

Investigation of Parameter Estimation of a Car-Trailer System Using Condition Numbers

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Abstract

In this research we investigate condition numbers obtained from least squares estimation for a car-trailer system to characterize estimation performance. In this case, we can select better parameter estimation methods or well-posed measured data sets for the car-trailer system using condition numbers. We calculate condition numbers from several different linear model-based least squares methods which use four linear regression models and three least squares methods to estimate trailer parameters. We also consider three different observed data sets in ideal and non-ideal sensor scenarios for simulation tests.

Keywords: Car-trailer, condition number, least squares, parameter estimation

1. Introduction

In this research we investigate condition numbers obtained from least squares estimation for a car-trailer system to characterize estimation performance. In this case, we can select better parameter estimation methods or well-posed measured data sets for the car-trailer system using condition numbers. Kinematic model parameters should be identified accurately since they are typically applied to derive robot control and planning commands. In particular, trailer backing critically depends on model parameters. However, model parameters change from their nominal values for many reasons such as manufacturing and assembly errors, clearance, backlash, and wear. Sensor noises may also provide significant estimation errors. Thus, estimation or calibration of model parameters is an important procedure for automatic controls of robot systems.

Least squares techniques have been widely used to identify or calibrate kinematic and dynamic parameters of robots and sensing systems by using a set of observed parameters [2-5]. Ordinary least squares are traditionally used for linear regression models. More recently, total least squares [6-8] were introduced for better accuracy. Further, condition numbers [9-11] or observability indexes [12-14] are applied to select best pose or configuration sets for robot calibration. Note the condition number indicates parameter sensitivity to disturbances such as sensor noises. In robotics research, least squares techniques were frequently applied to calibration of manipulators and parallel robots. However, less attention was paid to calibration of mobile applications. To the best of author's knowledge, least squares techniques were not used to estimate/calibrate trailer parameters. As a result, perfect model parameters were simply assumed in many control algorithms for the car-trailer system.

In this research we characterize parameter estimation methods in several different scenarios for the car-trailer system using condition numbers. As a result, we can use condition numbers to select better parameter estimation methods or better observed data for parameter estimation, which is our main contribution. We also contribute to

investigation of condition numbers when sensor noises are present. We estimate model parameters (*i.e.*, hitch and trailer lengths) for the car-trailer system using linear regression models and least squares techniques, which may easily be implemented with inexpensive sensors such as an odometry and a potentiometer. Toward this goal, we first derive closed form linear models considering forward steering kinematics of the car-trailer system. Several linear model-based least squares techniques [1] are then presented to estimate trailer parameters using linear model and least squares. Several observed data are also applied to parameter estimation of car-trailer systems. In our evaluation, we then compare condition numbers obtained from several different estimation methods and scenarios to investigate how observed data should be selected to provide better estimation results. We use the car-trailer model assuming no sensor noises to simulate ideal sensor measurements. We further use Gaussian noises to produce non-ideal sensor measurements similar to actual hardware implementation.

This paper is organized as follows; In Section 2, we present linear least squares methods. In Section 3, we provide closed form linear regression models for parameter estimation considering steering kinematics of a car-trailer system. Using linear models and least squares, we present linear model-based least squares estimation methods in Section 4. We then evaluate linear model-based estimation methods for several different curvature commands applying respective ideal and non-ideal sensor measurements in Section 5. Conclusions are finally provided in Section 6.

2. Least Squares Techniques

In this section we briefly describe ordinary least squares and total least squares methods for linear regression models, which will be used to estimate trailer parameters. We also present a condition number as an observability index.

2.1. Least Squares Techniques: OLS1, OLS2, and TLS

In general, a linear regression model can be written by,

$$y_i = f(\boldsymbol{\eta}_i, \boldsymbol{\beta}) = \sum_{j=1}^n \eta_{i,j} \beta_j ; \quad i=1, \dots, p \quad , \quad (1)$$

which is a linear combination of n input variables, $\boldsymbol{\eta}_i \in \mathfrak{R}^{n \times 1}$, and n unknown parameters, $\boldsymbol{\beta} \in \mathfrak{R}^{n \times 1}$. Note that y_i is the output variable, p is the number of input/output measurements. Alternatively, using matrix-vector form, this model can be expressed by,

$$\mathbf{y} = \mathbf{A}\boldsymbol{\beta} \quad , \quad (2)$$

where \mathbf{A} is the matrix of input variables. We can then estimate $\boldsymbol{\beta}$ easily using linear least squares methods for given input and output measurements.

In particular, considering a straight line in a plane, the linear model (1) can be simplified by,

$$y_i = ax_i + b ; \quad i=1, \dots, p \quad , \quad (3)$$

where x_i is the input variable and $\boldsymbol{\beta} = [a \ b]^T$. Given the output measurements, $\mathbf{Y} = [Y_1 \ \dots \ Y_p]^T$ and the input measurements, $\mathbf{X} = [X_1 \ \dots \ X_p]^T$, we can then estimate the unknown parameters, $\boldsymbol{\beta}$, applying least squares methods. In this research

we distinguish least squares methods according to applied fitting errors as follows; 1) Ordinary Least Squares based on only output errors, Δy_i , (OLS1), 2) Ordinary Least Squares based on only input errors, Δx_i , (OLS2), and 3) Total Least Squares based on both input and output errors (TLS). As illustrated in Fig. 1, we define three different fitting errors, $\mathbf{e} = [e_1 \dots e_p]^T$,

$$e_i = \begin{cases} \Delta y_i & \text{for OLS1} \\ \Delta x_i & \text{for OLS2,} \\ \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} & \text{for TLS} \end{cases} \quad (4)$$

where input and output errors are,

$$\Delta x_i = X_i - x_i, \quad \Delta y_i = Y_i - y_i. \quad (5)$$

2.1.1. Ordinary Least Squares based on only output errors (OLS1)

OLS1 refers to traditional ordinary least squares, which minimizes only output errors, Δy_i , between measurements and a model assuming $\Delta x_i=0$ (i.e., $x_i=X_i$), Figure 1 (a). Using the model (2) and the error definition (4)-(5), the fitting error is then,

$$\mathbf{e} = [\Delta y_1 \dots \Delta y_p]^T = \mathbf{Y} - \mathbf{y} = \mathbf{Y} - \mathbf{A}\boldsymbol{\beta}, \quad (6)$$

Now we can find the parameters, $\boldsymbol{\beta}$, by minimizing this fitting error,

$$\min_{\boldsymbol{\beta}} S = \min_{\boldsymbol{\beta}} \mathbf{e}^T \mathbf{e} = \min_{\boldsymbol{\beta}} (\mathbf{Y} - \mathbf{A}\boldsymbol{\beta})^T (\mathbf{Y} - \mathbf{A}\boldsymbol{\beta}). \quad (7)$$

Differentiating (7) with respect to $\boldsymbol{\beta}$ and solving for $\boldsymbol{\beta}$, the ordinary least squares estimate of $\boldsymbol{\beta}$ in OLS1 becomes,

$$\hat{\boldsymbol{\beta}}_{OLS1} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{Y}. \quad (8)$$

Note that we use $\hat{}$ to distinguish the estimate from true values.

Further, we consider the simple line model (3). Using $x_i=X_i$, the model and the fitting error become,

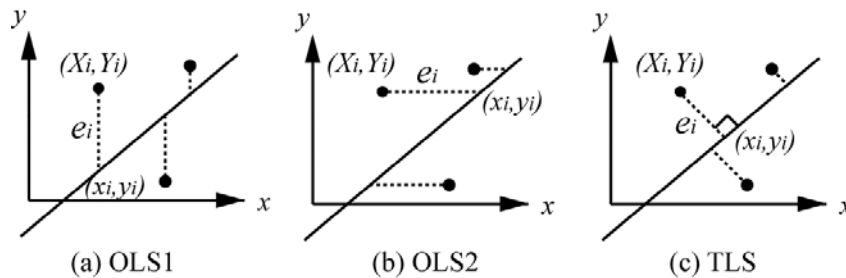


Figure 1. Fitting Errors for (a) Ordinary Least Squares based on only Output Errors (OLS1), (b) Ordinary Least Squares based on only Input Errors (OLS2), and (c) Total Least Squares based on both input and Output Errors (TLS)

$$\begin{aligned} y_i &= aX_i + b \\ e_i = \Delta y_i &= Y_i - y_i = Y_i - (aX_i + b) \end{aligned} \quad (9)$$

As a result, unknown parameters and input matrix become,

$$\begin{aligned} \beta &= [a \quad b]^T \in \mathfrak{R}^{2 \times 1} \text{ and } \mathbf{A} = [\mathbf{X} \quad \mathbf{I}_{p \times 1}] \in \mathfrak{R}^{p \times 2} \text{ if } b \neq 0 \\ \beta &= a \in \mathfrak{R}^1 \text{ and } \mathbf{A} = \mathbf{X} \in \mathfrak{R}^{p \times 1} \text{ if } b = 0 \end{aligned}, \quad (10)$$

where $\mathbf{I}_{p \times 1}$ is the p -by-1 identity matrix.

2.1.2. Ordinary Least Squares based on only input errors (OLS2)

In this research OLS2 is applied to only the line model (3) where an input and an output are easily invertible. As shown in Fig. 1(b), OLS2 is the same as OLS1 except the role of the input and the output are reversed in the model (3). Thus, we only consider input errors, Δx_i , assuming $\Delta y_i=0$. Thus, the fitting error becomes,

$$e_i = X_i - x_i = Xi - \left(\frac{1}{a} Y_i - \frac{b}{a} \right) \quad (11)$$

Applying matrix-vector form similar to OLS1, the ordinary least squares estimate of β in OLS 2 is determined by,

$$\hat{\beta}_{OLS2} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{X}, \quad (12)$$

where

$$\begin{aligned} \beta &= \left[\frac{1}{a} \quad -\frac{b}{a} \right]^T \text{ and } \mathbf{A} = [\mathbf{Y} \quad \mathbf{I}_{p \times 1}] \text{ if } b \neq 0 \\ \beta &= \frac{1}{a} \text{ and } \mathbf{A} = \mathbf{Y} \text{ if } b = 0 \end{aligned} \quad (13)$$

2.1.3. Total Least Squares based on both input and output errors (TLS)

TLS refers to total least squares (or orthogonal distance regression) which use both input and output errors to find unknown parameters. Using a singular value decomposition for the model (2) similar to [7], we have,

$$\mathbf{C} = [\mathbf{A} \quad \mathbf{Y}] = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T, \quad (14)$$

where \mathbf{U} is an p -by- p orthogonal matrix, $\mathbf{\Sigma}$ is an p -by- $(n+1)$ matrix of singular values, and \mathbf{V} is an $(n+1)$ -by- $(n+1)$ orthogonal matrix whose components are $\mathbf{V}_{11} \in \mathfrak{R}^{n \times n}$, $\mathbf{V}_{12} \in \mathfrak{R}^{n \times 1}$, $\mathbf{V}_{21} \in \mathfrak{R}^{1 \times n}$, $\mathbf{V}_{22} \in \mathfrak{R}^{1 \times 1}$,

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_{11} & \mathbf{V}_{12} \\ \mathbf{V}_{21} & \mathbf{V}_{22} \end{bmatrix}. \quad (15)$$

For non-singular \mathbf{V}_{22} , the total least squares estimate of β is,

$$\hat{\boldsymbol{\beta}}_{TLS} = -\mathbf{V}_{12} \mathbf{V}_{22}^{-1} \quad (16)$$

Further, considering both input and output errors in the line model (3), we can estimate $\boldsymbol{\beta}$ explicitly by minimizing the fitting errors,

$$\min_{\beta, \Delta x, \Delta y} S = \min_{\beta, \Delta x, \Delta y} \sum_{i=1}^p (e_i)^2 = \min_{\beta, \Delta x, \Delta y} \sum_{i=1}^p \{(\Delta y_i)^2 + (\Delta x_i)^2\} = \min_{a, b} \sum_{i=1}^p \frac{(Y_i - aX_i - b)^2}{1 + a^2} \quad (17)$$

2.2. Condition Number

The condition number [9, 10], $O(\mathbf{A})$, is an observability index, which may be used to investigate parameter sensitivity to disturbance and/or to select efficient pose commands for least squares estimation. The condition number in OLS1 and OLS2 is defined by,

$$O_{OLS}(\mathbf{A}) = \frac{\mu_1}{\mu_n}, \quad (18)$$

which is the ratio of the largest singular value, μ_1 , of $\mathbf{A} \in \mathbb{R}^{p \times n}$ to the smallest value, μ_n . Note that the singular value decomposition of \mathbf{A} is,

$$\mathbf{A} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}^T, \quad (19)$$

where \mathbf{U} is an p -by- p orthogonal matrix, \mathbf{V} is an n -by- n orthogonal matrix, and $\boldsymbol{\Sigma}$ is an p -by- n matrix of singular values. Also note these singular values are diagonals of $\boldsymbol{\Sigma}$, $\text{diag}(\boldsymbol{\Sigma}) = \text{diag}(\mu_1, \dots, \mu_n)$, where $\mu_1 > \dots > \mu_n$. The condition number satisfies the following inequalities for OLS1,

$$\frac{\|\delta \boldsymbol{\beta}\|}{\|\boldsymbol{\beta}\|} \leq O(\mathbf{A}) \frac{\|\delta \mathbf{y}\|}{\|\mathbf{y}\|}, \quad (20)$$

which illustrates parameter sensitivity to disturbances in the output \mathbf{y} . Thus, a low condition number indicates a well-conditioned estimation problem whereas a high condition number means an ill-conditioned problem. Further, the condition number for TLS [9] can be calculated explicitly by,

$$O_{TLS}(\mathbf{A}) = \|\mathbf{V}_{11}^{-T} \text{diag}(s_1, \dots, s_n)\| \sqrt{\|\boldsymbol{\beta}\|^2 + 1}, \quad (21)$$

where

$$s_i = \frac{\sqrt{\mu_i^2 + \mu_{n+1}^2}}{\mu_i^2 - \mu_{n+1}^2}, \text{diag}(\boldsymbol{\Sigma}) = \text{diag}(\mu_1, \dots, \mu_{n+1}) ; i = 1, \dots, n. \quad (22)$$

3. Linear Regression Model

In this section we provide linear car-trailer models, which will be combined with least squares presented in Section 2 to estimate trailer parameters. We first derive

closed form exact models assuming forward motion. We then propose prediction models simplified by linearization techniques.

3.1. Exact Model (EM)

In this section we derive a closed form linear Exact Model (EM), which can easily be applied to least square methods. Now we consider an Instantaneous Center of Rotation (ICR) and geometric configuration in Fig. 2. Assuming forward motion where the linear velocity is positive, $v > 0$, there exists the ICR, O , for rear and trailer axles. Considering two triangles, OQC_2 and QC_1P , the hitch angle, ψ , is correlated with trailer parameters and a path curvature,

$$\sin \psi = \frac{\overline{C_2Q}}{OQ} = \frac{L_2 + L_1 \sec \psi}{R + L_1 \tan \psi}, \quad (23)$$

where L_1 is the hitch length, and L_2 is the trailer length, and $R (=1/\kappa)$ is the radius of the path curvature, κ , at the rear axle center, C_1 . Applying $R=1/\kappa$, (23) can then be rewritten by,

$$\sin \psi - \kappa(L_2 + L_1 \cos \psi) = 0. \quad (24)$$

Choosing variables and unknown parameters in (24) as shown in Table 1, we can establish three linear Exact Models (EM1, EM2, and EM3) for car-trailer systems, which will be applied to least squares in the next section,

$$\begin{aligned} \text{EM1: } \sin \psi &= \kappa \cos \psi L_1 + \kappa L_2 \\ \text{EM2: } \kappa \cos \psi &= \sin \psi \left(\frac{1}{L_1} \right) - \kappa \left(\frac{L_2}{L_1} \right). \\ \text{EM3: } \kappa &= \sin \psi \left(\frac{1}{L_2} \right) - \kappa \cos \psi \left(\frac{L_1}{L_2} \right) \end{aligned} \quad (25)$$

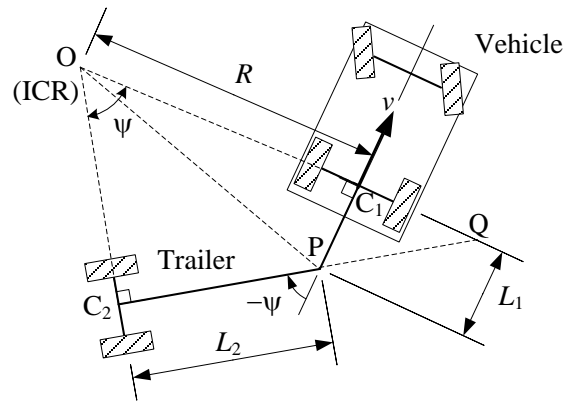


Figure 2. Kinematic Model for A Car-Trailer System

3.2. Prediction Model (PM)

Note that we have $R^2 + L_1^2 > L_2^2$ using two triangles, OC_1P and OC_2P , in Fig. 2. Thus, a curvature constraint can be determined by,

$$\kappa^2 = \frac{1}{R^2} < \frac{1}{L_2^2 - L_1^2}. \quad (26)$$

This constraint shows the curvature is bounded by a finite value. Note that $L_2 > L_1$ for actual car-trailer systems. The hitch angle is also physically constrained considering (24) and (26). In this research we thus choose $|\psi| < 0.785 \text{ rad}$ ($=45 \text{ deg.}$) considering typical trailer systems. As a result, we may assume small hitch angles without loss of generality.

Assuming small hitch angles and denoting $a = L_1 + L_2$, we can linearize (24) with respect to ψ such that we have,

$$\psi = a\kappa. \quad (27)$$

We call this model Prediction Model (PM) since this model can be used to predict input and output relations by estimating the combined parameter, $L_1 + L_2$. However, PM cannot be applied directly to estimate L_1 and L_2 , respectively. Fig. 3 shows respective curvature and hitch angle errors from small angle assumption comparing PM and EM for given nominal values of actual trailer parameters and physical constraints; $L_1 = 1.25 \text{ m}$, $L_2 = 2.48 \text{ m}$, $|\psi| \leq 0.785 \text{ rad}$, and $|\kappa| \leq 0.2 \text{ m}^{-1}$. This result then confirms that these errors are sufficiently small considering physical systems such that the PM (27) can be used to predict EM (24) or (25) without loss of generality. Contrary to EM, estimation problems combining PM and a least squares method are typically well-conditioned in the presence of sensor noises, which will be discussed later.

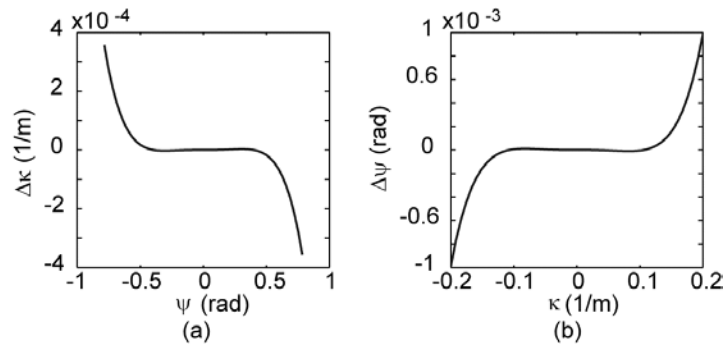


Figure 3. Errors Comparing the Prediction Model (27) to the Exact Model (24) as a Function of (a) ψ and (b) κ given Trailer Parameters and Physical Constraints ($L_1 = 1.25 \text{ m}$, $L_2 = 2.48 \text{ m}$, $|\psi| \leq 0.785 \text{ rad}$ ($= 45^\circ$), and $|\kappa| \leq 0.2 \text{ m}^{-1}$)

Table 1. Parameters and Variables for Exact Models (EM) and Prediction Models (PM)

Model	β	y_i	$\eta_{i,1}$	$\eta_{i,2}$
EM1	$[L_1, L_2]$	$\sin\psi_i$	$\kappa_i \cos\psi_i$	κ_i
EM2	$[1/L_1, L_2/L_1]$	$\kappa_i \cos\psi_i$	$\sin\psi_i$	$-\kappa_i$
EM3	$[1/L_2, L_1/L_2]$	κ_i	$\sin\psi_i$	$-\kappa_i \cos\psi_i$
PM	$[a]$	ψ_i	κ_i	—

4. Estimation Methods

Using the aforementioned least squares and linear models to estimate parameters, we present three estimation schemes; Exact Model-based Least Squares (EMLS), Prediction Model-based Least Squares (PMLS), and Combined Least Squares (CLS). In EMLS, we have six combinations using a least squares method (OLS1 or TLS) and an exact model (EM1, EM2, or EM3), which can estimate trailer parameters L_1 and L_2 . In PMLS, we have 3 combinations using a least squares method (OLS1, OLS2, or TLS) and a prediction model (PM), which can predict input and output relations. These combinations are illustrated in Fig. 4. Contrary to EMLS, PMLS methods are well-conditioned in the presence of input/output noises. Thus, we propose CLS estimation methods to apply well-conditioned input and output data to EMLS when noises are present. As shown in Fig. 4, we first predict a model applying input and output measurements to PMLS. We then estimate trailer parameters applying inputs and output relations predicted by PMLS to EMLS. In this case, we have total 18 combinations of PMLS and EMLS.

5. Simulation Results and Discussion

We evaluate our EMLS, PMLS, and CLS estimation methods in simulation. We use three different types of data by using Harmonic (H) data with $\kappa = 0.2\sin(0.1t)$, Curvilinear (C) data with $\kappa = 0.2\tanh(0.1t)$, and Linear (L) data, $\kappa = 0.2(0.1t/\pi - 1)$. In this case, the curvature is limited by $|\kappa| \leq 0.2 \text{ m}^{-1}$ considering an actual trailer system. Recall that parameters and input/output variables for each model are summarized in

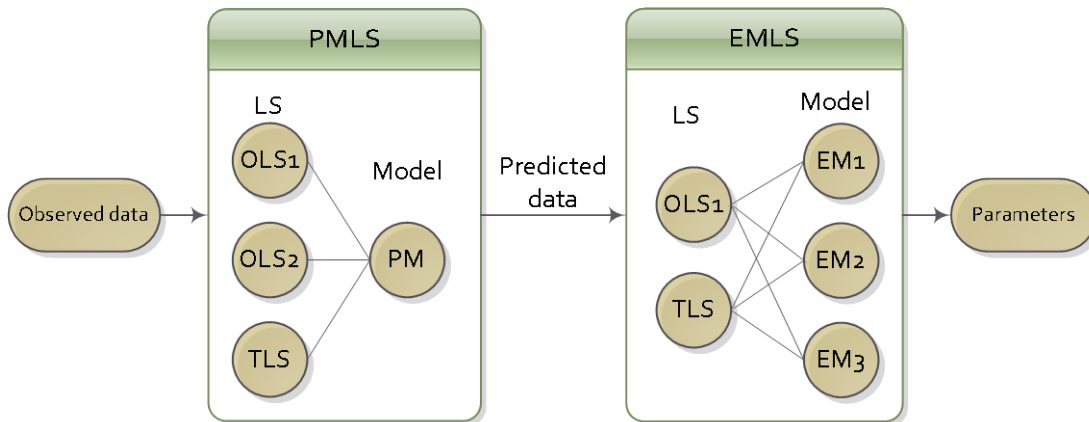


Figure 4. CLS Estimation using Model-based Least Squares (PMLS and EMLS)

Table 1. We further use Gaussian sensor noises to simulate non-ideal sensor measurements in a real car-trailer system. The magnitudes of sensor noises are selected to be 0.03 considering actual odometry and angular position sensors. For each non-ideal scenario where Gaussian sensor noises are applied, we repeat parameter estimation 10 times to obtain more rigorous estimation results.

Table 2 summarizes condition numbers and error norms of estimated parameters in ideal and non-ideal EMLS. These results show condition numbers from non-ideal scenarios with sensor noises are significantly different from ideal condition numbers. We observe parameter errors increase considerably as errors in condition numbers increase. As a result, the EMLS method cannot be applied to estimate parameters in an actual system where sensor noises are not negligible since EMLS methods are not robust. Further, we present CLS estimation results in Table 3. These results show CLS condition numbers are close to ideal EMLS condition numbers in all considered data scenarios. These results thus indicate CLS methods are robust to sensor noises such that parameter estimation errors are modest or small in CLS in the presence of sensor noises. Our simulation results confirm PMLS methods are also well-conditioned in ideal and non-ideal cases. Also note parameter errors from linear and harmonic data sets are relatively smaller than those from curvilinear data sets.

Table 2. Exact Model-based Least Square (EMLS) Estimation Results in Simulation Assuming true Trailer Parameters of $L_1=1.25\text{ m}$ and $L_2=2.48\text{ m}$

Data	Least Squares (Model)	No sensor noises.		With sensor noises	
		Cond. No.	$Norm(\Delta\bar{L}_1, \Delta\bar{L}_2)(m)$	Cond. No.	$Norm(\Delta\bar{L}_1, \Delta\bar{L}_2)(m)$
H	OLS1(EM1)	24.91	0	24.5	1.25
H	OLS1(EM2)	159.3	0	29.0	202.78
H	OLS1(EM3)	78.07	0	30.5	127.17
H	TLS(EM1)	16.2	0	1426.9	36.74
H	TLS(EM2)	29.3	0	3.8	36.74
H	TLS(EM3)	7.35	0	4.4	36.74
C	OLS1(EM1)	41.03	0	39.1	1.16
C	OLS1(EM2)	266.3	0	37.9	126.47
C	OLS1(EM3)	129.6	0	43.6	77.86
C	TLS(EM1)	21.0	0	3837.6	67.56
C	TLS(EM2)	38.4	0	3.42	67.56
C	TLS(EM3)	9.58	0	5.02	67.56
L	OLS1(EM1)	24.5	0	24.0	1.59
L	OLS1(EM2)	154.6	0	24.5	926.29
L	OLS1(EM3)	76.2	0	24.9	595.57
L	TLS(EM1)	19.1	0	1193.8	42.73
L	TLS(EM2)	34.3	0	3.7	42.73
L	TLS(EM3)	8.62	0	4.0	42.73

Table 3. CLS Estimation Results in Simulation Assuming True Trailer Parameters of $L_1=1.25$ m and $L_2=2.48$ m

Data	Applied EMLS	OLS1(PM)-based		OLS2(PM)-based		TLS(PM)-based	
		Cond. No.	$(\overline{\Delta L_1}, \overline{\Delta L_2})$ (%)	Cond. No.	$(\overline{\Delta L_1}, \overline{\Delta L_2})$ (%)	Cond. No.	$(\overline{\Delta L_1}, \overline{\Delta L_2})$ (%)
H	OLS1(EM1)	21.9	0.4,-2.6	24.7	1.8,-0.9	21.2	2.0,-1.17
H	OLS1(EM2)	135.4	0.4,-2.6	154.4	1.8,-0.9	132.7	2.0,-1.17
H	OLS1(EM3)	68.3	0.4,-2.6	77.8	1.8,-0.9	67.0	2.0,-1.17
H	TLS(EM1)	13.9	0.4,-26	16.0	1.8,-0.9	13.7	2.0,-1.17
H	TLS(EM2)	24.6	0.4,-26	27.8	1.8,-0.9	23.7	2.0,-1.17
H	TLS(EM3)	6.5	0.4,-26	7.4	1.8,-0.9	6.3	2.0,-1.17
C	OLS1(EM1)	27.5	2.1,-2.0	39.3	2.5,-0.8	26.9	3.0,-1.1
C	OLS1(EM2)	172.1	2.1,-2.0	249.3	2.5,-0.8	170.0	3.0,-1.1
C	OLS1(EM3)	87.1	2.1,-2.0	125.3	2.5,-0.8	86.1	3.0,-1.1
C	TLS(EM1)	13.9	2.1,-2.0	20.1	2.5,-0.8	13.8	3.0,-1.1
C	TLS(EM2)	24.2	2.1,-2.0	35.0	2.5,-0.8	23.7	3.0,-1.1
C	TLS(EM3)	6.5	2.1,-2.0	9.3	2.5,-0.8	6.4	3.0,-1.1
L	OLS1(EM1)	22.7	-0.7,-3.2	24.4	1.5,-0.8	21.7	1.6,-1.1
L	OLS1(EM2)	138.0	-0.7,-3.2	151.1	1.5,-0.8	134.1	1.6,-1.1
L	OLS1(EM3)	69.7	-0.7,-3.2	76.2	1.5,-0.8	67.9	1.6,-1.1
L	TLS(EM1)	17.0	-0.7,-3.2	18.9	1.5,-0.8	16.6	1.6,-1.1
L	TLS(EM2)	30.4	-0.7,-3.2	32.9	1.5,-0.8	28.9	1.6,-1.1
L	TLS(EM3)	8.0	-0.7,-3.2	8.7	1.5,-0.8	7.7	1.6,-1.1

6. Conclusion

In this research we investigate condition numbers to find more reliable estimation methods or observed data sets in the presence of sensor noises. Our simulation results indicate CLS methods are relatively robust for sensor noises. We also find ideal EMLS condition numbers can be used as a reference to examine estimation sensitivity or to predict estimation performance. Further, we may select linear or harmonic data sets as observed data to improve estimation accuracy rather than curvilinear data sets.

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