Neutrino-Lepton Masses, Zee Scalars and Muon $g - 2$

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Evidence for neutrino oscillations is pointing to the existence of tiny but finite neutrino masses. Such masses may be naturally generated via radiative corrections in models such as the Zee model where a singlet Zee-scalar plays a key role. We minimally extend the Zee model by including a right-handed singlet neutrino $\nu_R$. The radiative Zee-mechanism can be protected by a simple $U(1)_X$ symmetry involving only the $\nu_R$ and a Zee-scalar. We further construct a class of models with a single horizontal $U(1)_{FN}$ (à la Froggatt-Nielsen) such that the mass patterns of the neutrinos and leptons are naturally explained. We then analyze the muon anomalous magnetic moment $(g - 2)$ and the flavor changing $\mu \to e\gamma$ decay. The $\nu_R$ interaction in our minimal extension is found to induce the BNL $(g - 2)$ anomaly with a light charged Zee-scalar of mass $100 - 300$ GeV.

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The standard model (SM) of the electroweak and strong interactions has to be extended, in light of the existing neutrino oscillation data [1] which provides strong evidence for tiny but finite neutrino masses. The squared neutrino-mass differences are found to have two distinct ranges, $10^{-11}$ eV$^2 \leq \delta m^2_{21} \leq 10^{-5}$ eV$^2$ and $\delta m^2_{31} \simeq 10^{-3}$ eV$^2$, in order to interpret the solar and atmospheric neutrino anomalies, respectively. The very small neutrino masses may be explained by invoking the seesaw mechanism [2] where a heavy right-handed Majorana singlet neutrino is introduced at the grand unification scale $(10^{10-16}$ GeV). On the other hand, the radiative mechanism for neutrino mass generation, such as advocated in the Zee model [3], provides an important alternative, where the relevant new physics is expected to show up at or near the weak scale. Such low scale models, besides being able to explain some features of neutrino oscillations such as the bi-maximal mixing [4], are clearly of phenomenological relevance. The existing precision data have put some nontrivial constraints on the models [1] and further tests will be available at forthcoming collider experiments.

The minimal Zee model [3] contains the three active left-handed neutrinos of the SM and a bilepton singlet Zee-scalar which plays a key role for radiative generation of their Majorana masses. There is no underlying reason that forbids the existence of light right-handed singlet neutrinos ($\nu_R$), as $\nu_R$ can also be naturally contained in various extensions of the SM (such as the models with left-right symmetry or $SO(10)$). The introduction of $\nu_R$ thus provides the simplest possible extension of Zee model. However, a simple embedding of $\nu_R$ results in the loss of the prediction of neutrino masses as they are rendered arbitrary by the tree-level Dirac mass terms [5]. In this work, we first build a class of minimally extended Zee models including $\nu_R$ (called Type-I) and invoke a simple $U(1)_X$ symmetry (or its discrete subgroup $Z_n$) to effectively protect the radiative Zee-mechanism by forbidding the mixings between the $\nu_R$ and the active neutrinos. Such an extension is nontrivial since the successful embedding of a singlet $\nu_R$ requires the addition of a second singlet Zee-scalar in the minimal extension. Next we note that the original Zee model neither predicts the size of the Zee-scalar Yukawa couplings nor provides any insight on generating the lepton masses and their hierarchy. Though our simplest $U(1)_X$ in Type-I models protects the radiative Zee-mechanism, we can use this same $U(1)_X$ group as a horizontal symmetry involving both neutrinos and leptons (called Type-II), and thus explain the mass patterns of the neutrinos and leptons in a natural way, à la Froggatt-Nielsen [6]. In both Type-I and -II models, the $\nu_R$ can interact with the right-handed muon and Zee-scalar with a natural $O(1)$ Yukawa coupling, which has striking phenomenological consequences.

Finally, we apply the Type-I and -II models to analyze the Zee-scalar-induced contributions to the muon anomalous magnetic moment $g - 2$ and the lepton-flavor-violation decay $\mu \to e\gamma$. We find that the recent BNL $(g - 2)$ anomaly [7] can be explained with a light charged Zee-scalar of mass around $100 - 300$ GeV. Our models also have the $\mu \to e\gamma$ decay branching ratio around or below the current experimental limit.

**Minimal Extension of the Zee Model with a Right-handed Singlet Neutrino**

The minimal Zee model [3] introduces one extra singlet charged scalar ($S^+_1$) together with the usual two-Higgs-doublet sector. By assuming no right-handed $\nu_R$, as in the SM, this scalar only interacts with left-handed neutrinos and leptons. Thus, the Zee model contains the following additional Lagrangian,

\begin{equation}
\Delta \mathcal{L}_1 = \sum_{j,j'} \frac{f_{jj'}}{2} \bar{\epsilon}_{ab} T_{2j} L_{bj} S^+_1 + m_3 \epsilon_{ab} \tilde{H}^+_1 H^+_2 S^+_1 + \text{h.c.}
\end{equation}

\begin{equation}
= \left[ f_{12} (\bar{\nu}_e \mu L - \bar{\nu}_\mu \epsilon L) + f_{13} (\bar{\nu}_\mu \tau L - \bar{\nu}_\tau \epsilon L) + \right. \end{equation}
where $\ell_j, \ell_j' \in (\mu, \tau)$ and $L_j = (\nu_j, \ell_j)^T$ is the left-handed doublet of the jth family. The $(H_1, H_2)$ are the usual two-Higgs-doublets with hypercharge $(1/2, -1/2)$, where $H_1 = (-H_1^+, H_0^0)^T$, $H_2 = (H_2^0, H_2^0)^T$, and $H_j = \imath \eta_j H_j^+$.

The Yukawa sector can conserve total lepton number by assigning to $S_2^\pm$ the lepton numbers $\mp 2$. Thus, the total lepton number is only softly violated by the dimension-3 trilinear Higgs operator in Eq. (1). As such, the small Majorana neutrino masses are radiatively generated at one-loop and are automatically finite.

We minimally extend the Zee model by including a single right-handed Dirac neutrino $\nu_R$ with following Yukawa interactions,

$$\Delta L_2 \equiv \left[ f_{23} \nu_R^{\nu} \tau L - \nu_R^{\nu} \mu L \right] + m_3 (H_1^{0} H_2^+ - H_1^+ H_2^0) \left| S_2^+ \right| + \text{h.c.} \quad (2)$$

where $S_2^\pm$ is a second singlet Zee-scalar. The nontrivial issue with embedding $\nu_R$ is to avoid arbitrary tree-level Dirac mass terms generated by the Yukawa interactions $\mathcal{L}_1 H_2 \nu_R$ and $\mathcal{L}_2 H_1 \nu_R$ (which mix $\nu_j$ and $\nu_R$), so that the predictive power of the radiative Zee-mechanism can be effectively protected. We achieve this goal by noting that the Yukawa sector \(2\) of $\nu_R$ possesses a global $U(1)_X$ symmetry, which can properly forbid the neutrino Dirac mass terms once a Zee-scalar $S_2^\pm$ is included together with $\nu_R$. It can be shown, by assigning the most general $U(1)_X$ quantum numbers for the Zee-model with $\nu_R$, that the unwanted tree-level neutrino Dirac masses cannot be removed without $S_2^\pm$. We define our simplest Type-I models with $U(1)_X$ in Table 1, where only $\nu_R$ and $S_2^\pm$ carry $U(1)_X$ charges while all other fields are singlets of $U(1)_X$. Hence, the Type-I extension gives a truly minimal embedding of $\nu_R$ into the Zee model.

Table 1. Quantum number assignments for Type-I and -II models. The hypercharge is defined as $Y = Q - I_3$.

<table>
<thead>
<tr>
<th>$L_j$</th>
<th>$\ell_j R$</th>
<th>$\nu_R$</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$S_1^+$</th>
<th>$S_2^+$</th>
<th>$S^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U(1)_Y$</td>
<td>$-1/2$</td>
<td>$-1$</td>
<td>$0$</td>
<td>$1/2$</td>
<td>$-1/2$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$U(1)_X$</td>
<td>$0$</td>
<td>$0$</td>
<td>$x$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-x$</td>
</tr>
<tr>
<td>$U(1)_\nu^{\text{FN}}$</td>
<td>$0$</td>
<td>$y_j$</td>
<td>$x'$</td>
<td>$0$</td>
<td>$z$</td>
<td>$-z$</td>
<td>$-x-y$</td>
</tr>
<tr>
<td>$U(1)_\nu^{\text{bFN}}$</td>
<td>$u_j$</td>
<td>$y_j$</td>
<td>$x'$</td>
<td>$0$</td>
<td>$z$</td>
<td>$-z$</td>
<td>$-x-y$</td>
</tr>
</tbody>
</table>

Table 2 classifies all (dis-)allowed operators of Type-I up to dimension-4. It shows that, as long as $x \neq 0$, the radiative Zee-mechanism is protected and the $\nu_R$ remains massless. Such a massless $\nu_R$ does not contribute to the invisible Z-width as it carries no weak charge. A special case of our Type-I is to consider its discrete subgroup $Z_2$ under which $\nu_R$ and $S_2^\pm$ transform as, $\nu_R \rightarrow i \nu_R$, $S_2^\pm \rightarrow \mp i S_2^\pm$, while all other fields remain invariant. Other non-minimal variations of our Type-I can be easily constructed.

Table 2. Summary of $U(1)$ charges carried by the effective operators in Type-I and -II models.

<table>
<thead>
<tr>
<th>Operators</th>
<th>$U(1)_X$</th>
<th>$U(1)_\nu^{\text{FN}}$</th>
<th>$U(1)_\nu^{\text{bFN}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_R^{\nu} \nu_R^{\nu} \tau L$</td>
<td>$0$</td>
<td>$y_j$</td>
<td>$y_j - u_j$</td>
</tr>
<tr>
<td>$\nu_R^{\nu} \nu_R^{\nu} H_2$</td>
<td>$0$</td>
<td>$y_j - z$</td>
<td>$y_j - z - u_j$</td>
</tr>
<tr>
<td>$\nu_R^{\nu} \nu_R^{\nu} H_1$</td>
<td>$x$</td>
<td>$x'$</td>
<td>$x' - u_j$</td>
</tr>
<tr>
<td>$\nu_R^{\nu} \nu_R^{\nu} H_1$</td>
<td>$x$</td>
<td>$x' + z$</td>
<td>$x' + z - u_j$</td>
</tr>
<tr>
<td>$\nu_R^{\nu} \nu_R^{\nu} S_2^+$</td>
<td>$-x$</td>
<td>$-x - y$</td>
<td>$u_j + u_j' - x - y$</td>
</tr>
<tr>
<td>$\nu_R^{\nu} \nu_R^{\nu} S_2^+$</td>
<td>$x$</td>
<td>$x' + y_j - z$</td>
<td>$x' + y_j - z$</td>
</tr>
<tr>
<td>$\nu_R^{\nu} \nu_R^{\nu} S_2^+$</td>
<td>$0$</td>
<td>$x' + y_j - x - y$</td>
<td>$x' + y_j - x - y$</td>
</tr>
<tr>
<td>$\nu_R^{\nu} \nu_R^{\nu} S_2^+$</td>
<td>$2x$</td>
<td>$2x'$</td>
<td>$2x'$</td>
</tr>
<tr>
<td>$\nu_R^{\nu} \nu_R^{\nu} S_2^+$</td>
<td>$x$</td>
<td>$x + y - z$</td>
<td>$x + y - z$</td>
</tr>
</tbody>
</table>

**Neutrino Oscillations, Lepton Masses and Horizontal $U(1)_\nu^{\text{FN}}$ Symmetry**

While the above Type-I models give the most economic embedding of $\nu_R$ with all the good features of the original Zee-model retained, they do not provide any insight on two important issues: (i) There is no theory prediction on the size of the Zee-scalar Yukawa couplings $f_{23}$ in Eq. (1), but the neutrino oscillation data requires the following hierarchy \(3\):

$$\frac{f_{12}}{f_{13}} \approx \frac{m_2}{m_\tau} \approx 3 \times 10^2, \quad \frac{f_{13}}{f_{23}} \approx \frac{\sqrt{2} \delta m_{31}^2}{\delta m_{32}^2} \approx 10^2 \text{ or } 10^7.$$  \(3\)

where $f_{13}/f_{23} \approx 10^2 (10^7)$ corresponds to the MSW large angle solution (vacuum oscillation solution). (ii) The small lepton masses and their large hierarchy are not understood. Our goal is to construct this same $U(1)_Y$ group as a horizontal symmetry involving all the leptons so that these two issues can be naturally explained à la Froggatt-Nielsen (FN) \(4\). [This $U(1)$ will be called $U(1)_\nu^{\text{FN}}$.] The basic idea is to consider a horizontal $U(1)_\nu^{\text{FN}}$ spontaneously broken by the vacuum expectation value \(S^0\) of a singlet scalar \(S^0\). We can assign $U(1)_\nu^{\text{FN}}$ charges for relevant fields such that different mass terms are suppressed by different powers of $\epsilon \equiv (S^0)/\Lambda$ where $\Lambda$ is the scale at which the $U(1)_\nu^{\text{FN}}$ breaking is mediated to the light fermions. For instance, a low energy effective operator carrying a net $U(1)_\nu^{\text{FN}}$ charge $q$ (either $\geq 0$ or $< 0$) will
be suppressed by $\epsilon^{[q]}$. Though all mass terms are now allowed in the effective theory, we will build a class of FN-type models (called Type-II) in which the arbitrary tree-level neutrino Dirac-mass terms are suppressed to a level much below the one-loop radiative Zee-masses, and thus the predictive power of the Zee-mechanism remains. The role of the FN-scalar $S^0$ is to provide the spontaneous $U(1)_{FN}$ breaking and generate the relevant $U(1)_{FN}$-invariant effective operators that will give the desired neutrino Yukawa couplings and lepton masses at the weak scale. The heavy $S^0$ will be eventually integrated out from the low energy theory and our relevant particle spectrum of Type-II is the same as Type-I.

We provide two typical Type-II constructions, called Type-IIa and -IIb, respectively. The Type-IIa is the simplest extension of Type-I by further involving only the right-handed weak-singlet leptons in the $U(1)_{FN}$ (cf. Table 1). In the Type-IIb models, we further assign each lepton doublet $L_j$ a charge $u_j$. So, the lepton masses are determined by $\ell_{fR}$ charges in Type-IIa, while Type-IIb determines these masses by the charges of both $\ell_{fR}$ and $L_j$. The low energy effective operators up to dimension-4 (with the heavy $S^0$ integrated out) are classified in Table 2, from which we derive the general conditions for protecting the Zee-mechanism in Type-II,

$$10 > |x^r| \sim |x| \gg 1, \text{ and } |x - u_j|, |x + y| \gg 1, \quad (4)$$

with $x x^r > 0$ and $|y|, |z| \sim O(1)$. For the explicit analysis below, we choose a typical value of the suppression factor $\epsilon \simeq 0.1$. Thus, choosing leptons in mass-eigenbasis, we write their mass ratios as

$$m_e : m_\mu : m_\tau \simeq \epsilon^4 : \epsilon^1 : \epsilon^0, \quad (5)$$

which require,

$$(y_1 - u_1) - (y_3 - u_3) = \pm 4, \quad (y_2 - u_2) - (y_3 - u_3) = \pm 1. \quad (6)$$

The tau Yukawa coupling itself can be estimated as $\tau \simeq (m_\tau/m_\ell) \tan^3 \beta \simeq 10^{-2} \tan \beta \sim \epsilon^1$ (with $\tan \beta = (H_2/H_1)$), in the typical range of $\tan \beta \simeq 10 - 40$, and this restricts the $U(1)_{FN}$ charges of $\tau$ as $y_3 - u_3 = \pm 1$. Table 3 summarizes three explicit realizations of Type-II models. From Table 3 and Eq. (3), the Yukawa couplings of $\nu_R$ are predicted as

Type IIa : $(f_1, f_2, f_3) \sim (\epsilon^3, 1, \epsilon^1);$

Type IIb1 : $(f_1, f_2, f_3) \sim (\epsilon^5, 1, \epsilon^3);$

Type IIb2 : $(f_1, f_2, f_3) \sim (\epsilon^{10}, 1, \epsilon^3).$

From Table 3 and Eq. (3), we further predict the left-handed Yukawa couplings $f_{j'Y}$,

Type IIa : $(f_{12}, f_{13}, f_{23}) \sim \epsilon |z|;$

Type IIb1 : $(f_{12}, f_{13}, f_{23}) \sim (\epsilon^4 z^2, \epsilon^6 z^2, \epsilon^8 z^2);$

Type IIb2 : $(f_{12}, f_{13}, f_{23}) \sim (\epsilon^{3+} z^2, \epsilon^{5+} z^2, \epsilon^{12+} z^2);$

where the allowed values of $z$ are defined in Table 3. Thus, Type-IIa suppresses $f_{j'Y}$ couplings to $O(10^{-2} - 10^{-4})$. The models in Type-IIb1 (-IIb2), however, nicely accommodate the hierarchy (3) for the MSW large angle solution (vacuum oscillation solution), while the predicted size of $f_{12} \sim 10^{-3} - 10^{-6}$ is also of the right order (4). Finally, it is trivial to extend these models with more than one singlet $\nu_R$ (i.e., $\nu_{Rj}$ with $j = 1, \cdots, N_{\nu_R}$ and $N_{\nu_R} = 3$ for instance), by simply defining them to share the same $U(1)$ charges as in Tables 1 and 3.

<table>
<thead>
<tr>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
<th>$\epsilon_R$</th>
<th>$\mu_R$</th>
<th>$\tau_R$</th>
<th>$\nu_R$</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$S^{+}_1$</th>
<th>$S^{+}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IIa</td>
<td>0</td>
<td>0</td>
<td>-5</td>
<td>-2</td>
<td>-1</td>
<td>$x + 1$</td>
<td>0</td>
<td>$z - z$</td>
<td>1 - $x$</td>
<td></td>
</tr>
<tr>
<td>IIb1</td>
<td>-1</td>
<td>-3</td>
<td>-5</td>
<td>4</td>
<td>-1</td>
<td>-4</td>
<td>$x$</td>
<td>0</td>
<td>$z - z$</td>
<td>1 - $x$</td>
</tr>
<tr>
<td>IIb2</td>
<td>2</td>
<td>-5</td>
<td>-7</td>
<td>7</td>
<td>-3</td>
<td>-6</td>
<td>$x$</td>
<td>0</td>
<td>$z - z$</td>
<td>3 - $x$</td>
</tr>
</tbody>
</table>

Zee Scalars, Muon $g - 2$ and $\mu \rightarrow e \gamma$

The above minimally extended Zee-type models econonically incorporate the $\nu_R$ and naturally explain the mass patterns of the neutrinos and leptons. The Zee-scalar Yukawa couplings with the neutrinos/leptons also exhibit an interesting spectrum. Now we are ready to analyze their phenomenological impact. The Brookhaven E821 collaboration has announced a 2.6 standard deviation in the muon anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$, i.e., $\Delta a_\mu \equiv a_\mu^{Exp} - a_\mu^{SM} = (42.6 \pm 16.5) \times 10^{-10}$, which gives a 90% C.L. range for new physics,

$$15.5 \times 10^{-10} \leq \Delta a_\mu \leq 69.7 \times 10^{-10}. \quad (9)$$

Different authors have interpreted this anomaly in terms of supersymmetry, muon compositeness, extra $Z'$, leptoquarks and extended neutrino models. We attempt to explain it from the contribution of the Zee-scalars and the singlet $\nu_R$ in our minimal Type-(I,II) models.

The Zee-scalars $S^+_1$ and $S^+_2$ in Type-I/II contribute to $g_\mu - 2$ via the Yukawa couplings $f_{12,23}$ with $(\mu_R, \nu_R)$ and $f_{23}$ with $(\mu_L, \nu_R)$, respectively. Thus, we have,

$$\Delta a_\mu = \frac{m_\mu^2}{96 \pi^2} \left| \frac{|f_{12}|^2 + |f_{23}|^2}{M_1^2} + \frac{|f_{23}|^2}{M_2^2} \right| \approx 11.8 \times 10^{-10} \times \frac{|f_{23}|^2}{M_2^2}, \quad (10)$$

with $M_1^2 = (\cos^2 \phi/M_{S^2}^2 + \sin^2 \phi/M_{H^+}^2)^{-2}$. Here $(M_{S^2}, M_{H^+})$ are the mass-eigenvalues of the two charged scalars in Eq. (1) and $\phi$ is their mixing angle. Our models forbid or highly suppress the mixing between $S^2_1$ and $S^2_2$. 

3
Comparing this with Eq. (7), we see that Type-IIb1 has \( f_1 \) just below the current bound while Type-IIb2 is well below it. On the other hand, the \( f_1 \) coupling in Type-IIa lies slightly above the limit by a factor of 2 – 3; given the uncertainty of the parameters, it can be easily adjusted to stay within the bound. Also, a much weaker bound on \( f_3 \) can be derived from \( \tau \rightarrow \mu \gamma \) decay, i.e., \( |f_3| \lesssim 0.06 – 0.16 \sim O(0.1) \) at 90\% C.L., for \( 1.1 \lesssim |f_2| \lesssim 3 \), which is consistent with the Type-II predictions in [4]. Finally, if we include \( N_{\nu_R} \geq 2 \) singlet \( \nu_{1j} \) with the same Yukawa coupling \( f_2 \), the upper [lower] bound in Eqs. (11) and (10) [Eq. (12)] will be relaxed by a factor of \( \sqrt{N_{\nu_R}} \).

In summary, the Zee model naturally generates small neutrino Majorana masses by radiative corrections, but it neither predicts the Zee-scalar Yukawa couplings nor provides any insight on the lepton mass hierarchy. We have constructed a class of minimally extended Zee-models with the right-handed neutrino \( \nu_R \) embedded, where a \( U(1) \) symmetry protects the radiative neutrino masses while generating the lepton mass hierarchy, the hierarchy of the Zee-scalar Yukawa couplings required by the neutrino oscillation data, the hierarchy of Zee-scalar Yukawa couplings necessary for consistency with the \( \mu \rightarrow e\gamma \) bound, and the size of the Zee-scalar Yukawa coupling needed for the BNL \( g_\mu - 2 \) anomaly. Furthermore, a light Zee scalar \( S_2^\pm \) is predicted in our models, with a mass around 100 – 300 GeV.

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