Improved Bounds on the Electromagnetic Dipole Moments of the Tau Lepton

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Abstract
Using electroweak data and an effective Lagrangian approach we obtain stringent bounds on the tau anomalous magnetic moment, $-0.004 \leq a_\tau \leq 0.006$, and on its electric dipole moment, $|d_\tau| \leq 1.1 \times 10^{-17}$ e cm. This significantly improves our previous bounds. With the same method, we obtain limits on the dipole moments of other fermions.
1 Introduction

Tau lepton physics (see Ref.’s [1] and the report of K. Hayes in [2]) provides a laboratory to test ideas and models beyond the Standard Model such as compositeness and/or new sources of CP-violation.

In particular, the tau magnetic and electric dipole moments may play an important role in elucidating the nature of this heavy lepton. From the general electromagnetic matrix element describing the interaction of a $\tau$ lepton with photons

$$\langle p' | J_{\text{em}}^{\mu}(0) | p \rangle = e \bar{u}(p') \left( F_1 \gamma^\mu + \frac{i}{2m_\tau} F_2 + \gamma_5 \tilde{F}_2 \right) \sigma^{\mu\nu} q_\nu u(p),$$

where $q = p' - p$ and $F_1(q^2 = 0) = 1$, one defines the tau anomalous magnetic dipole moment

$$a_\tau = \frac{g_\tau - 2}{2} = F_2(q^2 = 0),$$

and the tau electric dipole moment (EDM)

$$d_\tau = e \tilde{F}_2(q^2 = 0).$$

In the Standard Model (SM), the tau is considered to be a spin-1/2 point-like Dirac particle. A theoretical SM calculation [3] predicts the value

$$a_\tau |_{\text{SM}} = 1.177 \times 10^{-3},$$

and a very small value for $d_\tau$, the precise value depending on unknown leptonic mixing angles and neutrino masses [4].

Up to now, the most stringent way to constrain the electromagnetic dipole moments $a_\tau$ and $d_\tau$ has shown to be the use of the method we developed in Ref. [5]. The idea is the following. The addition of new physics to the SM could lead to values for $a_\tau$ different from the value of Eq. (4) or to $d_\tau$ being larger than the SM predicts. We describe potential deviations from the SM induced by some kind of new physics by a set of effective Lagrangians that contain gauge-invariant operators contributing to the tau dipole moments. Such effective Lagrangians inevitably induce anomalous couplings of the $Z$ to the tau lepton, which would contribute to the partial width $\Gamma_\tau \equiv \Gamma(Z \rightarrow \tau\bar{\tau})$. The tight constraints of LEP1 data on deviations from the SM can be used to get model independent bounds on the electromagnetic dipole moments.

It is well known that the impressive accuracy of the LEP1 measurements makes necessary the inclusion of radiative corrections when comparing theory and experiment. The theoretical SM prediction for $\Gamma_\tau$ —which is an input in our analysis— depends on the top quark and Higgs masses, $m_t$ and $m_H$. When we worked out the above mentioned idea in 1992 there was only an experimental lower bound on $m_t$ [6]; therefore a two-parameter analysis was inevitable. (One of the parameters was $m_t$ and the other took into account the presence of new physics.) In the meanwhile, evidence for the top quark [7] has shown up and the corresponding $m_t$ measurements [8] allow us to considerably improve our previous bounds by directly comparing the SM
prediction for $\Gamma_\tau$ with its experimental value. In addition, we now have more data at our disposal including the 1995 run [9]. The substantial increase in the precision of the experimental measurements also improves our bounds. The main purpose of this letter is to show that the bounds on $a_\tau$ and $d_\tau$ are tightened significantly when we use the same ideas as [5] but employ the new experimental data. Using the same method, we find upper limits on the dipole moments of some other fermions. In this case we also achieve an improvement with respect to the limits presented in Ref. [10].

In this letter, we will show the formulae obtained in the framework of a linearly realized effective Lagrangian. In Ref. [10] it was shown that a non-linearly realized effective Lagrangian would lead to the same numerical limits for the dipole moments.

2 Bounds on tau lepton dipole moments

The lowest dimension gauge-invariant operators built from the SM fields that contribute to the tau anomalous magnetic dipole moment $a_\tau$ are:

$$\mathcal{O}_{\tau B} = \overline{L}_\tau \sigma^{\mu\nu} \tau_R \Phi B_{\mu\nu},$$  \hspace{1cm} (5)

and

$$\mathcal{O}_{\tau W} = \overline{L}_\tau \sigma^{\mu\nu} \bar{\sigma} \tau_R \Phi \bar{W}_{\mu\nu}. $$  \hspace{1cm} (6)

Here, $L_\tau$ is the $\tau$ left-handed isodoublet, $\tau_R$ its right-handed partner, $\bar{W}, B$ are the SU(2) and U(1) field strengths, and $\Phi$ is the scalar doublet. Contributions to $a_\tau$ appear below the weak scale due to spontaneous symmetry breaking,

$$\Phi \rightarrow \left( \begin{array}{c} 0 \\ v/\sqrt{2} \end{array} \right) + \ldots $$  \hspace{1cm} (7)

with $v^2 = 1/(\sqrt{2} G_F) \simeq (246 \text{GeV})^2$.

Let us examine the consequences of the operator $\mathcal{O}_{\tau B}$ in (5). An effective Lagrangian containing this operator, namely

$$\delta \mathcal{L} = \frac{\alpha_{\tau B}}{\Lambda^2} \mathcal{O}_{\tau B},$$  \hspace{1cm} (8)

where an unknown coupling constant $\alpha_{\tau B}$ and an unknown high-energy mass scale $\Lambda$ are introduced, would contribute to $a_\tau$ as

$$a_\tau = a_\tau |_{SM} + \delta a_\tau,$$  \hspace{1cm} (9)

and simultaneously would contribute to $\Gamma_\tau$ as

$$\Gamma_\tau = \Gamma_\tau |_{SM} + \delta \Gamma_\tau.$$  \hspace{1cm} (10)

Both contributions are related in the following manner:

$$\frac{e}{2m_\tau} |\delta a_\tau| = \cot \theta_w \left[ \frac{24\pi}{M_Z^3 \sqrt{1 - 4 \left( \frac{m_\tau}{M_Z} \right)^2}} |\delta \Gamma_\tau| \right]^{1/2},$$  \hspace{1cm} (11)
where $\theta_w$ is the weak mixing angle. It is important to realize that the relation (11) is independent of $\alpha_{\tau B}$ and $\Lambda$. This fact allows us to place a bound on $a_\tau$ in a model independent way.

The SM theoretical prediction is given by [12]

$$\Gamma_\tau|_{SM} = 83.78 \pm 0.05 \pm 0.05 \text{ MeV}, \quad (12)$$

where the first error comes from using the interval $m_t = 175 \pm 6 \text{ GeV}$ for the top mass [8] and the second corresponds to the interval $m_H = 150^{+150}_{-90} \text{ GeV}$ for the Higgs mass [9]. To be conservative, we add these two “theoretical errors” linearly. The prediction (12) has to be compared with the experimental value [9]

$$\Gamma_\tau|_{exp} = 83.72 \pm 0.26 \text{ MeV}. \quad (13)$$

The agreement between experiment and the SM prediction for $\Gamma_\tau$ restricts potential new physics contributions to the tau anomalous magnetic moment.

At the 95% CL we find the upper bound

$$|\delta a_\tau| \leq 0.0048. \quad (14)$$

The same analysis can be done using now the operator in (6). We get a stronger upper bound on $\delta a_\tau$ (by a factor of about 3). Thus, we conclude that our bound on the tau anomalous magnetic moment is the one given in (14). In so doing, we exclude unnatural cancellations between the effects of the two independent operators $O_{\tau B}$ and $O_{\tau W}$.

Combining (14) with the SM prediction (4), we finally obtain our 95% CL bound on the tau magnetic dipole moment

$$-0.004 \leq a_\tau \leq 0.006 \quad (2\sigma), \quad (15)$$

which improves our previous limit [5] by a factor of about two.

Now we turn our attention to the tau EDM, $d_\tau$. There are also two relevant operators

$$\tilde{O}_{\tau B} = \bar{\tau} \gamma^\mu \tau \Phi B_{\mu \nu}, \quad (16)$$

and

$$\tilde{O}_{\tau W} = \bar{\tau} \gamma^\mu \gamma_5 \tau \Phi \bar{W}_{\mu \nu}. \quad (17)$$

To place a bound on $d_\tau$, we could follow a similar procedure as we followed for $a_\tau$. Indeed, an effective Lagrangian containing the operator $\tilde{O}_{\tau B}$

$$\delta \mathcal{L} = \frac{\tilde{\alpha}_{\tau B}}{\Lambda^2} \tilde{O}_{\tau B}, \quad (18)$$

contributes to the EDM and modifies $\Gamma_\tau$. The limits on $\Gamma_\tau$ imply the bound $|d_\tau| \leq 2.7 \times 10^{-17} \text{ e cm} (95\% \text{ CL})$.

However, we get a much better bound if we use the results from the search for CP violation in the decay $Z \to \tau \bar{\tau}$ at LEP [13, 14]. Effects coming from a possible CP-violating $Z \tau \bar{\tau}$ vertex

$$\langle p' | J_2^\mu (0) | p \rangle = e \bar{u}(p') \tilde{F}_2^w \gamma_5 \sigma^{\mu \nu} q_\nu u(p), \quad (19)$$
have not been observed. This negative result implies a limit on the weak dipole moment of the tau lepton, $d^u_\tau$, defined as

$$d^u_\tau = e \tilde{F}^u_Z (q^2 = M^2_Z). \quad (20)$$

Using the effective Lagrangian (18) yields a simple relationship between the electric and the weak dipole moments:

$$|d_\tau| = \cot \theta_w |d^u_\tau|. \quad (21)$$

The most stringent limit obtained at LEP for $d^u_\tau$ is $|d^u_\tau| \leq 5.8 \times 10^{-18} \, e\, cm$ at 95% CL. Using now (21), we get the following upper limit on the tau EDM

$$|d_\tau| \leq 1.1 \times 10^{-17} \, e\, cm \quad (2\sigma), \quad (22)$$

which improves our previous bound [5] by a factor of about four.

The effective Lagrangian built with the second operator in (17) leads to a stronger limit on $d_\tau$ so that we keep the bound in (22) as our limit. In so doing, we do not allow for unnatural cancellations between the operators (16) and (17).

### 3 Bounds on other fermions dipole moments

The procedure used to constrain $a_\tau$ can obviously be extended to other fermions. For the magnetic moment of the tau neutrino we get an upper bound

$$|\mu(\nu_\tau)| \leq 2.7 \times 10^{-6} \, \mu_B \quad (2\sigma), \quad (23)$$

where $\mu_B = e/2m_e$ is the Bohr magneton. For the quarks, we obtain (at 95% CL)

$$|a_u| \leq 1.8 \times 10^{-5},$$

$$|a_d| \leq 7.3 \times 10^{-5},$$

$$|a_s| \leq 1.5 \times 10^{-3},$$

$$|a_c| \leq 4.8 \times 10^{-3},$$

$$|a_b| \leq 3.1 \times 10^{-2}, \quad (24)$$

where each anomalous magnetic moment is normalized to $e/2m_q$, where $m_q$ is the so-called current-quark mass of each quark. (We take $m_u = 5$ MeV, $m_d = 10$ MeV, $m_s = 200$ MeV, $m_c = 1.3$ GeV, $m_b = 4.3$ GeV.) The bounds on anomalous magnetic moments that we obtain for the other fermions are much weaker than the ones available in the literature (see [2, 10] and references therein) and so we do not quote them.

For the EDM’s of fermions other than the tau we may use the restrictions from the corresponding $Z$ partial width to constrain them. However, all the limits we obtain are much weaker than the ones reached by other methods (see [2, 10] and references therein) except for the $\nu_\tau$ EDM. Here we find

$$|d(\nu_\tau)| \leq 5.2 \times 10^{-17} \, e\, cm \quad (2\sigma). \quad (25)$$
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