Pseudo-Zernike Based Multi-Pass Automatic Target Recognition From Multi-Channel SAR

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Abstract

The capability to exploit multiple sources of information is of fundamental importance in a battlefield scenario. Information obtained from different sources, and separated in space and time, provide the opportunity to exploit diversities in order to mitigate uncertainty. For the specific challenge of Automatic Target Recognition (ATR) from radar platforms, both channel (e.g. polarization) and spatial diversity can provide useful information for such a specific and critical task. In this paper the use of pseudo-Zernike moments applied to multi-channel multi-pass data is presented exploiting diversities and invariant properties leading to high confidence ATR, small computational complexity and data transfer requirements. The effectiveness of the proposed approach, in different configurations and data source availability is demonstrated using real data.
I. INTRODUCTION

In the modern battlefield scenarios the availability of multiple sources of information, such as spatial, temporal or other diversities, allows improvements in sensor performance and capabilities. In particular, modern radar scenario involves different diversities, some provided by the sensor position in the space-time plane: spatial diversity given by multiple platforms observing from different positions and temporal diversity provided by multiple passes over the same area from the same platform, and their combinations; and others given by different sensor characteristics: frequency, waveform and polarization diversity. Of particular interest is the combination of these two categories of diversities that can be described as a Distributed Multiple-Input Multiple-Output Radar Sensor Network (DMRS).

In our work we investigate the possibility of exploiting DMRS environments for improving the performance compared to that achieved by a classic Single-Input Single-Output system. Other important aspects include the ability to achieve high performance with low cost algorithms and the capability to summarize the discriminating information thereby reducing the communication overhead between sensors.

A particular application of interest is Automatic Target Recognition (ATR) [1], [2], [3] and its lower level tasks (identification, characterization and fingerprinting) from Synthetic Aperture Radar (SAR) data. Moreover, the way in which targets scattered signals from different polarizations also contains information that can be exploited in target recognition, so the use of multi-polarization SAR data can lead to improved ATR performance. Specifically, the problem of civilian vehicle classification from SAR images has become of interest in the recent years, [4]. More specifically, in [4] a combination of polarimetric and frequency dependent features is exploited to distinguish among different targets in a SAR image. The approach is based on the link between the electromagnetic scattering of primitives geometry (such as cylinders, spheres, edges, top hats, etc.) and the physical geometry of the target, which can be seen as a combination of different elementary geometries. An analogous problem is the ATR of military vehicles, in fact in [5] a 2-D cepstrum-based feature is extracted with the aim of discriminating between clutter and man-made objects in a SAR image. Tests, on the MSTAR database [6] have shown good classification results using this technique. The use of polarization in SAR ATR has been already studied. Specifically, Lincoln Laboratory [7] has investigated both detection and classification of stationary ground targets using high resolution, fully polarimetric SAR images. Thus, in [7] a comparison of ATR performance for several polarization/resolution combinations has been provided (in particular, the single HH polarization is compared to the optimal combination of HH, HV, and VV polarizations). An approach exploiting AdaBoost algorithm is proposed in [8], while in [9] a framework to solve the ATR problem

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is proposed where a Genetic Algorithm based optimization was selected. A SAR image classification method based on global scattering center models was introduced in [10]. The main advantages of this method include the requirement a small amount of data small data amount, fast and flexible feature prediction, and fast image feature extraction and image-model matching. Finally, in [11] the problem of ATR in polarimetric ISAR is solved exploiting persistent polarimetric signatures.

In this paper a novel algorithm for ATR, with target identification capabilities, from multiple spatially separated, multi-channel SAR data is presented. The algorithm is capable of exploiting single or multi-channel information and, at low computational cost, extracts reliable and easy-to-share discriminating features based on the pseudo-Zernike moments [12]. Pseudo-Zernike moments belong to the family of geometric moments such as Hu and Zernike moments [13], [14], which were used both in image processing for pattern recognition and image reconstruction [15], [16], [17]. Some of the main advantages of these moments include position, scale, and rotational invariance. Another important property is that Zernike and pseudo-Zernike moments are independent moments, because they are computed from orthogonal polynomials. Moreover, pseudo-Zernike moments have a lower sensitivity to noise than Zernike moments as well as more moments for a given polynomial order. This last property is important as the availability of more independent moments provides more information (to be used for image reconstruction or classification purposes), with lower sensitivity to noise. Although the proposed frameworks applies to the general multi-channel SAR case, without loss of generality, our experimental analysis will focus on the case of multi-polarimetric SAR. The proposed algorithm is tested with the Gotcha dataset [18] that contains multiple observations of commercial vehicles. The results show that the proposed algorithm have shown good classification performance of the proposed algorithm as well as considerable improvement when using the full-polarimetric system in place of the single polarization one and with the use of spatial diversity.

The remainder of the paper is organised as follow. In Section II, the novel algorithm to extract the features from a multi-channel SAR observation is introduced together with two decision fusion frameworks for the case of multiple passes. Section III introduces the Gotcha dataset and presents the results obtained with different data training set for the case of 1, 2 and 3 sensors sharing the individual classification outputs. Finally, in Section IV, some conclusions and possible future directions are provided.

II. CLASSIFICATION ALGORITHM BASED ON PSEUDO-ZERNIKE MOMENTS

In this section, a novel algorithm for automatic target classification in SAR images is presented. Specifically, a new data representation is utilized for both single and multi-channel ATR from SAR.
The approach is based on the use of pseudo-Zernike moments [12], in order to obtain reliable feature vectors with relatively small dimension and low computational complexity. Notice that in [19] we have introduced the use the pseudo-Zernike moments for ATR applied to micro-Doppler signatures. This novel approach benefits from specific properties of the pseudo-Zernike moments such as invariance with respect to translation and rotation, and in addition scale invariance can be included if required by the specific application.

In the following subsections the background theory defining the pseudo-Zernike moments, is introduced; they are then compared with the Zernike moments; finally, the novel feature extraction algorithm and the decision fusion frameworks are illustrated in detail.

A. Pseudo-Zernike Moments

Let \( f(x,y) \) be a non-negative real image. The complex pseudo-Zernike moments [12] can be computed as

\[
\psi_{n,l} = \frac{n + 1}{\pi} \int_0^{2\pi} \int_0^1 W_{n,l}^* (\rho \cos \theta, \rho \sin \theta, \rho) f(\rho \cos \theta, \rho \sin \theta) \rho d\rho d\theta,
\]

where the symbol \((\cdot)^*\) indicates the complex conjugate operator and \(W_{n,l}\) are the pseudo-Zernike polynomials. The latter are a set of orthogonal functions that can be written in the form

\[
W_{n,l}(x,y,\rho) = W_{n,l}(\rho \cos \theta, \rho \sin \theta, \rho) = S_{n,l}(\rho) e^{i l \theta},
\]

with \(i = \sqrt{-1}\), \(x = \rho \cos \theta, y = \rho \sin \theta, l \) an integer, and \(S_{n,l}(\rho)\) a polynomial (called a radial polynomial) in \(\rho\) of degree \(n\) such that \(n \geq |l|\). Notice that the modulus of (2) is rotationally invariant [12]. Moreover, these functions form a complete basis and satisfy, on the unit disc (i.e. for \(x^2 + y^2 \leq 1\)), the orthogonality relation [12]

\[
\int \int_{x^2+y^2 \leq 1} W_{n,l}^* (x,y,\sqrt{x^2+y^2}) W_{m,k} (x,y,\sqrt{x^2+y^2}) \, dx \, dy = \frac{\pi}{n + 1} \delta_{mn} \delta_{kl},
\]

where \(\delta_{mn}\) is the Kronecker delta function, i.e. \(\delta_{mn} = 1\) if \(m = n\), and 0 otherwise. As given in [12], an explicit expression to compute the radial polynomials, \(S_{n,l}(\rho)\), is

\[
S_{n,l}(\rho) = \sum_{k=0}^{n-|l|} \frac{\rho^{n-k}(-1)^k (2n+1-k)!}{k!(n+|l|+1-k)! (n-|l|-k)!}.
\]

In a similar manner, the Zernike moments [14] are computed projecting \(f(x,y)\) on the Zernike polynomials, \(V_{n,l}(\cdot)\), namely

\[
\zeta_{n,l} = \frac{n + 1}{\pi} \int \int_{x^2+y^2 \leq 1} V_{n,l}^* (\rho(x,y), \theta(x,y)) f(x,y) \, dx \, dy = \zeta_{n,-l}^*.
\]
where
\[ V_{n,l}(x,y) = V_{n,l}(\rho \cos \theta, \rho \sin \theta) = R_{n,l}(\rho) e^{i\theta}, \] (6)
with \( R_{n,l}(\rho) \) the radial Zernike polynomials [12], [14]. Here
\[ R_{n,l}(\rho) = \sum_{k=0}^{(n-|l|)/2} (-1)^k \frac{(n-k)!}{k! \left( \frac{n+|l|}{2} - k \right)! \left( \frac{n-|l|}{2} - k \right)!} \rho^{n-2k}, \] (7)
where \( n \geq 0 \) and \( l \) are integers such that \( n - |l| \) is even and \( n \geq |l| \). Note that the only difference with respect to pseudo-Zernike moments is the form of the radial polynomial used i.e. \( R_{n,l}(\rho) \) vs \( S_{n,l}(\rho) \).

The main difference between Zernike and pseudo-Zernike moments is that, for a given \( n \), the number of linearly independent pseudo-Zernike polynomials is \((n+1)^2\) whereas those of Zernike polynomials is \( \frac{1}{2}(n+1)(n+2) \). This property is useful in many applications, such as image reconstruction and classification, as more independent moments can be obtained for a given order; Moreover, as previously stated an important characteristic of the Zernike and pseudo-Zernike moments is the simple rotational transformation property due to (2); indeed, the moment requires only a phase factor for the rotation [12].

B. Feature Extraction Algorithm

The feature extraction algorithm is summarized in the block diagram shown in Fig. 1, while a detailed explanation of the processing steps is described below.

The complex valued image for each channel from the \( j \)-th sensor is defined as \( X_j(x,y,h) \in \mathbb{C}^{B \times Z \times H} \) with \( x \) and \( y \) representing the range and cross-range pixel, respectively, of the \( B \times Z \) sub-image containing the target and \( h \) the index of the \( h \)-th channel in the set of \( H \) available channels (e.g. in the polarimetric SAR case, if \( H = 1 \) then \{HH\} is the polarization considered, while for \( H = 4 \) the set of polarizations is \{HH, VV, HV, VH\}).

The feature extraction algorithm begins with the generation of the multi-channel magnitude image of the target area
\[ \Omega_j(x,y) = \sum_{h=1}^{H} |X_j(x,y,h)|. \] (8)
Notice that, other fusion techniques exists in literature, particularly for the polarimetric case [20], [21], however the aim of this paper is to demonstrate the utility of the proposed framework and of the pseudo-Zernike moments not to compare them or to choose the best channel fusion algorithm, consequently the simplest fusion technique (8) is utilized in addition this will result in a lower computational burden of the entire ATR algorithm.
As $\Omega_j(x, y)$ can cover a very large range of values, its logarithm is used instead

$$\tilde{\Omega}_j(x, y) = \log_{10}(\Omega_j(x, y)).$$ (9)

In order to obtain features that are independent of different intensity levels, due to different observation angles and channel propagation properties, a normalization of $\tilde{\Omega}_j$ is required to restrict its magnitude to the interval $[0, 1]$

$$\overline{\Omega}_j(x, y) = \tilde{\Omega}_j(x, y) - \min[\tilde{\Omega}_j(x, y)],$$
$$\underline{\Omega}_j(x, y) = \overline{\Omega}_j(x, y) / \max(\overline{\Omega}_j(x, y)).$$
The next step of the algorithm (Fig. 1) is the projection of \( \hat{\Omega}_j(x,y) \) onto a basis of pseudo-Zernike polynomials. The polynomials can be pre-computed through (4) since it depends on the sub-image size \( B \times Z \) only (due to the dependencies of (4) only on \( \rho \)), and therefore may be used to populate a Look Up Table. As the pseudo-Zernike polynomials are defined on the unit disc, the support of the image \( \hat{\Omega}_j(x,y) \) is scaled, before the moments are computed, to avoid information loss. Applying (1) to \( \hat{\Omega}_j(x,y) \), the pseudo-Zernike expansion is obtained as

\[
\psi_{n,l} = \frac{n+1}{\pi} \int_0^1 \int_0^{2\pi} W_{n,l}^* (\rho \cos \theta, \rho \sin \theta, \rho) \hat{\Omega}_j(\rho \cos \theta, \rho \sin \theta) \rho d\rho d\theta.
\]

(10)

The output of this stage is the set of \( (n+1)^2 \) magnitudes of the pseudo-Zernike coefficients \( |\psi_{n,l}| \) \( 1 \leq l \leq (n+1)^2 \). From (4) the modulus of the pseudo-Zernike moments are rotationally invariant. This means that at a given observation angle the modulus of the moments are independent of the relative orientation of the target in the image plane. Hence, the feature vector is

\[
F = [|\psi_{0,0}|, \ldots, |\psi_{N,-N}|].
\]

(11)

Finally, the feature vector, \( F \), is normalized using the following linear rescaling

\[
\tilde{F} = (F - \mu_F)/\sigma_F,
\]

(12)

where \( \mu_F \) and \( \sigma_F \) are the mean and standard deviation of the feature vector. These values are then used as input to a classifier.

The last step of the algorithm consists of the classification procedure. The classification has been performed using a k-Nearest Neighbour (k-NN) classifier because of its low computational load and its capability of providing score values as an output [22], [23], however other classifiers with similar characteristics could also be selected.

The sum method is selected as fusion rule [22], [23]. Two strategies are considered for the fusion, maximum vote and maximum confidence. Let \( V \) be the number of possible classes, for each of the \( J \) sensors, the \( k \)-NN classifier returns as output a \( V \)-dimensional vector \( s_j \) containing the confidence levels for each cluster (or the value of the vote \([0,1]\) in case of the rule at maximum vote). The confidence levels (referred also as scores) are defined as the number of nearest neighbours belonging to the \( v \)-th class divided by \( k \), while the vote is defined as 1, if the observation is considered to belong to one of the \( V \) classes, and 0 otherwise. The sum of all the scores or votes is then computed as

\[
\lambda = \sum_{j=1}^J s_j,
\]

(13)
with \( \lambda = [\lambda_1, \lambda_2, \ldots, \lambda_V] \). This fusion strategy allows the exploitation of the information from multiple images and is known for its robustness [22], [23]. Additionally, it allows the definition of the unknown class if a draw occurs, when \( \lambda \) does not have a unique maximum element, or if the maximum value of \( \lambda \) does not satisfy a specific requirement, such as a sufficient score or vote. In particular, we define a threshold \( \eta \) (i.e. the minimum score or vote to be reached to permit a classification) and all observations with \( \lambda \) below \( \eta \) will be not classified and labelled as unknown. Thus the estimated class can be selected as

\[
\hat{v} = \begin{cases} 
\arg \max_v \lambda & \text{if } \exists! (\max \lambda) > \eta \\
\text{unknown} & \text{otherwise}
\end{cases}
\] (14)

Defining the unknown class is important as the number of unknowns provides a measure of the capability of the ATR system to decide for a class.

### III. Performance Analysis

In this section the performance analysis of the ATR algorithm described in Section II is presented. The algorithm is applied to real multi-polarimetric X-band SAR data. The dataset used in this analysis is the “Gotcha Volumetric SAR Data Set V1.0” [18], consisting of SAR phase history from a sensor with carrier frequency of 9.6 GHz and 640 MHz bandwidth, full azimuth coverage and 8 different elevation angles and full polarization. The imaged scene consists of numerous civilian vehicles and calibration targets.

For our analysis the aperture has been divided in sub apertures of 4 degrees in azimuth in leading to have approximately equal range-azimuth resolution cell of 23 cm. In this way 90 images (looks) for each of the 8 circular passes (different elevations) are available in four polarizations, \( \{HH, VV, HV, VH\} \), for each of the 9 commercial vehicles considered. In order to allow the reader to understand the imaged targets and scene, in Fig. 2 the 9 vehicles are shown; while in Fig. 3 the 360 degree full polarimetric image of the scene of interest is shown; the image is a multi-look image (adding all the 90 looks of one circular pass). As already mentioned, for testing a single look is used. In Fig. 3, the 9 vehicles are labelled with alphabetic letters. Specifically, the 9 vehicles are respectively: A) Chevrolet Prizm, B) Nissan Sentra, C) Nissan Maxima, D) CASE Tractor, E) Ford Taurus, F) Chevrolet Camry, G) Hyundai Santa Fe, H) Chevrolet Malibu, I) Hyster Forklift.

To perform the analysis equal sized sub-images (50 × 50 pixels) containing each vehicle have been selected. Specifically, of the 8 available passes (different elevations) a subset of the pass with lower altitude is used to train the classifier while all the other images (i.e. the unused images from the lowest
Fig. 2. Pictures of the 9 vehicles.

Fig. 3. Multi-Look (90 looks) and full polarimetric magnitude SAR image of the area of interest containing the 9 vehicles.

pass and all the images from the other seven, higher elevation, passes) are used to test the algorithm. In this way, different elevation and azimuth angles are considered for testing the images to provide independent training and validation sets.

The analyses have been conducted considering different choices of the training subset, specifically three training sets composed of images selected with azimuth spacing of 12°, 30° and 92° respectively.
The use of a limited number of aspect angles for training is meaningful in terms of a practical realization. Specifically, the acquisition of a database covering all the possible different aspect angles is expensive, time demanding and in some cases impossible. Thus having an under sampled database of training observation is a valid test for the proposed algorithm. The analysis is performed using 1, 2 and 3 randomly selected test data images to characterize the benefits of the multi-sensor framework and the classification fusion stages. Moreover, a comparison between the fusion techniques exploiting the output scores and votes of the k-NN classifier, respectively, have been considered. The analysis has been also conducted comparing the proposed algorithm based on both Zernike and pseudo-Zernike moments, and also a single polarization ($H = 1$) and full polarization ($H = 4$) analysis has been performed. Hence, to evaluate the performance of the classification algorithm, the correct classification in percentage, defined as the number of correctly classified sub-images over the total number of sub-images under test, is used as figure of merit. For the case of 1 test image all the available images have been used, whereas for the case of 2 and 3 test images 10000 pairs or triples are chosen randomly. For this reason the standard deviation of the correct classification rate for the cases of 2 and 3 sensors is also computed.

In Fig. 4 examples of the configurations considered are shown. Multiple acquisitions can be assumed to be done by multiple platforms or from the same platform in different instants of time. Moreover the analysis is performed for different orders $n$ of the pseudo-Zernike moments between 1 and 20 and using a k-NN classifier, analysing different values of $k$.

![Fig. 4. 1, 2 and 3 sensors acquisition examples.](image)

Fig. 5 shows the results obtained for 3 platforms using a training samples spacing of 12°, equivalent to 30 observations of a target with different equally spaced initial azimuth angles (e.g. 0°, 12°, 24°). The curves refer to both the single polarization (SP) and the full polarization (FP) cases, and both the
classifier based on the score and vote fusion rules have been analysed. Moreover, the subplots (a)-(c) of Fig. 5 refer to three different values of the parameter $k$ in the $k$-NN classifier, i.e. $k = 1, 3, 5$. Finally, in subplot (d) of Fig. 5 the number of unknowns obtained in the above cases are given versus the moment orders. Moreover, the $3\sigma$ confidence intervals are quite small (less than $1 - 2\%$). The curves show that in general, the FP case produces a higher level of correct classification with respect to the SP one, for the same $k$ and score/vote choice. However, this behaviour is not observed in the first case, $k = 1$, for the fusion rule based on the score. The curves also show that the correct classification increases as the moment order increases. As expected the number of unknowns reduces as the moment order increases, and the curves reflect the same behaviour observed for subplots (a)-(c). Finally, it is important to underline that the maximum correct classification is obtained in the FP case with $k = 3$ and with the use of the score-based rule.

Fig. 6 shows the results obtained for 3 platforms using a training samples spacing of $36^\circ$, equivalent to 10 observations of a target with different equally spaced initial azimuth angles. Again, the curves refer to both SP and FP cases. Again, both the classifier based on the score and vote fusion rules have been analysed. Also for this analysis, three different values of the parameter $k$ in the $k$-NN classifier, i.e. $k = 1, 3, 5$, have been considered (see subplots (a)-(c) of Fig. 5), whereas Fig. 5(d) shows the number of unknowns versus the moment orders. Also in this second case the $3\sigma$ confidence intervals are quite small (less than $1 - 2\%$). The analysis conducted here conducted is in agreement with the results obtained in the previous case, namely the FP system can reach a higher level of correct classification with respect to the SP one. Moreover, the classifier based on the score fusion rule outperforms that based on the vote fusion rule. However, comparing the curves of Fig. 6 with those of Fig. 5, it can be observed that increasing the azimuth spacing between the images causes the performance of the classifier to decrease because of the reduced number of training images. Also, in this case the number of unknown classifications still reflects the behaviour observed for subplots (a)-(c). Finally, it is important to underline that the maximum correct classification is obtained in the FP case with $k = 3$ and with the use of the score-based rule.

Fig. 7 shows the results obtained for 3 platforms using a training samples spacing of $92^\circ$, equivalent to 4 observations of a target with different equally spaced initial azimuth angles. Again, the curves refer to both SP and FP cases as for the previously analysed scenarios. As observed for the first two situations, also for this case the $3\sigma$ confidence intervals are quite small (more or less $1 - 2\%$). Again, the FP system produces a higher level of correct classification with respect to the SP one. The classifier based on the score fusion rule still outperforms the one based on the vote fusion rule, and a performance degradation with respect the previously analysed cases is observed. Also the number of unknowns is greater than
Fig. 5. Correct Classification (%) versus moments order of the proposed algorithm for training samples spaced 12° with 3 platforms. Both a full polarizations and a single polarization case have been considered with score and vote based fusion rules. The threshold is set to 1 and 2 for the score and vote rules respectively. Subplots (a)-(c) refer to \( k = 1, 3, 5 \) for the \( k \)-NN classifier. In subplot (c) the number of unknowns is given versus the moments order.

those obtained in the other cases. To conclude this analysis, it can be claimed that the value \( k = 3 \) with the FP system produces the best classification performances between the considered situations. In addition the score fusion rule is outperforms the vote one.

In Fig. 8 the correct classification expressed in percentage is given versus the moment orders for the classifiers based on Zernike and pseudo-Zernike moments, for the case of 12° of spacing between training samples in azimuth. Moreover, for comparison purposes the \( L_2 \) norm based algorithm has also been considered [24]. More precisely, the \( L_2 \) norm based algorithm extracts the sub-image containing the
Fig. 6. Correct Classification (%) versus moments order of the proposed algorithm for training samples spaced 36° with 3 platforms. Both a full polarizations and a single polarization case have been considered with score and vote based fusion rules. The threshold is set to 1 and 2 for the score and vote rules respectively. Subplots (a)-(c) refer to $k = 1, 3, 5$ for the $k$-NN classifier. In subplot (c) the number of unknowns is given versus the moments order.

object to classify and normalizes it in order to ensure unit norm. Then, this normalized image is given as input to the classifier. Moreover, Fig. 8 compares the results obtained using 1, 2 or 3 platforms, using a 3-NN classifier. As the vote rule was outperformed by the score decision rule in the previous analysis for $k = 3$, we consider for conciseness only the latter in this analysis. The number of unknowns reported in Fig. 8 subplot (b). Note that, the results shown in Fig. 8 are obtained considering a higher threshold with respect to those of Figs. 5-7, this selection is motivated in order to provide results showing the effectiveness of the algorithm with a different threshold set-up. For example a higher threshold is relative...
Fig. 7. Correct Classification (%) versus moments order of the proposed algorithm for training samples spaced $92^\circ$ with 3 platforms. Both a full polarizations and a single polarization case have been considered with score and vote based fusion rules. The threshold is set to 1 and 2 for the score and vote rules respectively. Subplots (a)-(c) refer to $k = 1, 3, 5$ for the $k$-NN classifier. In subplot (c) the number of unknowns is given versus the moments order.

to a scenario where higher confidence is required. In particular for the cases of 1 and 2 sensors a score of at least 1 was considered to be able to classify, while $4/3$ was the minimum score considered in the case of 3 sensors. The conducted analysis has shown that the algorithm based on pseudo-Zernike can achieve higher values of correct classification than those achieved by the one based on Zernike moments. Moreover, the $L_2$ based algorithm can achieve quite the same performance of the pseudo-Zernike moments of order 10. However, the former uses a $B \times Z$ dimensional space for the features, while the proposed approach uses a space of $(N+1)^2$ with $N \leq 20$, meaning that our approach for $N = 10$ on an image
of sizes $B = Z = 50$ requires 121 components of the feature vector while the $L_2$ approach requires 2500 components. This is a significant advantage in terms of computational complexity and bandwidth requirements. Finally, observing both the correct classification and the number of unknowns, it can be seen that the higher the number of platforms, the better the classification results.

![Graph](image)

(a) score

![Graph](image)

(b) unknowns

Fig. 8. Correct Classification (%) versus moments order of the proposed algorithm with Zernike (□-marked red curves) and pseudo-Zernike moments (○-marked blue curves) and the $L_2$ norm based algorithm (▽-marked black curves), for $12^\circ$ of spacing in azimuth between training samples, with 1, 2 and 3 platforms (represented with dashed, dot-dashed, and continuous lines respectively). A full polarizations case has been considered with score based fusion rule, subplot (a), whereas the threshold is set to 1 for the case of 1 and 2 sensors and $4/3$ for the 3 sensors case. Moreover a 3-NN classifier is utilized. In subplot (b) the number of unknowns is given versus the moments order.

Fig. 9 shows the correct classification versus the moment orders achieved by the classifiers based on Zernike and pseudo-Zernike moments, and on the $L_2$ norm, for the case of $36^\circ$ of spacing between training samples in azimuth. In Fig. 9 a comparison between the algorithms that exploit different numbers of platforms is provided. Again we consider the score decision rule and a 3-NN classifier. The results obtained in this analysis confirm those shown in Fig. 8. In general, compared with the results in Fig. 8, a performance degradation is obtained if fewer aspect angles are used as training samples. In addition the $L_2$ algorithm does not perform as well as the Pseudo-Zernike and Zernike based in this case, demonstrating a higher sensitivity to a smaller number of training samples.

Finally, in Fig. 10 the correct classification is plotted versus the moment orders, for the classifiers based on Zernike and pseudo-Zernike moments, and on the $L_2$ norm, for the case of $92^\circ$ of spacing between training samples in azimuth. Again, the behaviour analysed in Figs. 8-9 is still evident in this last case.
However, due to the large spacing between training samples, the performance degradation increases; all
the algorithms are unable to produce a correct classification score higher than 60% and they are also
not able to produce a number of unknowns less than 2500. Notice also, that in this last analysis the
performance of the $L_2$ based algorithm has deteriorated more than those of the Pseudo-Zernike and
Zernike approach.

To complete the analysis of the proposed ATR algorithm, in Tables I and II, two examples of confusion
matrices are shown. The analysis is conducted with 3 sensors, with $H = 4$ and an image spacing of 12°
for the results in Table I and 92° for the results in Table II. In both cases the pseudo-Zernike moments
of order 20 was used, $k = 3$ and the threshold was 4/3. From the results in the tables it can be seen how
for some targets (i.e. B, D and G) the increased number of aspects angles used for training improves the
classification capabilities, while for other targets (i.e. C, E and H) there are still few cases of incorrect
classification and unknowns.

In general the presented analysis demonstrates that the use of the score rule to assign the classes
provides better performances together with the use of 3 nearest neighbours in the classifier. Moreover
the better capability of the pseudo-Zernike moments to characterize the targets over the Zernike and $L_2$
Fig. 10. Correct Classification (%) versus moments order of the proposed algorithm with Zernike (□-marked red curves) and pseudo-Zernike moments (○-marked blue curves) and the $L_2$ norm based algorithm (▽-marked black curves), for 92° of spacing in azimuth between training samples, with 1, 2 and 3 platforms (represented with dashed, dot-dashed, and continuous lines respectively). A full polarizations case has been considered with score fusion rule, subplots (a), whereas the threshold is set to 1 for the case of 1 and 2 sensors and 4/3 for the 3 sensors case. Moreover a 3-NN classifier is utilized. In subplot (b) the number of unknowns is given versus the moments order.

TABLE I. Confusion Matrix for the 3 Sensors case, $H = 4$, image spacing 12°, pseudo-Zernike moments order 20, $k = 3$ and threshold $4/3$.

<table>
<thead>
<tr>
<th>class</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>Unknowns</th>
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Based approaches has been highlighted. Finally, in all the analysed cases the performance improves with the number of aspect angles available and with the number of observations from different aspect angles used to perform the classification.
TABLE II. Confusion matrix for the 3 Sensors case, $H = 4$, image spacing $92^\circ$, pseudo-Zernike moments order 20, $k = 3$ and threshold $4/3$.

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Clearly, the proposed algorithm appears to have multiple advantages: reliable target identification, multi-observation fusion capabilities without the requirement of a multi-platform training set, ability to provide good automatic target identification performance with a limited set of target observations as training, the capability to identify targets observed from an angle different from those used for training. The pseudo-Zernike moments’ properties of translation and rotation independence makes the algorithm robust with respect to the relative target orientation in the image plane and with no requirement for images to be registered between different platforms.

IV. Conclusion

In this paper a novel algorithm for automatic target recognition with the capability of target identification has been presented. The proposed algorithm exploits the pseudo-Zernike moments derived from multi-channel SAR images as features used to identify different targets. Moreover, the algorithm allows the fusion of the classification result of each of multiple observations from different aspect angles. A performance analysis using the “Gotcha Volumetric SAR Data Set V1.0” has been performed considering a different number of passes, polarizations and training aspect angles. Moreover the comparison with the Zernike moments based and the $L_2$ normalization approach was performed. In all the cases the proposed algorithms showed the capability to identify different vehicles and to take advantage of the multi-pass/multi-channel nature of the data. The results have indicated a high confidence target identification and multi-observation fusion capabilities without the requirement of a multi-platform training set. Moreover the pseudo-Zernike moments’ properties of translation and rotation independence makes the algorithm...
robust with respect to the relative target orientation in the image plane and unregistered images between different platforms. Future work will involve the exploitation of polarimetric decompositions in order to extract more information about the geometry of the targets and achieve roll independent features to be more robust with respect to the radar incidence angle.

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REFERENCES


