CERN-TH/95-188

Naturalness Constraints in Supersymmetric Theories with Non-Universal Soft Terms

S. Dimopoulos[1] and G.F. Giudice[2]

Theory Division, CERN, Geneva, Switzerland

Abstract

In the absence of universality the naturalness upper limits on supersymmetric particle masses increase significantly. The superpartners of the two light generations can be much heavier than the weak scale without extreme fine-tunings; they can weigh up to about 900 GeV — or even up to 5 TeV, if SU(5) universality is invoked. This supresses sparticle-mediated rare processes and consequently ameliorates the problem of supersymmetric flavor violations. On the other hand, even without universality, the gluino and stop remain below about 400 GeV while the charginos and neutralinos are likely to be accessible at LEP2.

CERN-TH/95-188
July 1995

[1]On leave of absence from Stanford University, Stanford, Ca. 94305, USA.

[2]On leave of absence from INFN, Sezione di Padova, Padua, Italy.

The naturalness problem of the standard model with fundamental Higgs particles [1] provides *la raison d'être* for supersymmetry at low energies [2]. Indeed supersymmetry cures the problem if the soft-supersymmetry breaking scale, which behaves as a cutoff for quadratic divergences, lies below the TeV range [2, 3]. This concept has been put on more quantitative grounds by the studies presented in refs. [4, 5], where upper bounds on physical supersymmetric particle masses were obtained. In this letter we wish to revisit the analysis of ref. [5] and extend it to models with non-universal boundary conditions at the unification scale.

We follow the procedure proposed in ref. [5] and quantify the naturalness criterion by requiring that

$$\left| \frac{a_i}{M_Z^2} \frac{\partial M_Z^2}{\partial a_i} \right| < \Delta, \tag{1}$$

if no fine-tunings larger than $1/\Delta$ are allowed[1]. In eq. (1) the a_i stand for all the soft supersymmetry-breaking parameters of the model which determine, via radiative symmetry breaking, the value of M_Z. We proceed by fixing one of the soft supersymmetry-breaking parameters from the equations

$$\frac{M_Z^2}{2} = \frac{m_1^2 - m_2^2 \tan^2\beta}{\tan^2\beta - 1}, \tag{2}$$

$$\sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}, \tag{3}$$

where m_1^2, m_2^2, and m_3^2 are the usual mass parameters in the Higgs potential, to be understood as functions of a_i. Next we impose eq. (1) in the explicit form

$$\begin{aligned} &\left| \frac{2a_i}{(\tan^2\beta - 1)M_Z^2} \left\{ \frac{\partial m_1^2}{\partial a_i} - \tan^2\beta \frac{\partial m_2^2}{\partial a_i} - \frac{\tan\beta}{\cos 2\beta} \times \right.\right. \\ &\left.\left. \left(1 + \frac{M_Z^2}{m_1^2 + m_2^2}\right) \left[2\frac{\partial m_3^2}{\partial a_i} - \sin 2\beta \left(\frac{\partial m_1^2}{\partial a_i} + \frac{\partial m_2^2}{\partial a_i} \right) \right] \right\} \right| < \Delta, \end{aligned} \tag{4}$$

and scan over the values of a_i which satisfy eq. (4).

Let us first consider the minimal supersymmetric standard model, defined by only five independent soft supersymmetry-breaking parameters at the unification scale: a common scalar mass ($\tilde{m}$), a common gaugino mass (M), a Higgs superfield mass term (μ), trilinear (A) and bilinear (B) coupling constants. We perform a scanning over the region of these five parameters searching for points which satisfy the naturalness criterion, eq. (4), under the requirement of a correct electroweak symmetry breaking, eq. (2), the stability of the potential, and the experimental constraint that all new charged particles must be heavier than $M_Z/2$.

The procedure followed here slightly differs from the calculation done in ref. [5], as the electroweak symmetry breaking condition is evaluated at the physical M_Z value, instead

[1] Anderson and Castaño [6] have recently proposed new naturalness criteria which avoid sensitivity on steep functional dependences. Since these new criteria lead to some dependence on the allowed variable range, we prefer to use eq. (1) which is an adequate fine-tuning measure in the case of cancellation among different terms.

of $M_Z = 0$, and as the actual upper bounds on the physical supersymmetric particle masses are derived from eq. (1), instead of inferring them from the maximum values of the soft supersymmetry-breaking parameters at the unification scale. Therefore the bounds presented here are more stringent than those obtained in ref. [5] since, given a choice of supersymmetric parameters, not all of them can simultaneously saturate their maximum values. Also, our results are presented as a function of the physical top-quark pole mass (as opposed to the top-quark Yukawa coupling h_t [5]) defined as:

$$m_t = h_t \frac{\sqrt{2} M_W}{g} \sin\beta (1 + \frac{4}{3\pi}\alpha_s), \tag{5}$$

where the right-hand side is computed at the scale $Q^2 = m_t^2$. This leads to the upper bounds on the supersymmetric particle masses shown by the solid lines in fig. 1, for $\Delta = 10$ (*i.e.* no more than 10% fine tuning), assuming the unification mass $M_{GUT} = 1.5\times10^{16}$ GeV and the unified gauge coupling constant $\alpha_{GUT}^{-1} = 24.5$. To obtain these bounds, we have solved the relevant renormalization group equations keeping only one-loop contributions from gauge coupling constants and the top-quark Yukawa coupling. This approximation is inadequate when $\tan\beta$ approaches m_t/m_b, since the bottom-quark Yukawa coupling h_b is no longer negligible. We have checked that all bounds are achieved for $\tan\beta << m_t/m_b$, and therefore we do not believe that the inclusion of h_b-effects can sizeably modify our results. We present the limits only for neutralinos, gluino, and lightest chargino and stop for reasons that will be clear in the following. We have included the one-loop QCD corrections on the pole gluino mass in the $\overline{\text{MS}}$ scheme [7], but used tree-level expressions for chargino and neutralino masses.

The limit on the lightest stop rapidly increases below $m_t = 170$ GeV, since in this range of m_t, the contribution proportional to $\tilde{m}$ in eq. (2) suffers an approximate cancellation, and the bound on $\tilde{m}$ disappears. The limits on gluino, chargino, and neutralino masses shown in fig. 1 are only weakly dependent on m_t, apart from the "knee" at $m_t < 165$ GeV, a remnant of the above-mentioned approximate cancellation. Below this "knee" we observe that the chargino mass is $m_{\chi^+} < 80$ GeV and the lightest and next-to-lightest neutralino masses are $m_{\chi^0} < 40$ GeV, $m_{\chi^{0\prime}} < 80$ GeV. Therefore, if minimal low-energy supersymmetry describes the world with no more than 10% fine tuning, then LEP2 has great chances to discover it.

These limits are very stringent largely as a consequence of the heavy top quark. The large top-quark Yukawa coupling has the tendency to drive m_2^2 to large negative values. Only by choosing smaller values of the soft-supersymmetry breaking parameters can one obtain the physical value of M_Z, without increasingly precise fine tunings.

It is well known [8] that the renormalization-group improved tree-level potential has a strong scale dependence, which is partly reabsorbed by the one-loop corrections to the effective potential. Indeed it has been claimed [9] that these corrections considerably relax the limits on supersymmetric particle masses from the requirement of no fine-tuned top-quark Yukawa coupling, as computed in ref. [10]. Although here we consider the top-quark mass as an experimental datum and not as an input variable, the question of scale dependence has to be addressed. We have therefore included in our calculation all one-loop corrections to the effective potential proportional to the top-quark Yukawa coupling, with the complete mixings in the stop sector. The bounds obtained in this way are about 20-30 % less stringent, as shown by the dashed lines in fig. 1.

It is interesting to understand whether the strong limits shown by the solid lines in fig. 1 depend on the specific assumptions of the form of the soft supersymmetry-breaking terms at the unification scale. We consider therefore the extreme case in which each possible gauge-invariant soft supersymmetry-breaking term at M_{GUT} is an independent parameter. We retain however the unification condition of a common gaugino mass, largely justified by the success of gauge-coupling-constant unification in supersymmetry [3].

In practice, in the approximation of keeping only one-loop renormalization effects from gauge coupling constants and the top-quark Yukawa, this generalization implies only two new free parameters a_i with respect to the previous case: m_1^2 and m_2^2 evaluated at M_{GUT}[2]. We can then identify $\tilde{m}^2$ with $(\tilde{m_{t_L}}^2+\tilde{m_{t_R}}^2)/2$, the average of the left- and right-handed stop squared masses at M_{GUT}, which is the only other new combination of soft supersymmetry-breaking terms appearing in the renormalization group equations. Apart from having seven free parameters instead of five, the upper bounds on supersymmetric particle masses are obtained with the same procedure followed above. For simplicity we have not included here the one-loop corrections to the effective potential.

Notice that in the non-universal case, within our approximation of keeping only leading one-loop effects, the parameter $m_1^2(M_{GUT})$ can be infinitely large, without implying large fine tunings since, in this limit, its value leaves M_Z unaffected ($M_Z^2/2 \to m_3^2 - m_2^2$ as $m_1^2 \to \infty$). This is in contrast to the previous case since, under universality conditions, $m_1^2 \to \infty$ implies $m_2^2 \to \infty$[3]. More generally, if the universality assumption is dropped, all of the first two generations of squark and slepton masses can become infinitely large, within our approximation, without causing fine-tuning problems in M_Z. Nevertheless, even if some of the soft supersymmetry-breaking parameters can become very large, not all of the upper bounds on supersymmetric particles are lost. For the particles considered in fig. 1, the mass bounds in the non-universal case, denoted by dot-dashed lines in the figure, are of the same order of magnitude as in the universal case. Actually they become much more constraining when accidental cancellations occur among the soft-breaking parameters correlated by universal boundary conditions, as it is the case for the stop, when $m_t < 170$ GeV, or for the gluino, when $m_t < 160$ GeV, see fig. 1. Light charginos and neutralinos are a signature of fine-tuning-free low-energy supersymmetry, independently of the specific assumptions on the soft supersymmetry-breaking terms at the unification scale.

These considerations suggest to divide the supersymmetric particles into two species: the "brothers of the Higgs", particles whose masses strongly influence the running of the Higgs mass parameters and consequently are severely constrained by the naturalness criterion; and the "cousins of the Higgs", particles which apparently are weakly constrained by naturalness.

Neutralinos, charginos, gluinos, and stops are "brothers of the Higgs" and their mass bounds, shown in fig. 1, weakly depend upon the model-dependent boundary conditions at M_{GUT}. The scalar partner of the left-handed bottom quark is also a "brother", because

[2] The boundary condition of the term coming from hypercharge D-term can be absorbed in the definition of the boundary conditions for m_1^2 and m_2^2. The effect of this term will discussed below.

[3] However, as mentioned above, for a particular value of the top-quark Yukawa, the dependence on $\tilde{m}$ disappears from the expression of M_Z, and $\tilde{m} \to \infty$ does not imply a large fine tuning.

of the weak SU(2) symmetry. On the other hand the partner of the right-handed bottom is not a "brother", but it may become it in the regime of large $\tan\beta$, where the bottom Yukawa coupling is no longer negligible. It may seem that the gluino is a "brother" only thanks to the unification assumption on gaugino masses. In reality, even if we relax this assumption, the strong gluino mass upper bound persists. Indeed the dominant gaugino contribution to the running of m_2^2 comes indirectly from the gluino influence on the evolution of the stop mass parameters.

Let us now turn to discuss the mass bounds of the "cousins of the Higgs", namely the first two generations of squarks and sleptons and, as previously discussed, some of the third generation sparticles, depending on the $\tan\beta$ regime. In order to obtain analytical expressions for their mass upper bounds, we simplify our procedure and use the following approximation. Notice that there exist values of $\tan\beta$ such that the coefficients multiplying $\frac{\partial m_1^2}{\partial a_i}$ and $\frac{\partial m_3^2}{\partial a_i}$ in eq. (4) vanish. This suggests that the best bounds will come from $\frac{\partial m_2^2}{\partial a_i}$, whose coefficient in eq. (4) has a non-vanishing minimum. We therefore convert eq. (4) into:

$$a_i^2 < \frac{\Delta}{4} M_Z^2 \left| \frac{1}{2a_i} \frac{\partial m_2^2}{\partial a_i} \right|^{-1} . \tag{6}$$

This expression is particularly convenient since, in the approximations used below, the term inside the absolute value is independent of a_i.

The most important dependence in M_Z on the "cousins" comes from a one-loop effect induced by the hypercharge D-term. Integrating the renormalization group equations in the usual approximation, we find:

$$m_2^2 = -\frac{1-Z_1}{22}\, \mathrm{Tr}(\bar{m}_Q^2 + \bar{m}_{\bar{D}}^2 - 2\bar{m}_{\bar{U}}^2 - \bar{m}_L^2 + \bar{m}_{\bar{E}}^2) + \hat{m}_2^2, \tag{7}$$

$$Z_1 = \left(1 + \frac{33}{20\pi}\alpha_{GUT} \log\frac{M_{GUT}^2}{Q^2}\right)^{-1} = 0.4 , \tag{8}$$

where $\bar{m}_A^2$ $(A = Q, \bar{U}, \bar{D}, L, \bar{E})$ are the boundary conditions of the soft-breaking masses of squarks and sleptons at M_{GUT}, all of which are independent parameters, and the trace in eq. (7) is taken over generation space. The term in eq. (7) denoted by $\hat{m}_2^2$ contains the usual dependence on the other soft supersymmetry-breaking terms.

Using eqs. (6) and (7), we obtain the bound[4]:

$$\bar{m}_A < \sqrt{\frac{11\Delta}{2(1-Z_1)}} M_Z = 900\sqrt{\frac{\Delta}{10}}\ \mathrm{GeV}. \tag{9}$$

This is approximately also the bound on the physical squark and slepton masses, since the renormalization effects are at most 10%, given the naturalness bounds on gaugino masses.

In theories with universal boundary conditions, the hypercharge D-term vanishes, as a result of anomaly cancellation. Actually it vanishes also in theories where the soft terms satisfy simple GUT relations like:

$$\bar{m}_Q^2 = \bar{m}_{\bar{U}}^2 = \bar{m}_{\bar{E}}^2 \equiv \bar{m}_{10}^2 , \qquad \bar{m}_{\bar{D}}^2 = \bar{m}_L^2 \equiv \bar{m}_5^2 . \tag{10}$$

[4] Of course heavy "cousins" require a certain degree of fine tuning not only to keep the Higgs light, but to keep the "brothers" light as well.

We refer to eqs. (10) as "SU(5) universality". Although it has been shown [11] that integration of heavy GUT particles can modify these relations already at the tree-level, eqs. (10) are still plausible conditions in certain GUT models. If they hold, in order to bound the masses of the "cousins of the Higgs", we have to rely on two-loop effects or one-loop effects suppressed by small Yukawa couplings.

We include these small effects as perturbations over the leading-order solutions. Let us take for simplicity the case in which all trilinear soft terms are zero, and write the solutions of the renormalization group equations for the stop and Higgs mass parameters as:

$$m_i^2 = \hat{m}_i^2 + \delta m_i^2, \tag{11}$$

where $\hat{m}_i^2$ are the solutions of the one-loop equations including only gauge and top Yukawa couplings. Similarly, the corresponding β-functions are separated into the part which depends on gauge and top Yukawa couplings ($\hat{\beta}_i$) and the perturbation ($\delta\beta_i$). At leading order, the evolution equation for δm_i^2 is

$$\frac{d}{dt}\delta m_i^2 = \sum_j \frac{\partial \hat{\beta}_i}{\partial m_j^2}\delta m_j^2 + \delta\beta_i \tag{12}$$

$$t \equiv \log\frac{M_{GUT}^2}{Q^2}. \tag{13}$$

Let us first choose as perturbation the two-loop dependence on $\bar{m}_{10}^2$ and $\bar{m}_{\bar{5}}^2$ not suppressed by small Yukawa couplings. Integrating eq. (12) using the appropriate two-loop β-functions [12], we find:

$$\delta m_2^2 = \frac{\alpha_{GUT}}{4\pi}\left[\left(\frac{1}{6}\log\frac{Z_3}{Z_1} - \frac{9}{56}\log\frac{Z_2}{Z_1} - \frac{1-Z_1}{66}\right)\mathrm{Tr}(\bar{m}_{10}^2 - \bar{m}_{\bar{5}}^2)\right.$$
$$\left. + \left(\frac{3}{2}Z_2 + \frac{1}{22}Z_1 - \frac{86}{99} - \frac{67}{99}\xi + \frac{1-\xi}{2}G\right)\mathrm{Tr}(3\bar{m}_{10}^2 + \bar{m}_{\bar{5}}^2)\right] \tag{14}$$

$$G = \frac{\int_0^t dt Z_3^{\frac{16}{9}} Z_2^{-3} Z_1^{-\frac{13}{99}}\left(\frac{16}{9}Z_3 - 3Z_2 - \frac{13}{99}Z_1\right)}{\int_0^t dt Z_3^{\frac{16}{9}} Z_2^{-3} Z_1^{-\frac{13}{99}}} \tag{15}$$

$$Z_i = \frac{\alpha_i(t)}{\alpha_i(0)}, \qquad \xi = \left(1 - \frac{m_t^2}{M_{FP}^2 \sin^2\beta}\right), \tag{16}$$

where $M_{FP} = 192$ GeV is the top-quark infrared fixed point value. Using eqs. (6) and (14), we obtain the upper bounds, which for m_t varying in the range between 160 and 190 GeV vary in the range

$$\bar{m}_{10} < 1.7 - 2.2\sqrt{\frac{\Delta}{10}}\ \mathrm{TeV}\ , \qquad \bar{m}_{\bar{5}} < 3.6 - 5.6\sqrt{\frac{\Delta}{10}}\ \mathrm{TeV}\ , \tag{17}$$

where the more stringent limit corresponds to the heavier top quark mass.

It is straightforward to repeat the calculation including as perturbation the one-loop dependence on Yukawa couplings different from the top one. In this case one finds bounds which are weaker that those obtained from the two-loop gauge effects.

In conclusion, the naturalness criterion sets very stringent bounds on the masses of the "brothers of the Higgs", particles whose existence strongly influence the evolution of the Higgs mass parameters. These bounds do not significantly depend on the particular boundary conditions of the supersymmetry-breaking terms at M_{GUT}. Neutralinos, charginos, gluinos, and stops belong to this class of particles. The new experimental evidence of a heavy top quark has made these bounds particularly constraining, suggesting that these particles should be soon discovered, independently of the particular model-dependent assumptions.

On the other hand, bounds on the masses of the "cousins of the Higgs" – first two generations of sleptons and squarks – depend much more on model assumptions. If one relaxes the usual universality assumption, their naturalness mass bounds can be significantly altered. Nevertheless, we have shown that a 10 % fine tuning criterion still requires them to be lighter than about 900 GeV, if there is no cancellation of the hypercharge D-term contribution, eq. (7). If this cancellation occurs, maybe caused by GUT universality relations, two-loop effects still provide limits in the range between 2 and 5 TeV. Although this ameliorates the problem of supersymmetric flavor violations [13], it does not completely solve it. A certain degree of degeneracy between sparticles or alignment of sparticle and particle masses [14] is still needed to adequately suppress all flavor violations.

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Figure Caption

Fig. 1. Upper bounds on gluino, lightest and next-to-lightest neutralino, and lightest chargino and stop masses based on the requirement of no fine tuning larger than 10 %, *i.e.* $\Delta = 10$ in eq. (1). The mass bounds approximately scale as $\sqrt{\Delta}$. The solid (dashed) lines refer to the minimal supersymmetric standard model with universal boundary conditions at M_{GUT} for the soft supersymmetry-breaking terms, without (with) the inclusion of the one-loop effective potential. The dot-dashed lines show the mass upper limits for non-universal boundary conditions at M_{GUT}, without the inclusion of the one-loop effective potential.