Gravitational lensing as a new probe of the dynamics of cluster cores

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ABSTRACT
We propose a new approach to study the dynamical implications of mass models of clusters for the velocity structure of galaxies in the core. Strong and weak lensing data are used to construct the total mass profile of the cluster, which is used in conjunction with the optical galaxy data to solve in detail for the nature of galaxy orbits and the velocity anisotropy in the central regions. We also examine other observationally and physically motivated mass models, specifically those obtained from X-ray observations and N-body simulations. The aim of this analysis is to understand qualitatively the structure of the core and test some of the key assumptions of the standard picture of cluster formation regarding relaxation, virialization and equilibrium. This technique is applied to the cluster Abell 2218, where we find significant velocity anisotropy in the core, with pre-dominantly tangential orbits, indicating the existence of a dynamically disturbed core. We find that the requirement of physically meaningful solutions for the velocity anisotropy places stringent bounds on the slope of cluster density profiles in the inner regions.

Key words: galaxy clusters: lensing – galaxy clusters: galaxy orbits – galaxy clusters: dynamics

1 INTRODUCTION

Studying the velocity structure of the cores of clusters of galaxies promises to provide new insights into the physics of the formation of clusters. The crucial physical consequence of the cumulative dynamical history of a cluster is its underlying mass distribution. We propose a new approach to study the dynamics of cluster galaxies using the mass profile measured from gravitational lensing. Lensing provides the most accurate determination of the mass profile, and is independent of assumptions as to the kinematics of the cluster.

The dramatic arcs and multiple images produced by rich clusters tightly constrains the mass of the cluster in the inner-most regions, on the scale of the Einstein radius, with typical range $30 < r_E < 200 h_{50}^{-1}$ kpc (Kneib & Soucail 1995). Current progress in observational techniques have made it possible to map the cluster mass out to large radii from the weak shearing of faint background galaxies. While uncertainties arise due to the correction for the point-spread function and the unknown redshift-distribution of background galaxies. While uncertainties arise due to the correction for the point-spread function and the unknown redshift-distribution of background galaxies, reliable mass profiles for an increasing number of clusters should be available in the near future (Kaiser, Squires, & Broadhurst 1995). Combining this with knowledge of the spatial distribution and line-of-sight component of the velocities of cluster galaxies, and assuming them to be good tracers of the cluster potential well, we can solve in detail for the variation of the velocity anisotropy parameter $\eta$ with distance from the cluster centre.

If a large number of galaxy redshifts ($N \sim 1000$) were available for a single cluster, then the velocity histogram and specifically its departure from gaussianity would give a direct measure of the anisotropy (Zabludoff, Franx, & Geller 1993). In practice, it is an observational challenge to sample any distant cluster, $z > 0.1$ in such detail and to efficiently disentangle the effects of substructure and axisymmetric infall in interpreting the deviation from gaussianity. However, securing 200 to 800 velocities for distant cluster galaxies is now becoming possible with the newly developed MOS multi-object imaging spectrographs (Yee, Ellingson, & Carlberg 1996).

Recent optical surveys by Colless & Dunn (1995) and X-ray studies of clusters by Briel & Henry (1995) seem to indicate that while the X-ray isophotes of cluster cores have an overall smooth appearance most of them are not only dynamically young but also quite disturbed. The nature of orbits is therefore an important indicator of the dynamical state of both the inner and outer regions of the cluster. Principally radial orbits, expected at the outskirts, are the signature of a region dominated by infall, whereas isotropic orbits imply the existence of a well-mixed region.

Previous studies of the velocity profile and estimates
of the degree of anisotropy in clusters have provided ambiguous results, primarily due to the lack of knowledge of the underlying mass distribution. The velocity structure of galaxies in clusters have been studied in detail (Coma and A2670) by Kent & Gunn (1982); The & White (1986); Merritt (1987); Sharples, Ellis & Gray (1988); Colless & Dunn (1995) and several other groups. These analyses used the observed galaxy positions and velocities to constrain the distribution of total mass and simultaneously find consistent and physically meaningful solutions for the velocity anisotropy.

The & White (1986) examined the uncertainties in the virial mass profiles derived for Coma from observational data. They showed that a wide range of mass models are consistent, consequently permitting a large range of orbital structures. Mass models that were more compact (but had low overall masses) implied circular orbits for the galaxies whereas the higher mass models implied predominantly radial orbits. Merritt (1987) examined the relative distribution of the dark and luminous matter in Coma and showed that it was impossible to distinguish between models where the galaxies trace the total mass and were on isotropic orbits versus those in which the dark matter was very concentrated and the galaxies were on primarily transverse orbits. Therefore, sufficient constraints on the total mass distribution and the velocity anisotropy cannot be obtained simultaneously using only the luminous tracers. In the recent survey of the Coma cluster by Colless & Dunn (1995), they find that the velocity distribution is highly non-Gaussian. The dynamics can be better interpreted in terms of an on-going merger between two sub-clusters, thus indicating that the system is not relaxed.

Similar studies of the cluster A2670 have also been inconclusive; in the faint photometric and spectroscopic survey by Sharples; Ellis & Gray (1988), both extremely anisotropic models and nearly isotropic ones were indistinguishable in terms of goodness of fit with respect to the available data.

In these clusters and others that have been studied, the largest uncertainty in the determination of the anisotropy arises from ignorance of the distribution of the total mass. In our analysis, independent data from lensing that constrains the overall mass distribution allows the elimination of this largest source of uncertainty.

The plan of this paper is as follows: in Section 2 we review observational probes of the internal dynamics of clusters and construct mass models from X-ray data, lensing data, and from N-body simulations. The mathematical formalism that forms the basis of our approach is presented in Sections 3 & 4. The robustness of the method is demonstrated for several fiducial forms of the total mass profile (Section 5), and the technique is then applied to the cluster A2218 (Section 6). Finally, we discuss our results and their implications for the physical state of the core of A2218. Throughout this paper, we assume $H_0 = 50 \text{ km s}^{-1}\text{Mpc}^{-1}$, $\Omega = 1$ and $\Lambda = 0$.

2 OBSERVED PROPERTIES OF THE CORE

2.1 Photometric and spectroscopic studies

With the rapid progress in photometric and spectroscopic techniques, the precision of observationally determined parameters for clusters has improved significantly in the past decade. The main limitations are the errors incurred in establishing cluster membership (Aragon-Salamanca et al. 1993) due to contamination from the field population for photometrically selected samples. On the spectroscopic front, while surveys are pushing down to fainter magnitude limits, a commensurate gain in the number of cluster galaxies sampled cannot be achieved since the number of faint background field galaxies grows rapidly. In the following subsections we examine the observationally determined quantities that we use in our present analysis.

2.1.1 Galaxy surface density profiles from optical data

Galaxy surface density profiles are generally determined by fitting the observed number density in a given cluster to physically motivated functional forms (Kent & Gunn 1982). Since to define cluster membership, one usually uses the color-magnitude relation of the E & S0 galaxies (given the limited spectroscopic samples available), the computed galaxy distribution profile preferentially probes the E & S0 cluster galaxies. The other morphological types in the cluster, could in principle have different profiles, but given the limitations of the available data we implicitly assume that the E & S0’s efficiently trace the overall mass distribution.

The modified Hubble-law profile, where the density drops off at large radius as $\rho \propto r^{-3}$ with a flat core fits the cluster number density fairly well,

$$\rho(r) = \frac{\rho_0}{(1 + \frac{r}{r_0})^\alpha}; \quad \alpha = 1.5.$$  \hfill (1)

The core radius $r_0$ for typical clusters ranges from $150 h_{50}^{-1}$ kpc to about $300 h_{50}^{-1}$ kpc. A least squares method is often used to determine the values of $\rho_0$ and core radius that simultaneously provide the best-fit to the data. An alternate method is to fit a non-parametric function to the surface number density as proposed by Merritt & Gebhardt (1995), which in principle gives better results. However, one needs copious good quality data in order to use these more mathematically sophisticated techniques.

2.1.2 Line-of-sight velocity dispersion profiles

The only measured component of the velocity dispersion of the galaxies is the projection along the line-of-sight. The interpretation of this measurement might be severely affected by substructure in the cluster and the presence of interacting sub-groups. For any distant cluster ($z > 0.1$), the line-of-sight velocity dispersion is not determined accurately enough at present to construct a secure radial profile. Even for nearby clusters, the rich region close to the centre and the outer regions, are not sampled adequately to infer conclusively the asymptotic behavior. For the well-studied cluster Abell 2218, the error bars in the measurements (see Fig. 6) are too large to perform any sensible fit to the available data and extract a radial profile.
2.1.3 X-ray observations

X-ray observations by Einstein, ROSAT and ASCA map the thermal bremsstrahlung emission from the hot intracluster gas at temperatures around \(10^7 - 10^8 \text{K}\); the typical observed X-ray luminosities range from \(10^{42} - 10^{45} \text{ergs s}^{-1}\) (in the 0.5 – 4.5 keV band). The measured gas mass within the central few Mpc is of the order of \(10^{14} M_\odot\), and the inferred cooling times for the gas in some clusters is of the order of 10^9 years (Fabian, Nulsen & Canizares (1982)). The surface brightness profiles of clusters are sharply peaked at the centre. From X-ray observations, with a few exceptions, most cluster cores appear smooth and uniform with very regular isophotes. Accurate measurements of the temperature profile in the cores are only now becoming feasible (for \(z < 0.1\) clusters) with the ASCA satellite.

2.1.4 Mass profiles from X-ray data

Standard deprojection analysis of the X-ray surface brightness profile gives the density profile of the gas, and the cluster potential within which the gas is confined (Sarazin 1988). The main limiting assumptions are spherical symmetry, hydrostatic equilibrium, the inability to take sub-structure into account and assumptions regarding the unknown radial temperature profile of the gas (Nulsen & Böhringer 1995). The latter assumption is particularly important when modelling the inner parts of cluster cores with either strong cooling flows, where one expects to have a multiphase ICM (Allen, Fabian, & Kneib 1995), or in clusters which have undergone recent mergers, resulting in a complex temperature structure. Preliminary results of measurements of the temperature structure of the core of A2256 (Briel & Henry 1995) seem to indicate that this cluster, which was believed to be one of the smoothest and most uniform from its X-ray image shows strong evidence for temperature gradients in the core (Markevitch et al. (1996)).

2.2 N-body simulations

N-body simulations provide the crucial link in understanding how the observed structure in clusters arises in the context of their evolution from the initial perturbations in the gravitational instability picture.

High-resolution simulations that incorporate gas dynamics and some of the important gas physics like shocks and radiative cooling are being used to study the formation, dynamics and evolution of galaxy clusters in the scenario where structure is built up hierarchically in a universe dominated by cold dark matter (Evrard (1990); Cen, Gnedin & Ostriker (1993); Navarro & White (1994); Frenk et al. (1995)). The evolution of both the dark matter and the baryonic component can be tracked to within the resolution limit. While the core of an individual cluster cannot be resolved in enough detail to understand relaxation processes, an ensemble of clusters can be studied for their ‘average’ properties (Navarro, Frenk, & White 1994).

The density profiles of clusters formed in these simulations are sensitive to the underlying cosmological model, the initial conditions, the accuracy of modeling gas dynamic processes and prescriptions for galaxy and star formation. Efstathiou et al. (1985) found that in simulations with only dark matter particles, the slope of the density profile on cluster scales steepens with increasing \(n\), where \(n\) is the spectral index of the scale-free, initial perturbations in an Einstein-de Sitter cosmology. It has been suggested by Crone, Evrard & Richstone (1994) and Navarro, Frenk & White (1994) that ensemble cluster properties, like abundance, clustering and density profiles might be a useful discriminant of cosmological parameters. However, properties of currently simulated clusters are not consistent in detail with their observed properties, primarily a reflection of the lack of understanding of the physics of galaxy formation and the role of non-gravitational processes coupled with the lack of knowledge of \(\Omega\). Qualitatively though, the morphology of simulated clusters is quite similar to ROSAT observations of X-ray clusters, and the physical effects of mass segregation due to dynamical friction and luminosity segregation seem to be borne out in the simulations - evidence to support our naive theoretical picture of the formation of clusters.

In a recent paper, Navarro, Frenk & White (1995) report the results of their N-body + SPH simulations, wherein a ‘universal density profile’ is found to be a good fit over a large range of scales for dark halos in standard CDM models. The halo profiles are more or less isothermal, shallower than \(r^{-2}\) near the central regions and steeper close to the virial radius. The density profile has the following form,

\[
\rho (r) = \frac{\rho_0}{r_s (1 + \frac{r}{r_s})^2} \quad (2)
\]

where \(r_s\) is a scale radius. The corresponding mass profile is given by:

\[
M(r) = M_0 [\ln(1 + \frac{r}{r_s}) + \frac{1}{(1 + \frac{r}{r_s})}] \quad (3)
\]

We examine this mass model and its consequences for the resulting dynamics of cluster cores in section 5.

2.3 LENSING BY CLUSTERS

Clusters of galaxies have the optimum cross-section for lensing the nearly isotropically distributed high redshift faint galaxy population. Lensing by an extended mass concentration can be understood in terms of a mathematical mapping (e.g. Blandford & Narayan (1987) and Fort & Mellier (1994)) from the source plane onto the image plane with the properties that it conserves surface brightness and is achromatic. The deflections produced are non-linear with impact parameter and therefore produce both amplification and distortion of the background sources. There are two important effects: the isotropic magnification and the non-isotropic distortion. The isotropic magnification is caused by mass interior to the beam and is pronounced in the region of the image plane where the local surface mass density \(\Sigma\) is of the order of the critical surface mass density, \(\Sigma_{\text{crit}}\), which occurs in the dense cores of rich clusters producing multiple images and arcs. \(\Sigma_{\text{crit}}\) depends on the angular distance to the source and lens and hence on cosmological parameters. The anisotropic distortion of images is caused by the gradient of the two-dimensional potential and characterizes the ‘weak lensing’ regime, the signal being arclets (single weakly sheared images) produced even at large distances from the cluster centre.
A composite mass profile for a cluster can be constructed using a variety of constraints from lensing effects over a range of scales. The strong lensing regime constrains the total mass enclosed within the ‘Einstein radius’, while weak shear effects (measured statistically from the ellipticities of the faint background galaxies) determine the slope of the mass profile at the outskirts.

### Constraints from Strong Lensing

The input from observations for the mass modeling are arc positions, the number of merging images and their parities, and the width, shape and curvature of the arcs. They are used to determine the location of the critical lines in the image plane which are then mapped back to the source plane in the method developed by Kneib (1993). The difference in parameters implied by each of the multiple images is then minimized in the $\chi^2$ sense in the source plane. In order to calibrate the lens model, at least one arc redshift needs to be measured. The usual mass profiles used in modeling the cluster mass distributions are the cored isothermal profile (Blandford & Kochanek (1987)), the pseudo-isothermal elliptical mass distribution (PIEMD) (Kassiola & Kovner 1993) or a linear combination of them. The PIEMD model has a 2-D surface mass density defined by:

$$\Sigma(r) = \frac{\Sigma_0}{\sqrt{1 + \frac{r^2}{r_0^2}}},$$

The corresponding 3-dimensional density profile and the mass are,

$$\rho(r) = \frac{\rho_0}{1 + \frac{r^2}{r_0^2}},$$

$$M(r) = M_0 \left[ \frac{r}{r_0} - \tan^{-1} \frac{r}{r_0} \right].$$

Modeling the arcs with this profile, the normalization and the core radius (which is a measure of the compactness of the mass distribution) are determined. The core radius of most lensing clusters is observed to be quite small, $30 h_{50}^{-1}$ kpc $\leq r_0 \leq 100 h_{50}^{-1}$ kpc.

### Constraints from Weak Lensing

The slope of the mass profile at large radii ($r > 200 h_{50}^{-1}$ kpc) is constrained by the observed weak distortion effects. The weak shear $\gamma$ induced by the cluster on the background images can be written for the circularly symmetric case as:

$$\gamma \propto \langle D(z_s) \rangle \left( \Sigma(<r) - \Sigma(r) \right),$$

where $\langle D(z_s) \rangle$ is the mean of the ratio of the angular distances $D_{\text{source}}/D_{\text{observer}}$ and $\Sigma$ the mean surface density within radius $r$. The Kaiser-Squires technique (Kaiser & Squires 1993) defines a mapping that relates the image ellipticities to the relative mass map $\Sigma(r)$ for a cluster. To construct the surface mass density profile one uses the statistic suggested by Fahlman et al. (1994) & Squires et al. (1995),

$$\Sigma(<r_1) - \Sigma(r_1 < r < r_2) = \frac{2\Sigma_{\text{crit}}}{1 - r_1^2/r_2^2} \int_{r_1}^{r_2} \langle \epsilon_\parallel \rangle \frac{dr}{r},$$

where $\langle \epsilon_\parallel \rangle$ is the mean tangential component of the image ellipticities. For this inversion, deep optical images under exquisite seeing conditions of a wide field over the lensing cluster are required. Details of correction and compensation for the anisotropy of the point-spread function and bad seeing conditions have been demonstrated by Kaiser, Squires & Broadhurst (1995) and Bonnet & Mellier (1996).

### Constraints from the cD galaxy

Dark matter in clusters is sharply peaked about the cluster centre around which the lensed images are seen. In most clusters with spectacular arcs, the centre of the brightest cluster galaxy and the centre of the dark matter distribution as determined both from X-ray and lensing studies seem to be coincident to within the errors - of the order of a few arcseconds.

The central bright elliptical galaxies are often cDs with diffuse halos extending out to beyond the Einstein radius. The orbits of the stars in these halos trace the overall dark matter potential. The density profiles of cD galaxies are fairly well-determined observationally (Kneib & Soucail et al. (1995), Broadhurst & Broadhurst (1995) and Bonnet & Mellier (1996)). The orbits of the stars in these halos trace the overall dark matter potential. The density profiles of cD galaxies are fairly well-determined observationally (Kneib & Soucail et al. (1995), Broadhurst & Broadhurst (1995) and Bonnet & Mellier (1996)).

### 3 DYNAMICAL EQUATIONS

We model the cluster as a collisionless system in which the individual galaxies move under the influence of the mean gravitational field $\phi$ generated by all the constituents. The system is characterized by its phase-space density $f(x,v,t)$, and a given configuration of the system is specified by $f(x,v,t) \, d^3x \, d^3v$ - the number of galaxies having positions in the infinitesimal volume $d^3x$, with velocities in the range $d^3v$. It should be noted here that in phase-space $x$ and $v$ are independent variables and the potential is not a function of $v$. With the assumption that the total energy is conserved, we obtain a continuity equation,

$$\frac{Df}{Dt} = \frac{\delta f}{\delta t} + v \cdot \nabla f - \nabla \phi \cdot \frac{\delta f}{\delta v} = 0,$$

which is the collisionless Boltzmann equation. Neglecting the explicit time derivative; taking the first velocity moment; integrating over all possible velocities for a spherical system, we obtain the Jeans equation:

$$\frac{d(\rho \sigma^2)}{dr} + \rho \frac{[2\sigma^2 - (\sigma^2 \phi + \sigma^2_v)]}{r} = -\rho \frac{d\phi}{dr},$$

where $\rho$ is the density profile and $\sigma^2$ are the components of the velocity dispersion. If, additionally, the velocities and the density are invariant under rotations about the cluster centre we have,
\[ \sigma_i^2 = \sigma_\phi^2 = \sigma_g^2, \]  
\[ \frac{d(\rho \sigma_r^2)}{dr} + 2\eta \frac{\sigma_r^2 \rho}{r} = -\rho \frac{d\phi}{dr}. \]  
The velocity anisotropy parameter \( \eta \) at a given point is,
\[ \eta(r) = (1 - \frac{\sigma_g^2}{\sigma_r^2}). \]  

3.1 The Isotropic Jeans equation
The isotropic Jeans equation is a special case of the more general equation above, wherein the galaxies are on isotropic orbits. Hence \( \eta = 0 \), and \( \sigma_i^2 = \sigma_t^2 \),
\[ \frac{d(\rho \sigma_r^2)}{dr} = -G M(r) \rho(r) \frac{r^2}{r^2} = -\rho \frac{v_c^2}{r}, \]  
where \( v_c(r) \) is defined to be the circular velocity. The solutions for the isotropic velocity dispersion are given by:
\[ \sigma^2(R) = \frac{G}{\rho} \int_R^\infty M(r) \rho(r) dr \frac{r^2}{r^2} = \frac{1}{\rho} \int_R^\infty \rho(r) v_c^2(r) \frac{dr}{r}. \]  
The velocity dispersion of both the galaxies and the total mass can therefore be computed given their respective density profiles and the underlying mass distribution. We plot the solutions obtained for a mass model of the form,
\[ M(r) = M_0 \left[ \frac{r}{r_0} - \tan^{-1} \frac{r}{r_0} \right], \]  
with \( r_0 = 50 \text{ kpc}, \) \( \sigma_0 = 1200 \text{ km s}^{-1} \), and generic density profiles for the tracers (galaxies or isothermal gas) of the form,
\[ \rho(r) = \frac{\rho_0}{(1 + \frac{r^2}{r_0^2})^\alpha}. \]  
The solutions for \( \alpha = 1.0, 1.2 \text{ and } 1.5 \) and for various values of \( r_c/r_0 = 1.0, 2.5 \text{ and } 5.0 \) are plotted in Fig. 1. The solutions have the following interesting properties:

(i) the velocity dispersion falls in the centre for small core radii,
(ii) the smaller the core radius of the tracer, the lower the central value of the velocity dispersion,
(iii) the mean value of the velocity dispersion is a weak function of \( r_c \), but depends on the slope \( \alpha \).

Therefore one can have different mean values of the velocity dispersion for different components if they do not have the same radial profile.

In Fig. 1, we also plot the ratio of the velocity dispersions of dark matter and galaxies, which provides a qualitative understanding of the velocity bias (as found in the numerical simulations by Carlberg (1994)). The difference in the asymptotic slope of the density profiles of dark matter and galaxies, and the ratio of the core radii is found to determine the velocity bias. To first order, the asymptotic behavior (regardless of core size) is:
\[ \left( \frac{\sigma_{DM}}{\sigma_g} \right)^2 \approx \frac{\alpha_{DM}}{\alpha_g}. \]  

3.2 Anisotropic Jeans equation
The Jeans equation is a mathematical statement of detailed pressure balance for an equilibrium stellar system. The dynamical evolution of clusters in N-body simulations has been studied using the distribution function formalism (Natarajan, Hjorth, & Van Kampen 1996). This analysis indicates that clusters evolve from one quasi-equilibrium state to another. A cluster in a quasi-equilibrium configuration is found to be virialized and has a smooth potential, which is traced by The quasi-equilibrium states are characterized by a smooth potential, which is traced by galaxies with one of the following orbital structures :

(i) \( \eta = 0 \), isotropic orbits
(ii) \( 0 < \eta \leq 1 \), orbits are mostly radial
(iii) \( \eta < 0 \), when the orbits are primarily tangential.

While \( \eta \) has no lower bound, it is strictly required to be less than 1 for any physically admissible solutions for the velocity dispersion. In this context, it is instructive to examine and compare with studies of the formation and evolution of elliptical galaxies (Hjorth & Madsen 1991). It has been shown that the observed uniformity in the properties of elliptical galaxies can arise from either of these two sets of initial conditions: dissipationless cold collapse or a ‘warm collapse’ (or merger) with dissipation. The evolution to the final state with a deep potential and significant radial anisotropy arises from the relaxation brought about by global potential fluctuations rather than two-body encounters (Aguilar & Merritt 1990) and Londrillo, Messina & Stiavelli (1991)). Therefore, radial anisotropy arises naturally in most models as a consequence of relaxation, although the data are at present consistent with isotropic velocity dispersion profiles. Conversely, for a galaxy cluster, the initial collapse conditions are different and additionally, many physical processes that can effect energy exchange are active and do occur in the dense core region. For a cluster core that has virialized and is partially relaxed, we expect the orbits in the core to reflect the efficiency of the energy exchange mechanisms, while outside the core region, we expect and do find that the orbits are largely radial, \( 0 < \eta \leq 1 \) (Natarajan, Hjorth, & Van Kampen 1996).

4 PROPOSED APPROACH
4.1 THE MATHEMATICAL FORMALISM
In our approach, we solve the full Jeans equation for the velocity anisotropy parameter \( \eta \) and for the radial component of the velocity dispersion \( \sigma_r^2 \), using the projected mass profile for the cluster as constructed independently from gravitational lensing.

From the observed projected galaxy positions, we fit to get a surface number density profile \( \Sigma_g(r) \) and use the Abel integral inversion to extract the three-dimensional density profile \( \rho_g(r) \). The key assumption made in the analysis below is that of spherical symmetry. Starting with the full Jeans equation,
\[ \frac{d(\rho_g \sigma_r^2)}{dr} + 2\eta(r) \rho_g \frac{\sigma_r^2}{r} = -G \frac{M_{tot}(r) \rho_g}{r^2}; \]  
where \( \rho_g(r) \) is three-dimensional galaxy density profile,
Figure 1. Left panel: Solutions of the isotropic Jeans equation. The dotted curves are for \( \alpha = 1.0 \), dashed curves for \( \alpha = 1.2 \) and solid curves for \( \alpha = 1.5 \); for each \( \alpha \), the smaller the core radius for the tracer, the smaller the central velocity dispersion (see text). Right panel: Ratio of the velocity dispersions for various values of \( \alpha \) and core radius \( r_0 \): solid curves - \( \alpha_{DM}/\alpha_{gal} = 1.0 \), \( r_g = 50, 125 \) and \( 250 \) kpc; dotted curves - \( \alpha_{DM}/\alpha_{gal} = 1/1.2 \), \( r_g = 50, 125 \) and \( 250 \) kpc; dashed curves - \( \alpha_{DM}/\alpha_{gal} = 1/1.5 \) for \( r_g = 50, 125 \) and \( 250 \) kpc

\( \sigma_r^2(r) \) the radial velocity dispersion of the galaxies, \( \eta(r) \) is the velocity anisotropy and \( \Sigma_{tot}(r) \) is the distribution of total mass (most accurately determined from gravitational lensing).

In addition, we have the equation that defines the observed line-of-sight velocity dispersion profile \( \sigma_{los}(R) \),

\[
\frac{1}{2} \left[ \Sigma_g(R) \sigma_{los}^2(R) \right] = \int_R^\infty \frac{r \rho_g(r) \sigma_r^2(r) \, dr}{\sqrt{r^2 - R^2}} - R^2 \int_R^\infty \frac{\eta(r) \sigma_r^2(r) \rho_g(r) \, dr}{r \sqrt{r^2 - R^2}}. 
\]  
(21)

We need to solve these two integro-differential equations numerically for \( \sigma_r^2 \) and \( \eta(r) \). For an individual galaxy with an assumed mass profile, these coupled equations have been solved by Binney & Mamon (1982) and Bicknell et al. (1989).

We truncate the integration at a large, finite truncation radius \( R_t \), defined strictly to be the radius at which both \( \Sigma_g(R_t) \) and \( \rho_g(R_t) \) tend to zero. Substituting the expression for \( \eta \) from equation (21) into equation (22),

\[
\frac{1}{2} \left[ \Sigma_g(R) \sigma_{los}^2(R) \right] = \frac{R^2}{2} \int_R^{R_t} \frac{GM_{tot}(r) \rho_g}{r^2 \sqrt{r^2 - R^2}} \, dr 
\]  
(22)

Integrating the first term on the right hand side by parts and substituting back we have,

\[
\frac{1}{2} \left[ \Sigma_g(R) \sigma_{los}^2(R) \right] = \frac{R^2}{2} \int_R^{R_t} \frac{GM_{tot}(r) \rho_g}{r^2 \sqrt{r^2 - R^2}} \, dr + \int_R^{R_t} \left( \frac{3R^2}{2} - r^2 \right) d\left( \rho_g \sigma_r^2 \right) \, dr. 
\]  
(23)

The equation can be further simplified and reduced after some algebra (for details see Bicknell et al. (1989)) to the following integrals,

\[
\rho_g(r) \sigma_r^2 = I_1(r) - I_2(r) + I_3(r) - I_4(r), 
\]  
(24)
\[ I_1(r) = \frac{1}{3} \int_r^{R_0} \frac{GM_{\text{tot}}(r)}{r^2} \rho_0 \, dr, \]
\[ I_2(r) = -\frac{2}{3\pi} \int_0^r GM_{\text{tot}}(r) \rho_0 \, r \, dr, \]
\[ I_3(r) = \frac{1}{r} \int_0^r R \, \Sigma_g(R) \sigma_{\text{los}}^2(R) \, dR, \]
\[ I_4(r) = \frac{2}{\pi^2} \int_r^{R_0} R \, \Sigma_g(R) \sigma_{\text{los}}^2(R) \left[ \frac{r}{\sqrt{R^2 - r^2}} - \sin^{-1} \frac{r}{R} \right] dR. \]

It is to be noted here that the explicit dependence on the mass profile and the observed line-of-sight velocity dispersion profile separate. All the above integrals are well-behaved with the exception of \( I_4 \), which has an integrable singularity which can be taken care of easily via a simple transformation of variables. Computing these integrals is nevertheless tricky as the final profile for \( \eta \) is sensitive to the precise asymptotic behavior of all the four terms. The numerical solution for \( \eta(r) \) is then substituted back into the Jeans equation to obtain \( \eta \).

\[ \eta(r) = -\frac{r}{2\rho_0 \sigma_r^2} \left[ \frac{GM_{\text{tot}}(r)}{r^2} \rho_0 + \frac{d}{dr} \left( \rho_0 \sigma_r^2 \right) \right]. \]

The variation of \( \eta \) with radius can be understood physically in terms of the relative importance of the mass term and the ‘galaxy pressure’ gradient term. Rewriting the above equation as follows,

\[ \eta(r) = -\frac{1}{2} \left[ \frac{\sigma_r^2}{\rho_0 \sigma_r^2} \right]^2 \left[ \frac{d}{dr} \ln \rho_0 \sigma_r^2 \right] \]

we find that the sign of \( \eta \) depends crucially on the asymptotic behavior of the mass model at large \( r \), and specifically for \( \rho_{\text{tot}} \) ranges between \( r^{-2} \) and \( r^{-3} \), it is found to be fairly insensitive to the slope of the assumed galaxy density profile. The sensitivity of the sign and magnitude of \( \eta \) to the slope of the mass profile enables its use as a discriminant between the various mass models.

5 RESULTS FOR VARIOUS MASS PROFILES

We consider several physically motivated fiducial density profiles for the total mass. In what follows, we assume a galaxy distribution as described in equation (1), but we have verified that our results are independent of this choice. The asymptotic slope of the density profile is defined to be \( \gamma \). All the mass profiles are normalized to have the same total projected mass enclosed within the Einstein radius \([\text{M}_{\text{arc}}(r_E = r_{\text{arc}}) = (5 \pm 0.1) \times 10^{13} M_\odot]\), as calibrated from strong cluster lensing in A2218. The density profiles and mass models studied are:

**MODEL I:**

\[ \rho(r) = \frac{\rho_0}{(r^2 + r_0^2)^{\alpha}} ; \quad \gamma = 2\alpha + 1, \]
\[ M(r) = 2\pi\rho_0 \ln (r^2 + r_0^2) ; \quad \alpha = 1. \]

**MODEL II:**

\[ \rho(r) = \frac{\rho_0}{(r^2 + r_0^2)^{\alpha}} ; \quad \gamma = 2\alpha, \]
\[ M(r) = 4\pi\rho_0 r_0 [\frac{r}{r_0} - \tan^{-1}(\frac{r}{r_0})] ; \quad \alpha = 1. \]

**MODEL III:**

\[ \rho(r) = \frac{\rho_0}{(r(r + r_0)^{\alpha})} ; \quad \gamma = \alpha + 1, \]
\[ M(r) = 4\pi\rho_0 r_0 [\frac{r}{r_0} - \ln (r + r_0)] ; \quad \alpha = 1. \]

5.1 Dependence on the slope \( \gamma \)

We analyse here the results for the various mass models with a specified asymptotic slope \( \gamma \), assuming a core radius of \( r_0 = 250 \) kpc for the galaxy distribution and \( r_0 = 60 \) kpc for the dark matter.

(i) For \( \gamma = -2.0 \) (Fig. 3 - top panel), and a range of input values of the line of sight velocity dispersion \((\sigma_{\text{los}} = 800, 1000, \text{and} 1400 \text{ km s}^{-1}\)), we obtain unphysical solutions \( \sigma_g(r) < 0 \) for the lowest \( \sigma_{\text{los}} \) for all the 3 models. On increasing \( \sigma_{\text{los}} \) to 1000 km s\(^{-1}\), the orbits are primarily tangential in the core progressing to more radial ones in the outer parts, so that \( \eta < 0 \). For the highest \( \sigma_{\text{los}} \) we find evidence for a small core region with mixed orbits, but with primarily radial orbits outside 200 kpc for all 3 models.

(ii) For \( \gamma = -2.5 \) (Fig. 3 - middle panel), unphysical solutions are obtained for the lower \( \sigma_{\text{los}} \) value for all models, but for \( \sigma_{\text{los}} = 1000 \text{ and} 1400 \text{ km s}^{-1} \) we do find physically admissible solutions. All 3 mass models have a finite core with mixed orbits leading on to largely radial orbits outside. Model I has the largest mixed region (of the order of 700 kpc) while Models II and III have smaller mixed regions which are of the order of 500 kpc. The highest value of \( \sigma_{\text{los}} \) produces primarily radial orbits right from the centre outward.

(iii) For \( \gamma = -3.0 \) (Fig. 3 - bottom panel), and the lowest value of \( \sigma_{\text{los}} \), we obtain unphysical solutions, but as \( \sigma_{\text{los}} \) is increased there is evidence for a core with tangential orbits. This seems to be consistent with the physical picture of an unrelaxed core with ongoing isotropization and regularization. A high \( \sigma_{\text{los}} \) is required to compensate for the sharp fall-off for the steeper profiles to produce physically meaningful solutions.

5.2 Dependence on the circular velocity

An important parameter for the dynamics of the galaxies in the global cluster potential is the circular velocity, \( v_c \) (Fig. 4), which measures the change in slope of the mass profile. For a given mass model with asymptotic slope \( \gamma \), increasing the core radius increases the circular velocity for all three models (where these were normalized to have the same projected mass within the radius of the arc).

Comparing different mass models that have the same asymptotic value of \( v_c \) (but different \( r_0 \) and \( \gamma \)), we find that the velocity structure of the core and anisotropy profiles are fairly similar. The qualitative behavior of \( \eta \) for fixed asymptotic \( v_c \) depends strongly on \( \sigma_{\text{los}} \); with increasing \( \sigma_{\text{los}} \) we find preferentially radial orbits. For a fixed \( \sigma_{\text{los}} \), increasing
the circular velocity increases the size of the mixed core region while lowering the value of \( \eta \) at large radius. For both high and low circular velocities, and \(-3.0 \leq \gamma \leq -2.0\); low line-of-sight velocity dispersion models \( \sigma_{\text{los}} < 1000 \ \text{km s}^{-1} \) are ruled out purely from the dynamical point of view.

5.3 Dependence on the central density profile

The best probe of the shape of the density profile at the very centre comes from the observed velocity dispersion of the stars in the cD halo (Miralda-Escude 1995). For the fiducial mass models of Section 5, we solve for the line-of-sight velocity dispersion of the cD halo stars using the isotropic Jeans equation (neglecting the contribution of the mass of the cD galaxy to the total mass of the cluster):

\[
\frac{d (\rho_{cD} \sigma^2)}{dr} = -\frac{GM_{\text{tot}}(r) \rho_{cD}}{r^2}.
\]

Assuming a scaling of \( \rho_{cD} \propto r^{-\delta} \) and \( \rho_{\text{tot}} \propto r^{-\gamma} \), close to the centre we obtain:

\[
\sigma_* = A \frac{r^{2-\gamma}}{(\delta + \gamma - 2)} + \text{const.}
\]

Therefore, for \( 0 \leq \gamma < 2 \) and \( \delta + \gamma > 2 \), we expect the velocity dispersion of the stars to rise. The cD profile from section 2.3.4 is used with a core radius \( r_1 = 0.05 \text{kpc} \) and \( r_{\text{cut}} = 35.0 \text{kpc} \). The three models studied in the previous section, predict profiles (Fig. 5) with low central values for \( \sigma_* \) that rise steeply with radius. Models I and III have central values \( \sim 200 - 400 \ \text{km s}^{-1} \), varying with \( \gamma \), such that the steeper the total mass profile the higher the central value. For the stars, \( \sigma_{\text{los}} \) rises to 700 km s\(^{-1} \) at \( r = 100 \text{kpc} \), which is consistent with the measurements of the cD galaxy IC1011 in A2029 by Dressler (1979) and Fisher, Illingworth & Franx (1995). Model II under-predicts the central value and is qualitatively incompatible with the data.

5.4 Summary of the important parameters

In the above analysis, there are several parameters to be kept track of in order to interpret the results for the computed velocity structure of the cluster core, namely the total mass distribution and the galaxy positions and velocities.

The mass profile for a given fiducial model is specified by three parameters, the central density, the core radius, and the slope. The velocity dispersion of stars in the cD halo favors mass models with a central ‘cusp’. Lensing constrains (i) the mass within the radius of the arc (strong regime), (ii) the circular velocity at large radius (weak-shear), and the generic shape of the profile. It favors compact cores and rules out steep slopes, \( \gamma < -2.5 \) for \( 50 \leq r \leq 600 \text{kpc} \). Our results show that the dynamics of the core can be recovered independent of the fine tuning of individual fiducial models, given the above constraints from lensing and an observationally well-determined line-of-sight velocity dispersion profile. Although currently limited by the errors in the observationally determined input quantities, this dynamical approach offers a better understanding of the physical state of the cluster core and can discriminate between various mass models for an individual lensing cluster.

6 APPLICATION TO A2218

We apply this technique to the Abell cluster A2218, at a redshift \( z = 0.175 \), with a mean measured velocity dispersion \( \sigma_{\text{mean}} \sim 1370^{+160}_{-120} \ \text{km s}^{-1} \) from 56 cluster members. A2218 is a cD cluster with a very peaked mass distribution and a compact core, hence a large number of gravitationally distorted arcs and arclets are observed. The mass model for this cluster was constructed using ground data (Kneib et al. 1995) and refined using HST data by (Kneib et al. 1996). Redshifts of two of the arcs were spectroscopically measured by Pello et al. (1992) and further redshifts of arclets have been determined by Ebbels et al. (1996) hence tightly calibrating the mass model.

6.1 OBSERVATIONAL DATA

The observational input for the galaxies in A2218 for our analysis comes primarily from the photometric and spectroscopic survey by Leborgne et al. (1992).
6.1.1 Line-of-Sight Velocity Dispersion Profile

In order to construct the line-of-sight velocity dispersion profile (Fig. 6), the galaxies were binned in 100 kpc annuli. The measurement errors preclude any fitting, and therefore we assumed a constant value for $\sigma_{\text{los}}$. This simplifying assumption is the largest source of error for our technique at present, but can be refined with the availability in the near future of more tightly sampled line-of-sight velocity dispersion profiles.

6.1.2 Galaxy density profile from optical data

The observed surface density of galaxies in A2218 was fitted to a modified Hubble law profile with a core radius $r_g = 250 h_{50}^{-1}$ kpc,

$$\Sigma(r) = \Sigma_0 \left(1 + \frac{r}{r_g}\right)^{-\gamma},$$

and the corresponding 3-D density profile from equation (1).

6.1.3 Constructing the mass profile

The lensing mass profile for the cluster was constructed from the strong lensing data (arcs, arclets and resolved multiple images) from ground-based observations and the HST image by Kneib et al. (1996), and the weak-lensing mass map published by Squires et al. (1995).

A2218 is best fit by the following functional form of MODEL III with $\alpha = 1.0$ (see Section 5),

$$M(r) = M_0 \left[\frac{r}{r_0} - \ln \left(1 + \frac{r}{r_0}\right)\right],$$

where we normalize $M_0$ to the mass enclosed by the arc at
Figure 4. Asymptotic behavior of the fiducial mass models - varying the core radius \( r_0 = 30, 60, 100 \) kpc; solid curves - Model I, dotted curves - Model II, dashed curves - Model III.

\[ r_{\text{arc}} = 78.5 \, h_{75}^{-1} \text{kpc} \] and \( r_0 \) the core radius is \( 60h_{75}^{-1} \) kpc. The corresponding three-dimensional density profile is as below,

\[ \rho(r) = \frac{\rho_0 r^2}{r(r + r_0)} \]

with \( \rho_0 = 1 \times 10^{-22} \text{ g cm}^{-3} \). The X-ray mass profile was obtained using the standard deprojection technique described by Fabian et al. (1981) to the archival ROSAT HRI map, assuming spherical symmetry and hydrostatic equilibrium for the intracluster gas. The integrated X-ray luminosity (in the 0.1 - 2.4 keV band) and central temperature of A2218 are measured to be respectively,

\[ L_x = 7 \times 10^{44} \text{ erg s}^{-1} \; ; \; T = 8 \text{ keV} \]

in good agreement with the Squires et al. (1995) results. The predicted circular velocity is,

\[ v_c^2(r) = \frac{GM(r)}{r} = -\frac{kT}{\mu m_p} \left( \frac{d \ln \rho_{\text{gas}}}{d \ln r} + \frac{d \ln T}{d \ln r} \right) \]

where \( M(r) \) is the total mass as inferred from the X-ray analysis.

6.2 Results

Using the total mass profile constructed from lensing as described above, we solve the equations to obtain solutions (see Fig. 8) for \( \sigma_r(r), \sigma_t(r) \) (the radial and tangential velocity dispersion profiles respectively) and the velocity anisotropy parameter \( \eta(r) \). The profile was also checked for consistency with the measured stellar velocity dispersion of the halo stars in the cD galaxy (Fig. 9). [It is to be noted here that the measured line-of-sight velocity dispersion profile for A2218 is inconsistent with an isotropic solution.] We find that the orbits in the core are primarily tangential, consistent with the picture of a virialized and an unrelaxed core. Both \( \sigma_r \) and \( \sigma_t \) fall within the inner 600 \( h_{75}^{-1} \) kpc, with \( \sigma_r \) declining more rapidly and then tending to flatten off. From the slope of \( \eta \), the trend with increasing \( r \) is that the nature of orbits changes from transverse to mainly radial at the outskirts, signaling the existence of a region dominated by infall. The physical picture that emerges for the description of the dy-
Figure 5. Computed line-of-sight velocity dispersion for the cD halo stars: solid curve - Model I, dotted curve - Model II, dashed curve - Model III.

Figure 6. The line-of-sight velocity dispersion profile for A2218
namical state of A2218 is one of a dynamically disturbed and unrelaxed cluster core. The predominance of tangential orbits in the central 400 kpc (which is precisely of the order of the distance between the 2 distinct optical clumps seen in the HST image) is an indication of the on-going energy exchange in the core en-route to a relaxed final state. Using the mass model from N-body simulations as the input (see Fig. 10), we find that the resultant predictions for $\eta(r)$ agree well with those calculated for the mass profile reconstructed from lensing. For the mass profile from X-ray data for A2218, we obtain qualitative agreement with the predictions from the lensing mass model.

7 CONCLUSIONS
Gravitational lensing provides the ‘cleanest’ way to construct the total mass profile for a cluster independent of the kinematic details; additionally, combining strong and weak lensing removes the scaling ambiguity allowing the calibration of other independent mass models. With ‘good’ data for an individual cluster, the requirements for consistency on the smallest to the largest scales are stringent enough to constrain generic density profiles for rich clusters. Accurate mass profiles are crucial to settling many important issues such as the baryon fraction problem and in understanding the discrepancies and biases arising in the X-ray, lensing and virial mass estimates for clusters.

In this paper, we have demonstrated that the dynamics and velocity structure of the core of galaxy clusters can be probed given an independently inferred total mass profile. The future applications of our method to study cluster cores are promising, given the prospect of collecting more spectro-photometric data of galaxies in cluster lenses (e.g. Yee, Ellingson & Carlberg (1996)).

With current data, we find strong evidence for the existence of a core region where the orbits of galaxies are primarily tangential. While this is consistent with the picture of on-going relaxation, it is in contrast to the strictly stellar dynamic case (in elliptical galaxies) where the core region is isotropic and radial anisotropy arises naturally in the outskirts as a consequence of the initial conditions and evolution. The important differences that we find in the case of clusters could arise from:
Figure 8. The best fit composite mass model for A2218: The radial and tangential components of the velocity dispersion and the velocity anisotropy parameter for $\sigma_{\text{los}} = 1100, 1200, 1300 \text{ km s}^{-1}$.
(i) the on-going strong potential fluctuations arising due to the presence of significant sub-structure
(ii) the Jeans equation not being an adequate description either due to the system not having attained equilibrium or the inability to incorporate all the relevant physics
(iii) the existence of physical processes that alter galaxy orbits, such as dynamical friction. Dynamical friction in an aspherical cluster can induce and amplify the velocity anisotropy as demonstrated by Binney (1977). This could be the possible origin for the inferred tangential anisotropy, given that in the specific case of A2218 the distribution of bright galaxies in the core region is elongated.
(iv) the frequent presence of a cD galaxy at the centre of the cluster potential, might influence the orbits in the core region.

Distinguishing between these possibilities will require detailed modeling of these more complex physical effects. Further extension of this analysis to incorporate the dynamics of the intra-cluster gas with the lensing model self-consistently, is required in order to understand the possible role of baryons in in the dynamics of cluster cores.

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