EARLY INDICATORS OF LONG TERM STABILITY IN HADRON COLLIDERS *

E. Todesco INFN, Sezione di Bologna

and

R. Bartolini, A. Faus-Golfe, M. Giovannozzi, W. Scandale, CERN, Geneva, Switzerland

Abstract

Early indicators of long-term stability, based on short-term tracking data, are considered in hadron colliders, such as the CERN-LHC. Two early indicators are analysed: the Lyapunov coefficient and the variation of the instantaneous nonlinear tunes. A threshold is associated to each indicator, by which a simple and automated procedure can be defined to select chaotic from regular trajectories. The methods are checked against long-term tracking for a linear lattice with a sextupole (Hénon map). The results show that rather precise long-term dynamic aperture estimates can be worked out using short-term tracking data. The method is successfully applied to identify the dynamic aperture of the CERN-LHC in realistic situations.

1 INTRODUCTION

Fast indicators of long-term stability are very useful to speed up the numerical simulations of the dynamic aperture in hadron colliders, that for a large machine such as the planned LHC should be carried out for more than 10^7 turns. One approach is based on the Lyapunov exponent, first applied to celestial mechanics [1, 2, 3] and then to accelerator physics [4, 5]. It allows one to select chaotic from regular motion with a limited number of turns: hence it provides a criterium for long-term stability under the assumption that all the chaotic particles are unstable. Another powerful indicator is the variation of the instantaneous tune [6] that can provide an analogous criterion. An alternative approach to the techniques presented in this paper, is based on the spirit of the Nekhoroshev theorem and on its generalization to symplectic mappings [7]: the basic idea [8] is to compute an invariant of motion with a high precision, and to numerically evaluate the drift in the invariant space for a limited number of turns, using this value to extrapolate a bound for a large but finite number of turns.

Our aim is to propose an automatic procedure to determine long-term particle loss. This approach relies on the definition of thresholds that depend on the number of turns. Moreover, we carry out an accurate check of the early indicators predictions against long-term particle loss, making a statistical analysis for a large ensemble of initial conditions. We considered a 4D Hénon map, the model of the SPS lattice used for diffusion experiments, and a four-dimensional model for the version 4.1 of the LHC lattice: we show that

one can establish thresholds for defining automatic procedures for long-term estimates, and that the indicators are predictive.

2 AUTOMATED EARLY INDICATORS

2.1 Lyapunov exponent

The Lyapunov exponent is an indicator related to the rate of divergence of two neighbour particles. Let $\mathbf{x} = (x, p_x, y, p_y)$ be an initial condition at a given section of the machine, and let $\mathbf{x}^{(n)}$ be its phase space position after n turns. If we consider a neighbour initial condition $\hat{\mathbf{x}} = \mathbf{x} + \delta$, with $|\delta| \ll 1$, then the estimate of the maximal Lyapunov exponent at the n-th turn reads

$$\lambda(n) = \frac{1}{n} \log \frac{|\hat{\mathbf{x}}^{(n)} - \mathbf{x}^{(n)}|}{|\delta|}. \quad (1)$$

If the orbit is regular, then $|\hat{\mathbf{x}}^{(n)} - \mathbf{x}^{(n)}|$ is linear with n, and therefore $\lambda(n)$ tends to zero for $n \to \infty$. If the orbit is chaotic, then $|\hat{\mathbf{x}}^{(n)} - \mathbf{x}^{(n)}|$ is exponential with n and therefore $\lambda(n)$ tends to a positive limit.

2.2 Tune variation

Another method to select regular from chaotic trajectories [6] is based on the variation of the instantaneous tune. Let $\nu_x(m : n)$ and $\nu_y(m : n)$ be the nonlinear tunes in the x and y plane respectively, computed over the $m, m+1, m+2, ..., n$ turns; then we define the variation of the tune $\tau(n)$ over two successive samples of turns $1, ..., n/2$ and $n/2 + 1, ..., n$ as

$$\tau(n) = \sqrt{\frac{1}{2} \sum_{i=x,y} [\nu_i(1 : n/2) - \nu_i(n/2 + 1 : n)]^2}. \quad (2)$$

For regular trajectories, the tunes are well defined and therefore $\tau(n)$ converges to zero in the limit $n \to \infty$. For chaotic trajectories, the tune is not well defined and therefore $\tau(n)$ is bounded away from zero. In order to use this method, it is crucial to have very precise tools to evaluate the tune also with a limited number of turns [6, 9, 10].

2.3 Thresholds for long-term prediction

We propose the following automatic procedure to forecast long-term particle loss: if the early indicator evaluated at n turns is greater than a threshold $\sigma(n)$, then we assume that the particle will be lost. Otherwise, we consider it as stable.

All the regular particles have a Lyapunov exponent $\lambda(n)$ that converges to zero according to the law $1/n \log(An)$, and the constant A is related to the derivative of the nonlinear tune with respect to the amplitude [11]. Therefore we define the threshold for the Lyapunov as

$$\sigma_\lambda(n) = \frac{1}{n} \log(A_\lambda n). \qquad (3)$$

The threshold for the tune variation has been fixed to

$$\sigma_\tau(n) = \frac{A_\tau}{n}; \qquad (4)$$

this can be justified by heuristic arguments: in fact, the dependence on the inverse of the number of turns is an upper bound to the precision associated to the tune estimate with n turns for generic signals.

We will optimize the choice of the constants A_λ and A_τ through the check with long-term tracking for the Hénon map, and we will show that the same values give good long-term estimates also for the LHC models.

3   ANALYSIS OF THE HÉNON MAP

We first analysed a linear lattice with a sextupolar kick (Hénon map), setting the linear tunes to 0.168 and 0.201 respectively, i.e., very close to a double resonance. We carried out an extensive sampling of initial conditions, and we computed the orbit for 10^7 turns. Both early indica-

intial conditions, for $n = n_1, n_2, n_3, n_4$. We marked in black the initial conditions that are lost before 10^7 turns. The distribution features a high peak with a sharp fall on the right part. The abscissa of the fall separates rather well the stable from the unstable particles, and therefore it appears to be the natural threshold of the Lyapunov. In fact, it turns out that the threshold evaluated numerically through the four histograms is very well interpolated by Eq. (3), with $A_\lambda = 0.15$. For low number of turns ($n = 128$ and $n = 512$), most of the particles whose early indicator prediction fails are unstable with Lyapunov lower than the threshold (intermittency). On the other hand, for higher number of turns ($n = 2048$ and $n = 8192$), most of the particles whose early indicator prediction is wrong are stable with large Lyapunov (stable chaos). This shows that for very large n the Lyapunov leads to a systematic underestimate of the dynamic aperture, since it assumes that all the chaotic particles will be lost. In Figure 2 the
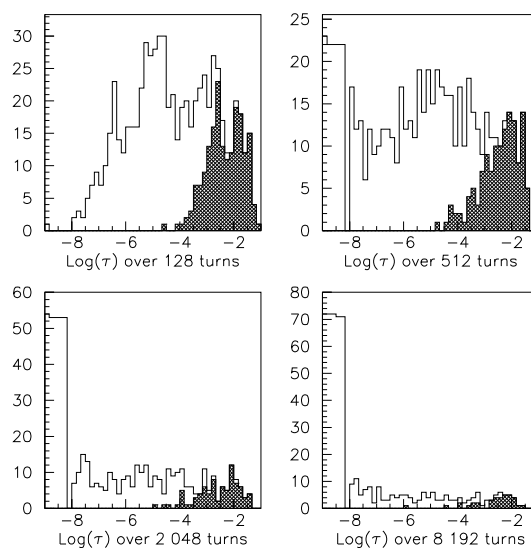
Figure 2: Distribution of the tune variation for the Hénon map; lost particles are marked in black.

same histograms shown in Figure 1 are plotted for the tune variation. The distributions are wider: this is a good feature since it implies that the long-term estimate is less sensitive on the threshold. On the other hand, there is no specific pattern that allows one to define a threshold without carrying out the long-term analysis. Using the long-term data, we empirically fixed the threshold using Eq. (4), with $A_\sigma = 0.1$. For $n = 2048$ and $n = 8192$ a very large fraction of the particles has a tune variation that is very small (less than 10^{-7}); no long-term loss is observed for these particles. This is another interesting feature that could allow one to define a conservative lower bound to long-term stability.

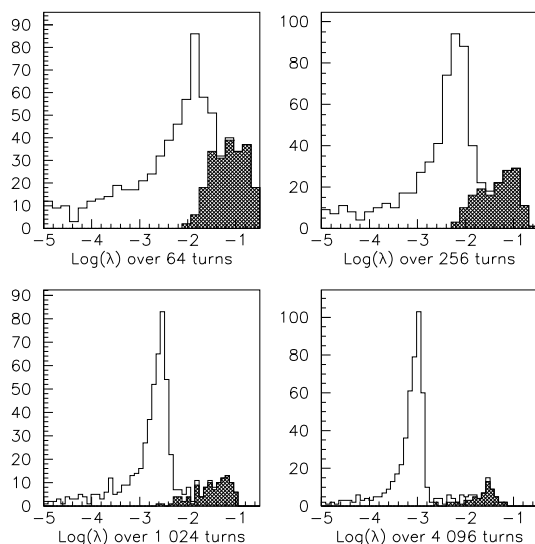It must be pointed out that, contrary to the Lyapunov anal-

Figure 1: Distribution of the Lyapunov for the Hénon map; lost particles are marked in black.

tors were evaluated for four different number of iterates: $n_1 = 128$, $n_2 = 512$, $n_3 = 2048$ and $n_4 = 8192$. In Figure 1 we plot the histogram of the distribution of the Lyapunov exponents $\lambda(n/2)$, computed over the grid of

ysis, the threshold parameter of the tune variation has been optimized through long-term tracking; hence the early indicator cannot be considered predictive for the Hénon map model. We will show in the next section that using the same threshold one obtains good estimates for the LHC.

4 APPLICATIONS TO LHC

We analysed a realistic model of the LHC that includes all field shape imperfections expected in the superconducting magnets. The only relevant approximation is that the synchrotron motion is neglected. Due to the complexity of the model, we had to limit the long-term tracking to 10^5 turns. The distributions of the Lyapunov and of the tune variation are shown in Figs. 3 and 4 respectively. Their similarity with the results for the Hénon map is impressive; it must be also pointed out that in this case the tune is far away from low-order resonances, whilst for the Hénon case it was set very close to resonances (6,0) and (0,5) to enhance the long-term effects. The same thresholds hold for the LHC case. In comparison with the results of the Hénon map, now a larger number of particles are stable but chaotic. This is probably due to the limited number of turns used in long-term tracking. Analogous simulations, carried out for the SPS lattice used for diffusion experiments, has shown very similar results (see Ref. [14] for a detailed discussion).
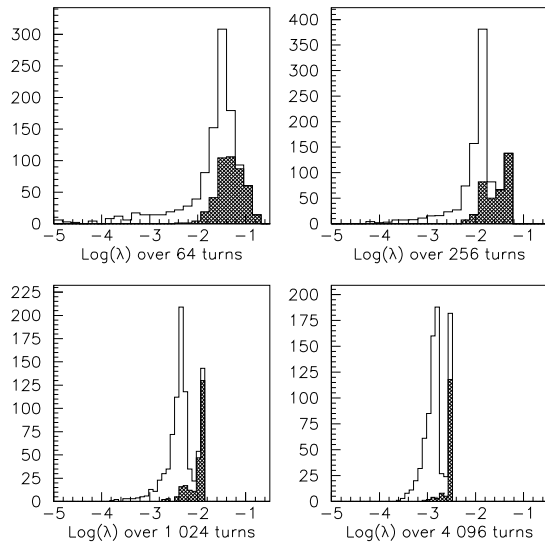

Figure 3: Distribution of the Lyapunov for a 4D model of LHC; lost particles are marked in black.

5 CONCLUSIONS

We have shown that it is possible to provide automated procedures to determine the long-term stability using either Lyapunov exponents or the tune variation. The thresholds
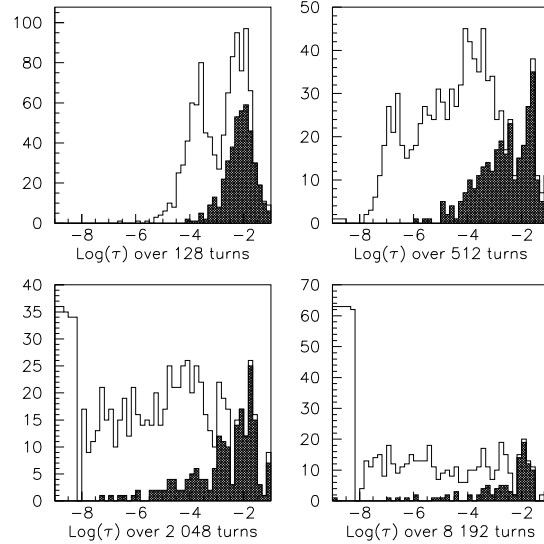

Figure 4: Distribution of the Tune variation for a 4D model of LHC; lost particles are marked in black.

for the early indicators are found to be the same in three different models: Hénon map, SPS and LHC. The use of these methods could be very effective for taking into account the long-term behaviour for situations where a lot of configurations have to be analyzed, for instance, in testing the validity of a sorting procedure [12, 13]. A check of the effectiveness of these techniques for the six-dimensional motion is in progress.

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