Health and Safety Executive

Improved generic strategies and methods for reliability-based structural integrity assessment

Summary report

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Improved generic strategies and methods for reliability-based structural integrity assessment

Summary report

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This report is a summary of the research undertaken during the EPSRC/HSE sponsored project entitled 'Improved generic strategies and methods for reliability-based structural integrity assessment'. Detailed findings are documented in a range of other publications, as listed in the references.

The research covers a wide range of topics including: the development of improved methods of reliability analysis which can be easily linked with standard methods of advanced structural analysis; a detailed study of the variability of fatigue crack growth in structural steels and the implications for fatigue reliability analysis; developments in the use of reliability-updating techniques in relation to the prediction of fatigue failure; applications of structural system reliability analysis to the behaviour of a North Sea jacket structure; and the development of a methodology for the reliability-based fracture assessment of pipelines containing cracks.

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CONTENTS

EXECUTIVE SUMMARY

This report is a summary of the research undertaken during the EPSRC/HSE sponsored project entitled ‘Improved generic strategies and methods for reliability-based structural integrity assessment’. Detailed findings are documented in a range of other publications, as listed in the references.

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1 INTRODUCTION

1.1 BACKGROUND

Over the last 30 or more years there has been a major transition from prescriptive regulations to the use of risk and reliability-based approaches in assessing and controlling human safety – both at work and in everyday life (e.g. medicine). Risk assessments are relatively commonplace, and are now required by law in many situations. In essence, a risk assessment requires an evaluation of the relative likelihoods of the complete spectrum of possible accidental or failure events that might occur, together with an estimate of the range of possible undesired consequences (e.g. fatalities and injuries, environmental impacts, financial losses) resulting from those failures. However, in a practical risk assessment, considerable simplifications are needed in order to make the approach tractable, especially in relation to modelling the influence of human factors on the performance of specific tasks (e.g. in design, and in real-time control functions).

In applications such as structural integrity, reliability-based methods have been studied since the early 1950's, and have been in practical use since the 1980's, firstly as a rational way of determining safety factors for design codes and standards (e.g. CIRIA Report 63 [1], BS5400: Part 3 [2]), and then more recently for the integrity assessment of complex structural systems, through the development of software packages such as PROBAN [3], STRUREL [4], RASOS [5-7] and many more. Of current importance are attempts to achieve better integration of structural reliability methods and advanced methods of structural analysis, as for example through the activities of organisations such as ASRANet, a technical network originally funded by the Engineering and Physical Sciences Research Council.

Reliability specialists will be aware, however, that there are a number of significant difficulties in conducting a probabilistic structural integrity analysis and in communicating the results to decision makers. Some of these difficulties relate to the interpretation of the calculations being performed, some to performing the relatively complex calculations themselves, and some to the modelling of the engineering system being analysed and the choice of data to be used. The main purpose of this research has therefore been to investigate these difficulties and to propose new generic strategies for reliability-based structural integrity assessment.

1.2 SUMMARY OF RESEARCH

1.2.1 Objectives

The objectives of the research were as follows:

1. To improve generic strategies and methods for reliability-based structural integrity assessment, focussing on the methodology required to justify the continued operation of specific ageing structures or items of plant.
2. To further develop reliability updating procedures, so that the maximum benefit can be obtained from data gathered during the service life, or from specially conducted in-service tests, so as to sharpen the predictions of the remaining life, or the magnitudes of the extreme loads that would cause failure.
3. To test the methodology by applying it to an existing offshore jacket structure, and to a pressure vessel or pipeline, in order to assess the practical difficulties encountered and the advantages likely to be gained in use.
4. To prepare a guidance document on the application of the improved methodology for general use.

These objectives remained valid throughout the research, but a number of additional and more specific objectives were included, as explained below. It was considered important, not just to propose improvements to reliability-based structural reliability methodologies, but to have some means of testing these improvements. A fundamental difficulty in validating the results of a reliability calculation is that the engineer is often dealing with high integrity structures with very small failure probabilities, with the result that the physical failures being predicted, or guarded against, only rarely or possibly never occur in practice.

To assist in resolving this difficulty, a number of laboratory tests were therefore proposed in which the ‘structure’ could be loaded to failure, and in which reliability-based predictions of the failure load could be made at different stages of the structure’s life. In all cases, the aim was to incorporate information gained at an early stage in the test, or life of the ‘structure’, to improve the probabilistic prediction of the remaining time to failure, or the actual load at which the structure would fail. In other words, a more detailed objective of the research was to study ways in which the uncertainties in the actual, but initially unknown, life of a structure can be reduced by making use of all the relevant information available to the analyst at the time the reliability calculation is performed. This work has focussed on the failure of steel structures by fatigue.

1.2.2 Research Activity

Research has been conducted in a number of related areas in order to achieve the goal of developing improved generic strategies for reliability-based structural integrity assessment. These are:

- Clarification of concepts and development of improved generic strategies and methods for the reliability assessment of engineering structures. This includes the refinement of Bayesian updating methods for structural integrity, and the related issue of choosing target reliability levels.
- Fatigue testing of welded steels to BS4360 Grade 50D to study the inherent spatial variability in crack growth rates through precision measurements, in order to develop better probabilistic models for crack growth parameters; and, in addition, to investigate the effect of specimen size and crack position (in relation to weld position) on this variability.
- The application of FORM/SORM methods [e.g. 8,9] to the prediction of failure by fatigue, and an investigation of the use of experimental data gained from the above tests in updating fatigue life predictions. Investigation of the fundamental differences between FORM/SORM and the use of Bayesian networks to update reliability predictions.
- The collection of material data during the construction of a new North Sea jacket structure to investigate the variability of the material properties, and the use of this data to recommend models for spatial variability in improved reliability analyses.
- System reliability analysis of the same jacket using the software package RASOS to investigate the influence of different assumptions in modelling material properties, and the benefits that can be achieved by using structure-specific and member-specific data.
- An investigation into the use of FORM/SORM reliability methods for the fracture assessment of nuclear piping systems, in which two distinctly different approaches have been used for the purposes of comparison. In the first, the safety margin equation for the fatigue and fracture analysis was obtained from a response surface model constructed directly from the output of a *J*-integral analysis of the cracked component using ABAQUS [10] linked to ZENCRACK [11]. In the second, the reliability problem was solved using an extended directional simulation approach. This work has been carried out as part of the original planned programme by Nahar Hamid [12].

2 SUMMARY OF KEY RESEARCH FINDINGS

2.1 IMPROVED GENERIC STRATEGIES

The research has shown that there needs to be a major thrust in re-educating engineers in the purpose of risk analyses and reliability assessments, and in the interpretation of the results obtained. The most common source of misunderstanding is the confusion between the observed frequency of accidents and failures (e.g. lack of containment of hydrocarbons in process pipe-work and vessels) and the assessed probability of such an event for a specific structure or item of plant, or for a defined group of such items. Actual failures occur in the real world in which the contributing factors such as poor material properties, high loads, accidental events and the influence of human factors are typically and separately inhomogeneous. The challenge to the risk analyst is therefore to capture sufficient of the underlying uncertainties that contribute to the occurrence of the failure event to be able to obtain reliability predictions that can be useful for decision-making. The key to this is appropriate modelling of the relevant uncertainties, and the correct modelling of relevant conditioning events.

Of particular importance is the understanding of the context in which the risk or reliability assessment is being undertaken and the definition of the relevant boundary conditions or conditioning events for the problem being studied. Failure to define these conditions with sufficient precision has led to misunderstandings in the past. In this research, considerable effort has been directed towards developing a suitable methodology for assessing the integrity of existing structures or items of plant. In this case, the probability that a structure will survive for a further period of time, or will fail before a stated load level is applied, is not fixed but varies with time as further information is gained about it, possibly through inspection or monitoring [13[1]]. It is shown that it is to be expected that this probability will tend to either zero or unity in virtually all circumstances – i.e. failure becomes either increasingly unlikely or increasingly probable. Methods for interpreting these assessed probabilities and changes in probability are therefore needed. In determining the effectiveness of inspection, or monitoring, it has been shown that this can be defined, in a reliability context, as the process that has the greatest effect in reducing the variance in the prediction of the failure load.

As structural integrity assessments are amenable to reliability updating, a procedure has been developed whereby non-standard structure-specific probability distributions for material properties can be utilised (and subsequently updated) in the application of FORM/SORM techniques [14].

2.2 STOCHASTIC MODELS FOR FATIGUE CRACK GROWTH

This part of the research was to investigate the most suitable way to model the uncertainties in fatigue crack growth for fatigue reliability analysis, as a particular example of the general methodology. Since the advent of reliability methods, a typical approach has been to fit probability distributions to the parameters C and m in the Paris Law relationship. Various proposals have been given in the literature for different materials based on available data. More recently, the results of extensive re-analyses of fatigue crack data for offshore steels has been given by King in report OTH511 [15]. However, it is clear that the type of statistical analysis that is required to determine basic design curves for fatigue crack growth is different from the analysis to establish the joint probability distribution of the C and m parameters required for reliability calculations. Although the data analysed in OTH511 is very extensive, it is considered that part of the variability exhibited therein is a result of within-data set inhomogeneity, resulting from the pooling of large amounts of data.

[1] This paper is reproduced as Appendix 1 to this report.

In the present research, it was therefore considered necessary to undertake a set of carefully controlled fatigue tests in which the crack growth rate could be precisely monitored and analysed, as a basis for establishing suitable probability distributions for crack growth parameters. To this end, a large 50mm thick steel plate to BS4360 Grade 50D was obtained, cut in half and then welded with a double-V butt weld. A total of 38 beam and compact tension fatigue specimens were prepared from the welded plate, with the fatigue cracks running either up through the parent plate or weld metal, or along the weld in the direction of welding. Fatigue tests were conducted under constant amplitude loading at an applied stress ratio $R = 0.2$.

The aim was to observe and model both the within-specimen crack growth rate variability and the variability from specimen to specimen. It was considered that the variability in growth rate could be influenced by the width of the crack front, as indeed was the case, so specimens of different thickness were also tested. Important findings from the prototype tests are reported in [16[2]]. Detailed results and findings from the main test series are given in [17].

The fatigue tests were planned in order to answer a number of specific questions:

- How does the rate of crack growth vary over short periods of time and over relatively small amounts of crack extension?
- What are the errors in assuming that fatigue cracking can be modelled by a linear or bi-linear law when considering a single crack growing under constant amplitude loading; and what is the corresponding model uncertainty?
- How does the crack growth rate vary between nominally identical specimens when loaded under nominally identical conditions?
- How does the thickness of the specimen and hence the length of the crack front influence the above?
- Is the rate of crack growth influenced by the direction of propagation in a plate?
- How are all the above influenced when the crack grows through regions of welded material where the crack front is likely to be sampling a range of micro-structures and where high magnitudes of residual stresses will be present?

The test programme revealed new insights into most of these issues. It was shown that for a single fatigue specimen (or structure), the variability in fatigue crack growth rate da/dN as a function of ΔK, the range of stress intensity, can be decomposed into local fluctuations about the mean rate at each value of ΔK, and changes in the mean rate with change in ΔK. In addition to this there are then random variations in the crack growth curves between nominally identical specimens, and systematic differences resulting from the effect of specimen thickness and direction of crack propagation, especially for welded specimens.

It has been shown that the short-term fluctuations in crack growth rate can be effectively eliminated by determining the average over crack extensions of about 0.5 mm. This source of variability has no influence on the prediction of fatigue life and will have an insignificant influence in any reliability assessment. However, the existence of these short-term fluctuations means that in the inspection of existing structures, crack extensions of less than about 0.5 mm (assuming that these can be measured) would be of little value is assessing crack growth rates.

To illustrate the variability in crack growth rate that can occur, Figure 1 shows test data from a total of 35 test specimens measured over consistent increments of 0.4 mm of crack extension at increasing values of ΔK. In comparison with this, Figure 2 shows the reduced variability for the subset of non-welded compact tension specimens, in which the only differences are in specimen thickness.

[2] This paper is reproduced as Appendix 2 to this report.

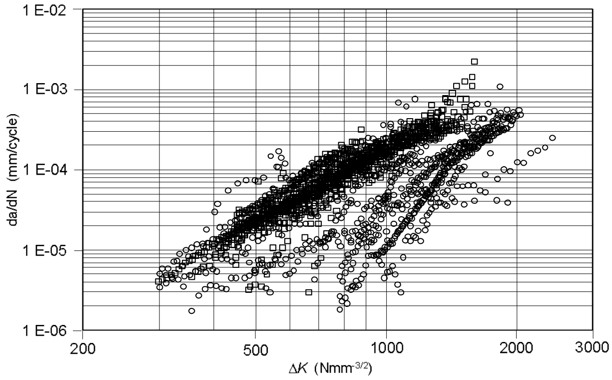

Figure 1 Pooled fatigue crack growth data from 35 test pieces

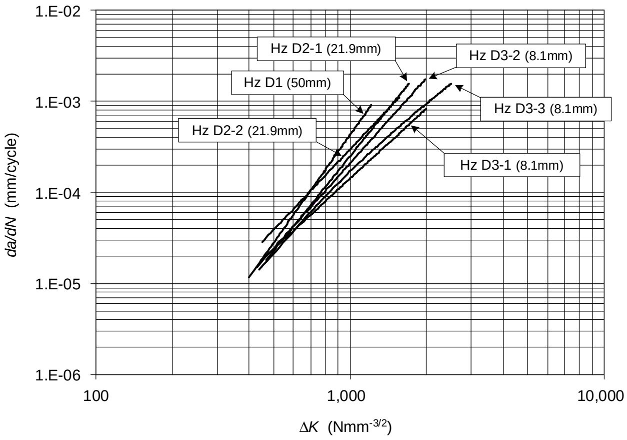

Figure 2 Fatigue crack growth rates for non-welded CT specimens of different thickness

The development of stochastic fatigue crack growth models has been investigated using data from tests of thin aluminium plates obtained by Virkler *et al* [18] and from the data obtained from the current experimental work on structural steels. It has been demonstrated that the probabilistic models for the parameters in the fatigue crack growth relationships are, not surprisingly, dependent on the physical model selected to represent the fatigue crack growth process. For cases where a linear crack growth relationship is a poor approximation to the real fatigue behaviour, the use of the Paris Law leads to additional uncertainty in computing the probability of fatigue failure and an increase in the variance of the fatigue life. However, it has been shown that even when a non-linear crack growth model is used, the uncertainty in predicting fatigue life is still relatively large.

It has been demonstrated, however, that information gained from early stages in the fatigue life of a structure can be used to update the reliability predictions, and that this additional information can lead to a significant reduction in the variance of the predicted fatigue life. These conclusions derive from the finding that for a given geometry and material the fatigue crack growth curves show a tendency to intersect, or approximately intersect, at a particular value of range of stress intensity. It must be emphasised, however, that the research reported in this study has been based on fatigue tests carried out under constant amplitude loading and that further uncertainties in crack growth behaviour arise as soon as non-constant amplitude loading conditions are present. These can only increase the overall uncertainty in fatigue life prediction.

2.3 USE OF RELIABILITY UP-DATING TECHNIQUES

A further reason for conducting the fatigue tests was to use the data obtained from individual specimens for experiments in reliability updating. Such calculations can be performed on real structures during their service life in order to improve the reliability predictions when new inspection data becomes available, as discussed under above. However, because real structures rarely fail, the results of successive reliability calculations performed after repeated inspections of real structures are effectively never available. In order to study the benefits of successive measurements of fatigue cracking on the reduction in variance of the predicted fatigue life, two basic procedures have been used. The first was based on computing the probability of the occurrence of complete fatigue failure conditioned on the events that the crack had particular measured sizes after known numbers of loading cycles. The calculations were performed using PROBAN. The second method was to use a Bayesian network approach with the software package Netica [14].

It has been shown that a significant factor in the ability to improve the prediction of fatigue life is the relative magnitude of the model uncertainty in the fundamental crack growth law; for example, the extent to which the relationship between $\log da/dn$ and $\log \Delta K$ for a particular structure and crack is indeed linear. Another factor that has been studied is the practical limit on the number of crack growth observations that can be incorporated into the reliability updating calculation, and their relative importance of each in improving the prediction of fatigue life. This is illustrated in Figure 3 (using data from Virker specimen #68) which shows the changes in the predicted number of cycles to cause failure (deemed here to be a crack size greater than 40.2 mm) for different levels of probability, when information on the actual crack size becomes available at different stages in the structure's life.

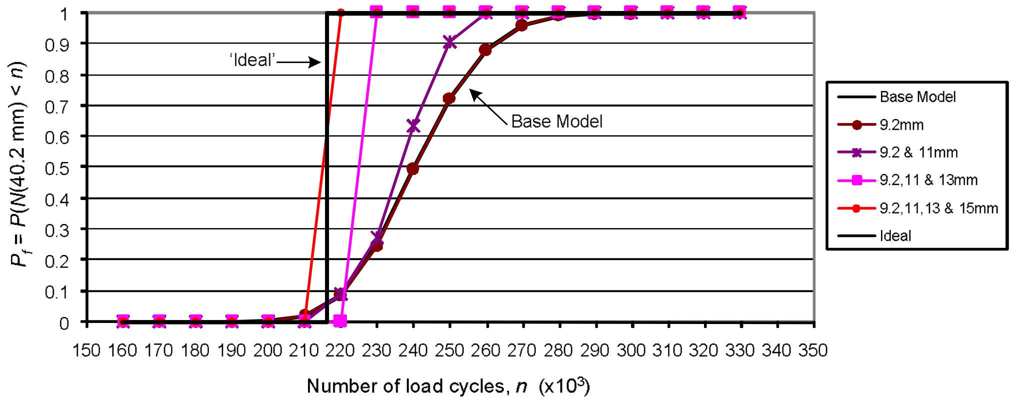

Figure 3 Illustration of the benefits of updating the base model predictions of the number of loading cycles to failure using crack growth information

2.4 MATERIALS DATA ANALYSIS FOR A NEW NORTH SEA JACKET STRUCTURE AND ASSOCIATED MODELLING

2.4.1 Data Analysis

A major factor in the systems reliability analysis of large structures is the appropriate stochastic modelling of the material properties, for example in the assessment of ageing offshore jacket structures. However, the spatial variations in properties such as fracture toughness and yield stress are difficult to model because the spatial information is rarely available. Historically, only the sample-to-sample variations obtained from mill testing have been known. The extreme assumptions that structural member properties are either statistically independent or fully correlated give rise to relatively large variations in system reliability predictions, and cannot be justified.

During the course of the project the opportunity arose to obtain and analyse materials data from a new jacket structure that was being constructed in the North Sea. This has been undertaken and information was gained on within-heat and between-heat variations for plates of different thickness. This data provides a basis for the improved modelling of material properties. The extent to which these findings can be generalised to different structures has also been studied [14]. The importance of these findings, in any given situation, depends on the extent to which the material property variables influence the computed reliability. For load dominated situations, which include many offshore applications, detailed modelling of the material properties may not be important, provided that large differences in mean properties can be avoided through appropriate quality control mechanisms.

2.4.2 Reliability analysis of a new North Sea jacket structure using various stochastic modelling assumptions

In this part of the research, a reliability-based non-linear collapse analysis of the jacket structure described above has been carried out in order to study the significance of different modelling assumptions for material properties on the assessed system reliability. The structure is a four-legged jacket comprising approximately 100 principal structural members, in about 100 metres of water. However, the structural model used in the reliability analysis consisted of 411 separate cans with known material property data.

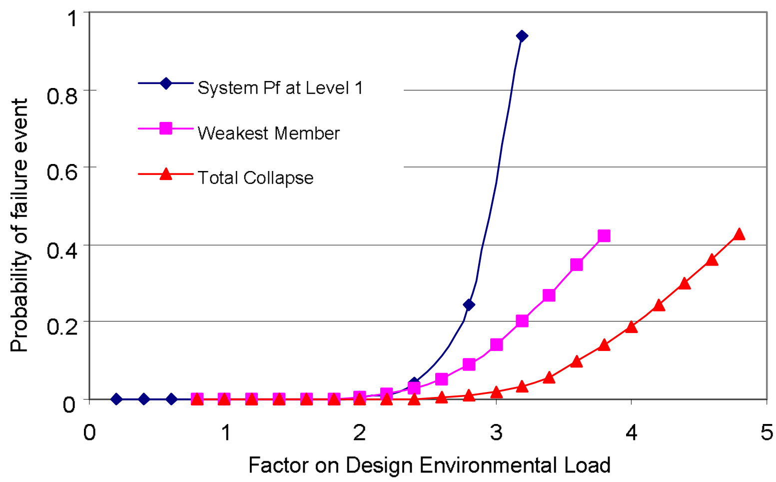

Figure 4 Results of typical systems reliability analysis

The reliability assessment was conducted using RASOS 6.15 [7]. This system reliability analysis takes account of the non-linear structural behaviour resulting from a sequence of member failures prior to final collapse. Using this approach, it has been possible to investigate the effects of different modelling assumptions for material properties on the assessed probability of structural failure. The relative likelihood of occurrence of different levels of structural damage, from a single plastic hinge up to total collapse, has been investigated. It has also been possible to determine the effects of obtaining member-specific property data for particular structural members (e.g. from in-service inspection and testing), and to study the benefits in terms of the reduction in the uncertainty of the predicted collapse load [14]. Typical basic results for a single set of modelling assumptions are shown in Figure 4. These graphs show the factor by which the design environmental load (wave forces) must be increased to achieve a particular probability of failure in a period of one year. The three curves represent: (i) failure of any single structural member (ii) failure of the structural member which has the highest probability of failing first and (iii) structural collapse by the formation of a mechanism.

2.5 FATIGUE AND FRACTURE ANALYSIS OF PROCESS PIPE-WORK USING RESPONSE SURFACE ANALYSIS AND DIRECTIONAL SIMULATION

One of the central features in developing an improved generic methodology for reliability-based structural integrity analysis is the full integration of the engineering analysis and the structural reliability calculations. For any analysis to have practical validity, it is necessary for the following to apply:

- The method of structural analysis used must be compatible with the failure mode (limit state) being analysed.
- Account should be taken of the model uncertainty associated with the method of structural analysis used (e.g. implicit simplifications, bias, etc)
- The input random variables should be chosen to represent the uncertainty in the state of knowledge of each variable at the time of the analysis.
- The method of reliability analysis must be valid for the problem being analysed, and be capable of including system effects where these are present.

Increasingly, tools such as finite element analysis (FEA) and computational fluid dynamics (CFD) are being used to analyse engineering systems. Their main advantage is that they can accommodate complex geometry and realistic constitutive relations for the materials, with the result that model uncertainty associated with the engineering analysis is being progressively reduced as knowledge grows and computing power increases. However, these methods of analysis are inherently deterministic and are computationally expensive and/or time-consuming. For structural reliability problems modelled in terms of, say, only 20 random variables the problem arises of undertaking several hundred FE runs for a single reliability computation, even if the most efficient type of FORM/SORM analysis is used. This may be prohibitive, and the direct linking of FEA and reliability analysis is not straightforward. If Monte Carlo simulation is used in place of FORM, the number of analyses required to obtain a stable solution depends on the reliability of the component or system and this generally precludes the use of basic Monte Carlo analysis.

Various solutions to this problem are available, including response surface analysis [19] linked to an efficient form of simulation such as importance sampling [20], and directional simulation [21, 22]. However, although many of these methods work well when the failure surface is well-behaved in the region of the so-called FORM ‘design point’, it can easily be demonstrated that some methods become unstable when the failure surface is irregular or has a high curvature. These difficulties have been investigated in the present research.

In this work the aim has been to develop a robust methodology for determining the reliability of duplex stainless steel pipelines containing minor crack-like defects subjected to loading resulting from internal pressure and thermal stressing, using the most realistic physical modelling assumptions. Unlike the jacket structure example above, where the major uncertainties are in the environmental loading, the major challenge in this case is the integration of the fracture analysis with the reliability prediction. Figure 5 shows an FE model of a surface-breaking semi-elliptical defect on the internal wall of a pipe subject to internal pressure, thermal stresses, axial forces and bending moments. The model was built in ABAQUS with the crack-tip elements being generated by ZENCRACK. ABAQUS was then used to determine the *J*-integral value at various positions around the crack front for sets of values of the loading variables.

The stress-strain curve used in the analysis was a four-stage piecewise continuous model aimed at representing typical stress-strain curves for this material as closely as possible in the non-linear range. The stress-strain curve was modelled by a linear segment up to the limit of proportionality which was treated as a random variable, a modified Ramberg-Osgood relationship from the limit of proportionality up to a strain of 0.2%, a further Ramberg-Osgood relationship with different parameters up to strain limit of 6.3%, and finally a cubic polynomial up to strains of about 50%. The stress-strain curve for the material was additionally taken to be temperature dependent, with the maximum operating temperature being a random variable. The complete stress-strain was scaled to take into account the effect of temperatures above the reference temperature. Full details are given in [12].

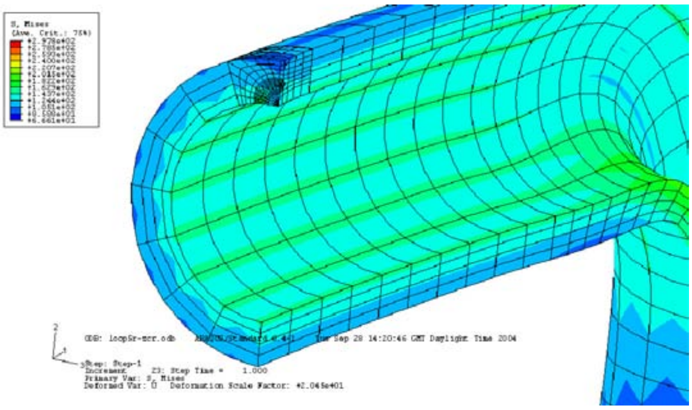

Figure 5 Finite element mesh for *J*-integral analysis of pipe with planar defect [12]

The failure of the pipe by fracture initiation was assessed by comparing the maximum value of the computed *J*-integral with the material toughness J_{Ic} also modelled as a random variable.

The reliability calculations have been performed both by importance sampling and directional simulation. Of these methods the second has been found to perform in the most stable manner, with least computational effort.

For directional simulation, the approach is to generate a number of random trials by generating random vectors in *n*-dimensional standard normal space, where *n* represents the number of random variables involved in modelling the uncertainties in the physical problem. If the failure surface can be mapped into this space by the development of an explicit algebraic expression, the *n* random distances r_i to the failure surface from the origin can be determined algebraically. In this case, and in general, the failure probability is then given by

$$P_f \approx \frac{1}{m}\sum_{i=1}^{i=m}\left(1-\chi_n^2(r_i^2)\right) \qquad (1)$$

where $\chi_n^2(r_i^2)$ is the chi-squared distribution function with n degrees of freedom, and m is the number of trials. However, when the failure surface is defined only in terms of a numerical procedure, as in the case of a J-integral based FE computation, an efficient scheme for selecting which random vectors should be analysed and which should be discarded is required. In order to minimise the number of unnecessary FE analyses, only those vectors which intersect the failure surface at distances of approximately $r < 2\beta$ need be considered, where β is the initially unknown FORM reliability index. This is because the contributions from other vectors to the summation in Eq. (1) will be zero or negligible.

The procedure that was developed was to undertake an initial J-integral analysis at the mean values of the input random variables, corresponding approximately to the origin in standard normal space, and then to analyse only those directions in which there was a significant reduction in the value of the safety margin $(J_{Ic} - J)$ with distance from the origin, as illustrated in Figure 6. It can be seen that the approximate distance r to the failure surface in this direction can easily be found by interpolation, from only three additional J-integral analyses, and is thus very efficient in terms of computing effort.

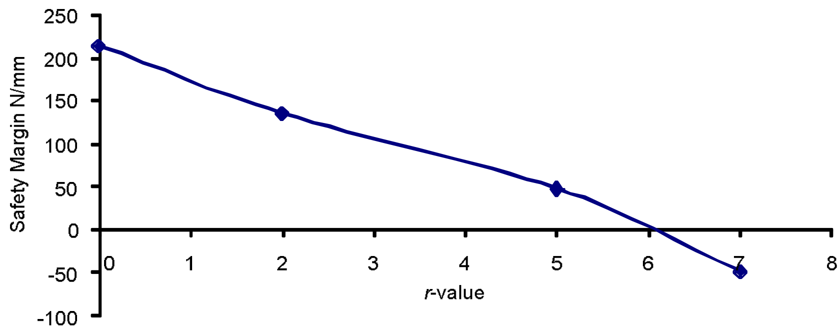

Figure 6 Change in safety margin with radial distance from origin in standard normal space showing 'failure' at $r \approx 6$ [12]

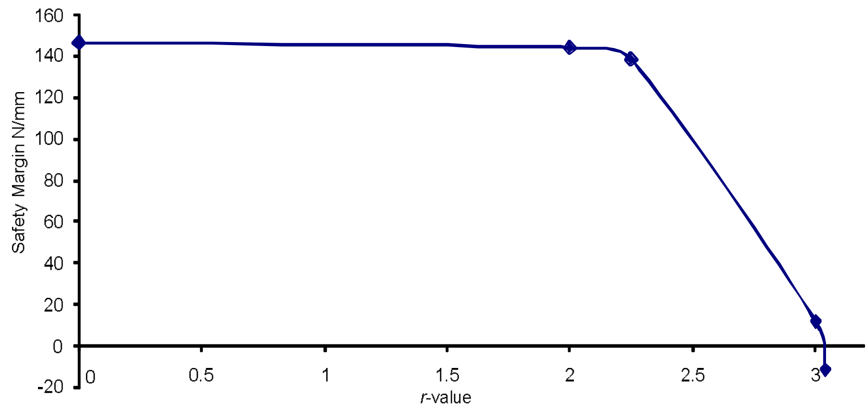

Figure 7 Change in safety margin with radial distance from origin of standard normal space showing 'failure' at $r \approx 3.05$ [12]

However, care has to be taken to include situations such as shown in Figure 7 in which the highly non-linear nature of the failure surface in standard normal space leads to conditions in which there is a rapid change in gradient in the *M* versus *r* curve. Indeed, Figure 7 demonstrates why instability is likely to occur in an adaptive response surface approach.

Finally, Figure 8 illustrates the convergence of the estimated probability of fracture using the directional simulation approach for a number of crack depths and lengths for the problem defined in Figure 5.

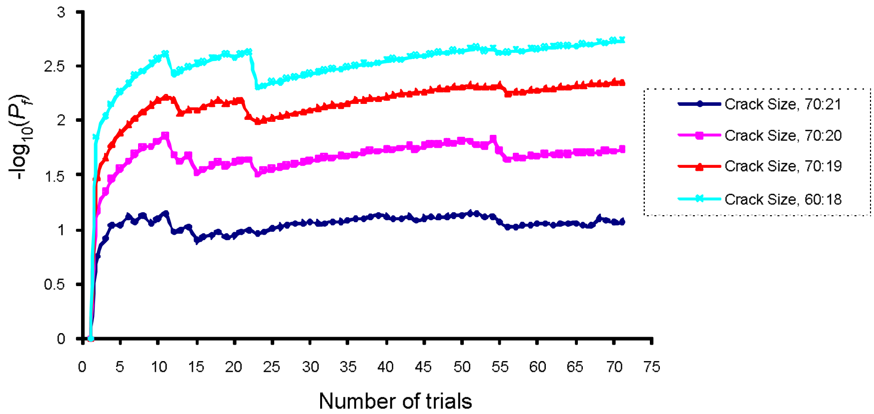

Figure 8 Directional simulation results for probability of fracture

Although the directional simulation approach described above has been used to investigate a particular fracture reliability problem, it has been demonstrated to be a method with wide-ranging potential in any situation where structural behaviour is analysed by numerical methods such as finite element analysis.

3 DISCUSSION AND CONCLUSIONS

The project described in this summary report has covered a wide range of research topics all of which were planned to contribute to the advancement of reliability-based structural integrity analysis in different ways.

In using these approaches, however, distinction must be made between the use of structural reliability methods at the design stage of a structure, including their use in the determination of safety factors for design, and their use in assessing the reliability of existing structures. In the former, the probability of failure can be thought of as a property of the structure or the ensemble of structures and can be set to some target value. This target value should depend on the consequences of failure – for example, 10^{-6} per year for a reasonably high integrity structure. It should be noted, however, that nominally identical structures built to such a target reliability level will, by definition, not be identical and will therefore vary in their ability to carry loads. However, assuming that the probabilistic models used in the reliability analysis are truly representative of the uncertainties that are present, the target failure probability (e.g. 10^{-6} per year) will be the average failure probability for that ensemble of structures and could be used to determine the overall failure frequency.

In contrast to the above, in the reliability analysis of unique existing structures, some of the uncertain quantities are fixed but unknown (e.g. most of the material properties). In this case the assessed reliability or probability of failure is a number that changes with time as further information about the structure becomes available. As mentioned earlier in this report, the assessed probability of failure of an existing structure in a specified period of time will typically tend to either zero or one with the progression of time. Further work is required in setting logical acceptance criteria for such situations and will depend on the relative influence of the time-variant and time-invariant uncertainties.

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APPENDIX 1

The Integration of Structural Reliability Assessment and Structural Monitoring

ASRANet International Colloquium, Barcelona, July 2004

THE INTEGRATION OF STRUCTURAL RELIABILITY ASSESSMENT AND STRUCTURAL MONITORING

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Abstract

Over the last 20 years, major applications of structural reliability analysis have been seen in the development of structural codes and standards, especially in relation to the evaluation of safety factors and in reliability-based design, but it has also been used extensively in the assessment of existing structures. At the design stage, most design variables are inherently uncertain because they relate to future events where the controls are imperfect – e.g. in manufacture, construction and use of the structure. For existing structures, however, some of the uncertainties are of a different nature since they represent lack of knowledge of what is actually present in the structure, and the development of appropriate stochastic models for these uncertainties is equally, if not more, challenging for the analyst than in the case of design.

The paper describes work that has been undertaken to increase the benefits that can be achieved by the reliability-based assessment of existing structures through using information gained during the structure's service life, the aim being to reduce the variance in predicted failure load(s), in order to avoid unnecessary repair or premature termination of the structure's useful life.

The paper focuses on the experience gained in using Bayesian updating techniques to analyse the behaviour of two types of structures, reinforced concrete slab bridge decks failing in shear under cyclic loading, and various types of steel specimens failing by fatigue. In both cases, laboratory testing has been used in place of field measurements on full-scale structures to obtain extensive information during the service lives of these structures and components to produce increasingly better estimates of the remaining life. The objective has been to study the relationship between the amount of information obtained from the structural monitoring and the improvements in the reliability prediction achieved. It has been found that of critical importance are the models and the corresponding model uncertainties associated with both the structural behaviour being observed and monitored, and the ultimate limit states being predicted.

The results of this work should lead to improvements in the reliability assessment of existing structures where field measurements are possible during the service life.

Keywords: Structural Integrity, Structural Monitoring, Failure Modes, Reliability Updating, Fatigue, Shear Failure, Erosion, Bayesian Networks.

INTRODUCTION

Background

Over the last 20 years, major applications of structural reliability analysis have been seen in the development of structural codes and standards, especially in relation to the evaluation of safety factors and in reliability-based design, but it has also been used extensively in the assessment of existing structures [1]. At the design stage, most design variables are inherently uncertain because they relate to future events where the controls are imperfect – e.g. in manufacture, construction and use of the structure. For existing structures, however, some of the uncertainties are of a different nature since they represent lack of knowledge of what is actually present in the structure, and the development of appropriate stochastic models for these uncertainties is equally, if not more, challenging for the analyst than in the case of design.

Most engineering structures never fail in service, and would typically collapse at loads considerably in excess of their design loads if the loading were to be increased progressively until failure occurred. Some structures, however, may be found to be under-designed for their current usage, either because of changes in conditions during their service life (e.g. bridges where traffic loading has increased over a period of time), or because of erroneous design assumptions. Other structures and engineering systems may be subject to various deterioration mechanisms, such as fatigue, corrosion, or, in the case of pipelines and vessels by erosion, or erosion-corrosion. In all these situations the engineer has to decide whether or not to take the structure out of service, or in the case of bridges whether to impose load restrictions. For marine structures and vessels the possibility of reducing the loading may not be feasible, especially if this is principally of environmental origin – e.g. waves and currents – so the question of whether a structure should be repaired or strengthened may arise.

For all structures, knowledge of how each is performing at various stages of its life is likely to be of benefit to the owner or operator in deciding whether it is safe to continue using it without intervention. To aid this decision-making process some structures and systems are subjected to either periodic inspections or continuous condition monitoring. Examples of the latter are: acoustic emission monitoring for aircraft structures to detect fatigue cracking; vibration monitoring of offshore structures to detect member failures; wall thickness monitoring in pipelines and vessels to detect erosion and corrosion which might lead to leakage of toxic or flammable material; and the displacement monitoring of piles, rock faces and dam crests to provide warning of movements which could precede catastrophic collapse. Indeed, a whole new discipline known as 'structural health monitoring' [2] is growing up with the aim of improving the ease with which structures can be monitored, and to develop and use new types of sensor.

The question arises therefore as to how structural monitoring data can best be used to improve structural decision-making and the 'management' of structures. In many cases structural monitoring systems are installed simply to provide advanced warning of impending failure or of accelerating deterioration, thereby allowing some action to be taken to prevent the failure or to minimise the undesired consequences associated with it – e.g. fatalities. The monitoring itself can be continuous, intermittent, triggered to take place only if other conditions are met (e.g. wind speeds above a certain magnitude, or vehicles above a certain weight approaching a bridge), or linked to regular or irregular periods of inspection.

In this paper, however, consideration is given to how monitoring data can be used in the context of the reliability assessment of existing structures in order to improve reliability predictions.

Reliability-based Design and Assessment

Reliability analysis is concerned with making predictions about the possible range of behaviour of structures and structural systems during their intended design life, and occasionally for extended periods of time beyond this. It is of course also applicable to a wide range of non-structural problems. In a reliability-based design, the analyst may envisage his/her particular structure as a single sample from a notional population of nominally similar structures, and that the probability of failure in a specified time T is the assessed proportion of those structures that would 'fail' if they were all constructed. Failure in this context is exceedance of any specified limit state or performance limit.

In the reliability-based assessment of an existing structure, the reliability calculations may or may not be the same as in a reliability-based design, but the interpretation of the assessed probability will be different. In such situations, the uncertainties that exist relate to the state of knowledge that the analyst has about the structure at the time the reliability assessment is carried out. A wide range of possibilities exists, including the following:

- The reliability analyst may have *less* knowledge of the structure than the original designer, for example if design drawings, calculations, material specifications and material test data have been lost with time – a not unusual situation with old bridges, and for other assets which have been sold to new owners or operators.
- The analyst may have only *slightly more* information than the original designer, for example when a structure has been built but when no construction-phase data has been made available. In this case the only additional knowledge may be that the structure is able to support its own self-weight without exceeding any observable limit state.
- The analyst may have *considerably more* information than was available at the design stage as a result of possessing extensive construction-phase and commissioning data, possibly including proof-load test information.
- The above situation may apply, but the structure may be well into its service life and visible signs of deterioration by mechanisms such as fatigue and corrosion may be apparent.

The availability or unavailability of different types and quantities of information about any specific structure clearly raises issues in assessing its fitness for purpose for the remainder of its intended design life, or for any other period of time – e.g. until the next inspection. However, the problem of how best to use all the information available exists regardless of whether deterministic or probabilistic methods are used in the assessment. Historically, and indeed up to the present time, engineers have used their experience and what has commonly been called 'engineering judgement' to 'weigh' all available evidence before reaching a decision in relation to structural adequacy. However, regardless of the method used, the assessment of an existing structure is a challenging task in the face of incomplete information, and this is the situation that generally exists.

It is now necessary to look more closely at the interpretation of the probabilistic measures used in the assessment of existing structures. Again, a number of special case scenarios can be identified:

- Case A: Consider the situation when a load of known type and magnitude is going to be applied to a structure at some future date – in practice this could be an extremely heavy vehicle of known weight passing over a bridge. In this case the bridge will either collapse, or reach some other limit state, when the load is applied, or it will not. If it is further assumed that there will be no change in the mechanical properties of the structure, or the structure itself, up to the time the load is applied, and that the magnitude of the future load is known precisely, the assessed probability that the structure will fail is dependent only on the uncertainties in the relevant mechanical properties and the uncertainty in the behaviour of that structure with those properties under the known load (typically known as model uncertainty).

- Case B: Now consider a somewhat similar situation where the unknown material properties and the structure itself are invariant with time, but where the future load to be applied is of uncertain magnitude, or where many different loads of unknown magnitude will be applied over a period of time. In this case, the additional uncertainties associated with the loading as well as the uncertainties in the fixed but unknown material properties will influence the value of the assessed probability of failure.

- Case C: Finally, consider the situation where instead of the structure having properties that are invariant with time, it is subject to various deterioration mechanisms that proceed at uncertain rates. In this case, the assessed probability of failure will depend on the initially unknown material properties, the unknown deterioration rates and the uncertain magnitude of the future loading.

In all these idealised situations, the assessed probability of failure needs careful interpretation. In a Bayesian sense it represents the analyst's 'degree-of-belief' that the structure will fail, given the information available at the time the reliability calculations are performed. It is therefore a conditional probability, where the available information about the structure represents the conditioning events. However, if there is complete agreement on the information available, and there is an agreed best way of taking this into account, then the assessed probability is a robust number that will not change from analyst to analyst and so is not a 'personal degree-of-belief', but a collective one.

In case A above, both the material properties and the load to be imposed on the structure are fixed in magnitude (although not known) at the time that the reliability calculation is performed, and the computed or assessed failure probability depends only on this lack of knowledge, or uncertainty, about 'these facts'. Further knowledge of the material properties, structural dimensions, etc. would lead to the assessed probability of failure changing to a value either close to zero or close to one. If complete information were to become available the assessed probability would be either exactly zero or exactly one.

In case B above, the assessed probability of failure for the unique structure being analysed would change if some of the actual material properties or structural dimensions became known, but since the magnitude of the maximum loading remains uncertain (until after it has occurred), the assessed probability of failure will always be non-zero. However, with increasing information about the fixed but unknown material properties, it will tend to the value of the cumulative distribution function of the load corresponding to the computed load-carrying capacity of the structure based on the known properties. Of course, for structures subjected to multiple time-varying loads the situation is clearly more complex.

In case C above, the assessed probability of failure will be influenced additionally by the time-dependent deterioration processes that are taking place. This probability will change as more information becomes available. In this case, the assessed probability of failure is itself a stepwise continuous function that will tend to either zero or one as the structure approaches the end of its design life.

In all the idealised cases discussed above it can be seen that for the assessment of a unique structure, the computed or assessed failure probability will change according to the information available about it, even though there may be no physical change to the structural system itself. For more practical scenarios the situation is the same, and the assessed probability is simply the analysts assessed 'degree-of-belief' that the structure will fail, based on all the information that is available at the time the analysis is undertaken. If this does indeed include all the information that is available, then the assessed probabilities can be used as a rational basis for decision-making – for example, in deciding whether the structure is safe for continued use.

It must be noted at this point that these assessed probabilities should not be given relative frequency or frequentist probability interpretations, simply because the underlying populations do not physically exist for the conditional probabilities that are being computed. The reason for this is that even though a number of structures may be built to a particular design and are thus nominally identical, as soon as any further information is gained about each post-construction, the structures become distinct and are no longer part of the same homogeneous population. In other words, although a set of structures may form part of a homogenous set at the design stage, as soon as any information relating to any individual structure becomes available post-construction, this structure becomes part of a sub-population with a membership of one. This does not mean, however, that such a population cannot be envisaged, but it generally does not physically exist. The assessed probability of failure can, however be thought of as the proportion of this sub-population that would fail if the population were to exist.

A consequence of the above is that statistics of historical failure rates will generally bear little similarity to the assessed probabilities of failure. There are also other reasons for such differences.

In summary, it is therefore to be expected that, for a specific existing structure, the assessed probability of failure during any specified period of time will change as more information pertaining to each random variable affecting its behaviour becomes available. In the sections below, this is explored in the context of data obtained from structural monitoring.

Objectives of this Paper

In the light of the discussion above, the objectives of this paper are to review the way in which data obtained from structural monitoring can be incorporated into structural reliability analyses in order to refine the calculations and improve decision-making. In practice, especially for safety-critical structures the failure of which would result in unacceptable losses of a human or financial kind, such decisions might relate to whether the structure (e.g. bridge, aircraft, oil-rig, pressure vessel, etc) should kept in service, whether it should be inspected more frequently or conversely not at all, or whether it should be strengthened.

These discussions are illustrated with the results of experiments that have been carried out in the laboratory in three distinct areas: the fatigue failure of steels, the shear failure of concrete highway bridges and the erosion of pipelines.

RELIABILITY PREDICTION FOR EXISTING STRUCTURES

The methods available for structural reliability calculations are now well known [3],[4],[5] and will not be discussed in detail here, and for many problems the necessary calculations can be performed by a range of commercially available software packages such as PROBAN and STRUREL. However, some aspects do need to be considered.

In a typical component or system reliability analysis, it is first necessary to define the relevant failure modes and corresponding limit state equations. These are typically re-arranged into the form of a so-called safety margin equation which in generic form is usually written as:

$$M_i = g_i(X_1, X_2, \cdots, X_n) = g_i(\underline{X}) \qquad (1)$$

where M_i is the safety margin, $g_i(.)$ is the limit state function and $\underline{X}$ is a vector of input quantities which may be random variables or stochastic processes, or a combination of these. Provided that the precise form of $g_i(.)$ is known and that agreement is reached on the type of probability distributions and corresponding parameters to be used for modelling each random variable X, the numerical value of the probability $P(M \le 0)$ can be determined in a number of different ways, either by FORM/SORM methods, Monte Carlo simulation, or by other numerical schemes. The exact numerical answers may differ somewhat from each other depending on the numerical approximations being made in the solution technique. These difference are generally acceptably small – i.e. much less than an order of magnitude.

In the case of system reliability calculations (e.g. for the analysis of offshore jacket structures) the number of possible failure paths can be enormous, and it is generally necessary to truncate many of the less probable failure paths and to compute upper and lower bounds on the system failure probability. However, given enough computing power, even that would not be necessary.

Nevertheless, it is clear than any structural reliability calculation will only be as good as the underlying limit state models and probabilistic models for the corresponding input quantities.

The influence of model uncertainty

Throughout the development of structural reliability analysis, the main focus has been on the probabilistic aspects of the problem and on the reliability computations themselves. Nevertheless, it was recognised from an early stage that additional uncertainties over and above the natural variability of the input parameters was likely to be present in many situations, especially where the limit state equations are empirically-based, rather than being derived from fundamental engineering principles. A good example of this are the equations used in most Codes for design against shear failure in concrete. However, these so-called model uncertainties are extremely difficult to model and quantify, as they generally require a comparison of the results of full-scale tests with the corresponding theoretical predictions of structural behaviour. In most cases this is not feasible because of the costs involved in testing. That this is a difficult area is self-evident as otherwise better models would already have been adopted in design and analysis.

For situations where the model uncertainty is explicitly recognised, a modified form of limit state equation may be used as follows:

$$M_i = g_i(\underline{X}, \underline{X}_m) \qquad (2)$$

where $\underline{X}_m$ represents one or more model uncertainties corresponding to, say, uncertainties in the distribution of forces throughout the structure (e.g. originating from construction sequence effects), and uncertainties in the computed structural capacity of a structural component (e.g. shear strength). However, in this case, the parameters $\underline{X}_m$ are being used to account for lack of knowledge of the true structural behaviour. These are unlikely to be random in nature and are likely to be made up of a number of systematic effects which vary in an unknown way over the design parameter space.

In some applications of engineering design and analysis, the issue of accounting for model uncertainty can be partly overcome simply by the use of more accurate modelling – e.g. detailed 3-d finite element analysis of large parts of the structure – but this is beneficial only when the structural behaviour is well understood and the relevant constitutive relationships can be assumed to be known. For structures in which the relevant material properties cannot be assumed homogeneous and isotropic, or where the properties can only be sensibly modelled by a random field, more detailed forms of conventional structural analysis cannot overcome the model uncertainty problem. Such problems exist with structures which are subject to failure by fatigue and fracture, where local brittle zones in welds or heat affected zones, and the presence of residual stresses, and stress concentrations due to local geometrical effects can have an important influence on the occurrence of failure. Similar situations arise where the structural behaviour is sensitive to local geometrical imperfections, as in the buckling of thin-walled structures such as box girders and stiffened cylinders; in the analysis of shear cracking in concrete and in the treatment of structures under the influence of corrosion. In these cases, model uncertainty is present because the normal structural analysis does not capture the complexity of the real behaviour.

The analyst is therefore faced with a dilemma. He/she has the choice of using a knowingly simplified model of the true structural behaviour in which the model uncertainties are likely to be large and difficult to quantify, or of using a more complex behaviour model with more input variables whose joint probability distributions will be difficult to define.

STRUCTURAL MONITORING AND RELIABILITY ANALYSIS

With the situation described above, it may be extremely difficult to guarantee high levels of structural reliability (small failure probabilities) when the model uncertainties are large. This will certainly be the case when there is no structure specific information available at the time the reliability analysis is undertaken. Continuous or intermittent structural monitoring, however, provides a means of obtaining more data about the structure and its behaviour under normal operating conditions; for example, the horizontal displacement of the deck of an offshore jacket structure, the extension of a fatigue crack, and the stresses in the flanges of a bridge deck.

In some situations, the response variables that could be measured during structural monitoring – for example: strains, displacements, vibration frequencies and amplitudes, crack widths and lengths, etc – are directly related to the quantity of interest. For example, the measurement of crack growth rates in a welded joint can be used in a direct way in the prediction of the joint's fatigue life. In other situations, however the quantities that can be measured are only indirectly related to the limit state of interest. For example, the mid-span deflection of a bridge deck under normal traffic loading, which can be measured relatively easily, provides only limited information about the magnitude of the traffic loading that would cause collapse in bending or shear.

The question arises therefore as to what benefit can be gained from structural monitoring in terms of reducing the uncertainty that a particular limit state will be exceeded during the remaining design life, or in any other specified period of time. In mathematical terms, the updated probability of failure, given that a particular structural response has been observed during a period of monitoring is:

$$P_{f\,|M_i \le 0} = P(M_i \le 0 \,|\, M_j \le 0) = \frac{P[(M_i \le 0) \cap (M_j \le 0)]}{P(M_j \le 0)} \qquad (3)$$

where $M_i \le 0$ is the principal failure event against which the structure is being assessed, and $M_j = 0$, or $M_j < 0$, is the event that that has been observed by monitoring the structure, and where:

$$M_i = g_i(\underline{X}, \underline{X}_{mi}) \qquad (4)$$

$$M_j = g_j(\underline{X}, \underline{X}_{mj}) \qquad (5)$$

with $\underline{X}$ being the complete vector of random variables with an effect on either M_i or M_j or both. Note that if a particular random variable has no influence on a particular limit state, it can still be retained in the limit state equation and associated with a zero coefficient. Finally, the variables $\underline{X}_{mi}$ and $\underline{X}_{mj}$ are the model uncertainties associated with the two safety margin equations $g_i(.)$ and $g_j(.)$ respectively.

The extent to which the data obtained from monitoring is effective in modifying $P(M_i \le 0)$ from its unconditional value is of major interest, and will depend on a number of factors: (a) the number of common variables shared by the two limit state equations, (b) the extent to which the variables associated with M_i are correlated with those influencing M_j and (c) the variances and covariances of the respective model uncertainties.

Equation (3) may, of course, be generalised to include the effect of incorporating structural monitoring data from a number of different sources and sensors, and information over a period of time. It should be stressed, however, that the use of monitoring data should have the effect of driving the value of the assessed probability of failure of a particular structure either towards zero or towards one, depending on whether the data indicate that the structure is behaving better or significantly worse than might be expected. If the data that is gathered during the monitoring of the structure has little bearing on the limit state of interest then the assessed failure probability will not change significantly.

In the following sections, these concepts have been applied to explore the effectiveness of structural monitoring in two different areas, but with data obtained from laboratory experiments rather than field measurements on full-scale structures.

APPLICATION TO ASSESSMENT OF FAILURE BY FATIGUE

As is well known, variations in loading on a structure with time may induce fatigue cracking that can lead to sudden failure. The consequences of this may be a relatively trivial matter if alternative load paths exist, although the subsequent failure of other structural members may eventually lead to complete failure with more severe consequences. However, the progressive failure of such systems will not be considered here.

Instead, we report on the some of the results from an experimental programme in which a total of 35 test pieces, 5 beams and 30 compact tension (CT) specimens, were tested under fatigue loading with the aim of examining the variability in crack growth rate under different conditions, with a view to the development of improved models for the reliability assessment of fatigue failure.

The test pieces were all cut from the same slab of 50mm thick Grade 50D steel which had been cut and then welded with a full-thickness butt weld on a double v-notch preparation. The objective was to explore the variations in fatigue crack growth rate and the influence of specimen thickness and orientation on both the mean growth rate and its variability. Some test pieces were prepared so that the fatigue cracks were allowed to grow either up through the thickness of the parent plate itself or parallel

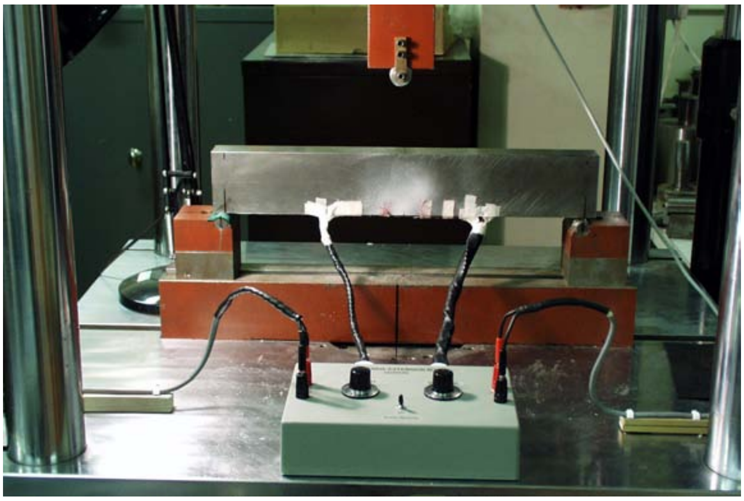

Figure 1: Simple test rig used prior to use of auto-scanning ACPD equipment

to one of its edges. Other test pieces were cut so that the fatigue crack grew either up through the weld or in a direction parallel to the weld runs. In all the tests, the growth of the crack was continuously monitored using ACPD equipment, using three channels wherever possible spread across the crack front. Full details of these tests are reported in [6],[7]. In the present paper, however, some of the test data have been analysed to determine the benefits that can be achieved by monitoring the crack growth and using this information to predict the time to fatigue failure. Some preliminary results from pilot tests were reported in [8].

The aim of the work was to replicate in the laboratory, Figure 1, the scenario in which the presence of a crack might be detected in a major structure, and that this information would then be used to update the probability of complete fatigue failure within a specified period of time. Whilst this idea appears relatively simple, and an early methodology was set out in [9], the situation has been found to be much more complex in practice, which probably explains why few case studies have been reported in the literature. Some of the issues are as follows:

- In order to carry a reliability-based fatigue calculation, and to update this prediction with crack growth or crack size data taken from the structure, it is necessary to have a suitable fatigue crack growth model. The Paris Law is often used for this purpose, but is not suitable in all situations, as discussed below.
- A decision must be made as to whether the structural monitoring process will yield instantaneous crack growth rates or simply approximate crack sizes. This will be influenced by the measurement uncertainties, which will have a disproportionately large effect when the crack growth rate is small.
- Even if the instantaneous crack growth rate can be determined, the crack growth parameters may be subject to spatial variability and therefore may not be valid over the length of the complete fatigue crack.

These factors and others must be taken into account in using crack monitoring data in updating reliability predictions for fatigue failure.

Some Findings from the Test Programme

The most obvious findings from the test programme were that large variations in crack growth rate occurred both within and between the different specimens. In Figures 2(a) and (b) the crack growth rate *da/dn* is plotted against the range of stress intensity factor ΔK for a single welded beam specimen, showing that there are large fluctuations in the 'instantaneous' crack growth rate. The specimens were tested under constant amplitude loading with the crack size measurements being taken at regular intervals of about 300 cycles. The raw data are shown in Figure 2(a). These have been re-plotted in Figure 2(b) to show the corresponding variations when the growth rate is averaged over successive increments of about 0.05 mm of growth. The observed variability about the mean trend is therefore clearly dependent on the averaging length selected.

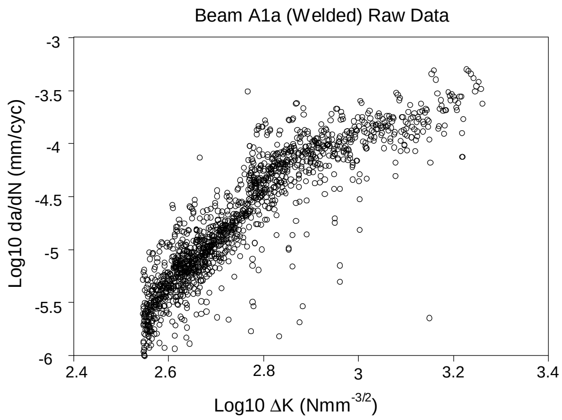

Figure 2(a): Raw fatigue crack growth data

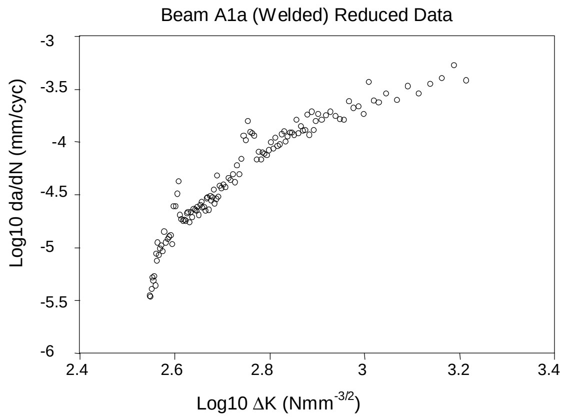

Figure 2(b): Reduced data based on 0.05 mm intervals of crack size

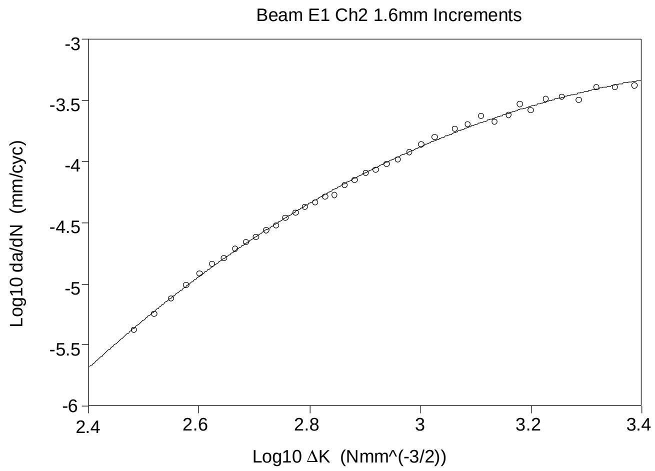

Figure 3: Fatigue crack growth data for a single non-welded beam

Similar data are shown in Figure 3 for a single non-welded beam based on crack growth increments of 1.6mm. In this case there is almost no variability about the mean crack growth curve. However, the latter differs significantly from a straight line, showing that the equivalent *m* in the Paris Law relationship changes as the crack grows. This has been further investigated by plotting the crack growth rate data against $\Delta J^{1/2}$ where *J* is the *J*-integral determined from a full non-linear analysis of the cracked specimen using the measured stress-strain curve for the steel. However, this relationship is even less linear than the curve in Figure 3.

Figure 4 shows some further data for a number of horizontal compact tension specimens in which the curves shown are the mean crack growth relationships. This illustrates that there are large differences between specimens, corresponding to an order or magnitude or more in the crack growth rates. Some of these differences appear to be attributable to the different stress ranges at which the tests were conducted, with those subjected to higher stress ranges cracking at a slower rate than those tested at lower stress ranges for the same value of the range of stress intensity ΔK. However, this effect is yet to be fully investigated.

Reliability Assessment and Use of Crack Monitoring Data

As previously mentioned, the aim of this work was to investigate how crack monitoring data could best be used to improve the assessed probability that a structure would survive under fatigue loading for specified periods of time.

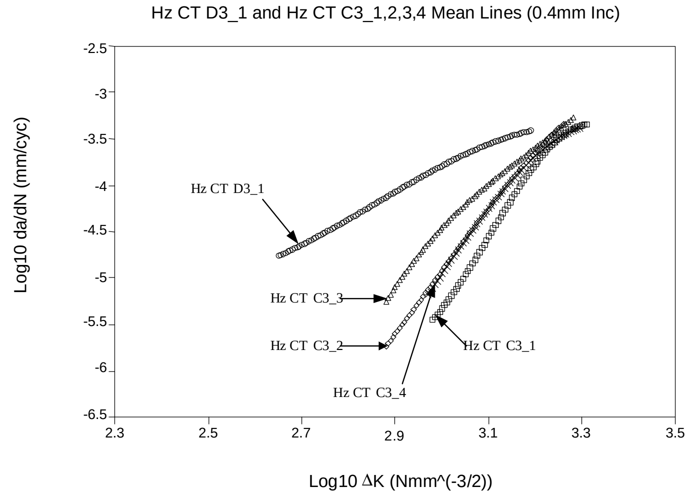

Figure 4: Mean fatigue crack growth curves for non-welded CT specimens

Consideration of the test data shows that the random variations in fatigue crack growth rate about the mean curve become small as the crack growth increments become larger, as might be expected if the variability is effectively white noise, as first suggested by Ditlevsen [10]. Furthermore, many of the crack growth curves are non-linear, showing that the normally assumed Paris law relationship

$$\log\left[\frac{da}{dn}\right] = \log C + m\log(\Delta K) \tag{6}$$

is not really valid over the most significant and practical range of stress intensity ΔK. In developing a model for fatigue crack growth one therefore has the choice of using an approximate linearised model, which as previously discussed will have a significant model uncertainty associated with it, or of using a more complex non-linear crack growth model for which more parameters need to be determined. In the work that has been undertaken, a number of different models have been developed.

Of major significance is the finding, first noted by Gurney [11], that fatigue crack growth curves when plotted on a log-log plot tend to pass through approximately the same point, which hereafter will be referred to as the Gurney point. This tendency is evident in Figure 4 for the five CT specimens shown. Taking all the data from the tests conducted and fitting a best straight line to each gives the results shown in Figure 5 in which there is shown to be a closely linear relationship between the Paris m and $\log_{10} C$. It can easily be shown that such a linear relationship between m and $\log_{10} C$ corresponds to series of straight lines on the fatigue crack growth plot as shown in Figure 6.

This relationship forms the basis for the first, *a priori*, model for the total number of cycles of fatigue loading N to cause n increments of crack growth of size da:

$$N = \sum_{i=1}^{n} dN_i = \sum_{i=1}^{n} \left(\frac{da}{C(\Delta K)^m} \right)_i \tag{7}$$

where for non-welded beams

$$C = \frac{1.56965 \times 10^{-4}}{(968.398)^m} \tag{8}$$

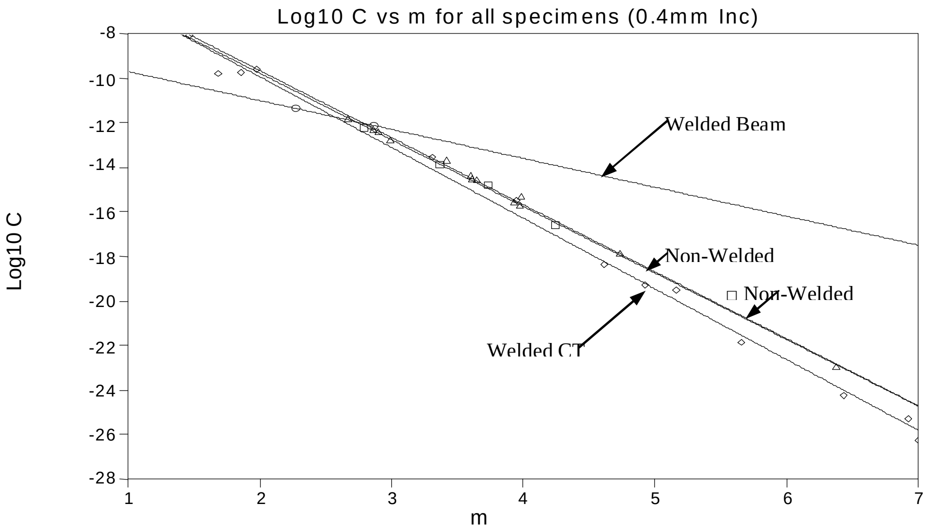

Figure 5: Plot of Paris Law parameters for experimental data

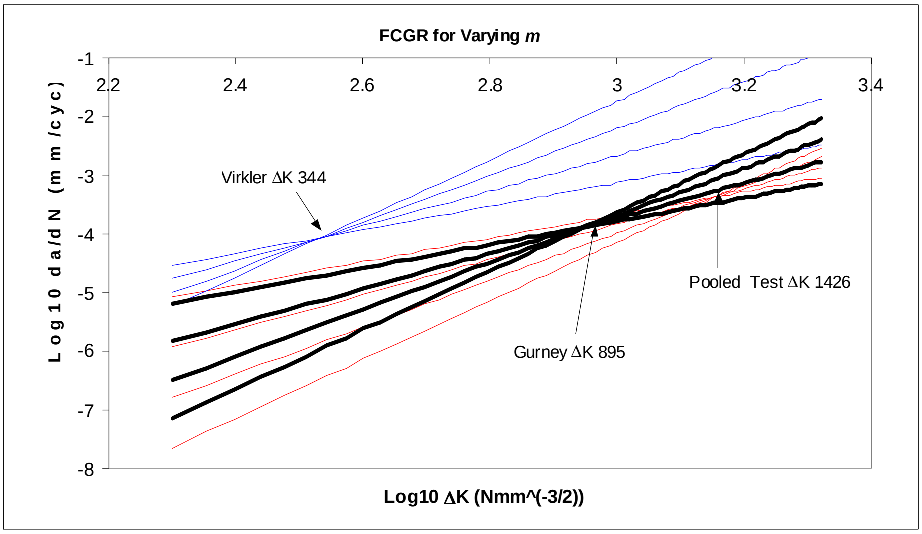

Figure 6: Linear fatigue crack growth relationships for different data sets

The safety margin corresponding to the *a priori* probability that a test specimen will fail before N_{spec} cycles of fatigue loading are applied is then given by:

$$M = \left(\sum_{i=1}^{30} \left(\frac{0.4}{10^{\left(-3.6829+(m(\log_{10}(\Delta K)))-3.0017+\varepsilon_1+\varepsilon_2\right)}} \right)_i \right) - N_{spec} \qquad (9)$$

where m and ε_1 and ε_2 are random variables with parameters derived from the general data. This is illustrated in Figure 7 for non-welded beam B1 which grew to a critical crack size in about 2.8×10^5 cycles of loading. This shows that without taking account of any monitoring data, the beam could reasonably be expected to fail at anywhere between about 1×10^5 and 7×10^5 cycles.

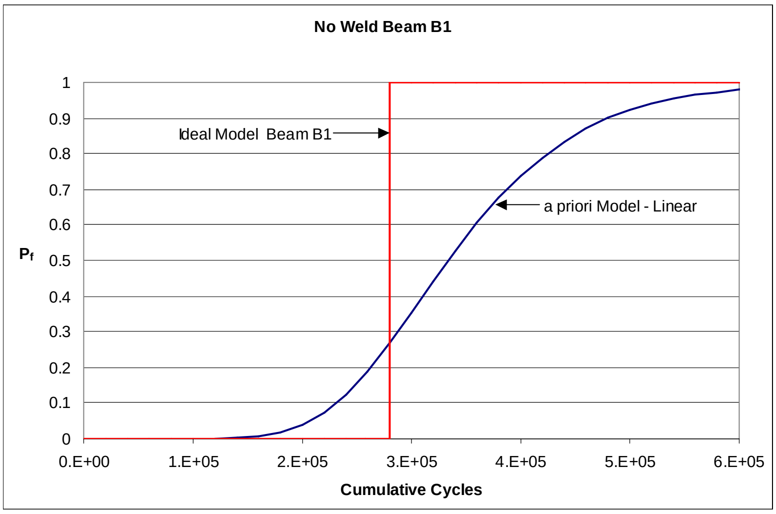

Figure 7: Assessed probability of failure versus number of fatigue cycles for Beam B1

It is now necessary to create a revised model that can incorporate knowledge gained from structural monitoring. As can be seen very easily from Figure 3, any attempt to determine the value of the Paris *m* parameter by monitoring the fatigue crack growth in the early stages of growth would lead to a serious overestimate of *m* and consequently a large underestimate in the number of cycles that would cause failure.

Recognising that all the beams have non-linear fatigue crack growth relationships, an improved model for crack growth would be of the form:

$$dN = \frac{0.4}{10^{(a(\log_{10}\Delta K)^3 + b(\log_{10}\Delta K)^2 + c(\log_{10}\Delta K) + d)}} \qquad (10)$$

where the coefficients *a, b, c,* and *d* need to be fitted. This leads to a corresponding safety margin equation.

The final problem remains of how to make use of monitoring data to improve the reliability predictions. Recognising that the fatigue crack growth curves all pass through or close to a single point as shown in Figures 4 and 6, it is proposed that knowledge of fatigue crack growth rates obtained from monitoring the structure at early stages in the life can then be used to fit equation (10) to this data. This will have the effect or rotating equation (10) about the Gurney point.

The result of applying this model to the failure of Beam B1 is shown in Figure 8. This illustrates the progressive improvements that can be achieved. Indeed, the assessed probability of failure of the beam in fatigue for specified numbers of cycles less that the number that actually caused failure is shown to decrease as the models are improved and monitoring data is obtained, and the probability of failure under specified numbers of cycles greater than the actual number become close to unity.

APPLICATION TO ASSESSMENT OF CONCRETE BRIDGE DECKS FAILING IN SHEAR

In addition to the work described above on the failure of steel components in shear, work has been progressing for a number of years on the use of monitoring techniques for reinforced concrete bridges having an unacceptably high probability of failure in shear – see Figure 9. However, unlike the monitoring of fatigue crack growth, the direct monitoring of shear failure is not possible until it is too late. However, a method was devised as illustrated in Figure 10 to monitor cracking in the shear zone as the total bridge load is increased from zero up to its failure load, and it has been possible to produce a model which relates the shear stiffness to parameters such as the ultimate shear capacity of the bridge

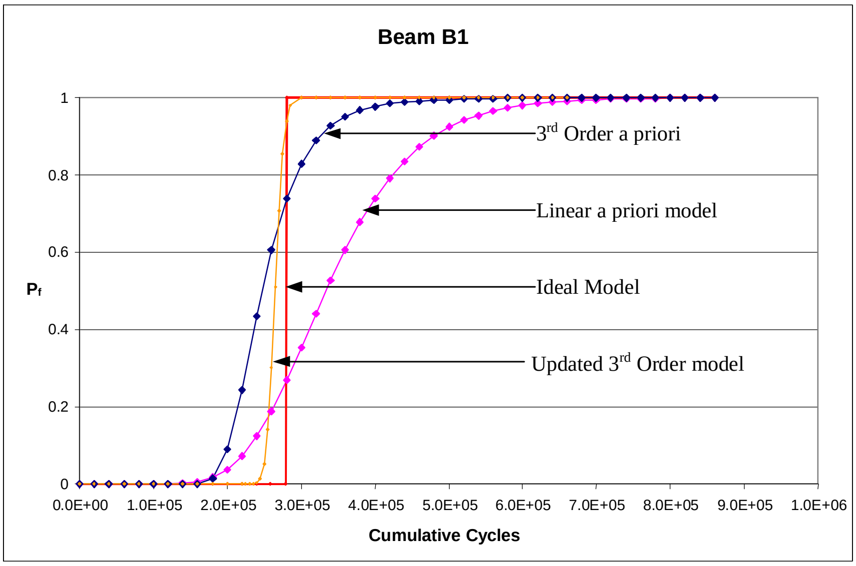

Figure 8: Improved models for reliability prediction using monitoring data

and the historical maximum load that the bridge has experienced. This model can then be used to incorporate knowledge gained from monitoring the bridge to update the probability of shear failure in a similar way to that described above for fatigue. Both the models for predicting shear failure and the new model linking the amount of cracking and the associated shear stiffness are associated with relatively large model uncertainties, but it is hoped to show that the combined use of the two models in a single calculation can reduce the uncertainty in the predicted failure load. However, this work is still under development.

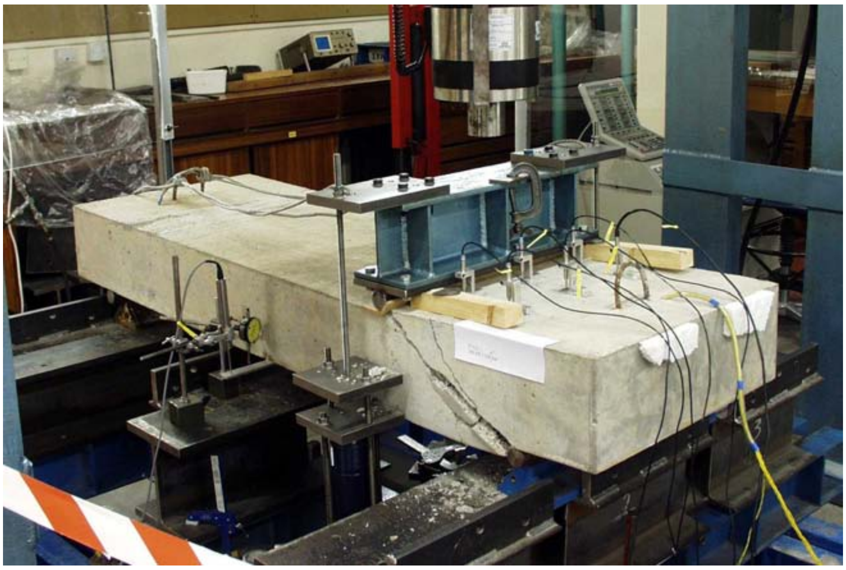

Figure 9: Bridge deck – shear test rig

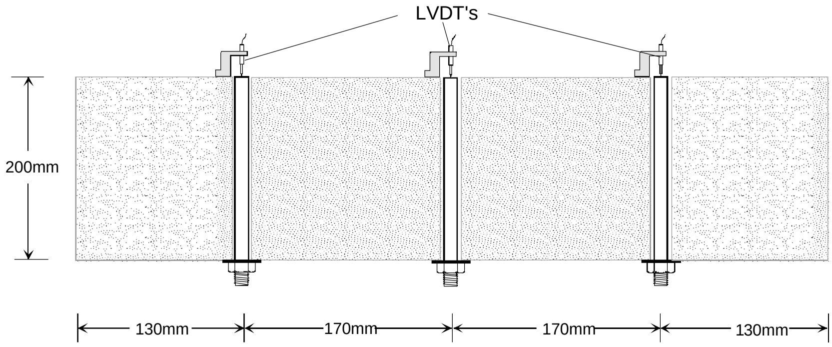

Figure 10: Monitoring of shear cracking

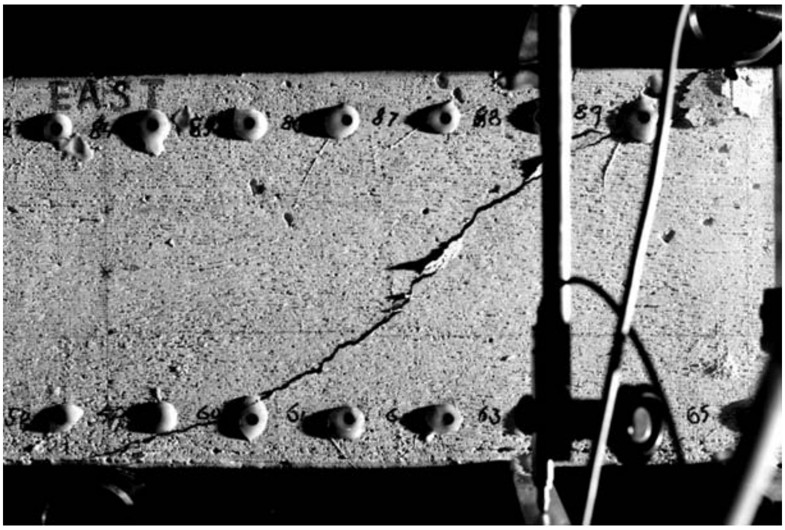

Figure 11: Fully-developed shear crack in model bridge deck

ACKNOWLEDGEMENTS

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APPENDIX 2

An Evaluation of Reliability Updating Techniques for Fatigue Crack Growth Using Experimental Data

ASRANet International Colloquium, Glasgow, July 2002

AN EVALUATION OF RELIABILITY UPDATING TECHNIQUES FOR FATIGUE CRACK GROWTH USING EXPERIMENTAL DATA

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Abstract

This paper reviews some of the fundamental differences between reliability-based design and the reliability-based assessment of existing structures. Consideration is given to the differences in the types of modelling assumptions that need to be made in the two cases. To illustrate these issues, the paper gives some preliminary results from experimental work in which fatigue crack growth measurements have been studied in the laboratory, and how this information along with other data has been used to investigate changes in reliability predictions with time throughout the service life. The paper also outlines a novel approach to performing reliability updating calculations that might prove to be beneficial in some circumstances. This work forms part of on-going research into improved methods for reliability-based structural integrity assessment.

Keywords: Structural Integrity, Failure Modes, Reliability Updating, Fatigue, Bayesian Networks.

INTRODUCTION

Background

Many decisions taken during the design, construction and maintenance of engineering structures are based on the requirements of national or international Codes of Practice and Standards. From a safety and reliability perspective, the main aims are to ensure, as far as is reasonably practicable, that structures will not fail at any time during their intended lifetime in a way that will endanger life, adversely affect the environment, or cause financial losses. The assessment of such risks is part of a decision-making process, parts of which are regulated by law in most countries – for example, those aspects of structural performance affecting human safety. In addition to satisfying any formal requirements, a good measure of professional experience and 'engineering judgement' is also required in such decision making.

Assume that, at the design stage for a given structure, agreement has already been reached on the basic structural form and the construction materials to be used. The types of decisions that are then needed are those that affect its long-run structural integrity. In basic terms, these decisions relate to the margins between system 'capacity' and system 'demand' in each of the various failure modes that can be envisaged for that structure, including serviceability failures, and will typically be highly structure specific. The localised failure of individual structural components in modes such as shear, buckling, fatigue, fracture, plastic bending, etc., or a combination of these, will influence the behaviour of the structure as a whole and will affect its ability to meet the specified performance requirements for the complete structural system throughout its lifetime.

The decisions that need to be taken at the design stage to reduce the risks associated with undesired forms of structural behaviour to a sufficiently low level are much more wide-ranging than just the choice of safety factors for each of the various failure modes. For example, the possibility exists: for improving (or possibly reducing) control over the quality of the basic construction materials; for improving (or reducing) control over fabrication processes (e.g. welding); for obtaining more (or less) post-fabrication data on the completed structure (e.g. by NDT techniques); for introducing some form of active or passive corrosion control and monitoring; for modifying the frequency with which the structure is inspected whilst in service, and hence changing the times at which it is possible to observe progressive deterioration (e.g. cracking or corrosion) and take remedial action; for load monitoring and possible load control (e.g. bridges); and finally the possibility of proof load or pressure testing prior to commissioning, and possibly again during the service life (e.g. in pipelines and pressure vessels). Some of these measures are currently employed for some structures.

In many situations, the standard approach is simply to follow previous so-called ‘normal best practice’ for the type of structure being designed, without explicitly quantifying the benefits of using any of the additional measures mentioned above. However, it is clear that some of these will have an important, possibly dominant, effect on structural integrity, especially where time-dependent deterioration effects are important (e.g. corrosion and fatigue). The practical question arises therefore as to the cost-effectiveness of imposing a more rigorous approach to the overall ‘design’ process, where the effects of some or all of the above measures are quantified. It is clear that any measures that have a significant effect on reducing the likelihood of unexpected structural failure are of benefit, especially when the consequences of failure are severe in human or economic terms (e.g. large bridges, aircraft structures, nuclear plant, etc.). However, the difficulties in quantifying these benefits in a rigorous way are non-trivial, especially when there are complex interactions between the measures (e.g. when the localised corrosion of a steel shell may influence its buckling behaviour).

The preceding paragraphs relate mainly to decisions that need to be taken at the design stage of a structure. However, there are also situations where it is necessary to verify the integrity of a specific structure during its service life – for example, when a structure has originally been under-designed for some reason, when the service loading has increased, or when a structure has been damaged. In these cases, the question arises as to whether there is a significant likelihood that the structure will fail before the end of the intended service life, either with or without planned intervention (e.g. strengthening or repair).

Although some codified procedures have been developed for the assessment of existing structures (e.g. bridges), calculations to predict remaining life are typically more difficult than the original design calculations, because structure specific information has to be obtained and included. Often the form of this data is incompatible with the original design assumptions and is therefore difficult to include. For example, if the strength of concrete in a particular bridge deck, as determined from tests on cores is found to vary significantly over the width of the deck, there is no easy way of including this information in a conventional structural analysis in which it is implicitly assumed that the concrete properties are uniform throughout the structure, or major parts of it. Hence, the rigorous assessment of existing structures is typically a much more complex process than the original structural design, in which a range of simplifying and generally conservative assumptions can be made.

Reliability-based Design and Assessment

A review of the methods for assessing the reliability of structures and structural systems would show that structural reliability theory has been used for two main categories of problem. The first of these is in aiding decisions at the design stage of a structure, and in particular in the evaluation of suitable safety factors for conventional design Codes. The second main area is in the assessment of particular structures, often with a view to justifying their continued operation, with bridges and offshore structures being common targets for study [1]. However, in many cases, the procedures used for the assessment of existing structures have differed very little from corresponding reliability-based design calculations. In such cases, this is often because very little additional data is available to the analyst, for example on material properties. Indeed, the transfer of ownership of many structures has resulted is a more or less complete loss of design and construction data (e.g. for some ships, bridges and offshore installations). Ironically, there may therefore be less information available to the reliability analyst of an existing structure than to the original designer, since the latter is in a good position to specify and then control material properties and standards of workmanship during construction. By contrast, the analyst of an existing structure may, in extreme cases, be in an initial position of knowing little more than that the structure exists and has not yet failed. This raises the question of determining what data and how much data should be obtained prior to undertaking the assessment, and how this information should be incorporated into the calculations. This applies to both deterministic and reliability-based calculations.

However, there are also more subtle problems. As discussed above, traditional design calculations frequently rely on a range of simplifying assumptions to make the calculations tractable. For example, the high residual stresses remaining in a structure after particular construction sequences are almost never taken into account in the design calculations, reliance being placed on the redistribution of stresses through plasticity, if yield stresses are locally exceeded.

Let us assume therefore that, in an attempt to gain further information about a structure for the purposes of reliability updating and integrity assessment, some strain measurements are taken in a number of locations, and that these are then compared with the theoretical strains based on normal design assumptions. If some of the measured strains are higher than those predicted, the conclusion might be reached that the structural component is in a more critical state than it is, and this might lead to inappropriate action (e.g. unnecessary strengthening). The converse may, of course, also occur. In other words, discrepancies between observed structural performance (e.g. strains, deflections, etc.) and corresponding theoretical predictions may arise more from the limitations of the conventional methods of structural analysis (i.e. high model uncertainties) than from other factors. It is therefore clear that model uncertainty, especially bias, arising from simplifications in structural calculations, has a complex influence on the reliability prediction of existing structures in which some of the information about the structure is gained from observations of real behaviour.

Objectives of this Study

In order to explore some of the issues mentioned above, data have been collected on the fatigue behaviour of a relatively simple steel beam tested in the laboratory, the aim being to use all information available to improve the predictions about when the beam will fail. Failure in this case, has been deemed to be the development of a fatigue crack of a specified size, and not complete loss of load-carrying capacity, thus using a very general definition of ‘failure’. Strictly, the objective of the research has been to study the reduction in the variance of the predicted number of cycles to cause failure, as additional information about the ‘structure’ is gained during the test.

To generalise, the relevance of this study is therefore in trying to determine how to gather data and information that will provide the greatest reduction in the uncertainty of reliability predictions for specific existing structures, at reasonable cost. The problem then is to identify, at a sufficiently early stage, those structures that will not meet performance requirements so as to avoid failure or minimise the failure consequences. These issues are discussed in more detail in the following.

RELIABILITY PREDICTION FOR EXISTING STRUCTURES

Methods for computing the reliability of single structural components and structural systems are now well defined, and in many situations the computation is relatively straightforward once the relevant safety margin equations, or limit state functions, have been established and the uncertain quantities (random variables or random fields) have been defined [e.g. 2,3,4]. First-order reliability methods (FORM), second-order reliability methods (SORM), and various Monte Carlo techniques combined with variance reduction strategies, either used separately or in combination, provide a powerful suite of computational tools.

The current challenges, however, lie with: the physical definition of the failure event and evaluation of the corresponding failure ‘consequences’ or losses; the definition of the limit state functions; the definition of the random variables or random fields representing the various uncertainties and their associated correlation structures; the problem of defining the relevant model uncertainties; and the eventual interpretation of the computed reliability measures.

Only this last point will be discussed here. As illustrated by Baker [5], distinction must be made between the probability of failure (or relative failure frequency) of effectively mass-produced structural or mechanical components manufactured and operated under precisely defined conditions, and the assessed reliability of particular (one-off), structures or components.

In the first situation, the computed reliability should be relatively close to the observed failure frequency, provided the limit state equations and the probabilistic models used in the reliability predictions are a sufficiently good representation of the physical circumstances. In practice, however, this may not be the case because there may be insufficient control over the 'operating environment' for the conditions to be deemed stationary. For example: material properties may be inhomogeneous as a result of poor quality control, or a decision by a manufacturer to supply 'above-specification' material; nominally similar components may be subjected to somewhat different loading regimes; and some failures will be influenced by the occurrence of human error or deliberate violations of Code Rules. Although there is no reason why all these influences should not be taken into account in the probabilistic modelling of a particular structural problem, this is rarely done for practical reasons. Calculated failure probabilities can therefore be expected to match observed failure frequencies only in very well-controlled circumstances.

By contrast, for the reliability assessment of existing (one-off) structures, one is dealing with an object that currently exists in physical form. In this situation, the reliability predictions must have a different interpretation. At the design stage of such a structure, the computed reliability has the same meaning as for mass produced structures or components because although only a single structure is to be constructed, one can envisage an ensemble of nominally similar structures subjected to the same nominal loading environment, a small proportion of which would fail during their design lives.

However, when built, the structure becomes unique, and, as further information is gained about it, the assessed reliability of the structure will change. Indeed, the assessed reliability will change throughout the design life and will eventually reach zero or one (corresponding to failure or non-failure). This logic applies if the structure (say an offshore installation) is initially designed for a fixed lifetime of say 40 years, and the assessed reliability relates to its probability of survival, with or without maintenance, for this period of time. The computed reliability is therefore as much a function of the analysts knowledge of the structure as it is a basic property of the structure itself. In general, the extent of the change in the computed reliability with increases in information will depend on the relative importance of the uncertainties in the time-invariant, but possibly largely unknown, material properties and in the future and hence random loading environment that the structure will experience.

In the following, attention is focussed on the special case of failure by fatigue and the situation where the future fatigue loading is assumed known, but where the material properties governing fatigue cracking are initially uncertain. This is illustrated in Figure 1 in which the computed probability that a fatigue crack will have reached a critical depth of 30 mm in a specimen of total depth of 75 mm after n cycles of fatigue loading is shown. The curve shown as the 'base case' has been computed using prior distributions for the initial crack size and fatigue crack growth parameters, as discussed later. This can be compared with the curve referred to on the diagram as 'perfect updating' in which it has been assumed that sufficient information has been gained about the crack growth mechanism to predict that the crack has a negligibly small probability of reaching the critical size in less than 9×10^6 cycles of loading, and yet has a very high probability of reaching the same critical size when the number of cycles exceeds 9×10^6 cycles by only a small amount. This ideal curve corresponds to perfect information about the crack growth mechanism and is of course impossible to achieve in practice. However, it provides a baseline by which to compare the effects of practical updating strategies.

EXPERIMENTAL INVESTIGATION OF FATIGUE CRACKING

The aim of this investigation was to obtain data gathered under carefully controlled experimental conditions to investigate the effectiveness of reliability updating techniques for fatigue cracking in relation to the reduction in variance of the estimated fatigue life, and hence determine the benefits that might be derived in practice for more realistic structures. This section of the paper describes the experimental procedures and the crack growth data obtained.

Test Specimen and Crack Growth Measurement

The test specimen selected for the experimental investigation was deliberately chosen to be as simple as possible, so that difficulties in calculating stress intensity factors would be minimal and the crack growth could be easily monitored. The specimen (Fig. 2) consisted of a 75mm deep by 38mm wide rectangular mild steel beam of uncertain pedigree which was loaded at mid-span in 3-point bending in blocks of constant amplitude loading. The rationale for choosing a specimen with initially unknown mechanical properties lay in the attempt to replicate the situation likely to be encountered in practice when unique structures are subjected to fatigue loading and cracks are found during an inspection.

With the aim of initiating crack growth without an excessive amount of fatigue loading, a 2mm deep V-shaped notch was cut in the bottom of the beam at mid-span. Crack development and subsequent growth was monitored by ACPD (Alternating Current Potential Drop) equipment [6,7], using three measurement channels equally spaced across the bottom face of the beam (Fig. 3), and this was carefully installed to minimise electronic noise during testing. Current focussing wires were used in order to obtain, as far as possible, highly localised readings of crack growth at the three positions, as growth was not expected to be precisely symmetric. In addition, three reference channels were used, each placed in line with one of the main measurement channels, so that crack growth could be determined by comparing the potential drop between the active channel and the reference channel, as this was considered to lead to the most accurate crack measurements. A number of other precautions were also taken to try to limit any drift and noise arising from thermal effects and extraneous inductive pickup.

Following the eventual failure of the specimen (Fig. 4), standard tensile tests, Charpy tests and residual stress measurements were carried out to determine the stress-strain characteristics of the material, the fracture properties and the distribution of residual stresses [8].

Fatigue Loading

The specimen was loaded in blocks of between 50,000 and 100,000 cycles of constant amplitude sinusoidal loading, at R values close to zero ($R = \sigma_{\min} / \sigma_{\max}$). Alternate loading blocks were given relatively high and relative low amplitudes, in an attempt to create distinct beachmarking of the specimen, for subsequent comparison with, and if necessary calibration of, the ACPD crack growth measurements. During the later stages of the test, as the crack size became large, the maximum loads were reduced somewhat to avoid excessively high stress intensities. The loading frequencies were kept well below 20 Hz in order to minimise dynamic effects and to prevent clipping of the load signal.

Between each block of loading, the test was stopped and crack growth measurements were taken. This continued until failure of the specimen by fracture after about 60mm of crack growth and 10^7 cycles of loading (Fig. 4).

Crack growth data

Since the aim of this experimental study was to use crack growth measurements obtained during the test, together with other data, to explore the reduction in the uncertainty of the number of stress cycles that would cause 'failure' of the specimen, considerable care was taken with all the measurements. From the ACPD readings there was some evidence of crack growth from a relatively early stage in the test (10^6 cycles), but this was not uniform across the specimen, as can be seen from the beachmarks in Figure 4. Although the test specimen was loaded symmetrically, the main fatigue crack grew from one end of the initial notch. This propagated reasonably steadily, but there is clear evidence from the failure surface that some very localised fracturing occurred when the crack front reached various positions. The largest of these can be clearly seen on the extreme left-hand side at the bottom of the test piece in Figure 4, after which the crack front became almost parallel with the bottom face of the beam. The main reason for the asymmetric cracking during the first part of the test was later found to be due to the presence of high values of residual stress in the beam (up to 120 N/mm^2), which would have influenced the maximum values of the stress intensity experienced (although not the range of stress intensity, ΔK). The material was also found to have reasonably low Charpy energy characteristics in the region of 10 Joules, thus being a relatively brittle material.

The change in crack size with the number of cycles of load is shown in Figure 5 for each of the three measurement channels and, even at this scale, crack growth in the early part of the test can be observed. However, detailed examination of the raw data shows that the ACPD signals were not totally immune from noise, which had the effect of introducing measurement error in the crack growth readings when taken over relatively small numbers of loading cycles. This measurement error can be important for very small crack sizes and of course should be considered as a further source of uncertainty in the analysis. When the crack reached a size of about 15mm, further anomalies were also encountered, with an apparent reduction in crack size after a number of the loading stages. This was attributed to partial crack closure under the influence of residual stresses (see Fig. 5). However, at all stages of crack growth, the averaging of the measurements across all three measurement channels has the beneficial effect of increasing the signal to noise ratio, and most of the subsequent analyses were carried out with this pooled estimate of crack size at each stage of the fatigue loading. The beachmarkings on the failure surface were also used to validate the ACPD readings.

DETERMINISTIC FATIGUE CRACK GROWTH MODEL

The prediction of fatigue crack growth for the specimen under test was based on the basic Paris Erdogan equation

$$\frac{da}{dn} = C(\Delta K)^m \tag{1}$$

with

$$\Delta K = Y \Delta\sigma \sqrt{\pi a(n)} \tag{2}$$

where C and m are the crack growth parameters, ΔK is range of stress intensity, $\Delta\sigma$ is the stress range given by $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$, $a(n)$ is the current crack depth after n cycles of loading, and Y is the shape factor given by

$$Y = (1.12 - (1.39\frac{a(n)}{W}) + (7.3\frac{a(n)^2}{W^2}) - (13\frac{a(n)^3}{W^3}) + (14\frac{a(n)^4}{W^4})) \tag{3}$$

for the crack geometry concerned, where W is the beam depth.

COMPUTATION OF RELIABILITY

The 'failure event' that was the subject of investigation was taken to be the growth of the fatigue crack to a value greater than a specified critical size after n cycles of loading. In the results reported here, this critical size was taken to be 30mm – an arbitrary, but relatively large, value. The safety margin for this failure event is therefore simply

$$M = 30 - a(n) \tag{4}$$

where $a(n)$ is the predicted crack size after n cycles of loading. Hence, the unconditional probability of 'failure' is $P(M \le 0)$. However, because the loading that was applied to the beam consisted of various blocks of constant amplitude loading, but with each block having a somewhat different number of cycles n_i and different stress range $\Delta\sigma_i$, the calculation of the probability distribution function for $a(n)$, using equations (1-3) is not completely straightforward. However, it may be evaluated numerically by the use of any FORM/SORM package.

Reliability Updating

For the purposes of updating the reliability predictions by making use of crack growth and crack size information from the early stages of the test, a minor complication arises. As the crack front was not completely straight, and indeed far from straight in the early stages of crack growth, the crack size used in the reliability updating was taken to be the average value, $\bar{a}$, of the crack depth at the three ACPD measurement positions.

When such information has been gained by observing the crack growth over the first block of loading, the conditional probability P_f that the crack will reach the 'critical size' during subsequent loading cycles is then given by

$$P_f = P\big[(M \le 0) \mid a(n_1) = \bar{a}_1\big] \tag{5}$$

where $\bar{a}_1$ is the average value of the crack depth observed after n_1 cycles of stress range of magnitude $\Delta\sigma_1$, and $a(n_1)$ is the predicted crack depth obtained by the application of equations (1-3) for the same number of cycles of loading.

This approach can be easily generalised to include the effects of measuring the crack size on a number of occasions at the end of each block of loading, and then using this additional information to revise the prediction that 'failure' will occur during subsequent loading cycles, namely

$$P_f = P\big[(M \le 0) \mid a(n_1) = \bar{a}_1,\ a(n_2) = \bar{a}_2, \ldots\ldots, a(n_m) = \bar{a}_m\big] \tag{6}$$

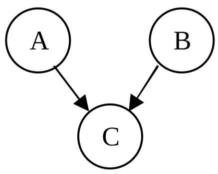
Equation (6) is then determined from

$$P_f = \frac{P\big[(M \le 0) \cap (a(n_1) = \bar{a}_1) \cap\ (a(n_2) = \bar{a}_2) \cap \ldots\ldots \cap (a(n_m) = \bar{a}_m)\big]}{P\big[(a(n_1) = \bar{a}_1) \cap\ (a(n_2) = \bar{a}_2) \cap \ldots\ldots \cap (a(n_m) = \bar{a}_m)\big]} \tag{7}$$

The normal approach to evaluating Eq (7) would be to use a package such as PROBAN [9], but in the present study a Bayesian Belief Network approach has been used to compute the fatigue crack growth and the updated reliabilities. A direct numerical comparison between the two approaches has not yet been completed.

Bayesian Belief Networks

In a departure from the traditional FORM/SORM approach to performing reliability calculations and subsequent reliability updating, a Bayesian Belief Network (BBN) was constructed for determining the 'failure' probabilities. As discussed by Pearl [10], BBNs are directed acyclic graphs modelling probabilistic dependencies among variables. The graphical part of the Bayesian network reflects the structure of a problem, while conditional probability tables quantify local interactions among neighbouring variables. Nodes represent variables and arcs (arrows) represent causality or functional relationships between 'parent' and 'child' nodes. For example, the following network is used to model and compute $P(C \mid A, B)$



In a situation where variables A, B and C are discrete and binary, the computation of $P(C \mid A, B)$ using Bayes' Rule can be readily performed manually. However, as the number of variables increases, and the number of states for each variable also increases, the computation of conditional probabilities becomes increasingly difficult and onerous. BBNs, through the use of conditional probability tables (CPTs) offer a convenient and pragmatic method of performing multi-variable multi-state Bayesian computations.

When CPTs are used to quantify the variables, a discretisation process is utilised. Where a variable is discrete in nature, each discretised 'bin' for that variable represents a possible realisation state. In the situation where a variable is continuous in nature, each discretised bin represents all possible values for the variable within the upper and lower bounds of the bin, and hence the variable is modelled as a series of discrete bins that are continuous over the defined hypothesis space.

In the situation where random variables are either probabilistically or functionally related, the computation of joint probabilities is performed utilising a multi-dimensional integration and interpolation process. This process involves random sampling from within each discretised bin.

Prior distributions are encoded into CPTs either manually for the discrete case or from the use of distribution functions for the continuous case. Probabilistic inference (and belief propagation) over a compiled network is performed using the Probability Propagation in Tree Clusters (PPTC) algorithm, as discussed by Lauritzen and Spiegelhalter [11] and Cowell *et al.*[12].

When performing Bayesian learning or the updating of discretised prior distributions when data becomes available, Dirichlet distributions are used to represent the probability of occurrence within each of r bins. The conjugate prior used for multinomial sampling is

$$P(\theta) = Dir(\theta \mid \alpha_1, \ldots, \alpha_r) \equiv \frac{\Gamma(\alpha)}{\prod_{k=1}^{r}\Gamma(\alpha_k)} \prod_{k=1}^{r} \theta_k^{\alpha_{k-1}} \tag{8}$$

where $\alpha = \sum_{k=1}^{r} \alpha_k$ and $\alpha_k > 0,\ k = 1,\ldots.,r$.

The posterior distribution is given by

$$P(\theta \mid D) = Dir(\theta \mid \alpha_1 + N_1, \ldots, \alpha_r + N_r) \tag{9}$$

Given this conjugate prior and data set D, the probability distribution for the next observation is given by

$$P(X_{N+1} = x^k \mid D) = \int \theta_k Dir(\theta \mid \alpha_1 + N_1, \ldots, \alpha_r + N_r) d\theta = \frac{\alpha_k + N_k}{\alpha + N} \tag{10}$$

The updating of random variables (which is observation and/or frequency based) in this manner is a true Bayesian approach. However it differs from the traditional approaches as currently espoused, for example in [1].

Application to Fatigue Crack Growth

The Bayesian Belief Network constructed for the reliability analysis was developed using Netica [13]. A graphical representation of the base network is shown in Figure 6. As a discretisation process is used, it is necessary to make a prior declaration of the hypothesis space over which all random variable are applicable. This model treats the crack growth rate as a random variable dependent upon the uncertainties in the Paris law variables C, m and the initial defect size a_0. The increment of computed crack growth G for each loading block is then given by

$$G = \frac{da}{dN} \times n \tag{11}$$

where n is the number of cycles in the block, and crack length becomes

$$a(n_1) = a_0 + G \tag{12}$$

For pragmatic reasons, the distribution for the C parameter in this model has been based on Gurney's relationship [14]

$$C = \frac{1.315e^{-4}}{(895.4^m)} \tag{13}$$

Therefore, only the distribution for the m parameter is required in Eq.(1). The prior distribution used for m in this model was a normal distribution with parameters $\mu_m = 3.0, \sigma_m = 0.25$.

Reference was made to BS7910 [15] for the modelling of fatigue crack growth thresholds. For surface breaking defects in air, the standard recommends that the crack growth rate becomes zero when

$$\Delta K \leq 170 - (214R)\ \ \text{N/mm}^{3/2} \quad \text{for } 0 \leq R < 0.5 \tag{14}$$

where R is the stress ratio ($R = \sigma_{\min} / \sigma_{\max}$), or

$$\Delta K \leq 63 \quad \text{for } a \leq 1\text{mm}. \tag{15}$$

These threshold values could be modelled probabilistically, but have been treated here as being deterministic. The reliability at the end of each loading block is then computed using Eq.(4).

As illustrated in Figure 6, the BBN model developed for this analysis recognises that a loading regime has been established prior to the commencement of fatigue loading. This information is entered as *DLoad* (to compute $\Delta\sigma$), *Stress Ratio* (to compute R) and N. The initial crack size *AN* can be assigned either a deterministic or probabilistic value. The prior for *AN* in this model was taken as a fixed value of 0.2mm, based upon on initial ACPD readings. The subsequent loading block information is utilised to forward predict the reliability.

The ACPD measurements of the crack growth were subsequently used as ‘evidence’ to perform the reliability updating calculations. This evidence is used to convert the distributed values of crack size (*AN*) to a fixed value with magnitude equal to the observed value.

ANALYSIS OF RESULTS

As previously discussed, the experimental work produced large quantities of data for the single test specimen, namely the crack size measured in three positions across the lower face of the beam after every block of constant amplitude loading. This information is plotted in Figure 5 against the cumulative number of stress cycles, but it should be noted that, because different blocks of loading used different stress ranges, the crack growth should not be expected to form a smooth curve against the number of cycles, even if random effects were to be removed.

Independently of the physical testing, reliability analyses were carried out to examine the probability that the fatigue crack would exceed 30mm in depth after n cycles of the applied loading, where n corresponds to the cumulative number of cycles of load used in the physical test after each loading block. The calculations were carried out using the actual stress ranges used in the experiment, and the results of these calculations are plotted in Figures 7(a) and 7(b) (with the caption ‘No updates’). The curve is of a typical sigmoidal shape, but again it should be noted that this is not a smooth curve, for the same reasons as mentioned above.

Following these preliminary calculations, the observed crack size after each block of loading was then used to update the reliability predictions for the further crack growth beyond that loading stage. Thus, although the actual crack growth characteristics for the complete test were already known, information beyond the loading stage used for each updating calculation was effectively ignored. Figures 7(a) and 7(b) show the results of this process, although data are plotted only for every tenth updating calculation, except close to failure.

The results of these updating calculations can be compared with the idealised curves in Figure 1. Under the assumption of ‘almost perfect’ information about the structure’s crack growth characteristics, the lower probability distribution curve (in Fig. 1) rises sharply in the vicinity of the number of cycles of loading that will cause failure (i.e. there is almost no uncertainty in the random time to ‘failure’). However, for the experimental results shown in Figure 7, this effect can be seen only when the specimen has a large crack present and is relatively close to failure. The effect of the updating procedure has therefore been mainly to shift the distribution curves to the right, without significant changes in shape. In other words, the bias in the prediction is slowly reduced with successive updating calculations, but the variance in the estimate remains roughly constant. The immediate conclusion is that the reliability updating process is not very useful in improving the prediction about the number of cycles that will cause ‘failure’.

In order to investigate the reasons for the behaviour seen above, a more careful analysis of the crack data was undertaken. It was decided to study the sample statistics of the effective number of cycles to cause differing amounts of crack growth. The term effective is used here, because the physical testing was not carried out under constant ΔK conditions, and so it was necessary to adjust the actual number of cycles used to allow for this. Some of the findings (based on a fixed ΔK of 200 N/mm$^{3/2}$) are shown graphically in Figures 8 and 9 and demonstrate that after the crack initiation and coalescence phase (taken to correspond to the first 1mm of recorded growth), the cracking occurs reasonably uniformly with increasing numbers of cycles at fixed ΔK, but with an some degree of scatter. This scatter has been analysed to determine the effect of spatial averaging of the numbers of cycles over different increments of growth from 0.1mm to 2.0mm, as shown in Table 1.

This analysis confirms that there is considerable spatial variability in the crack growth rates, which is reduced by spatial averaging. The low values of the sample autocorrelation, even between growth increments as small as 0.1mm confirm that crack growth should ideally be modelled as a random process. The random nature of the crack growth in this single test piece has been investigated further by plotting the crack growth rate da/dn from different stages of the test against the corresponding stress intensity ΔK, as shown in Figure 10. The large scatter in the data points, especially early in the test, may be partly due to measurement noise from the ACPD equipment, but largely confirms the irregularity in the crack growth throughout the test.

Growth Increments (mm)	Sample C.O.V.	Sample Autocorrelation
0.1	0.692	-0.285
0.2	0.572	0.197
0.5	0.487	0.146
1.0	0.461	-0.585
2.0	0.440	0.079

Table 1: Sample statistics of number of cycles to cause crack growth increments of different sizes

Of greater importance, however, is the implication for reliability computations and reliability updating. It is clear that the probability distributions that are used for modelling the Paris law parameters *C* and *m* cannot be sensibly defined without consideration of spatial averaging.

Further work is continuing on the issues raised above.

ACKNOWLEDGEMENTS

This paper forms part of the ongoing research being undertaken at the University of Aberdeen under the EPSRC's Structural Integrity Managed Programme. The research is co-funded by the EPSRC and the UK Health and Safety Executive. PROBAN has been used to perform some of the reliability calculations.

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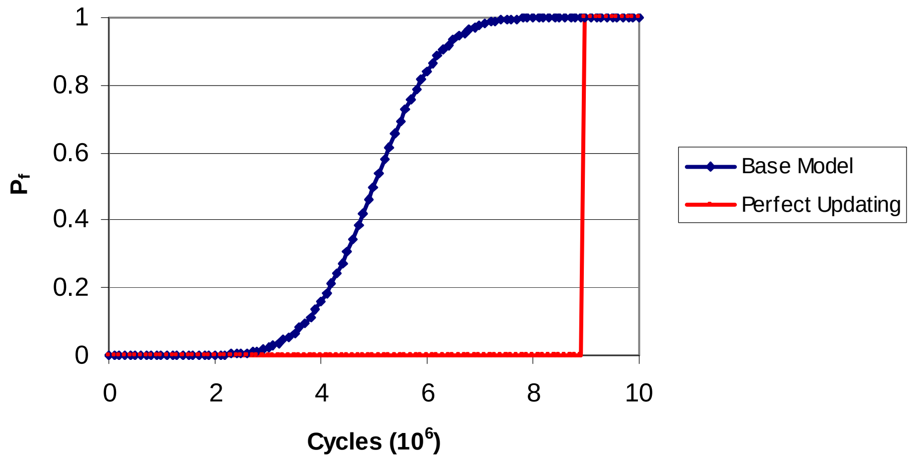

Figure 1: Reliability predictions using (a) base model and (b) with perfect updating

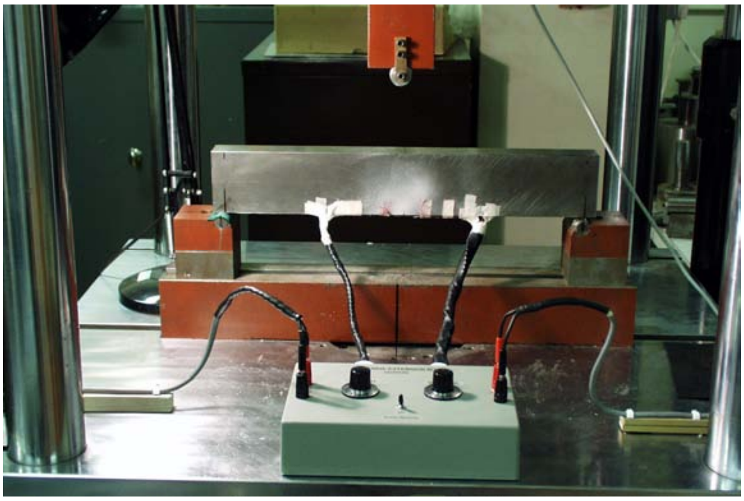

Figure 2: Fatigue specimen under test

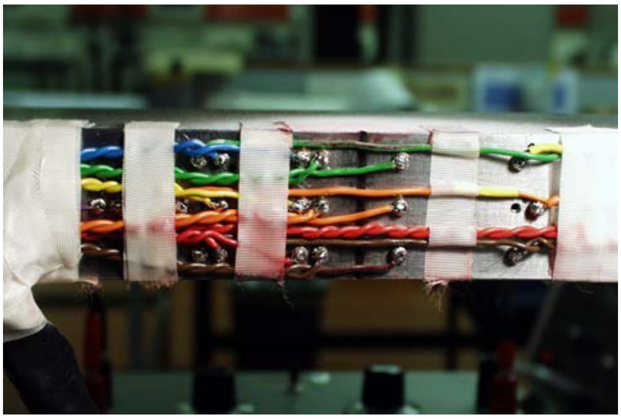

Figure 3: ACPD wiring arrangement across machined notch

Ch3 Ch2 Ch1 Ch1 Ch2 Ch3

Figure 4: Opposite faces of fatigue crack plane

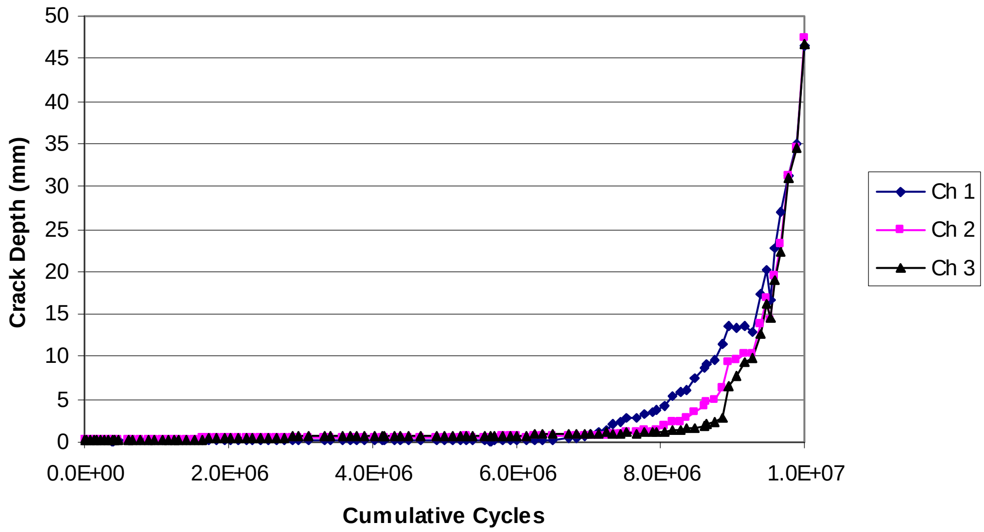

Figure 5: Crack growth curves

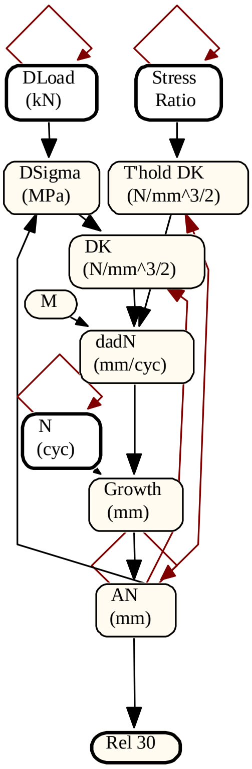

Figure 6: Simplified Bayesian Network

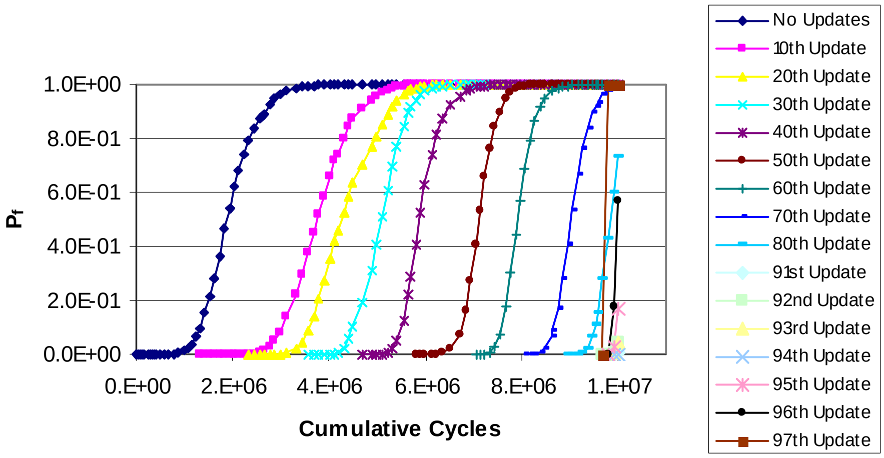

Figure 7(a): Reliability predictions after various stages of cracking (natural scale)

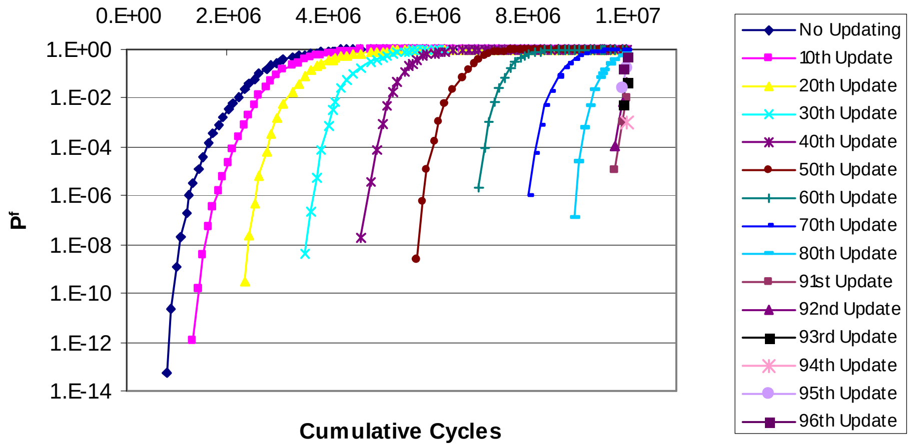

Figure 7(b): Reliability predictions after various stages of cracking (logarithmic scale)

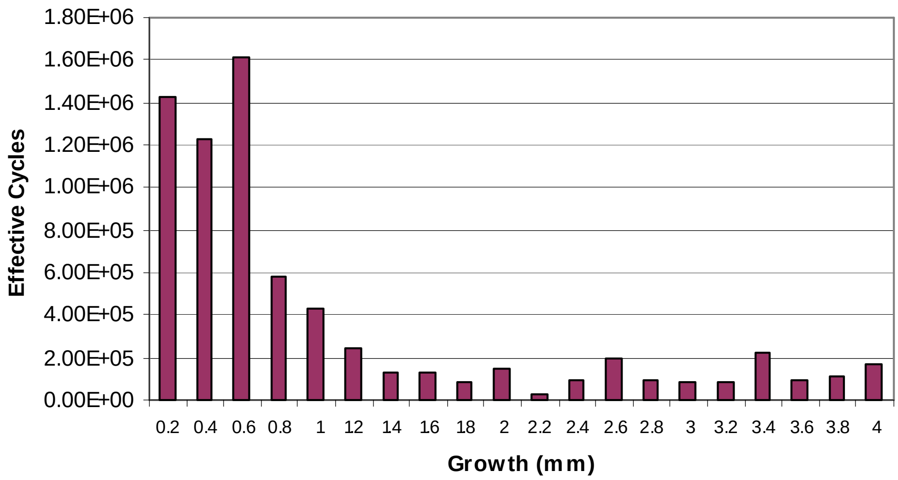

Figure 8: Effective number of cycles for 0.2mm of growth (averaged over all channels)

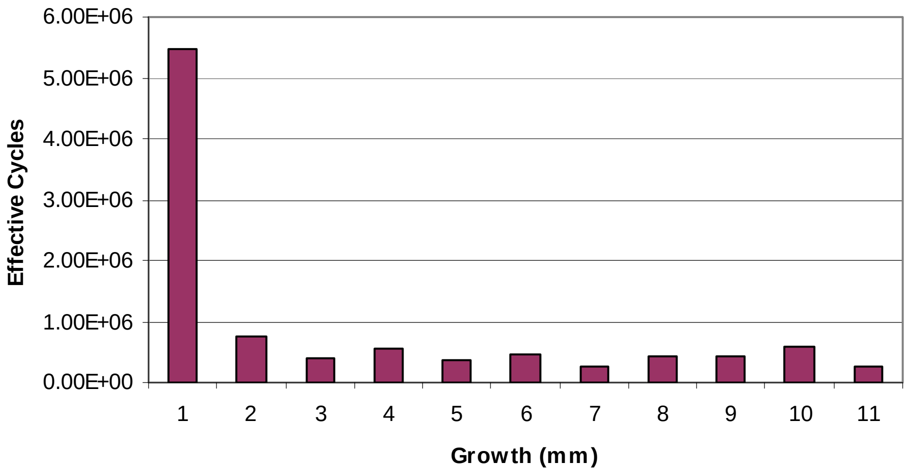

Figure 9: Effective number of cycles for 1.0mm of growth (averaged over all channels)

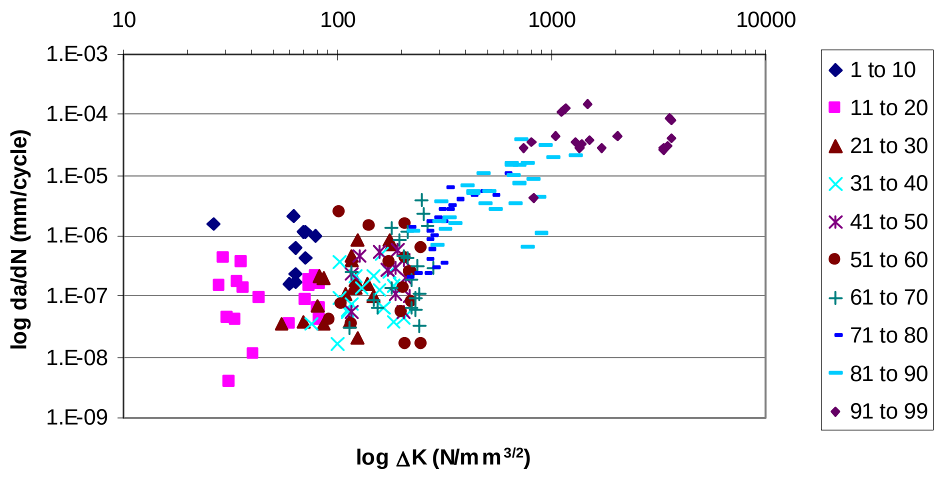

Figure 10: Experimentally derived log da/dN versus logΔK relationships across channels 1, 2 and 3 for different loading blocks

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Improved generic strategies and methods for reliability-based structural integrity assessment

Summary report

This report is a summary of the research undertaken during the EPSRC/HSE sponsored project entitled ‘Improved generic strategies and methods for reliability-based structural integrity assessment’. Detailed findings are documented in a range of other publications, as listed in the references.

The research covers a wide range of topics including: the development of improved methods of reliability analysis which can be easily linked with standard methods of advanced structural analysis; a detailed study of the variability of fatigue crack growth in structural steels and the implications for fatigue reliability analysis; developments in the use of reliability-updating techniques in relation to the prediction of fatigue failure; applications of structural system reliability analysis to the behaviour of a North Sea jacket structure; and the development of a methodology for the reliability-based fracture assessment of pipelines containing cracks.

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