QCD Phase Transition at High Temperature in Cosmology

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The cosmological QCD phase transition is studied in terms of the color confinement at finite temperature using the dual Higgs theory of QCD. The confinement force is largely reduced at high temperature, which leads to the swelling of hadrons. We derive analytical formulae for the surface tension and the boundary thickness of the mixed phase from the effective potential at $T_\ast$. We predict a large reduction of the glueball mass near $T_\ast$. We investigate also the process of the hadron-bubble formation in the early Universe.

1. QCD, Color Confinement and Dual Higgs Mechanism

Quantum chromodynamics (QCD) is the fundamental theory of the strong interaction. In spite of the simple form of the QCD lagrangian, it miraculously provides quite various phenomena like color confinement, dynamical chiral-symmetry breaking (D\textsubscript{\chi}SB), non-trivial topologies, quantum anomalies and so on. Therefore QCD can be regarded as an interesting mimic of the history of the Universe, where a quite simple Big Bang also created various things including galaxies, stars, lives and thinking reeds. This is the most attractive point of the QCD physics.

As a modern progress in QCD, confinement physics is providing an important current of the hadron physics in ’90s, since recent lattice QCD studies shed a light on the confinement mechanism. In the ’t Hooft abelian gauge \cite{1}, QCD is reduced into an abelian gauge theory with the QCD-monopole, which appears from a hedgehog configuration corresponding to the non-trivial homotopy group $\pi_2(\text{SU}(N_c)/U(1)^{N_c-1}) = Z_8^{N_c-1}$ on the nonabelian manifold. In this gauge, the nonperturbative QCD (NP-QCD) vacuum is described as the dual Higgs phase with QCD-monopole condensation. Due to the dual Meissner effect, color-electric flux is squeezed like a one-dimensional flux-tube, which leads to the linear quark confinement potential \cite{1}. Thus, the origin of color confinement can be recognized as the dual Higgs mechanism by monopole condensation.

Many recent studies based on the lattice QCD \cite{4} show QCD-monopole condensation in the confinement phase and abelian (monopole) dominance for NP-QCD, e.g., linear confinement potential, $D\text{\chi}SB$ and instantons \cite{4}. Abelian (monopole) dominance means that the essence of NP-QCD is described only by abelian (monopole) variables in the abelian gauge \cite{4}. Therefore the condensed monopole in the ’t Hooft abelian gauge is nothing but the relevant collective mode for NP-QCD, and the NP-QCD vacuum can be identified as the dual-superconductor in a realistic sense.

As a remarkable fact in the duality physics, these are two “see-saw relations” between the electric and magnetic sectors.

1. Due to the Dirac condition $eg = 4\pi$ \cite{1} in QCD between the electric charge $e$ and the magnetic charge $g$, a strong-coupling system in one sector corresponds to a weak-coupling system in the other sector.

2. The long-range confinement system corresponds to a short-range interaction system in the other sector.

As the most attractive point in the dual Higgs theory, the highly-nonlocal confinement system can be described by a short-range interaction theory with the dual variables.

2. QCD Phase Transition in Dual Ginzburg-Landau Theory

The dual Ginzburg-Landau (DGL) theory is the effective theory based on the dual Higgs mechanism, and can be derived from the QCD lagrangian and the QCD nature (monopole condensation and abelian dominance). The DGL lagrangian \cite{1-3} for the pure-gauge system is described with the dual gauge field $B_\mu$ and the QCD-monopole field $\chi$,

$$\mathcal{L}_{\text{DGL}} = \text{tr} \left\{ -\frac{1}{2} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + [\hat{D}_\mu, \chi] [\hat{D}_\mu, \chi] - \lambda (\chi^\dagger \chi - v^2)^2 \right\},$$

where $\hat{D}_\mu = \partial_\mu + igB_\mu$ is the dual covariant derivative.

The dual gauge field $B_\mu \equiv \vec{B}_\mu \cdot \vec{H} = B^3_\mu T^3 + B^8_\mu T^8$ is defined on the dual gauge manifold $U(1)_m^3 \times U(1)_m^8$ \cite{1-3}, which is the dual space of the maximal torus subgroup $U(1)_m^3 \times U(1)_m^8$ embedded in the original gauge group $SU(3)_\text{c}$. The abelian field strength tensor is written as $F_{\mu\nu} = *(\partial \wedge B)_{\mu\nu}$, so that the role of the electric and magnetic fields are interchanged in comparison with the ordinary $A_{\mu}$ description.

The QCD-monopole field $\chi$ is defined as $\chi \equiv \sqrt{2} \sum_{\alpha=1}^{3} \chi_\alpha E_\alpha$ \cite{3}. In the QCD-monopole condensed vacuum with $|\chi| = v$, the dual gauge symmetry $U(1)_m^3 \times U(1)_m^8$ is spontaneously broken. Through the dual Higgs mechanism, the dual gauge field $B_\mu$ acquires its mass $m_B = \sqrt{3}gv$, whose inverse provides the
hadron-flux-tube radius. The dual Meissner effect causes the color-electric field excluded from the QCD vacuum, which leads to color confinement. The QCD-monopole fluctuations \( \tilde{\chi}_\alpha = \chi_\alpha - v \) \((\alpha=1,2,3)\) also acquire their mass \( m_\chi = 2\sqrt{\lambda} v \) in the QCD-monopole condensed vacuum. As a relevant prediction, one QCD-monopole fluctuation \( \tilde{\chi} = \sum_{\alpha=1}^3 \tilde{\chi}_\alpha \) appears as a color-singlet scalar glueball in the confinement phase, although the dual gauge field \( B_\mu \) and the other two combinations of the QCD-monopole fluctuation are not color-singlet and cannot be observed.

In the DGL theory, the QCD phase transition is characterized by QCD-monopole condensate \( \tilde{\chi} \equiv |\chi_\alpha| \), which is an order parameter on the confinement strength. For the study of the QCD phase transition, we formulate the effective potential \( V_{\text{eff}}(\tilde{\chi}; T) \) at finite temperature as the function of \( \tilde{\chi} \) using the quadratic source term to avoid the imaginary scalar-mass problem [1-3].

We study the relation between \( V_{\text{eff}}(\tilde{\chi}; T) \) and the surface tension \( \sigma \) [1,3], which characterizes the strength of the first order in the phase transition, and is very important for the shape of the boundary surface in the mixed phase. Using the sine-Gordon (SG) kink ansatz [1,3] for the boundary profile, \( \tilde{\chi}(z) = \tilde{\chi}_c \tan^{-1} e^{z/\delta} \), we derive simple formulae for the surface tension \( \sigma \equiv \frac{4\sqrt{\pi}}{\pi} \sqrt{\bar{\chi}_c} \simeq (112 \text{MeV})^3 \), and the phase-boundary thickness \( 2\delta \simeq \frac{2\sqrt{3}}{\pi} \tilde{\chi}_c/\sqrt{\bar{\chi}_c} \simeq 3.4 \text{fm} \), where \( h \) is the “barrier height” between the two minima in \( V_{\text{eff}}(\tilde{\chi}; T_c) \).

At high temperature, the hadron string tension \( k(T) \) becomes smaller and drops rapidly near \( T_c \) [2,3]. Therefore, the slope of the inter-quark potential is reduced and the inter-quark distance inside hadrons increases at high temperature. In addition, the color-electric field spreads according to the decrease of \( m_B \). Thus, the reduction of the confinement force leads to the swelling of hadrons at high temperature [3].

We predict also a large mass reduction on the QCD-monopole (a scalar glueball) near \( T_c \). The violent excitation of QCD-monopoles (scalar glueballs) with a reduced mass would promote the QCD phase transition.

### 3. Hadron Bubble Formation in the Early Universe

Finally, we study the hadron bubble formation [3,5] in the early Universe using the DGL theory [3]. As Witten pointed out, if the QCD phase transition is of the first order, the hadron and QGP phases should coexist in the early Universe. During the mixed-phase period, there appears the inhomogeneity on the baryon density distribution [5], which can strongly affects the primordial nucleo-synthesis.

The hadron bubble created in the supercooling QGP phase is described by the SG-kink type profile of the QCD-monopole condensate [3], \( \tilde{\chi}(r; R) = C \tan^{-1} e^{(R-r)/\delta} \), where \( R \) and \( 2\delta \) correspond to the hadron-bubble radius and the phase-boundary thickness, respectively. The total energy of the hadron bubble with radius \( R \) can be estimated using \( V_{\text{eff}}(\tilde{\chi}; T) \), \( E(R; T) = 4\pi \int_0^\infty dr r^2 \{3(\frac{d\tilde{\chi}(r; R)}{dr})^2 + V_{\text{eff}}(\tilde{\chi}; T)\} \), which is roughly estimated as the sum of the positive surface term and the negative volume term [3]. The hadron-bubble energy \( E(R; T) \) takes a maximal value at a critical radius \( R_c \). Hence, the hadron bubbles with \( R < R_c \) collapse, and only large hadron bubbles with \( R > R_c \) grow up with radiating the shock wave [3,5]. On the other hand, the creation probability of large bubbles is strongly suppressed [3] because of the thermodynamical factor \( P(T) \equiv \exp\{-\frac{12}{\pi^2} R_c(T)^3 h(T)/T\} \), with a large barrier height \( h(T) \) in \( V_{\text{eff}}(\tilde{\chi}; T) \). Thus, the only small bubbles are created practically, although its radius should be larger than \( R_c \) [3].

As the temperature decreases, the smaller hadron bubbles are created, while the bubble formation rate becomes larger [3]. Then, we can imagine how the QCD phase transition happens in the Big Bang scenario: (a) Slightly below \( T_c \), only large hadron bubbles appear, but the creation rate is quite small. (b) As temperature is lowered by the expansion of the Universe, smaller bubbles are created with much formation rate. During this process, the created hadron bubbles expand with radiating shock wave, which reheats the QGP phase around them. (c) Near \( T_{\text{low}} \), many small hadron bubbles are violently created in the unaffected region free from the shock wave. (d) The QGP phase pressured by the hadron phase is isolated as high-density QGP bubbles [3], which provide the baryon density fluctuation [5]. Thus, the numerical simulation using the DGL theory would tell how the hadron bubbles appear and evolve quantitatively in the early Universe.

### References