and LEP1 experiments [11].

In SUSY GUT scenarios it is generally expected that the third generation sleptons and squarks are lighter than sfermions of the first two generations. Moreover, due to the large mass of the top quark, the flavor violation might be expected maximal in the third generation. Low-energy limits for the third generation sfermions are not very restrictive. Therefore we will concentrate on effects due to third generation sleptons. In Ref. [6] the possibility of \( \tilde{\nu}_\tau \to b\bar{b} \) decays, induced by the \( \lambda'_{333} \) coupling, has been discussed in the context of the process \( e^+e^- \to b\bar{b} \). However, for analyses at the Tevatron, the case \( \lambda'_{311} \neq 0 \) is more interesting since it allows for sneutrino resonance formation in valence quark collisions.

<table>
<thead>
<tr>
<th>Process</th>
<th>Coupling</th>
<th>Limit</th>
<th>For j = 1, 2</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K \to \pi \nu \nu )</td>
<td>( \lambda'_{ij} )</td>
<td>( 0.012 r_{d_R} )</td>
<td></td>
<td>[12]</td>
</tr>
<tr>
<td>( K\bar{K} ) mixing</td>
<td>( \lambda'_{ij} )</td>
<td>( 0.08 [r_{d_R}^{-2} + r_{d_R}^{-2}]^{-1/4} )</td>
<td>j = 1, 2</td>
<td>[12]</td>
</tr>
<tr>
<td>( \tau \to \pi \nu_\tau )</td>
<td>( \lambda'_{31k} )</td>
<td>( 0.16 r_{d_R} )</td>
<td></td>
<td>[13]</td>
</tr>
<tr>
<td>( D\bar{D} ) mixing</td>
<td>( \lambda'_{ij} )</td>
<td>( 0.16 [r_{l_i}^{-2} + r_{d_R}^{-2}]^{-1/4} )</td>
<td>j = 1, 2</td>
<td>[12]</td>
</tr>
</tbody>
</table>

Table 1: Limits relevant for the \( \lambda'_{311} \) coupling (see text); \( r_i = m_i/100 \) GeV.

Individual bounds\(^2\) relevant for \( \lambda'_{311} \) are summarized in Table 1. The limits from \( K \) decay and \( K\bar{K} \) mixing have been derived for the basis in which \( NDD^c \) is diagonal; if \( EUD^c \) is diagonal, limits can be derived from \( D\bar{D} \) mixing.

The bound on the coupling \( \lambda_{131} \), which determines the \( \tilde{\nu}_\tau \to e^+e^- \) and \( \tilde{\tau} \to e\nu_e \) decay rates, has recently been updated to the value \( \lambda_{131} \leq 0.08 \) for a mass scale of 200 GeV [7]. For the coupling \( \lambda_{232} \), which determines the \( \tilde{\nu}_\tau \to \mu^+\mu^- \) and \( \tilde{\tau} \to \mu\nu_\mu \) decay rates, an upper limit of 0.08 has been established in Ref. [4]. In summary, present low-energy data allow the product of couplings relevant for the processes (3) and (4) to be of the order \( \lambda_{33i}\lambda'_{311} \leq (0.05)^2 \) for \( i = 1 \) and 2. In the following analysis we will consider a specific scenario in which we take \( \lambda_{131}\lambda'_{311} = (0.05)^2 \) to illustrate the impact of possible sneutrino and charged slepton resonance formation on \( e^+e^- \) or \( e\nu_e \) production at the Tevatron. The same results hold for \( \mu^+\mu^- \) or \( \mu\nu_\mu \) production if \( \lambda_{232}\lambda'_{311} = (0.05)^2 \) is assumed.

\(^2\)The couplings \( \lambda'_{ijk} \) derived from loop induced neutrino masses are strongly constrained only for large \( \tan \beta \) [14].
3. In supersymmetric theories with $R$-parity breaking of the type (1), lepton pairs are produced in $pp$ collisions, in addition to the standard Drell-Yan processes, primarily through $s$-channel sneutrino and charged slepton exchanges if the masses are small enough, Fig. 1:

Figure 1: Mechanisms in the scenario $\lambda_{131}^{'} \lambda_{311}^{'} \neq 0$ (first column) and $\lambda_{131}^{'} \neq 0$ (second column) for (a) $d\bar{d} \rightarrow e^+e^-$ scattering including $s$-channel exchange of $\tilde{\nu}_\tau$ and $t$-channel exchange of $\tilde{t}_L$; (b) $u\bar{u} \rightarrow e^+e^-$ scattering including $u$-channel exchange of $\tilde{b}_R$; and (c) $u\bar{d} \rightarrow e^+\nu$ scattering including $s$-channel exchange of $\tilde{\tau}$ and $u$-channel exchange of $\tilde{b}_R$. The Drell-Yan processes are shown in the third column.

The $t$- and/or $u$-channel exchanges of squarks also contribute to the cross sections; they interfere with the Standard Model $\gamma, Z$ exchanges. However the Yukawa coupling $\lambda_{131}^{'}$ is expected to be so small ($\lesssim 0.05$) that these mechanisms will not play an important role in practice\(^3\). Since sleptons are produced in collisions of quarks and antiquarks with the same helicities while the helicities are opposite in the Drell-Yan processes, the amplitudes for the slepton signals and the Drell-Yan backgrounds do not interfere and the total cross sections are the incoherent sum of the individual cross sections. For electron-pair production they

\(^3\)The complete expressions for the cross sections, including the contributions from squark exchange, are given in the appendix.
read at the parton level

\[
\frac{d\hat{\sigma}}{d\cos \theta}[q\bar{q} \rightarrow e^+e^-] = \frac{d\hat{\sigma}[\tilde{\nu}_\tau]}{d\cos \theta} + \frac{d\hat{\sigma}[\gamma, Z]}{d\cos \theta} \tag{5}
\]

The sneutrino and antisneutrino resonances contribute only to \(d\bar{d}\) annihilation

\[
\frac{d\hat{\sigma}[\tilde{\nu}_\tau]}{d\cos \theta} = \frac{1}{3} \frac{\pi \alpha^2 \hat{s}}{4} \frac{(\lambda_{131}\lambda'_{311}/e^2)^2}{(\hat{s} - m_{\tilde{\nu}_\tau}^2)^2 + \Gamma_{\tilde{\nu}_\tau}^2 m_{\tilde{\nu}_\tau}^2} \tag{6}
\]

The second term in (5) is due to the standard Drell-Yan \(\gamma, Z\) exchange diagrams. The factor \(\frac{1}{3}\) in front of (6) [and (8)] accounts for the matching of the initial quark/antiquark colors when the standard luminosity distributions are used later in Eq. (9). \(\sqrt{\hat{s}}\) is the center-of-mass energy of the \(q\bar{q}\) system.

For electron-neutrino final states, the cross section at the parton level can be written in a similar form

\[
\frac{d\hat{\sigma}}{d\cos \theta}[u\bar{d} \rightarrow e^+\nu_e] = \frac{d\hat{\sigma}[\tilde{\tau}]}{d\cos \theta} + \frac{d\hat{\sigma}[W]}{d\cos \theta} \tag{7}
\]

where the \(\tilde{\tau}\) contribution is given by

\[
\frac{d\hat{\sigma}[\tilde{\tau}]}{d\cos \theta} = \frac{1}{3} \frac{\pi \alpha^2 \hat{s}}{8} \frac{(\lambda_{131}\lambda'_{311}/e^2)^2}{(\hat{s} - m_{\tilde{\tau}}^2)^2 + \Gamma_{\tilde{\tau}}^2 m_{\tilde{\tau}}^2} \tag{8}
\]

while the second term is due to the Drell-Yan \(W\)-exchange diagram. The angular distribution of the final state leptons is isotropic for scalar \(\tilde{\nu}_\tau\) or \(\tilde{\tau}\) production in the slepton center-of-mass frame.

For \(p\bar{p} \rightarrow e^+e^-\) and \(e\nu_e\) the differential cross sections are obtained by combining the parton cross sections with the luminosity spectra for quark-antiquark annihilation

\[
\frac{d^2\sigma}{dM_{\ell\ell}dy}[p\bar{p} \rightarrow \ell_1\ell_2] = \sum_{ij} \frac{1}{1 + \delta_{ij}} \left( f_{i/p}(x_1)\bar{f}_{j/\bar{p}}(x_2) + (i \leftrightarrow j) \right) \hat{\sigma}[ij \rightarrow \ell_1\ell_2] \tag{9}
\]

where \(\ell_1\ell_2 = e^+e^-\) or \(e^+\nu\), \(x_1 = \sqrt{\tau}e^y\), \(x_2 = \sqrt{\tau}e^{-y}\). \(M_{\ell\ell} = (\tau s)^{1/2} = (\hat{s})^{1/2}\) is the mass and \(y\) the rapidity of the lepton pair. The probability to find a parton \(i\) with momentum fraction \(x_i\) in the (anti)proton is denoted by \(f_{i/p(\bar{p})}(x_i)\). When the cross sections are compared with the data from the CDF Collaboration [15], the CTEQ3L parametrization [16] is used together with a multiplicative \(K\) factor for the higher order QCD corrections to Drell-Yan pair production; this follows the procedure of Ref. [15]. Since the corresponding \(K\) factor for slepton production has not been determined yet, the couplings \(\lambda\lambda'\) are theoretically uncertain at a level of about 10% which is tolerable at the present stage of the analysis.

The total cross sections for sneutrino and charged slepton production in the \(e^+e^-\) and \(e\nu_e\) channels at the Tevatron are shown as a function of the slepton mass in Fig. 2. The
Figure 2: The cross sections for sneutrino ($\tilde{\nu}_\tau$) and antisneutrino ($\bar{\tilde{\nu}}_\tau$) and stau ($\tilde{\tau}$) production at the Tevatron, including branching ratios to lepton-pair decays, as a function of the slepton mass. Parameters: $\lambda_{131} \lambda'_{311} = (0.05)^2$, $\Gamma_{\tilde{\nu}_\tau} = \Gamma_{\tilde{\tau}} = 1$ GeV.

Scalar particles are assumed to belong to the third generation, $\tilde{\tau}$ and $\tilde{\nu}_\tau$, and the two Yukawa couplings are taken to be $\lambda_{131} \lambda'_{311} = (0.05)^2$ for illustration, compatible with the bounds derived at low energies. The widths of $\tilde{\nu}_\tau$ and of $\tilde{\tau}$ have been set to the typical value of $\Gamma = 1$ GeV, corresponding to branching ratios of $O(0.01)$ for leptonic decays.

The size of the peak in the distribution of the di-electron invariant mass $M_{ee}$ is depicted in Fig. 3. Following again CDF practice, the differential cross section has been integrated over the range $|y| < 1$ and subsequently divided by 2 to account for two units of the rapidity. While the full curve represents the distribution for an ideal detector, the dashed curve demonstrates the smearing of the peak by experimental resolution, characterized by a Gaussian with a width of 5 GeV. The transverse momentum distributions of the electrons in the decay processes $\tilde{\nu}_\tau \to e^+e^-$ and in $\tilde{\tau} \to e\nu$ develop Jacobian peaks, which however are smeared out by the non-zero widths of the sfermions and QCD radiative corrections in the initial state.

In the same figure the dilepton spectrum for $p\bar{p} \to e^+e^-$, measured by the CDF Collaboration at the Tevatron [15], is compared with the prediction of the Standard Model modified by the presence of a hypothetical sneutrino resonance at 200 GeV. Assuming the sneutrino contribution to be smaller than the experimental error of the data points, the
product of the Yukawa couplings can be estimated to be

\[(\lambda_{131}\lambda'_{311})^{1/2} \lesssim 0.08 \gamma^{1/4}\]  

(10)

for sneutrino masses in the range 120 – 250 GeV. The decay width of the sneutrino enters the estimate only weakly through the fourth root of the parameter \(\gamma\) which denotes the sneutrino width in units of GeV.

4. The same product \(\lambda_{131}\lambda'_{311}\) can also be studied in \(e^+e^-\) annihilation to hadrons at LEP2. Neglecting the small contribution from the \(t/u\)-channel squark exchange the cross section can be written as the incoherent sum of the sneutrino and the Standard Model \(\gamma, Z\) contributions:

\[
\frac{d\hat{\sigma}}{d\cos\theta}[e^+e^- \rightarrow q\bar{q}] = \frac{d\hat{\sigma}[\tilde{\nu}_\tau]}{d\cos\theta} + \frac{d\hat{\sigma}[\gamma, Z]}{d\cos\theta}
\]  

(11)
The sneutrino and antisneutrino resonances contribute only to the $d\bar{d}$ final state:

$$\frac{d\sigma[\tilde{\nu}]}{d\cos \theta} = \frac{3\pi\alpha^2 s}{4} \frac{(\lambda_{131}'\lambda_{311}/e^2)^2}{(s - m_{\tilde{\nu}}^2)^2 + \Gamma_{\tilde{\nu}}^2 m_{\tilde{\nu}}^2}$$  \hspace{1cm} (12)$$

Figure 4: Cross section for the process $e^+e^- \to$ hadrons as a function of the total energy. The prediction of the Standard Model $\gamma,Z$ exchange is given by the dotted line. The contribution of a potential $\tilde{\nu}_\tau$ (and $\tilde{\nu}_\tau$) resonance is superimposed in the full curve. Parameters: $\lambda_{131}\lambda_{311}' = (0.05)^2$, $\Gamma_{\tilde{\nu}} = 1$ GeV.

The total hadronic cross section for $e^+e^-$ annihilation is presented in Fig. 4, again for $\lambda_{131}\lambda_{311}' = (0.05)^2$ and for a sneutrino mass of $m_{\tilde{\nu}} = 200$ GeV. The angular distribution of the $d$ and $\bar{d}$ jets is nearly isotropic on the sneutrino resonance. As a result, the strong forward-backward asymmetry in the Standard Model continuum, $A_{FB} \sim 0.65$ at $\sqrt{s} = 200$ GeV, is reduced to $\sim 0.03$ on top of the sneutrino resonance. If the total cross section $\sigma[e^+e^- \to hadrons]$ can be measured to an accuracy of about 1% at $\sqrt{s} = 184$ (192) GeV, the Yukawa couplings for a 200 GeV sneutrino can be bounded to

$$(\lambda_{131}\lambda_{311}')^{1/2} \lesssim 0.072 \ (0.045)$$  \hspace{1cm} (13)$$

This bound compares well with the bound Eq. (10) which has been derived from the Tevatron data. Since the limit (13) is estimated for energies much below the resonance, $|\sqrt{s} - m_{\tilde{\nu}}| \ll \Gamma_{\tilde{\nu}}$, it is independent of the sneutrino width. If the light sneutrino exists, the
Yukawa coupling $\lambda_{131}$ can be measured separately in Bhabha scattering $e^+e^- \rightarrow \nu_\tau \rightarrow e^+e^-$ at LEP2, cf. Ref. [7], which in turn would allow to derive limits on $\lambda'_{311}$.

5. In conclusion, the simultaneous presence of lepton number violating couplings $\lambda$ and $\lambda'$ can manifest itself in a large number of different reactions. We have shown that taking the product of couplings $\lambda_{131}\lambda'_{311}$ according to the present limits from low-energy experiments, an excess in lepton-pair production at the Tevatron could be observed for sufficiently low masses of sleptons. Conversely, if no excess of events in $p\bar{p} \rightarrow e^+e^-$ or $e\nu$ are observed, stringent bounds on products of the $\lambda\lambda'$ Yukawa couplings can be extracted. Similar sensitivities to the size of Yukawa couplings are expected in $e^+e^-$ annihilation to hadrons. Combining these limits with bounds on $\lambda$ couplings from purely leptonic processes in $e^+e^-$ collisions, individual constraints on $\lambda$ and $\lambda'$ couplings can be established in an experimentally direct way.

Acknowledgments

We are grateful to G. Bhattacharyya, J. Conway, S. Eno, A.S. Joshipura and Y. Sirois for discussions. Communications by K. Maeshima and W. Sakumoto concerning the CDF data points are gratefully acknowledged.

Appendix

a) $q\bar{q} \rightarrow \ell_1\ell_2$

The differential cross section for the process $q\bar{q}^{(c)} \rightarrow \ell_1\ell_2$ in the quark-antiquark rest frame can be written most transparently in terms of helicity amplitudes

$$\frac{d\hat{\sigma}}{d\cos\theta}(q\bar{q}^{(c)} \rightarrow \ell_1\ell_2) = \frac{\pi\alpha^2s}{24} \left\{ 4 \left[ |f_{LL}^t|^2 + |f_{RR}^t|^2 \right] 
+ (1 + \cos\theta)^2 \left[ |f_{LR}^s|^2 + |f_{RL}^s|^2 + |f_{LR}^t|^2 + |f_{RL}^t|^2 + 2\text{Re}(f_{LR}^s f_{LR}^t) + 2\text{Re}(f_{RL}^s f_{RL}^t) \right] 
+ (1 - \cos\theta)^2 \left[ |f_{LL}^t|^2 + |f_{RR}^t|^2 \right] \right\}$$

(14)

The complete cross section for $q\bar{q} \rightarrow e^+e^-$ is built up by the $s$-channel exchange of $\gamma, Z$ bosons and by additional contributions due to sfermion exchanges: the $s$-channel $\tilde{\nu}_\tau$ and the $t$-channel $\tilde{t}_L$ exchanges in $d\bar{d} \rightarrow e^+e^-$, and the $u$-channel $\tilde{b}_R$ exchange in the $u\bar{u} \rightarrow e^+e^-$ subprocess (see Fig. 1a, b).

While the $s$-channel $\gamma, Z$ amplitudes in the Standard Model involve the coupling of vector currents, the sfermion exchange is described by scalar densities. By performing appropriate Fierz transformations, the $s$-channel $\tilde{\tau}$ exchange amplitudes can formally be rewritten as $t$-channel vector amplitudes, and $t/u$-channel $\tilde{q}$ exchange amplitudes as $s$-channel vector amplitudes [17]; for the operators: $(\tilde{f}_R f_L^*) (\tilde{F}_L F_R^*) \rightarrow -\frac{i}{2}(\tilde{f}_R \gamma_\mu F_R^*) (\tilde{F}_L \gamma_\mu f_L^*)$. 

9
The independent $s$-channel amplitudes $f^{s}_{h,h'}$ for $q\bar{q} \to e^+e^-$ are therefore given by

\begin{align}
    f_{LR}^s &= -\frac{Q^q}{\hat{s}} + \frac{g^q_\mu g^\mu_L}{\hat{s} - m_Z^2 + i\Gamma_Z m_Z} \tag{15} \\
    f_{RL}^s &= -\frac{Q^q}{\hat{s}} + \frac{g^q_\mu g^\mu_R}{\hat{s} - m_Z^2 + i\Gamma_Z m_Z} \tag{16} \\
    f_{LL}^s &= -\frac{Q^q}{\hat{s}} + \frac{g^q_\mu g^\mu_L}{\hat{s} - m_Z^2 + i\Gamma_Z m_Z} - \frac{1}{2} \frac{(\lambda'_{131}/e)^2}{\hat{u} - m^2_{br}} \delta_{qu} \tag{17} \\
    f_{RR}^s &= -\frac{Q^q}{\hat{s}} + \frac{g^q_\mu g^\mu_L}{\hat{s} - m_Z^2 + i\Gamma_Z m_Z} + \frac{1}{2} \frac{(\lambda'_{131}/e)^2}{\hat{t} - m^2_{it}} \delta_{qd} \tag{18}
\end{align}

where $\hat{t} = -\hat{s}(1 - \cos \theta)/2$, $\hat{u} = -\hat{s}(1 + \cos \theta)/2$; the $t$-channel amplitudes $f^t_{h,h'}$ read

\begin{align}
    f^t_{LR} &= 0 \tag{19} \\
    f^t_{RL} &= 0 \tag{20} \\
    f^t_{LL} &= \frac{1}{2} \frac{\lambda_{131} \lambda'_{311}/e^2}{\hat{s} - m_Z^2 + i\Gamma_Z m_Z} \delta_{qd} \tag{21} \\
    f^t_{RR} &= \frac{1}{2} \frac{\lambda_{131} \lambda'_{311}/e^2}{\hat{s} - m_Z^2 + i\Gamma_Z m_Z} \delta_{qd} \tag{22}
\end{align}

Note the relative sign between the $\bar{b}$ and $\bar{t}$ contributions due to the different ordering of fermion operators in the Wick reduction. To simplify the notation, we have defined the indices $L, R$ to denote the helicities of the ingoing antiquark and the outgoing electron are fixed by the $\gamma_5$ invariance of the vector interactions: they are opposite to the helicities of the quark and positron in $s$-channel amplitudes and the same in $t$-channel amplitudes.

The left/right $Z$ charges$^4$ of the fermions are defined as

\begin{equation}
    g^f_L = \left( \frac{\sqrt{2} G_\mu m_Z^2}{\pi\alpha} \right)^{1/2} \left[ I_3 - s^2_W Q^f \right] \quad \text{and} \quad g^f_R = \left( \frac{\sqrt{2} G_\mu m_Z^2}{\pi\alpha} \right)^{1/2} \left[ - s^2_W Q^f \right] \tag{23}
\end{equation}

where $s^2_W = \sin^2 \theta_W$.

The analysis of the process $q\bar{q}' \to ev$, Fig. 1c, proceeds analogously. The helicity amplitudes for $ud \to e^+\nu_e$ can be written as follows:

\begin{align}
    f^s_{LR} &= \frac{1}{2 s^2_W} \frac{1}{\hat{s} - m_W^2 + i\Gamma_W m_W} \tag{24} \\
    f^s_{RL} &= -\frac{1}{2} \frac{(\lambda'_{131}/e)^2}{\hat{u} - m^2_{dr}} \tag{25} \\
    f^s_{LL} &= -\frac{1}{2} \frac{\lambda_{131} \lambda'_{311}/e^2}{\hat{s} - m_\tilde{t}^2 + i\Gamma_\tilde{t} m_\tilde{t}} \tag{26}
\end{align}

$^4$Note that in Eqs. (15-22) the outgoing positron with the helicity $L(R)$ couples with the charge $g_R(g_L)$. 

10
while all other helicity amplitudes vanish.

b) \( e^+e^- \rightarrow q\bar{q} \)

For completeness we include the explicit expressions for the process \( e^+e^- \rightarrow q\bar{q} \) (the contributing Feynman diagrams are the time-reversed versions of the diagrams in Fig. 1a, b). The cross section can be written as

\[
\frac{d\hat{\sigma}}{d\cos\theta}[e^+e^- \rightarrow q\bar{q}] = \frac{3\pi\alpha^2 s}{8}\{4 |f_{LL}^t|^2 + |f_{RR}^t|^2 + (1 + \cos\theta)^2 [|f_{LR}^s|^2 + |f_{RL}^s|^2 + |f_{LL}^t|^2 + |f_{RR}^t|^2 + 2\text{Re}(f_{LR}^s f_{RL}^t) + 2\text{Re}(f_{RL}^s f_{LR}^t)]
+(1 - \cos\theta)^2 [|f_{LL}^s|^2 + |f_{RR}^s|^2]\}
\]

(27)

The indices \( L,R \) in the helicity amplitudes \( f \) denote the helicity of the incoming electron (first index) and of the outgoing antiquark (second index). Again, after applying Fierz transformations, the amplitudes can be cast in the following form:

\[
f^s_{LR} = -\frac{Q^q}{s} + \frac{g^e_L g^q_L}{s - m^2_Z + i\Gamma_Z m_Z}
\]

(28)

\[
f^s_{RL} = -\frac{Q^q}{s} + \frac{g^e_R g^q_R}{s - m^2_Z + i\Gamma_Z m_Z}
\]

(29)

\[
f^s_{LL} = -\frac{Q^q}{s} + \frac{g^e_L g^q_R}{s - m^2_Z + i\Gamma_Z m_Z} - \frac{1}{2}\left(\lambda^{131}/e^2\right)^2 \frac{\delta_{uq}}{u - m^2_{br}}
\]

(30)

\[
f^s_{RR} = -\frac{Q^q}{s} + \frac{g^e_R g^q_L}{s - m^2_Z + i\Gamma_Z m_Z} + \frac{1}{2}\left(\lambda^{131}/e^2\right)^2 \frac{\delta_{qd}}{t - m^2_{lt}}
\]

(31)

\[
f^t_{LR} = 0
\]

(32)

\[
f^t_{RL} = 0
\]

(33)

\[
f^t_{LL} = \frac{1}{2}\left(\lambda^{3131}/e^2\right) \frac{\delta_{q\bar{q}}}{s - m^2_{\bar{v}v} + i\Gamma_{\bar{v}v} m_{\bar{v}v}}
\]

(34)

\[
f^t_{RR} = \frac{1}{2}\left(\lambda^{3131}/e^2\right) \frac{\delta_{q\bar{q}}}{s - m^2_{\bar{v}v} + i\Gamma_{\bar{v}v} m_{\bar{v}v}}
\]

(35)

The gauge couplings \( g^{e,q}_{L,R} \) have been defined in Eq. (23).

References


[11] For recent and comprehensive reviews see *e.g.* G. Bhattacharyya, talk at the Workshop, “Beyond the Desert – Accelerator and Non-Accelerator Approaches”, Ringberg Castle, June 1997; H. Dreiner, in Ref. [2].


