MODE TRANSIT TIMES IN NEAR-PARABOLIC-INDEX OPTICAL FIBRES

Indexing terms: Fibre optics, Guided light propagation, Optical waveguides

Calculations are presented of the transit times of guided and tunnelling rays in graded-index optical fibres. It is shown that 25% of the power launched by an incoherent source into a parabolic-index fibre is carried by leaky rays, which may increase the r.m.s. width of the impulse response by up to 41%. The profile which most effectively equalises the combined transit times of both guided and tunnelling rays is given by $\alpha = 2-4\Delta$

Introduction: In a recent publication,[1] it was shown that tunnelling leaky rays[2] are present on all multimode guiding structures having circular symmetry, and the plane-wave decomposition technique[3] was used to delineate the angular region in which they exist. For a near-parabolic-index fibre, this angular region was found to be within the meridionally defined numerical aperture. Thus, in contrast to the step-index fibre, leaky rays are excited not only by an incoherent source, but also by other extended sources which may fill the numerical aperture, such as a broad-contact semiconductor laser. Consequently, these rays assume some importance in graded-index fibres, and may be expected to have a significant effect on both attenuation and pulse-dispersion measurements.

We show here that for a parabolic-index fibre 25%* of the power launched by an incoherent source is contained within the leaky modes, and that if all these modes propagate unattenuated the r.m.s. width of the impulse response may be increased by up to 41%. It is further shown that the index profile may be adjusted to equalise the transit times of both leaky and guided modes, leading to a somewhat different result from that obtained by equalising the transit times of guided modes alone.[3]

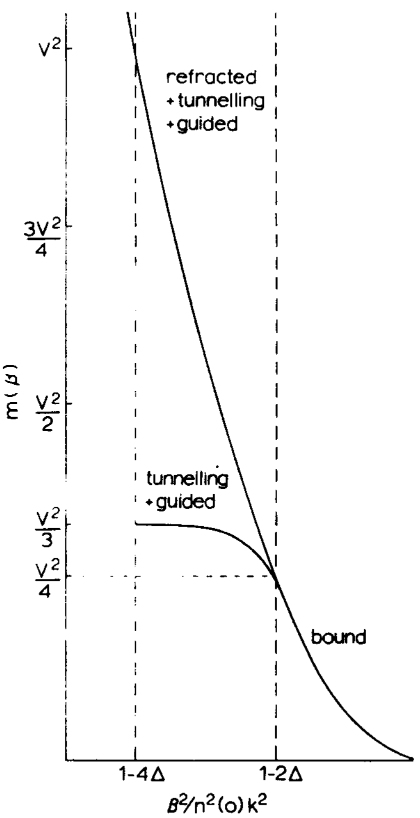

Fig. 1 *Number of modes $m(\beta)$ in parabolic-index fibre having propagation constant β greater than value specified on abscissa*
Below the bound mode cutoff the Figure shows both the total number of modes (upper curve) and the number of modes having some bound energy (lower curve). No tunnelling modes exist below $\beta^2/n^2(0)k^2 = 1-4\Delta$

* As this letter was being prepared, we learnt that the same result has been derived independently by J. A. Arnaud using an entirely different method. We are indebted to Dr. Arnaud for a preprint of his paper 'Pulse broadening in multimode optical fibres' to appear in *Bell Syst. Tech. J.*, Sept. 1975

Number of leaky modes: As a first step towards determining the pulse dispersion, we may calculate the number $m(\beta)$ of all modes having a propagation constant β greater than a certain specified value, including those below cutoff. Modes below cutoff have $\beta^2 \leq n^2(0)k^2(1-2\Delta)$, where k is the wavenumber, Δ the normalised maximum refractive-index difference and $n(0)$ the index at the core centre. In this case, the modes are of two types: tunnelling and refracted.[2] The total number summed over all modes may be determined by the plane-wave decomposition technique,[1,3] and is given by the analytic continuation of Gloge and Marcatili's[3] expression below cutoff. The result is shown by the upper curve in Fig. 1. The lower curve is obtained by summing only the guided and tunnelling modes, and represents the subset of modes assumed here to be propagating unattenuated. The Figure is drawn for a parabolic-index fibre for which $\alpha = 2$, where α is defined in Reference 3.

Whereas refracted modes may have $\beta \to 0$, there is a critical value of β below which no further tunnelling modes are found. The limit occurs when the ν^2/r^2 curve is tangential to the $k^2n^2(r)-\beta^2$ curve (Fig. 2, Reference 1), so that there is no

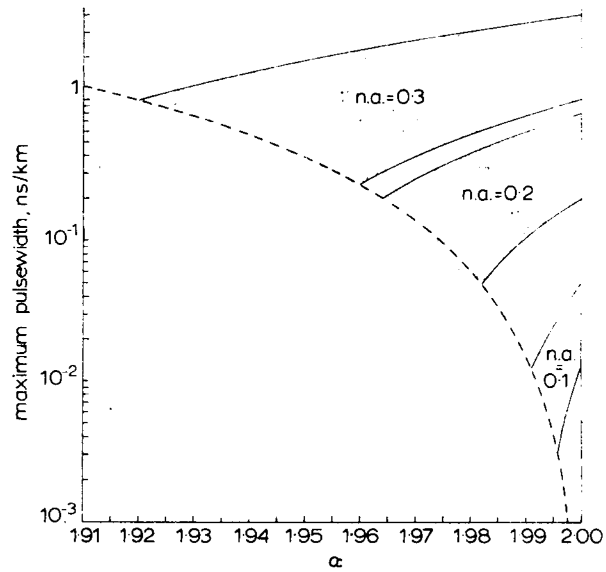

Fig. 2 *Pulse dispersion as function of index-profile parameter α for fibres having numerical apertures of 0·1, 0·2 and 0·3*
For a given α the pulsewidth is determined either by the broken line or by the limits of the shaded regions

longer an oscillatory region within the core. For this condition to apply it can be shown that

$$\beta^2 \geq n^2(0)k^2[1-(2+\alpha)\Delta] \geq 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It is clear from Fig. 1 that there are $V^2/4$ guided modes in a parabolic-index fibre, since this number has a propagation constant greater than the bound-mode cutoff value of $\beta^2 = n^2(0)k^2(1-2\Delta)$ (V is the normalised frequency). Similarly, the number of guided plus tunnelling modes is $V^2/3$, these modes having $\beta^2 \geq n^2(0)k^2(1-4\Delta)$. Thus 25% of the power launched by an incoherent source will be contained within the tunnelling modes, since the ratio of the number of guided to tunnelling modes is 3 : 1.

An alternative method of obtaining the above result is to integrate the near-field intensity profile shown in Fig. 3 of Reference 1 over the radius r, from which the ratio of the number of bound modes to leaky modes may be calculated. We note in passing that Snyder's result[4] for the total number of tunnelling modes in a step index fibre may be derived by either method.

Differential mode delays: If, for the present, we ignore the attenuation of leaky tunnelling modes, we may obtain an upper limit for the dispersion effects on near-parabolic fibres. Following the method of Reference 3, we differentiate the $m(\beta)$ curve shown in Fig. 1 (using the curve for all modes) to obtain the dispersion $d\beta/dk$.

Fig. 2 shows the calculated pulse dispersion, as a function of the index-profile parameter α. For a given numerical aperture the shaded area represents a region of allowed pulse dispersion, the actual pulsewidth depending on the power remaining in the leaky modes at the fibre output. Each shaded region is bounded by the calculated curves for guided rays[3] (lower bound) and for leaky-plus-guided rays (upper bound). Outside the shaded regions the pulsewidth is unaffected by the presence of leaky rays, and is given unambiguously by the broken curve. It should be noted that the pulse dispersion referred to here is the total spread in transit times between the fastest and slowest modes, and represents an upper limit on the pulsewidth.

For a near-parabolic-index variation in the form of a power law,[3] we assume

$$\alpha = 2 - K\Delta \qquad (2)$$

It has been shown[3] that the optimum choice of K to minimise the group-delay differences between guided modes is given by $K = 2$, resulting in a dispersion per unit length of $n(0)\,\Delta^2/8c$. However, Fig. 2 shows that this choice of profile is no longer optimal if all tunnelling modes are present. In this case the pulse length may be increased to $9n(0)\,\Delta^2/8c$. A new optimum value of $K = 4$ is now appropriate and the maximum pulse length is reduced to $n(0)\,\Delta^2/2c$. We note that this result has been obtained by Geckeler[5] for helical rays only.

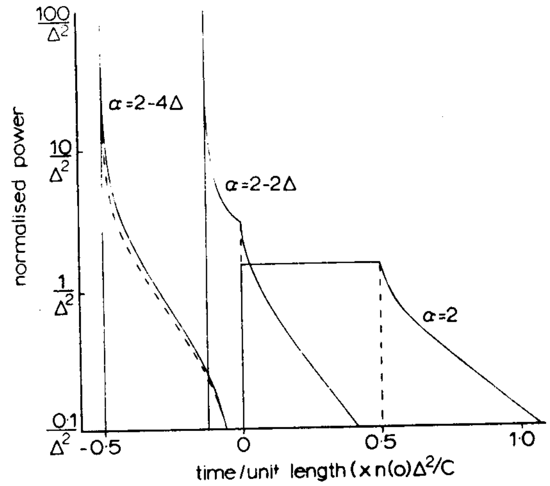

Fig. 3 *Impulse response per unit length for index profiles described by $\alpha = 2, 2-2\Delta$ and $2-4\Delta$*

The solid lines show the pulse dispersion with both bound and tunnelling modes present and the broken lines show the effect of bound modes only

Impulse response: Having obtained the number of tunnelling modes and the transit times for all modes below cutoff, it remains to determine the impulse response, assuming that the guided and tunnelling modes propagate unattenuated. The results are shown in Fig. 3 for K of 0, 2 and 4 in eqn. 2, corresponding, respectively, to the perfect parabolic index, the optimised distribution for guided modes and the optimised distribution for guided-plus-tunnelling modes. It is seen that for $K = 0$ and 2 leaky rays have the effect of adding a tail at the end of the pulse, since their transit times are all greater than that of the slowest bound mode. For the perfect parabola the r.m.s. width of the response is increased by 41% from $0{\cdot}5n(0)\,\Delta^2/c$ to $0{\cdot}704n(0)\,\Delta^2/c$ when all these rays are present. For the new optimum condition $K = 4$, the effect of leaky rays is to increase the amplitude of the response, rather than the overall width, since this choice of profile has the effect of adjusting the transit times of the leaky rays to lie within the normal range of those of the bound rays. Note that it is this effect which gives the unambiguous pulse dispersion shown by the broken line in Fig. 2. The complete overlap of the transit times of both leaky and bound rays is only possible in fibres with close-to-parabolic index profiles.

In conclusion, it must be reiterated that the results herein ignore the attenuation of leaky tunnelling modes, so that they represent a worst-case estimate. However, there are indications that these modes[2] may persist for long distances and carry a significant proportion of the power. In addition Stewart[6] has shown that many of the modes treated here as leaky no longer radiate if the fibre has a cladding of finite thickness. In this case they may be expected to have low loss and, consequently, to have a significant effect on pulse broadening in long fibres.

Acknowledgments: The authors wish to express their thanks to Prof. W. A. Gambling for his advice and comments on the manuscript. Acknowledgements are made to the UK Science Research Council, the UK Ministry of Defence and the Pirelli General Cable Co. for the award of research fellowships.

M. J. ADAMS
D. N. PAYNE
F. M. E. SLADEN

Department of Electronics
University of Southampton
Southampton SO9 5NH, England

2nd July 1975

References

1 ADAMS, M. J., PAYNE, D. N., and SLADEN, F. M. E.: 'Leaky rays on optical fibres of arbitrary (circularly symmetric) index profiles', *Electron. Lett.*, 1975, **11**, pp. 238–240
2 SNYDER, A. W.: 'Leaky-ray theory of optical waveguides of circular cross-section', *Appl. Phys.*, 1974, **4**, pp. 273–298
3 GLOGE, D., and MARCATILI, E. A. J.: 'Multimode theory of graded-core fibres', *Bell Syst. Tech. J.*, 1973, **52**, pp. 1563–1578
4 PASK, C., SNYDER, A. W., and MITCHELL, D. J.: 'Number of modes on optical waveguides', *J. Opt. Soc. Am.*, 1975, **65**, pp. 356–357
5 GECKELER, S.: 'Dispersion in optical waveguides with graded refractive index', *Electron. Lett.*, 1975, **11**, pp. 139–140
6 STEWART, W. J.: 'A new technique for determining the V values and refractive index profiles of optical fibres'. Conference on optical fiber transmission, Williamsburg, USA, 1975