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MHD Flow Past An Oscillating Infinite Vertical Plate With Variable Temperature Through Porous Media

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***Abstract**: This paper analyzes MHD flow with heat and mass transfer on flow past an oscillating infinite vertical plate with variable temperature in a porous media. The dimensionless governing partial differential equations are solved by using Laplace transform method. Exact analytical solutions satisfy governing equations and the conditions are obtained. The velocity profiles are studied for different physical parameters. It is observed that the solutions are significantly affected by the phase angle (ωt), magnetic field (m), thermal Grash of number (Gr), mass Grash of number (Gc), permeability parameter (K), Schmidt number (Sc) and time t.*

***Keywords**: Porous Medium, Magnetohydrodynamic (MHD), oscillating infinite, vertical plate, Variable temperature*

I. Introduction

MHD flow has many applications in geophysical and astrophysical dynamics. Effect of heat and mass transfer plays vital role, in space craft design, in the cooling of liquid metal of nuclear reactors, pollution of environment etc. Flow through porous media have numerous engineering problems, for example, in the underground water resources ,rain water harvesting ,the movement of oil and natural gas through oil sandstone reservoirs ,purification of crude oil, paper and pulp industry, membrane separation process, flow of blood . The purpose of present study is to study the MHD flow with Heat and Mass Transfer Effects on Flow Past an Oscillating Infinite Vertical Plate with Variable Temperature through Porous Media. Soundalgekar [1] presented an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the Laplace-transform technique. Heat transfer effects on flow past an impulsively started infinite vertical plate in the presence of constant heat flux and variable temperature are studied by Soundalgekar and Patil [2,3]. Muthucumaraswamy et al. [4] studied the unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. Muthucumaraswamy and Valliamal [5] considered first order chemical reaction on exponentially accelerated isothermal vertical plate with mass diffusion. Muthucumaraswamy et al. [6] studied heat transfer effects on flow past an exponentially accelerated vertical plate with variable temperature. Basant kumar Jha [7] analyzed MHD free convection and mass transform flow through a porous medium. Recently R. Muthucumaraswamy, and A. Vijayalakshmi [8] studied the effects of heat and mass transfer on flow past an oscillating vertical plate with variable temperature. V. Rajesh [9] studied MHD effect of free convection and mass transform flow through a porous medium with variable temperature.

Here the flow past an infinite vertical plate with variable temperature and mass diffusion through a porous medium has been considered. x* axis is taken in vertically positive direction and flow is taken along x* axis .y* axis is taken in the direction normal to the plate .A uniform magnetic field B is transversely applied to the plate. Initially in the stationary condition the temperature T^*_∞ of the plate and the fluid is same at all points with concentration C^*_∞ .The plate starts oscillatory motion in its own plane at time $t^* > 0$ with velocity $u = u_0 \cos\omega^* t^*$.The temperature and the concentration of the plate raised linearly with time. The magnetic lines of force relative to the plate are fixed. Then by usual Boussinesq's approximation the equation of unsteady flow governed is given by

$$\frac{\partial u^*}{\partial t^*} = g\beta(T^* - T^*_\infty) + g\beta^*(C^* - C^*_\infty) + \nu\frac{\partial^2 u^*}{\partial y^{*2}} - \nu\left(\frac{u^*}{K^*}\right) - \frac{\sigma B^2}{\rho}(u^* - u_0\cos\omega^* t^*) \tag{1}$$

$$\rho C_p \frac{\partial T^*}{\partial t^*} = \kappa\frac{\partial^2 T^*}{\partial y^{*2}} \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \tag{3}$$

With initial and boundary conditions

$t^* \le 0,\quad u^* = 0,\quad T^* = T^*_\infty \quad C^* = C^*_\infty \quad$ for all y*

$t^* > 0,\ u^* = u_0\cos\omega^* t^*,\ T^* = T^*_\infty + (T^*_w - T^*_\infty)\frac{u_0^2}{\nu}t^*\ ,\ C^* = C^*_\infty + (C^*_w - C^*_\infty)\frac{u_0^2}{\nu}t^*$ at y*= 0

$u^* = 0, \quad T^* \to T^*_\infty \quad C^* \to C^*_\infty \quad \text{as} \quad y^* \to \infty$ (4)

Normalizing the above equations by using following non-dimensionless parameters,

$$u = \left(\frac{u^*}{u_0}\right), \; t = \left(\frac{t^* u_0^2}{\nu}\right), \; y = \left(\frac{y^* u_0}{\nu}\right), \; \theta = \left(\frac{T^* - T^*_\infty}{T^*_w - T^*_\infty}\right), \; \omega = \left(\frac{\nu\omega^*}{u_0^2}\right), \; Sc = \frac{\nu}{D} \; Gr = \left(\frac{g\beta^*\nu(T^*_w - T^*_\infty)}{u_0^3}\right), \; Gc = \left(\frac{g\beta^*\nu(C^*_w - C^*_\infty)}{u_0^3}\right),$$

$$C = \left(\frac{C^* - C^*_\infty}{C^*_w - C^*_\infty}\right), \; Pr = \frac{\mu C_p}{k}, \; \frac{1}{K} = \frac{u_0^2 K^*}{\nu^2}, \; M = \frac{\sigma B^2 \nu}{\rho u_0^2} \tag{5}$$

Equations (1) to (4) become

$$\frac{\partial u}{\partial t} = Gr\theta + GcC + \frac{\partial^2 u}{\partial y^2} - M(u - \cos\omega t) - Ku \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{8}$$

With dimensionless initial and boundary conditions,

$u = 0, \quad \theta = 0, \quad C = 0, \quad$ for all $\quad y \le 0, \quad t \le 0$

$u = \cos\omega t, \quad \theta = t, \quad C = t, \quad$ at $\quad y = 0, \quad t > 0$

$u = 0, \quad \theta \to 0, \quad C \to 0 \quad$ as $\quad y \to \infty$ (9)

All physical variables are defined in nomenclature.

SOLUTION OF THE PROBLEM

The governing equations (6),(7) and (8) with boundary condition are solved by Laplace transform method. On taking laplace transform of the equation (6),(7),(8) and (9) we get,

$$\frac{d^2\bar{u}}{dy^2} - (s + M^*)\bar{u} = -Gr\bar{\theta} - Gc\bar{C} - \frac{Ms}{s^2 + \omega^2} \tag{10}$$

$$\frac{d^2\bar{\theta}}{dy^2} - s\,Pr\,\bar{\theta} = 0 \tag{11}$$

$$\frac{d^2\bar{C}}{dy^2} - sSc\bar{C} = 0 \tag{12}$$

$\bar{u} = 0, \quad \bar{\theta} = 0, \quad \bar{C} = 0 \quad$ for all y, $t \le 0$

$\bar{u} = \frac{s}{s^2 + \omega^2}, \quad \bar{\theta} = \frac{1}{s^2}, \quad \bar{C} = \frac{1}{s^2} \quad$ at $\quad y = 0, t > 0$

$\bar{u} = 0, \quad \bar{\theta} \to 0, \quad \bar{C} \to 0 \quad$ as $y \to \infty$, $t > 0$ (13)

Where s is the parameter of laplace transform.

On solving above equations, we get,

$$\bar{u} = \left[\frac{s}{s^2 + \omega^2} - \frac{sM}{(s^2 + \omega^2)M^*} + \frac{1}{s^2}\left\{\frac{Gr}{s(Pr-1) - M^*} + \frac{Gc}{s(Sc-1) - M^*}\right\}\right]e^{-y\sqrt{s+M^*}}$$
$$-\frac{1}{s^2}\left\{\frac{Gre^{-y\sqrt{s\,Pr}}}{s(Pr-1) - M^*} + \frac{Gce^{-y\sqrt{sSc}}}{s(Sc-1) - M^*}\right\} + \frac{sM}{(s^2 + \omega^2)(s + M^*)} \tag{14}$$

$$\bar{\theta} = \frac{e^{-y\sqrt{s\,Pr}}}{s^2} \tag{15}$$

$$\bar{C} = \frac{e^{-y\sqrt{sSc}}}{s^2} \tag{16}$$

Taking inverse laplace transform of equation (14),(15)and(16),we get

$$u = \frac{(N+i\omega M)}{4(\omega^2+M^{*2})} e^{-i\omega t}\left[e^{y\sqrt{M^*-i\omega}}\text{erfc}\left\{\eta+\sqrt{(M^*-i\omega)t}\right\}+e^{-y\sqrt{M^*-i\omega}}\text{erfc}\left\{\eta-\sqrt{(M^*-i\omega)t}\right\}\right]$$
$$+\frac{(N-i\omega M)}{4(\omega^2+M^{*2})} e^{i\omega t}\left[e^{y\sqrt{M^*+i\omega}}\text{erfc}\left\{\eta+\sqrt{(M^*+i\omega)t}\right\}+e^{-y\sqrt{M^*+i\omega}}\text{erfc}\left\{\eta-\sqrt{(M^*+i\omega)t}\right\}\right]$$
$$+e^{y\sqrt{M^*}}\text{erfc}(\eta+\sqrt{M^*t})\left[\frac{Gr}{Pr-1}\left(\frac{-1}{2c^2}+\frac{t}{2c}+\frac{y}{4c\sqrt{M^*}}\right)+\frac{Gc}{Sc-1}\left(\frac{-1}{2b^2}+\frac{t}{2b}+\frac{y}{4b\sqrt{M^*}}\right)\right]$$
$$+e^{-y\sqrt{M^*}}\text{erfc}(\eta-\sqrt{M^*t})\left[\frac{Gr}{Pr-1}\left(\frac{-1}{2c^2}+\frac{t}{2c}-\frac{y}{4c\sqrt{M^*}}\right)+\frac{Gc}{Sc-1}\left(\frac{-1}{2b^2}+\frac{t}{2b}-\frac{y}{4b\sqrt{M^*}}\right)\right]$$

$$-\frac{Gre^{-ct}}{2(Pr-1)c^2}\left[e^{y\sqrt{-cPr}}\operatorname{erfc}\left(\eta\sqrt{Pr}+\sqrt{-ct}\right)+e^{-y\sqrt{-cPr}}\operatorname{erfc}\left(\eta\sqrt{Pr}-\sqrt{-ct}\right)\right]$$
$$-\frac{Gce^{-bt}}{2(Sc-1)b^2}\left[e^{y\sqrt{-bSc}}\operatorname{erfc}\left(\eta\sqrt{Sc}+\sqrt{-bt}\right)+e^{-y\sqrt{-bSc}}\operatorname{erfc}\left(\eta\sqrt{Sc}-\sqrt{-bt}\right)\right]$$
$$-\frac{Gc}{(Sc-1)}\left[\operatorname{erfc}\left(\eta\sqrt{Sc}\right)\left\{\frac{-1}{b^2}+\frac{1}{b}\left(t+\frac{y^2Sc}{2}\right)\right\}-\frac{y}{b}\sqrt{\frac{Sct}{\pi}}e^{-\eta^2 Sc}\right]-\frac{Gr}{(Pr-1)}\left[\operatorname{erfc}\left(\eta\sqrt{Pr}\right)\left\{\frac{-1}{c^2}+\frac{1}{c}\left(t+\frac{y^2Pr}{2}\right)\right\}-\frac{y}{c}\sqrt{\frac{Prt}{\pi}}e^{-\eta^2 Pr}\right]$$
$$+\frac{Gre^{-ct}}{2(Pr-1)c^2}\left[e^{y\sqrt{M^*-c}}\operatorname{erfc}\left(\eta+\sqrt{(M^*-c)t}\right)+e^{-y\sqrt{M^*-c}}\operatorname{erfc}\left(\eta-\sqrt{(M^*-c)t}\right)\right]$$
$$+\frac{Gce^{-bt}}{2(Sc-1)b^2}\left[e^{y\sqrt{M^*-b}}\operatorname{erfc}\left(\eta+\sqrt{(M^*-b)t}\right)+e^{-y\sqrt{M^*-b}}\operatorname{erfc}\left(\eta-\sqrt{(M^*-b)t}\right)\right]+\frac{MM^*\operatorname{erfc}(\eta)}{\omega^2+M^{*2}} \quad (17)$$

$$\theta = t\left[(1+2\eta^2 Pr)\operatorname{erfc}(\eta\sqrt{Pr})-\frac{2}{\sqrt{\pi}}e^{-\eta^2 Pr}\eta\sqrt{Pr}\right] \quad (18)$$

$$C = t\left[(1+2\eta^2 Sc)\operatorname{erfc}(\eta\sqrt{Sc})-\frac{2}{\sqrt{\pi}}e^{-\eta^2 Sc}\eta\sqrt{Sc}\right] \quad (19)$$

Where $M^* = M + K$, $N = M^{*2} + \omega^2 - M M^*$, $c=-\frac{M^*}{Pr-1}$, $b=-\frac{M^*}{Sc-1}$, $\eta=\frac{y}{2\sqrt{t}}$

II. Results and Discussion

Velocity profiles has been discussed for various physical parameters namely thermal Grash of number (Gr), mass Grash of number (Gc), Prandtl number (Pr), Schmidt number (Sc), time (t), Permeability parameter (K). Magnetic parameter (M), phase angle (ωt). Different velocity profile has been presented in the figures 1-8. In the figure-1 velocity profile for different magnetic parameter M = 0, 1,2,3 has been presented .it is observed that velocity decreases as magnetic parameter M increases. In the figure-2 we studied the velocity profile for different values of permeability parameter K=0.25,0.5,0.75,1. It is observed that the velocity profile decreases near the surface of the plate as K increases and then increases away from the plate as K increases.

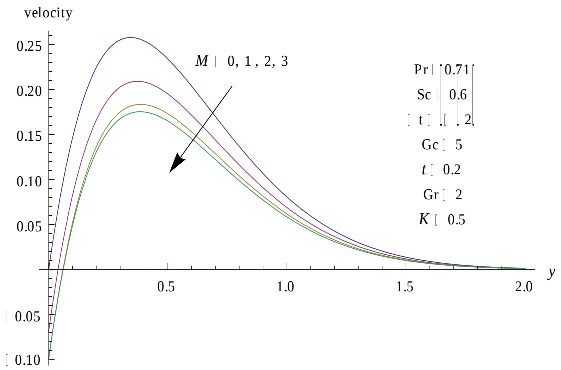

Figure -1

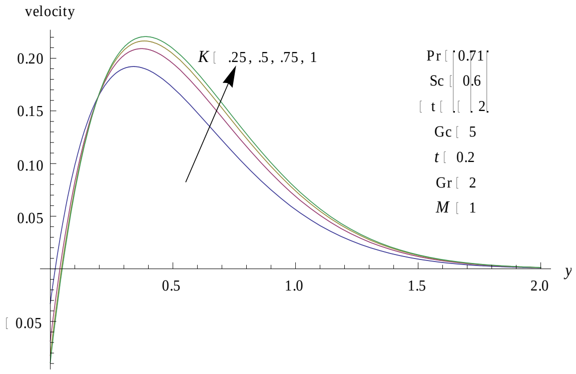

Figure-2

In the Figure-3 the velocity profile for different phase angle (ωt =0,π/6, π/4, π/2) has been studied and it is observed that velocity decreases as phase angle increases. In figure -4 the velocity profile for different time t= 2,3,4,5,6 has been presented and it is observed that velocity increases as t increases.

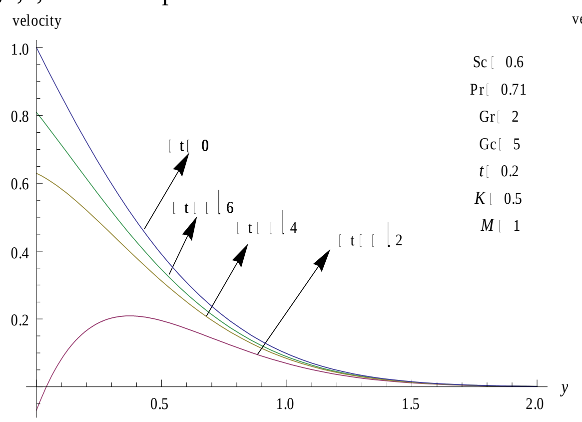

Figure-3

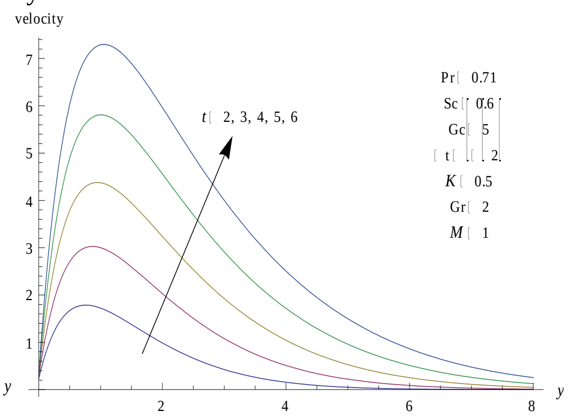

Figure-4

In figure-5 velocity profile for different thermal Grash of number Gr= -10,-5,5,10 has been studied and it is observed that velocity increases as Gr increases.
In figure-6 velocity profile for different mass Grash of number Gc= -10,-5,5,10 has been studied and it is observed that velocity increases as Gc increases.

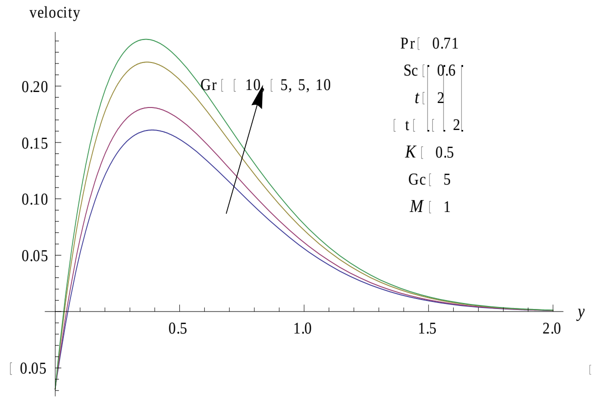

Figure-5

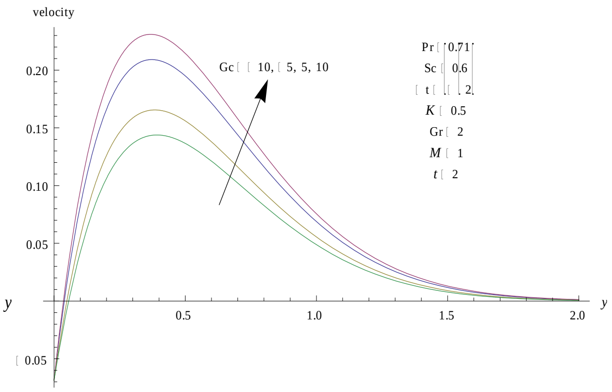

Figure-6

In figure -7 velocity profile for different Schmidt number Sc= 0.2,0.4,0.71,5 has been studied and it is observed that velocity decreases as Sc increases.

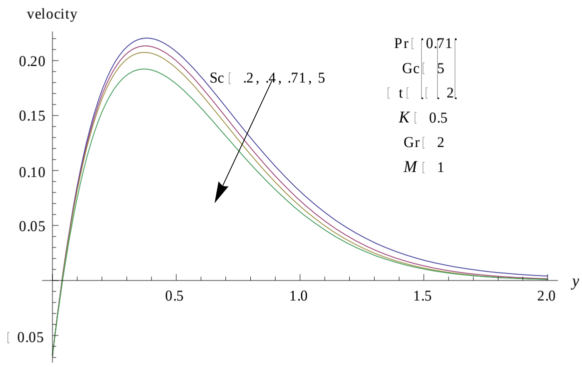

Figure-7

III. Conclusion

The following results have been concluded from the study:
The velocity increases as thermal Grash of number (Gr), mass Grash of number (Gc), time (t), Permeability parameter (K) increases. The velocity decreases as Magnetic parameter (M) , Schmidt number (Sc), phase angle (ωt) increases.

Appendix

Nomenclature

C^* - Concentration in the fluid far away from the plate, C^*_w - Concentration of the plate, y^* -Coordinate axis normal to the plate , t^* - Time, u^* -Velocity of the fluid in the x^* -direction, u_0 -V elocity of the plate, C - Dimensionless concentration, y- Dimensionless coordinate axis normal to the plate, u - Dimensionless velocity, B - External magnetic field, Gm- Mass Grash of number, Gr- Thermal Grash of number, Pr -Prandtl number, Sc- Schmidt number, C^* -Species concentration in the fluid, Cp- Specific heat at constant pressure,T^*_∞ - Temperature of the fluid far away from the plate, T^* -Temperature of the fluid near the plate, T^*_w - Temperature of the plate, κ - Thermal conductivity of the fluid, D - Chemical Molecular diffusivity, g - Acceleration due to gravity, K - permeability parameter, M - Magnetic field parameter, t - Dimensionless time,

Greek symbols

μ - Coefficient of viscosity , erfc - Complementary error function, ρ - Density of the fluid, τ- Dimensionless skin friction, θ- Dimensionless temperature, σ- Electric conductivity, erf - Error function, υ - Kinematic viscosity, α -Thermal diffusivity

Subscripts

w - Conditions on the wall, ∞ - Free stream conditions

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