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Constructive Volumetric Modeling

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***Abstract*—In this article we intend to present a method of obtaining high complexity sinthetic scenes by using simple volumes as the building blocks. The below described method can be used to obtain both homogenous and heterogenous volumes. This is done by combining volumes of different voxel densities.**

***Index Terms*—volumetric data, voxel, constructive solid geometry, volume modelling, constructive volume geometry.**

I. Introduction

THE VOLUMETRIC data imaging technology has greatly improved sice the 1990's. Before this many fields had to work with data images that used the depth or field effect known from 2D screens.

Because of the improved graphical representation, the volumetric data has found extensive use in medical applications such as 3D ultrasound, CAT (Computed Axial Tomography) or MRI (Magnetic Resonance Imaging). Other fields putting the technology to use include geological surveying, security scanning and, potentially, 3D gaming.

Given the importance of volumetric data, a lot of research has been done lately in the field, ranging from volumetric generation and rendering to volumetric segmentation, indexing and compression. This research was mainly done using data that resulted from real medical cases. Only recently did researchers start to use data obtained by scanning physical objects using lasers. Even so, volumetric data is stil scarce and not readilly available.

Because volumetric data is in general obtained through medical imaging devices it is usually hard to come by. Given the ease with wich sinthetic scenes can be manufactured, the sinthetic senes are somtimes prefered for volumetric analysis. It must be said that the sinthetic scenes offer less diversity than the real medical data but they can be custom made to the precise needs of the desired field of analysis. Volume segmentation and indexing can greatly take advantage of sinthetic scence tailormade for its needs.

II. Constructive volume modeling

The constructive volume generation technology is not a new thing and many articles have been written on this topic. For example the Constructive Volume Geometry (CVG) [1] article presents an algebraic framework for modelling complex spatial objects using combinational operations. In this article we present another approach for volume generation. We will use basic volumes like spheres, prisms, cylinders, cones, tori, etc. as building blocks for more complexe volumes. These volumes are combined using boolean operators like union and intersection. The resulting volumes can be combined with other volumes to form more complex objects.

The volumes that we are using are made up of voxels that have a position in the volumetric spece they are defined in and a density, with values in the interval [0,1]. The density can later on be interpreted as a color in a given spectrum. For the examples given in this article we chose a palette consisting of shades of green ranging from light green for density 0 to dark green for density 1.

A volume is stored in a 3D matrix of densities, where a voxel is represented by its position in the matrix and the density stored at that position.

Fig. 1. Sphere and cone union.

In order to combine their volumetric data, all volumes must be defined in the same subspace.For example we consider a 256x256x256 cube made up of voxels. Each voxel in the cube has a density between 0 and 1. Let us take two such cubes, A and B, which contain the volumetric data for a homogenous sphere of radius 100, centered in (100, 100, 100) and a homogenous cone of radius 100 and height 50, centered in (100, 200, 100) respectively. Both volumes have the density equal to 1. The union of A and B will be another 256x256x256 cube of voxels as presented in Fig. 1.

When we make a union between two volumes, we actually add the densities of corresponding voxels in the cubes containing these volumes, caping the densities at a maximum of 1, thus obtaining a valid new cube, with all densities between 0 and 1. The resulting cube can be used in subsequent operations.

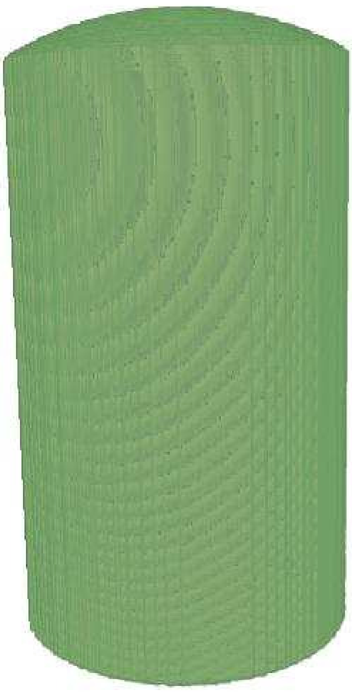

Fig. 2. Sphere and cylinder intersection.

For the intersection of two volumes we compute the product of all corresponding densities in the cubes containing the volumes. Given that the densities of voxels have values in the interval $[0, 1]$, after multiplication the resulting densities also have values in the $[0, 1]$ interval. An example of an intersection between a sphere and a cylinder is given in Fig. 2.

The difference of two volumes is obtained by subtracting the corresponding densities of voxels in the cubes containing the volumes. Because the resulting densities can fall below 0, we need to limit these values at 0. An exemple of a difference is given in Fig. 3.

The complement of a volume can be obtained by subtracting from 1 the voxel densities of the cube containing the volume. The resulting cube has all densities in the $[0, 1]$ interval.

Fig. 3. Sphere and torus difference.

These operations have the potential of creating realy complex volumes. In Fig. 4. there is a volume created from a sphere and three cylinders.

The basic volumes used in these operations are created using their parametrized equations. For example, a sphere is defined by the equation (1).

$$\begin{aligned} x &= x_0 + r \sin\theta \cos\phi \\ y &= y_0 + r \sin\theta \sin\phi \quad (0 \le \phi \le 2\pi \ and \ 0 \le \theta \le \pi) \\ z &= z_0 + r \cos\theta \end{aligned} \tag{1}$$

By taking discrete values from the intervals $[0, 2\pi]$,$[0, \pi]$, and $[0, r]$ we are able to build our sphere voxel by voxel. The

Fig. 4. Union of three cylinders.

algorithm for constructing a homogenous sphere is given in Algorithm 1.

Algorithm 1 Generating voxels for a homogenous sphere of density 1.

Require: $r > 0$
  $r \leftarrow 1$;
  while $r <= radius$ **do**
    $theta \leftarrow 0$;
    while $theta \leq 2\pi$ **do**
      $phi \leftarrow 0$;
      while $phi \leq \pi$ **do**
        $x \leftarrow x_0 + r * \sin(theta) * \cos(phi)$;
        $y \leftarrow y_0 + r * \sin(theta) * \sin(phi)$;
        $z \leftarrow z_0 + r * \cos(theta)$;
        $volume_{x,y,z} \leftarrow 1$;
        $phi \leftarrow phi + \arcsin(1/r)$;
      end while
      $theta \leftarrow theta + \arcsin(1/r)$;
    end while
    $r \leftarrow r + 1$;
  end while

In order to obtain a heterogenous volume we can combine homogenous volumes using the operations defined previously. For example a sphere with three layers of density d_1, d_2 and d_3 can be obtained by making a union between a homogenous sphere with density d_1, a homogenous sphere shell with density d_2 and another homogenous sphere shell with density d_3. A section through the resulting sphere can be seen in Fig. 5.

Fig. 5. Sphere and cylinders difference.

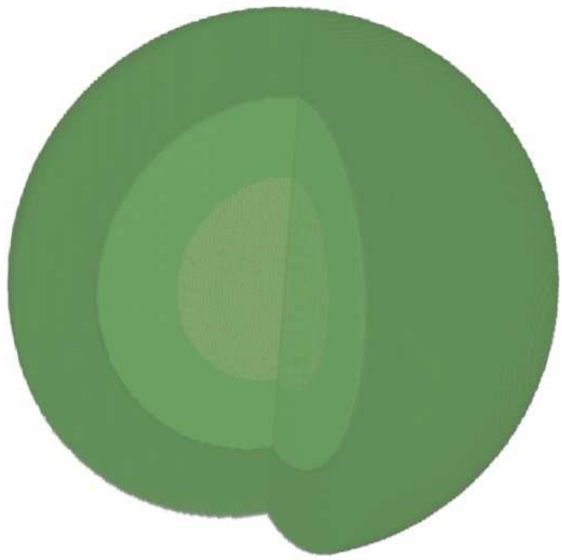

Fig. 6. Heterogenous sphere obtained by combining three basic homegenous volumes through union. The section through the sphere was made by means of a difference with a prism.

III. Conclusion

We have shown that more complexe volumetric objects can be obtained by combining basic volumes using boolean operators. This is very helpful in obtaining synthetic data for other volume related fields of research like volumetric segmentation and indexing, and volume compression.

The simple sinthetic data can be combined into more complex forms thus giving a large colection of objects from where to choose when performing volumetric analysis.

The method that we have used is a very simple but an ingenious one as presented above. We will use this method in our future work.

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