Possible Evidence for Relativistic Shocks in Gamma-Ray Bursts

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ABSTRACT

Relativistic shock models of gamma-ray bursts may be tested by comparison of their predicted low energy asymptotic spectral indices $s$ to observations. Synchrotron radiation theory predicts that the instantaneous spectrum has $s = -1/3$ and the spectrum integrated over the radiative decay of the electrons' energies has $s = 1/2$, with other cases lying between these limits. We examine the spectra of 11 bursts obtained by the Large Area Detectors on BATSE. One agrees with the predicted instantaneous spectrum, as does the initial portion of a second, and three are close to the predicted integrated spectrum. All the observed asymptotic spectral slopes lie in the predicted range. This evidence for relativistic shocks is independent of detailed models of bursts and of assumptions about their distances. Radiation observed with the predicted instantaneous spectrum has a comparatively smooth time dependence, consistent with the necessarily long radiation time, while that with the predicted integrated spectrum has a spiky time dependence, consistent with the necessarily short radiation time.

Subject headings: Gamma Rays: Bursts — Gamma Rays: Theory

1. Introduction

The high intensities, likely large distances, short time-scales and inferred small source sizes of gamma-ray bursts (GRB) have led most astrophysicists to conclude that they involve relativistic...

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motion, and that their radiation is produced when the kinetic energy of this motion is thermalized in a relativistic shock. Katz 1994a and Tavani 1995 argued that acceleration by a relativistic shock produces a characteristic distribution of particle energies. Because all the particles are accelerated, a simple phase space argument implies that the energy distribution function (in the frame of the shocked fluid) is peaked about the mean internal energy per particle (in that frame). This characteristic energy is defined by the ratio of the proper energy density to the proper number density. Thus, nearly all the particles are relativistic. Relativistic shock acceleration is characterized by a paucity of low energy particles and by a synchrotron spectrum which, below the characteristic synchrotron frequency (of a particle with the characteristic energy), resembles that of a monoenergetic distribution of particles. Therefore, the instantaneous synchrotron spectrum of particles accelerated by a relativistic shock should always (in the absence of absorption) have a low frequency asymptotic spectral index \( s = -\frac{1}{3} \) (\( F_\nu \propto \nu^{-s} = \nu^{1/3} \)). This result explains the well known “X-ray paucity” problem of GRB, and is independent of the presence of a high energy “tail” to the particle distribution function.

These predictions may be compared to those of stochastic acceleration in non-relativistic shocks, in which no such characteristic energy can be defined because there is an effectively infinite reservoir of unaccelerated thermal particles. The proper number density of accelerated particles is indeterminate, because additional thermal particles can be recruited to the accelerated distribution (if the total number density, including thermal particles, were used to define the characteristic energy, the result would be nonrelativistic—we use this definition of a non-relativistic shock—and would tell nothing about the distribution function of the relativistic particles). In the absence of a characteristic energy a power-law (rather than a peaked) particle distribution must be obtained, extending down to a very low thermal, rest mass or injection energy. Radiation by these intermediate energy particles dominates the lower frequency part of the synchrotron spectrum of a non-relativistic shock, and gives it a spectral index which depends on the distribution function of accelerated particles.

Estimates of the synchrotron radiation efficiency and electron lifetime, assuming magnetic equipartition (Katz 1994b, Sari, Narayan & Piran 1996, Sari & Piran 1997a) in GRB suggest that the radiating electrons lose most of their energy by synchrotron radiation in a time much shorter than the duration of a typical burst, and especially of long bursts (the data reported in this paper all come from long bursts, with \( T_{90} > 20 \) s). Therefore, for comparison with spectra integrated over the duration of a GRB the theoretical spectrum should be integrated over the spectrum emitted by a single electron as its energy decays. Noting that the characteristic synchrotron frequency varies \( \propto E^2 \), where \( E \) is the electron’s energy, we have \( F_\nu \propto dE/d\nu \propto \nu^{-1/2} \); the predicted spectral index \( s = \frac{1}{2} \). This result is also applicable to Compton scattering if the photon field upon which the electrons scatter does not change as the electrons lose their energy.

In general, a real astronomical object (probably heterogeneous in both space and time) will have a spectrum somewhere between these two extremes. Relativistic shock theory thus predicts a
spectral index

\[-\frac{1}{3} \leq s \leq \frac{1}{2}\]  

below the characteristic synchrotron frequency.

In a given magnetic field, an electron’s radiative loss time increases \( \propto E^{-1} \) as \( E \to 0 \), so that at sufficiently low frequency any observation of specified duration becomes effectively instantaneous, with a predicted asymptotic spectral index

\[ s = -\frac{1}{3}. \]  

(2)

It is only necessary that the duration of emission be limited by radiative energy losses and that the length of integration not increase along with the (expected) increase of duration with decreasing frequency. For comparison, non-relativistic shocks can lead to any \( s \geq -\frac{1}{3} \) (this lower bound is set by the synchrotron spectrum of monoenergetic electrons).

The purpose of this paper is to compare the predictions of relativistic shock theory to observed spectra of GRB. Characteristic synchrotron frequencies (the peaks of \( \nu F_{\nu} \) or breaks in a log-log plot of \( F_{\nu} \)) are typically in the range 300–1000 KeV. The behavior of GRB spectra at yet higher energies reflects the high energy “tail” of the particle distribution function, and is not relevant here. We instead concentrate on observations at lower energies, and ask the question: Do the observed spectra asymptotically approach the predicted power laws of spectral indices \( s = -\frac{1}{3} \) or \( s = \frac{1}{2} \)? We compare these predictions to spectral data obtained by BATSE’s Large Area Detectors on CGRO. To the extent that the predictions are confirmed, this may be the first empirical evidence for relativistic shocks in astrophysics.

In §2 we report and describe data from 11 GRB. §3 contains a comparison of these data to the asymptotic power laws described above. In §4 we show that, within our sample, bursts with \( s \approx -\frac{1}{3} \) have smoother time profiles than those with \( s \approx \frac{1}{2} \), in agreement with expectations about their radiation times. §5 contains a brief summary, and discusses relativistic shocks in other astronomical objects, such as pulsar winds, AGN and double radio sources.

2. Spectra of 11 GRB

The BATSE LAD is a NaI(Tl) scintillator, whose intrinsic energy resolution for soft gamma-rays is not high. The fractional dispersion in pulse height of such scintillators for 100 KeV gamma-rays is about 10\% (O’Kelley 1961). It is larger at lower photon energies, although not by as much as a naive \( \nu^{-1/2} \) dependence would imply. The LAD has a FWHM (2.35 times the dispersion for a Gaussian profile) of 27\% at 88 KeV (Horack 1991), close to O’Kelley’s estimate. Attempts to deconvolve the measured count distribution to obtain the source photon spectrum by multiplying the count vector by the inverse of the known Detector Response Matrix (DRM) (Pendleton, et al. 1995) fail because they amount to attempts to recover information lost in convolving the spectrum with the DRM.
The usual procedure in spectral studies of GRB (Schaefer, et al. 1994, Briggs 1996), which we adopt, is first to fit a standard spectral form to the data. We use the four parameter model described by Band, et al. 1993. This model consists of two asymptotically power law segments smoothly joined and parameterized so that the energy of the peak in $\nu F_\nu$ characterizes the break between the two. It can accommodate a considerable amount of curvature in the spectrum, and has been successfully fit to a large number of GRB. The forward-folding fitting and calibration procedures used here are described elsewhere (Preece, et al. 1997) and incorporate the full detector response, as well as effects of scattering of gamma-rays in the spacecraft and the Earth’s atmosphere (Pendleton, et al. 1995).

This fitted spectral form is then folded through the DRM to obtain a predicted distribution of counts in each energy channel. The ratio of the model photon flux to the predicted counts is then an estimate of the reciprocal detector efficiency in each channel. The observed count rate in each channel is then multiplied by this reciprocal efficiency to produce an estimate of the incident photon spectrum, which we plot.

This procedure would be exact (independent of any assumed spectral model) if the DRM were diagonal, and provides an estimate of the diagonal elements of the inverse matrix. It is stable. It accounts for the gross variation of detector efficiency with photon energy without performing an unstable deconvolution. However, it is not a unique resolution of the ambiguity introduced by a detector response function of finite width; it cannot reverse the spectral broadening. There can be no unique reversal because information has been irretrievably lost. Unfortunately, the necessary assumption of a spectral model introduces an irremediably bias into the estimated reciprocal efficiencies, and this and the implicit approximation of the DRM as diagonal introduce bias into the estimated incident photon spectrum.

Figures 1a–l show the spectra of 11 GRB observed by the Large Area Detectors on BATSE. In general, most of the fluence was integrated to obtain these spectra. They are shown in chronological order. These bursts were chosen on the basis of their total fluence ranking, as determined in the BATSE 3B Catalog (Meegan, et al. 1996). The calculation of fluence assumes that certain BATSE data types exist for each burst which cover the entire event; this is true for most of the bursts in the Catalog. For two of the brighter bursts, the total spectrum was used for calibration of the Large Area Detectors and thus cannot be used for any other purpose; these were omitted from our list.

In the case of GRB910601 two spectra are shown. The spectrum denoted GRB910601rise was obtained from the initial rising part of the burst, not including its main peak (see Figure 2a). This was initially done inadvertently, but turns out to be illuminating, as discussed in §4.

In each figure the solid line corresponds to $F_\nu \propto \nu^{1/3}$ and the dashed line to $F_\nu \propto \nu^{-1/2}$. These straight lines have been, somewhat arbitrarily, normalized to the geometric mean of the four lowest energy data points; of course, it is their slope, and not their normalization, which is of interest. The horizontal error bars show the nominal widths of the spectral bins, with the data
points plotted at the geometric mean energies. The vertical error bars represent 1-\(\sigma\) counting statistics only, and do not include any systematic errors, including the uncertainty introduced by the assumption of a specific functional form in the fits. The absolute sensitivity calibration is irrelevant to testing our asymptotic models, but differential calibration errors between spectral bins would be important; the calibration procedures were described by Pendleton, et al. 1995. The data shown represent integrations over most of the gamma-ray fluences of the bursts.

3. Testing the relativistic shock model

We are testing the predictions of the low energy asymptotic behavior of the GRB spectrum. Because we cannot specify the complete spectrum (we make no predictions as to the shape of the electron distribution around its peak) we cannot predict at what energy, or with what shape, the spectrum approaches the asymptotic power law. As a result, the test of the predicted asymptotic behavior is qualitative: do the spectra, to the eye, appear to approach the predicted asymptotic power laws?

GRB910601rise and GRB921123 closely approach the \(F_\nu \propto \nu^{1/3}\) asymptote at low energies. An additional eight spectra, GRB910503, GRB910601, GRB920406, GRB920622, GRB930201, GRB930916, GRB940206 and GRB940302, may be approaching that asymptote, but the evidence is suggestive rather than persuasive. The remaining two bursts, GRB930506 and GRB940217, show no evidence of approaching this asymptote, but even these data are consistent with the prediction that \(s \geq -1/3\).

The instantaneous radiation of a relativistic shock was predicted to have an asymptotic low energy spectral index of \(-1/3\). However, the predicted synchrotron energy loss times are much shorter than the burst durations and integration times. The confirmation of the predicted instantaneous asymptotic spectrum in GRB910601rise and GRB921123 suggests that the synchrotron loss times may have been underestimated (for example, if the magnetic field is below its equipartition value), or that the radiation is terminated (at the price of reduced radiative efficiency) by some other process, such as adiabatic expansion of a dense clump of radiating particles and field, before the particles’ energy is degraded by radiative energy loss and their emitted spectrum is softened.

The effects of adiabatic energy loss are easy to estimate. For \(d\)-dimensional expansion \((d = 1\) corresponds to an expanding sheet or slab, \(d = 2\) to an expanding line or filament and \(d = 3\) to expansion from a point in all directions), assuming the particle distribution function remains isotropic, a particle’s energy scales with expansion distance \(r\) and time \(t\) as \(E \propto r^{-d/3} \propto t^{-d/3}\). We assume that the magnetic field also remains statistically isotropic, so that its energy density adiabatically declines like that of a relativistic fluid, and \(B \propto r^{-2d/3} \propto t^{-2d/3}\). If, instead, a flux conservation argument were used then for \(d = 1\) or \(d = 2\) the field would become strongly anisotropic and the total magnetic energy would diverge; this is impossible, and magnetic
reconnection probably limits the magnetic energy and isotropizes the field.

Given the assumptions of isotropy, integration of the radiation emitted by a particle as its energy and the magnetic field undergo adiabatic decay, until radiation is cut off when the particle’s characteristic synchrotron frequency equals the frequency of observation, leads to a spectral index

\[ s = \frac{3 - 2d}{4d}. \]

(3)

For \( d = 3 \) \( s = -\frac{1}{4} \), essentially indistinguishable from the instantaneous spectrum; for \( d = 2 \) \( s = -\frac{1}{8} \); for \( d = 1 \) \( s = \frac{1}{4} \), not far from the spectrum integrated over radiative energy losses. All these spectral indices lie between those obtained for the instantaneous spectrum and for the spectrum integrated over the radiative decay of the electrons’ energy.

GRB930201 closely approaches an asymptotic spectral index of \( \frac{1}{2} \) (except perhaps for its lowest energy point). This may be evidence for the observation of radiation from a relativistic shock in which the radiating electrons are observed throughout their loss of energy (by radiation rather than by adiabatic expansion), as their characteristic synchrotron frequency passes through the band of observation. GRB940302 similarly, but not so closely, approaches an asymptotic spectral index of \( \frac{1}{2} \) (again, except perhaps for its lowest energy point), and GRB940217 may also show this behavior.

The deviations of the lowest energy points, if real, may be explained by a gradual transition to an asymptotic spectral index of \( -\frac{1}{3} \), expected at lower energies, where the radiative lifetimes are longer. As in the case of the asymptotic spectral index of \( -\frac{1}{3} \) found for GRB910601rise and GRB921123, this interpretation would require unexpectedly long radiative decay times. We note, however, that these points are close to the I K-edge, where deconvolution is particularly difficult.

Most of the GRB show asymptotic low energy spectral indices between \( -\frac{1}{3} \) and \( \frac{1}{2} \). These indicate intermediate cases between the index of \( -\frac{1}{3} \) obtained for negligible radiative loss and that of \( \frac{1}{2} \) obtained when the radiating electrons lose all their energy during the observation, and are also consistent with adiabatic energy loss. It is not surprising that the time-integrated spectrum of a complex multi-peaked (and probably spatially heterogeneous) event like a GRB should lie between these two limiting cases. Such intermediate behavior is consistent with the relativistic shock acceleration model, although it does not point to it unambiguously as a value \( s = -\frac{1}{3} \) would. It is additional evidence in favor of relativistic shocks that values of \( s > \frac{1}{2} \), although frequently found in other astronomical synchrotron sources and readily produced by nonrelativistic shock acceleration processes (as well as others), have never been found on the low energy side of the spectral peak in GRB.

4. Time Structure

In a spiky, multi-peaked GRB the synchrotron radiation cooling time (or adiabatic expansion time) must be less than the spike duration, and hence much less than the burst duration. Thus
spiky bursts are predicted to have \( s = \frac{1}{2} \), unless adiabatic expansion is dominant, in which case \( s \) assumes the appropriate value from Equation (3). Conversely, bursts with \( s = \frac{1}{2} \) must have short radiation times and may have spiky time structure.

In a smooth, single peaked GRB the radiation time can be comparable to the burst duration because the intensity need not drop to zero again and again during the burst. Hence the predicted instantaneous \( s = -\frac{1}{3} \) may approximate the observed spectrum, despite its finite length of integration.

These predictions may be tested by examining the time structures of the GRB whose spectra we study. In Figures 2a–d are shown the time histories of GRB910601 and GRB921123, which show the best fit to \( s = -\frac{1}{3} \), GRB930201, which shows the best fit to \( s = \frac{1}{2} \), and GRB940217, which resembles \( s = \frac{1}{2} \), but less closely; GRB940302 is not shown because of gaps in the data. In each case the 25–55 KeV count rates are plotted, as recorded in 64 ms bins; we chose this lowest energy range because only in it do the spectra approach their low energy asymptotic forms. The dashed lines denote the fitted backgrounds and the vertical lines delimit the intervals over which the spectra shown in Figure 1 were integrated. GRB910601rise is the spectrum obtained between the left and middle vertical lines, and GRB910601 between the left and right vertical lines.

It is evident that the bursts with \( s = -\frac{1}{3} \) are comparatively smooth, while those with \( s = \frac{1}{2} \) have complex structure, with many clearly separated peaks. In the case of GRB910601 the smooth initial rising portion of the spectrum has \( s = -\frac{1}{3} \), while the peak of the burst, intermediate in spikiness between a smooth burst like GRB921123 and very spiky bursts like GRB930201 and GRB940217, has an intermediate spectrum. This confirms the predicted qualitative behavior.

It is desirable to quantify the distinction between spiky and smooth bursts. We modified the algorithm of Li & Fenimore 1996 to identify maxima (shown by circles) and minima (shown by asterisks) at the “7 \( \sigma \)” level (meaning that minima and maxima must be separated by at least 7 standard deviations, so each need be significant only at the 3.5 \( \sigma \) level). Our modified algorithm searches for broad peaks of comparatively low amplitude by smoothing the data between the maxima which it has already found, searching again, smoothing further, etc. This algorithm is not perfect, but is useful in demonstrating that the apparent fine structure seen, for example, near the maxima of GRB910601 and GRB921123 is probably only statistical fluctuation, but that most of the maxima seen by the eye in GRB930201 and GRB940217 are real. The number of maxima found is a quantitative measure of spikiness.

5. Discussion

The spectra we find are all consistent with those predicted by relativistic shock theory, and we find evidence for its two limiting cases. In these limiting cases we confirm the predictions that bursts with \( s = -\frac{1}{3} \) should have long radiation times, and therefore smooth profiles, while bursts with \( s = \frac{1}{2} \) should have short radiation times and therefore may have spiky multi-peaked profiles.
Of course, this division of GRB into two classes is not absolute. It is apparent that even the smoother $s = -\frac{1}{3}$ profiles are not strictly single peaked. This does not contradict the suggestion that the radiation time in them is long: even a burst dominated by smoothly varying radiation may also contain weaker but more rapidly varying contributions from other regions, which do not have a large effect on its spectrum.

The simplest interpretation of our data concerning the correlation between spectrum and time structure is that the electron radiation time is comparable to the width of a spike or subpeak within a burst, or of the burst itself if it is smooth. Longer radiation times are excluded by the observed time dependence (unless adiabatic loss takes the place of energy loss by radiation). Shorter radiation times could be consistent with the observed time structure, if the electrons were continually resupplied at a steady rate, but this is not consistent with the spectra we observe for smooth bursts.

The characteristic geometrical time $t_g \sim r/(c\gamma^2)$, where $\gamma$ is the Lorentz factor of bulk motion of the radiating matter and $r$ its distance from its origin, must in general be comparable to or shorter than the observed time scale of variation (Sari & Piran 1997b). In bursts in which the instantaneous spectrum ($s = -\frac{1}{3}$) is observed the time structure is determined geometrically, and $t_g$ is directly measured. In bursts in which the integrated spectrum ($s = \frac{1}{2}$) is observed the time structure only sets an upper bound on $t_g$.

Fenimore, et al. 1995 reported that the characteristic observed subpulse width and the width of the autocorrelation function of GRB vary as the $\approx -0.4$ power of the photon energy of observation. This is in reasonable agreement with expectations for a population of electrons losing energy by synchrotron radiation, for which an exponent of $-\frac{1}{2}$ is predicted, and supports our suggestion that subpulse widths are determined by radiative energy loss. The same hypothesis also can explain the frequent observation (Nemiroff, et al. 1994) of fast rise, slow (usually exponential) decay time structure in GRB (and the accompanying spectral softening), if the rise time is attributed to geometry and the temporal and spectral decay to radiative energy loss. However, it is not possible to explain in this manner the few bursts in which the envelope of a rapidly varying intensity shows slow decay.

The model-dependent procedure (§2) we use for extracting the source spectrum from the observed count distribution is unavoidable, given the physics of NaI(Tl) detectors. A skeptic might argue that this invalidates our conclusions, and we would be unable to prove him wrong. It is possible, of course, to use different model spectra in the data reduction, but this would not solve the problem: if essentially the same results were obtained for different spectral models, the skeptic could argue that we had not been sufficiently diabolical in our choice of models, while if different results were obtained we might argue that that model was unlikely for some reason. This controversy cannot be resolved without data from detectors of much higher intrinsic energy resolution, or over a much broader energy band (from gamma-rays to visible light, for example). If these were available then deconvolution would not be an issue in determining the broad energy
distribution of GRB.

Our procedure differs from simply extrapolating the four parameter model spectra directly. The asymptotic form of a spectrum is not generally the same as the asymptotic form of a model fitted to the spectrum. The difference may be large when the spectrum does not closely approach the asymptote in the region in which the model is fitted to data, as in this work. It is for this reason that we have attempted to determine the source photon spectrum, rather than just the parameters of the fitted model. For the same reason, the result (Crider, et al. 1997) that the fitted low frequency asymptotic slope of the Band, et al. 1993 model varies with time within a GRB is consistent with our results: the fitted value of this parameter depends on the higher frequency spectrum (as is shown by the correlation they find between the slope and the spectral break energy), and does not uniquely determine the actual low frequency asymptote.

Tavani 1996ab compared the spectra of five GRB to a theoretical model of the instantaneous spectrum which has the same low frequency asymptote as ours, but which also describes the higher energy spectrum, and found satisfactory fits. His more complex model necessarily has more free parameters; in our work the only free parameter is an overall flux normalization. We tested a different hypothesis than his, using data from different detectors on a larger sample of GRB; to the extent to which they may be compared, our conclusions are consistent with his.

Preece, et al. 1996, using data from the BATSE Spectroscopy Detector, have found in some GRB a statistically very significant excess flux at photon energies around 10 KeV, lower than the lowest energy for which we have data. These excess fluxes are not consistent with our power law spectra (as shown by their fitted parameters), or with any simple extrapolation from higher energies. An additional radiation mechanism may be required, as discussed by Preece, et al. 1996; this problem is beyond the scope of this paper.

Relativistic shocks are encountered elsewhere in astrophysics, for example, in the termination shocks of pulsar winds and relativistic jets. It is unclear whether the spectra of these objects have the predicted slopes. If their radiation is emitted by particles accelerated in their central sources then their spectra need not resemble those we have discussed; these particles flow out into the wind and radiate there (their adiabatic expansion “losses” are only the conversion of an isotropic distribution of velocities to directed motion—collimation—preserving their mean energy and spectrum). If, however, the radiation is emitted by particles accelerated in relativistic termination shocks, as is plausible in the lobes of double radio sources and in supernova remnants which contain pulsars, then the characteristic spectrum of relativistic shock acceleration is expected. If these sources are observed in steady state, as would be expected for a relativistic gas with nonrelativistic bulk velocity, with ages much greater than the synchrotron lifetimes of the radiating electrons, then a spectral index $s = \frac{1}{2}$ is predicted. This may account for the frequent observation of spectral indices close to this value.

A spectral index of $\frac{1}{2}$ is obtained for any energy distribution of accelerated electrons, if they are observed throughout the radiative decay of their energy, so long as their initial characteristic
synchrotron frequencies exceed the range over which the spectral index is measured. This value of $s$ is therefore expected even when the accelerating mechanism is not a relativistic shock. It is only necessary that the condition discussed by Katz 1994a be met—that the distribution of accelerated electron energies be flat enough ($dN/dE \propto E^{-p}$ with $p < 1/3$). Observed spectral indices $\approx 0.5$ therefore need not imply acceleration by nonrelativistic shocks with $p \approx 2$.

The spectral index $s$ affects the extrapolation of GRB spectra to lower frequencies, and particularly their predicted visible magnitudes. For example, replacement of $s = -1/3$ by $s = 1/2$ in extrapolating from 300 KeV gamma-rays to visible light would increase the visible brightness by about 10 magnitudes. For the $10^{-5}$ erg/cm$^2$s “burst of the month” discussed by Katz 1994a the predicted visible magnitude would then be about 8, rather than 18. Similarly, Ford & Band 1996 extrapolated the fitted four-parameter model either all the way to visible frequencies or to 1 KeV, below which an $s = -1/3$ extrapolation was assumed, and found a very broad range of predicted visible magnitudes, including some comparatively bright values.

Magnitudes much less than 18 are unduly optimistic. The synchrotron lifetime increases as the electron energy declines. Therefore, even if the gamma-ray region of the spectrum had $s = 1/2$, the spectral index would be predicted to approach the instantaneous value $s = -1/3$ at lower frequencies, probably long before the visible region. Similarly, if the spectral index in the gamma-ray region reflects adiabatic expansion ($-1/4 \leq s \leq 1/4$), as the expansion proceeds and the electron energies decline the expansion time scale becomes longer and the spectral index will again approach its instantaneous value $s = -1/3$. In either case, the consequence is that the predicted visible magnitude is less than 18, but probably by a few magnitudes rather than by 10. The same conclusion can be reached without any theoretical assumptions at all by noting that the spectrum has not approached its asymptotic form at $\hbar \nu = 300$ KeV, but may at X-ray energies.

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Fig. 1.— Spectra of 11 GRB. Solid lines correspond to \( s = -\frac{1}{3} \) and dashed lines to \( s = \frac{1}{2} \). GRB910601rise denotes the spectrum of the initial (pre-maximum) rise of GRB910601; see Figure 2.

Fig. 2.— Intensity histories of four GRB in the 25–55 KeV band. The dashed lines are fits to the backgrounds. Circles denote maxima and asterisks minima. The spectra in Figure 1 were integrated over the periods between the vertical lines. The spectrum labeled GRB910601rise was integrated from the first to the middle line, and that labeled GRB910601 from the first to the last line.