# Part 16: Group Recommender Systems Rank Aggregation and Balancing Techniques

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### Content

- Group recommendations
- Rank aggregation optimal aggregation
- Rank aggregation for group recommendation
- Dimensions considered in the study
  - Group size
  - Inter group similarity
  - Rank aggregation methods
- Sequential Group Recommendations
- Balancing
- User study

#### **Group Recommendations**

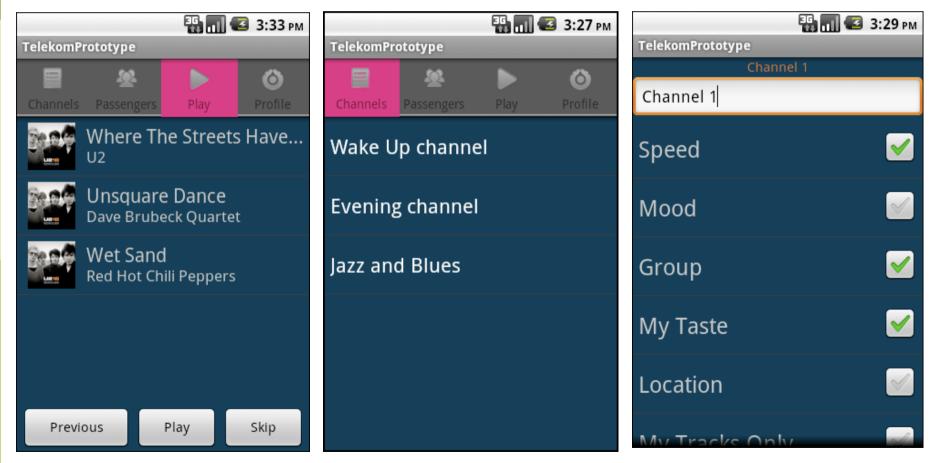
- Recommenders are usually designed to provide recommendations adapted to the preferences of a single user
- In many situations the recommended items are consumed by a group of users
  - A travel with friends
  - A movie to watch with the family during Christmas holidays
  - Music to be played in a car for the passengers



# **Mobile Application**



# Recommending music compilations in a car scenario



[Baltrunas et al., 2011]

# **Effects of Groups on User Satisfaction**

#### Emotional Contagion

- Other users being satisfied may increase a user's satisfaction (and viceversa)
- Influenced by your personality and the social relationships with the other group members

#### Conformity

- The opinion of other users may influence your own expressed opinion
- Normative influence: you want to be part of the group
- Informational influence: opinion changes because you believe the group must be right.

#### **Group Recommendation Model**

Items will be experienced by individuals together with the other group members: the evaluation function depends on the group:

$$r: U \times I \times \mathcal{O}(U) \to E$$

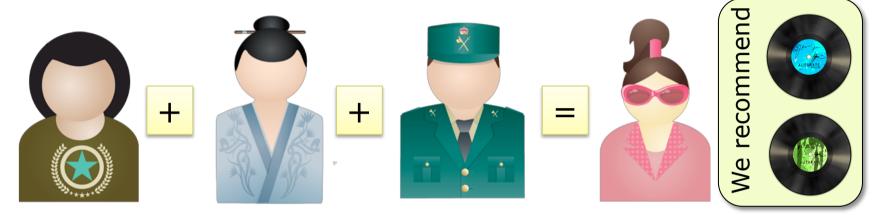
- U is the set of users, I is the set of Items, P(U) is the set of subsets of users (groups), E is the evaluation space (e.g. the ratings {?, 1, 2, 3, 4, 5}) of the rating function r
- Normally researchers assume that r(u,i)=r(u,i,g) for all groups g∋u
- But users are influenced in their evaluation by the group composition (e.g., emotional contagion [Masthoff & Gatt, 2006]).

#### **Recommendation Generation**

- Having identified the best items for each group member how we select the best items for the group?
- How the concept of "best items" for the group can be defined?
- We could introduce a fictitious user g and be able to estimate r(g,i)
- But how?
- Two approaches have been considered [Jameson & Smyth, 2007]
  - Profiles aggregation
  - Recommendations aggregation

### **First Mainstream Approach**

Creating the joint profile of a group of users



We build a recommendation for this "average" user

#### Issues

- The recommendations may be difficult to explain individual preferences are lost
- Recommendations are customized for a "user" that is not in the group
- There is no well founded way to "combine" user profiles – why averaging?

# **Second Mainstream Approach**

Producing individual recommendations



Then "aggregate" the recommendations:



#### Issues

- How to optimally aggregate ranked lists of recommendations?
- Is there any "best method"?

# **Optimal Aggregation**

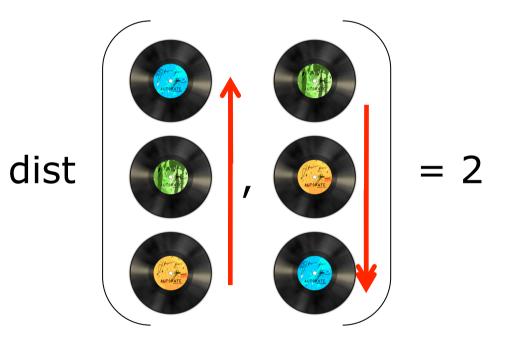
- Paradoxically there is not an optimal way to aggregate recommendations lists (Arrows' theorem: there is no fair voting system)
- [Dwork et al., 2001] introduced the notion of Kemeny-Optimal aggregation:
  - Given a distance function between two ranked lists (Kendall tau distance)
  - Given some input ranked lists to aggregate
  - Compute the ranked list (permutation) that minimize the average distance to the input lists.

#### **Arrow's Theorem**

- No rank-order voting system can be designed that satisfies these three fairness criteria:
  - If every voter prefers alternative X over alternative Y, then the group prefers X over Y
  - If every voter's preference between X and Y remains unchanged when Z is added to the slate, then the group's preference between X and Y will also remain unchanged
  - There is no *dictator*: no single voter possesses the power to always determine the group's preference.

#### **Kendall tau Distance**

The number of pairwise disagreements



1 One item is preferred to the other

# Why Kendall tau distance?

- Kemeny optimal aggregation has a maximum likelihood interpretation:
  - 1. Assume that there is a "correct" ordering *t*
  - 2. Assume that there are  $t_1, ..., t_k$  ordering that are obtained by randomly swapping two elements (with probability < 0.5)
  - 3. Then a Kemeny optimal aggregation of  $t_1, ..., t_k$  is maximally likely to have produced these orderings.

# **Kemeny Optimal Aggregation**

- Kemeny optimal aggregation is expensive to compute (NP hard – even with 4 input lists)
- There are other methods that have been proved to approximate the Kemeny-optimal solution
  - Borda count no more than 5 times the Kemeny distance [Dwork et al., 2001]
  - Spearman footrule distance no more than 2 times the Kemeny distance [Coppersmith et al., 2006]
    - SFD: the sum over all the elements of the lists of the absolute difference of their rank
  - Average average the predicted ratings and sort
  - Least misery- sort by the min of the predicted ratings
  - Random 0 knowledge, only as baseline.

#### **Average Aggregation**

- Let r\*(u,i) be either the predicted rating of u for i, or r(u,i) if this rating is present in the data set
- **\square** Then the score of an item for a group g is

□  $r^*(g,i) = AVG_{u \in g} \{r^*(u,i)\}$ 

- Items are then sorted by decreasing value of their group scores r\*(g, i)
- Issue: the recommended items may be very good for some members and less convenient for others
- Hence ... least misery approach

#### **Borda Count Aggregation**

- Each item in the ranking is assigned a score depending on its position in the ranking: the higher the rank, the larger the score is
- The last item  $i_n$  in the ranking of user u has  $score(u,i_n) = 1$  and the first item has  $score(u,i_1) = n$
- Group score for an item is calculated by adding up the item scores for each group member:

$$score(g,i) = \sum_{u \in g} score(u,i)$$

Items are then ranked according to their group score.

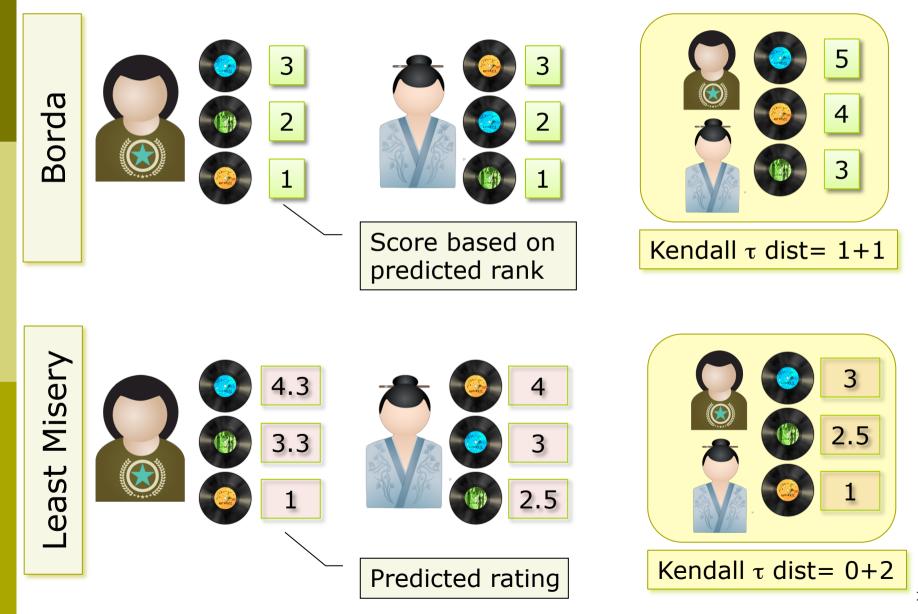
#### **Least Misery Aggregation**

- Let r\*(u, i) be either the predicted rating of u for i, or r(u, i) if this rating is present in the data set
- Then the score of an item for a group g is:

 $\Box r^*(g, i) = MIN_{u \in g} \{r^*(u, i)\}$ 

- Items are then sorted by decreasing value of their group scores r\*(g, i)
- The recommended items have rather large predicted ratings for **all** the group members
- May select items that nobody hates but that nobody really likes (shopping mall case).

#### **Borda Count vs. Least Misery**



# **Evaluating Group Recommendations**

- Ask the users to collectively evaluate the group recommendations
- Or use a test set for off-line analysis:
  - But how to compare this best "group recommendation" with the **true** "best" item for the group?
  - What is the ground truth?
- We need again an aggregation rule that computes the **true** group score for each recommendation

•  $r(g,i) = Agg(r(u_1, i), ..., r(u_{|g|}, i))$ 

■ *U<sub>i</sub>* ∈ *G* 

How to define Agg?

#### **Circular Problem**

If the aggregation function used in the evaluation is the same used in the recommendation generation step we have "incredibly" good results

#### Example

- If the items with the largest average of the predicted ratings AVG<sub>u∈g</sub> {r\*(u,i)} are recommended
- Then these will score better (vs. items selected by a different aggregation rule) if the "true best" recommendations are those with the largest average of their true ratings AVG<sub>u∈g</sub> {r(u,i)}

# **Other Online Studies**

- [Masthoff 2004] studied how people aggregate users' preferences
- She showed to subjects the following data and asked them to generate recommendations for this group

	А	В	С	D	Е	F	G	Н	Ι	J
Peter Jane		4			10 7				10 3	8 8
Mary	1 10	-			9				3 7	•

- Participants cared about fairness and their behavior reflected several strategies (least misery, average without misery) while others were not used (Borda count)
- But a recommender system cannot simply mimic users they have limited computational power
- When users evaluate recommendations they can prefer those generated by totally different strategies!

# **Other Online Studies**

 In [Masthoff 2004] the subjects were also asked to evaluate recommendations generated by a range of aggregation strategies

	А	В	С	D	E	F	G	Н	Ι	J
Peter	10	4			10					8
Jane	1	9	8	9	7	9	6	9	3	8
Mary	10	5	2	7	9	8	5	6	7	6

- Multiplicative Strategy (multiplies the individual ratings) performed best
- Borda count, Average, Average without Misery and Most
   Pleasure also performed quite well
- It confirms the observations made in the previous slide users may like recs that they are not capable to generate
- Still this is a very simple recommendation scenario: imagine that each user in the group rated 100 items ...

# **Evaluating Group Recommendations**

- Our off-line approach [Baltrunas, Mackcinskas, Ricci, 2010]
- Given a group of users including the active user
- Generate two ranked lists of recommendations using a prediction model (matrix factorization) and some training data (ratings):
  - a) Either based only on the active user **individual preferences**
  - b) Or **aggregating** recommendation lists for the **group of users** (including the active user)
- Compare the recommendation list with the "true" preferences as found in the **test set** of the user
- We have used Movielens data
- Comparison is performed using Normalize Discounted
   <sup>23</sup> Cumulative Gain.

#### **Normalised Discounted Cumulative Gain**

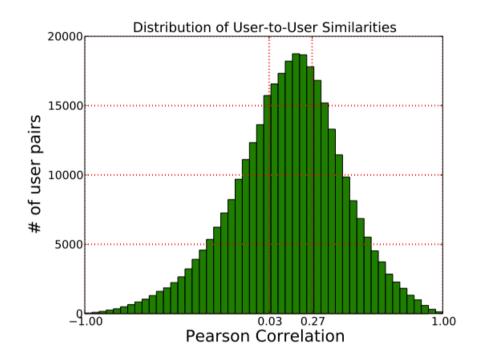
It is evaluated over the k items that are present in the user's test set

$$nDCG_{k}^{u} = \frac{1}{Z_{uk}} \sum_{i=1}^{k} \frac{r_{up_{i}}}{\log_{2}(i+1)}$$

- r<sub>upi</sub> is the rating of the item in position i for user u as it is found in the test set
- $Z_{uk}$  is a normalization factor calculated to make it so that a perfect ranking's NDCG at k for user u is 1
- It is maximal if the recommendations are ordered in decreasing value of their true ratings.

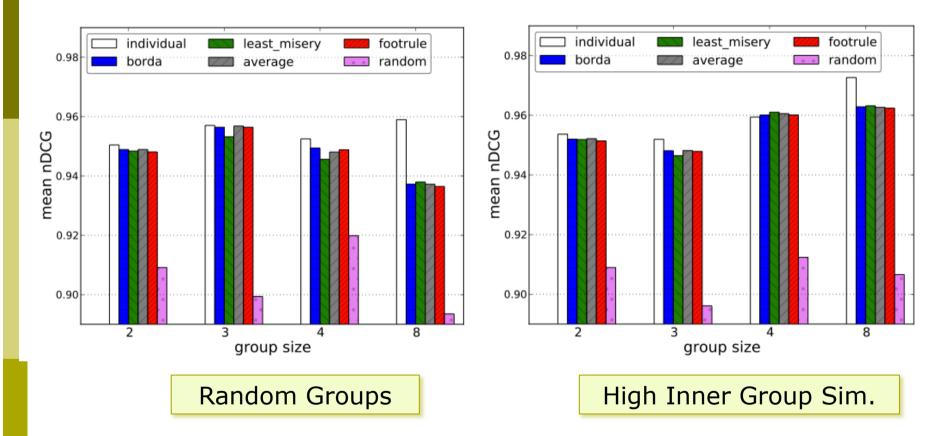
### **Building pseudo-random groups**

- Groups with high inner group similarity
- Each pair of users has Pearson correlation larger than 0.27
- One third of the users' pairs has a similarity larger that 0.27
- We built groups with:2, 3, 4 and 8 users



Similarity is computed only if the users have rated at least 5 items in common.

## **Random vs Similar Groups**



- For each experimental condition a bar shows the average over the users belonging to 1000 groups
- Training set is 60% of the MovieLens data

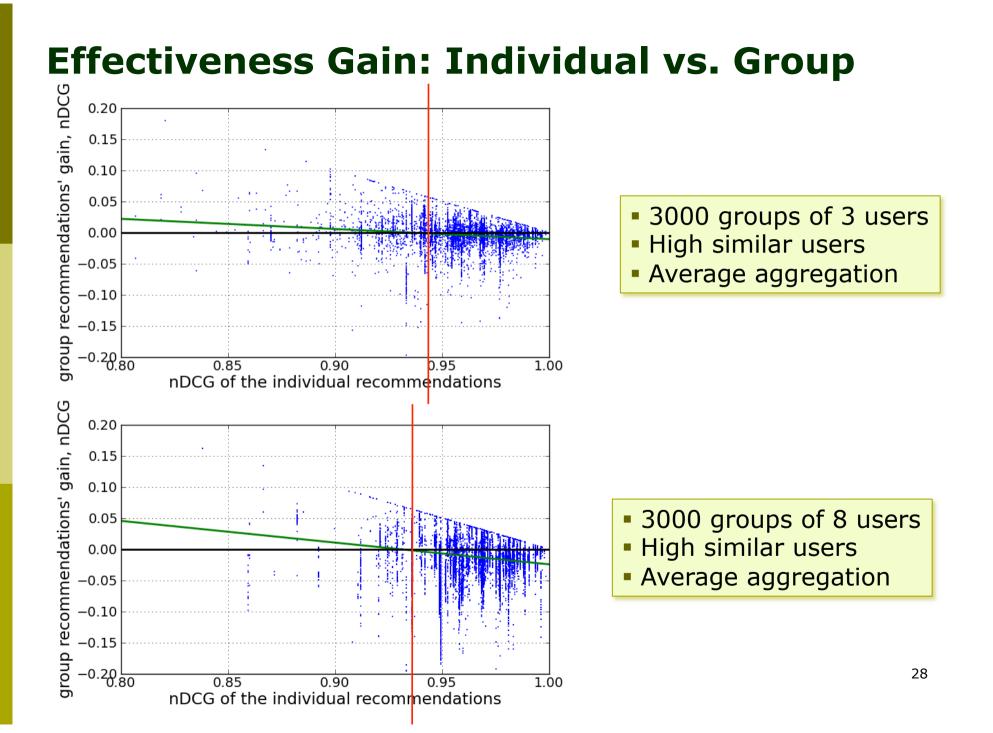
#### **Group Recommendation Gain**

Is there any gain in effectiveness (NDCG) if a recommendations is built for the group the user belongs to?

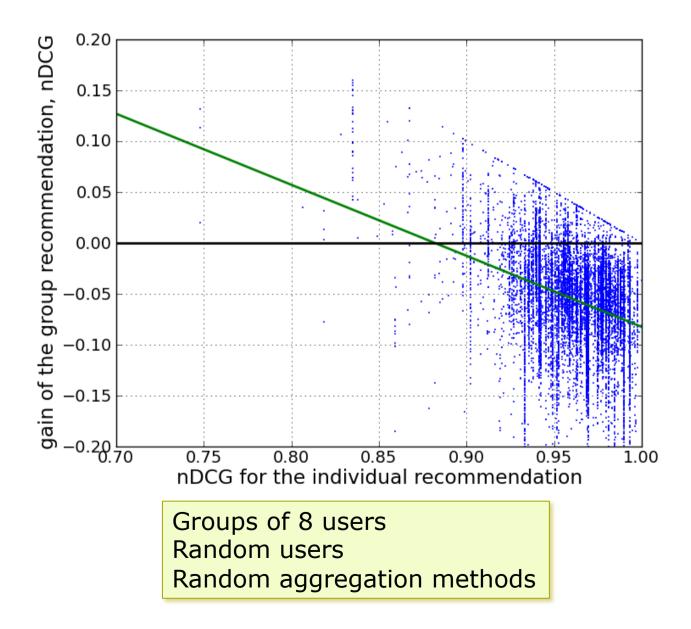
Gain(u,g) = NDCG(Rec(u,g)) - NDCG(Rec(u))

#### When there is a positive gain?

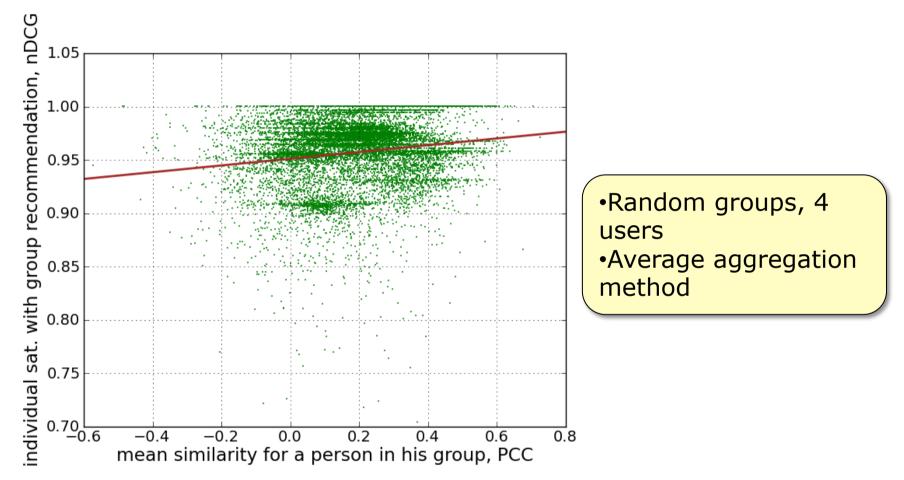
- Does the quality of the individual recommendations matter?
- Inner group similarity is important?
- Can a group recommendation be better (positive gain) than an individually tailored one?



#### **Gain vs. Effectiveness of Individual Recs**



#### **Effectiveness vs. Inner Group Sim**



The larger the inner group similarity is the better the recommendations are – as expected.

#### **Sequential Recommendations**

- How these techniques tackle sequential recommendation problems?
- The goal is to compile a sequence of recommendations that receive a large evaluation as a whole
- Examples:
  - A sequence of songs
  - A sequence of meals for the next week
  - A sequence of movies one for each time a group of friends will meet

#### **Facets of Sequential Recommendations**

- One can re-use the previous techniques and select the top-N recommendations to generate a sequence of length N
- But a sequence of recommendations can be built using other heuristics:
  - The recommendations should go well together in a given sequence: e.g., uniform mood or genre
  - If a user is not totally satisfied with one element of the sequence then he can be made happier with a next element
  - User satisfaction for an item is influenced by the previous items (aggregated satisfaction) [Mastoff & Gatt 2006]
- The recommended sequence must be evaluated as a single recommendation.

#### **Interface: initial track rating**



#### Thank you for registering.

This web application is providing music track recommendations. Recommendations can be made either individually or for a group of people that would like to listen to music together.

In order to make good recommendations we need you to leave as many track ratings as possible. To begin with we ask you to rate 30 tracks. The tracks can be rated in multiple sessions. You can leave the page and return as many times as you like.

You have rated 6 songs, 24 more left!



\* (very bad)
 \*\*
 \*\*\*
 \*\*\*\*
 \*\*\*\*
 \*\*\*\*\* (very good)

Next

[Piliponyte, 2012]

#### **Interface: recommendation making**

Home	Tracks	Groups	Rec	commendations	Predictions							
group_a4	3 🗘											
As the first task you have to make your own recommendation of a music track sequence for this group.												
Imagine that you and the other group members want to find 10 music tracks to which you could listen together (e.g. when travelling together in a car). Your goal is to find such track sequence that would make each group member sufficiently happy.									d			
When making track selection you have to take into account your own music preferences as well as the preferences of other group members. A good overview of preferences can be found in 'Groups' tab.												
	You can search for tracks via the interface below. Tracks are presented in a random order, but you can sort them by user. In this way tracks that are rated by a certain user are ordered according to the ratings.											
You have	chosen 3 sor	ngs out of	10 require	ed:								
X AC/D	× AC/DC 'Highway To Hell'											
× Nirva	na 'Come As	You Are'										
× Aeros	smith 'Walk '	This Way'										
Keyword	:			Genre: rock	🕈 Rating: a	II 🗘 So	t by user: 🔽	irga Pleskaitė 🛟	Search			
Artist		Τ	itle			Gen	e Eilwerijus Kondratas	Virga Pleskaitė	Knoedi Sechs			
AC/DC		Hi	ghway To	o Hell		rock	****	* *****		remove	play	
Pink Flo	yd		ne Happie all	est Days Of Our Live	es/Another Brick	In The rock		<b>*****</b>		add	play	
Bob Dyl	an	K	Knockin' On Heaven's Door			rock	<b>**</b> inin	<b>* ****</b> *		add	play	
Queen		Ci	Crazy Little Thing Called Love			rock	<b>**</b> @@	<b>* ****</b> *		add	play	34
Nirvana		C	Come As You Are				****	* *****		remove	play	

Home	T

н.

Predictions

#### group\_a61

This is the last phase of the experiment. We ask you to evaluate two recommendations for music track sequences that were made for your group. One of the recommendations is made by one of your group members and the other by the program. The two recommendations are put in a random order.

When evaluating the recommendations take into consideration the fact that recommendations were made for the whole group and should sufficiently satisfy each group member.

#### Recommendation 1:

cks

Artist	Title	Genre	
Nirvana	Come As You Are	rock	play
Green Day	Holiday	рор	play
John Mayall & The Bluesbreakers	Kokomo	blues	play
Rockmafia	The Big Bang	рор	play
Linkin Park	Numb	рор	play
Tom Petty	I Won't Back Down	rock	play
Jet	Are You Gonna Be My Girl	рор	play
Gonzalo Rubalcaba	The Hard One	jazz	play
Stevie Ray Vaughan	08 - Little Wing	blues	play
Steve Miller Band	The Joker	rock	play

Q1: How good is this recommended sequence for your group?	****
Q2: How good is this recommended seqence for you personally?	× _
Q3: To what extend does this recommended sequence contain interesting and unexpected tracks that you think you group would like?	**
Q4: How good is this recommended sequence compared to the one that you have suggested?	similar

show your recommendation sequence

#### **Recommendation 2:**

Artist	Title	Genre	
Muddy Waters	Long Distance Call	blues	play
Nirvana	Come As You Are	rock	play
Police	Message In A Bottle	rock	play
Police	Roxanne	rock	play
Eagles	Hotel California	rock	play
Gorillaz	Feel Good, Inc	рор	play
The Blues Label	Leadbelly - Pig meat papa	blues	play
The Dave Brubeck Quartet	Three To Get Ready	jazz	play
Pink Floyd	Hey You	rock	play
Pink Floyd	Wish You Were Here	rock	play

Q1: How good is this recommended sequence for your group?	<b>•</b>
Q2: How good is this recommended seqence for you personally?	•
Q3: To what extend does this recommended sequence contain interesting and unexpected tracks that you think you group would like?	•
Q4: How good is this recommended sequence compared to the one that you have suggested?	:

show your recommendation sequence

#### **Recommendation Techniques**

- User built: each group member builds a recommended compilation for his group
- Averaging: the tracks with the largest average predicted (or actual) ratings are selected

#### Balancing:

- the compilation is generated incrementally
- at each step a new track is added: that one minimizing the differences of the accumulated satisfactions of the users

#### Balancing with decay:

 Similar to balancing but in the computation of the user satisfaction at one step the older tracks count less.

## Balancing

If S is a sequence of tracks and M is the sequence of tracks of equal length with the highest ratings (either predicted or actual) then the satisfaction of u for S is:

$$sat(u,S) = \frac{\sum_{i \in S} r^*(u,i)}{\sum_{j \in M} r^*(u,j)}$$

If S<sub>+i</sub> is the sequence extending S with track i then the item added to S by the Balancing rule is such that

$$\operatorname{Arg\,min}_{u,v \in g} \left| \operatorname{sat}(u, S_{+i}) - \operatorname{sat}(v, S_{+i}) \right|$$

	Track1	Track2	Track3	Track4	Track5	Track6
John	3	2	5	4	5	2
Peter	4	5	2	2	1	4
Ann	5	4	3	3	4	5
Group	4	3.67	3.33	3	3.33	3.67
average:						



	Track1	Track2	Track3	Track5	Track6
John	3	2	5	5	2
Peter	4	5	2	1	4
Ann	5	4	3	4	5
Group	4	3.67	3.33	3.33	3.67
average:					

**Candidate set:** contains tracks with large average predicted ratings

#### Candidate set:

	Track1	Track2	Track3	Track5	Track6
John	3	2	5	5	2
Peter	4	5	2	1	4
Ann	5	4	3	4	5
Group	4	3.67	3.33	3.33	3.67
average:					

Sequence: track1 is the best initial option because has the largest average rating.

	Track1	Track2	Track3	Track5	Track6
John	3	2	5	5	2
Peter	4	5	2	1	4
Ann	5	4	3	4	5
Group	4	3.67	3.33	3.33	3.67
average:					

<Track1, Track3> minimizes the satisfaction differences among group members

Sequence	Sat(John,s)	Sat(Peter,s)	Sat(Ann,s)	Sat differences
Track1, Track2	5/10	9/9	9/10	1
Track1, Track3	8/10	6/9	8/10	0.267
Track1, Track5	8/10	5/9	9/10	0.689
Track1, Track6	5/10	8/9	10/10	0.999

	Track1	Track2	Track3	Track5	Track6
John	3	2	5	5	2
Peter	4	5	2	1	4
Ann	5	4	3	4	5
Group	4	3.67	3.33	3.33	3.67
average:					

<Track1, Track3, Track2> is the balancing sequence with 3 tracks

Sequence	Sat(John,s)	Sat(Peter,s)	Sat(Ann,s)	Sat differences
Track1, Track3, Track2	10/13	11/13	12/14	0.176
Track1, Track3, Track5	13/13	7/13	12/14	0.539
Track1, Track3, Track6	10/13	10/13	13/14	0.318

## Comparison

#### **•** Rank aggregation with average:

	Track1	Track2	Track3	Track4	Track5	Track6
John	3	2	5	4	5	2
Peter	4	5	2	2	1	4
Ann	5	4	3	3	4	5
Group	4	3.67	3.33	3	3.33	3.67
average:						

#### **B**alancing:

	Track1	Track2	Track3	Track4	Track5	Track6
John	3	2	5	4	5	2
Peter	4	5	2	2	1	4
Ann	5	4	3	3	4	5
Group	4	3.67	3.33	3	3.33	3.67
average:						

## **Experimental setup**

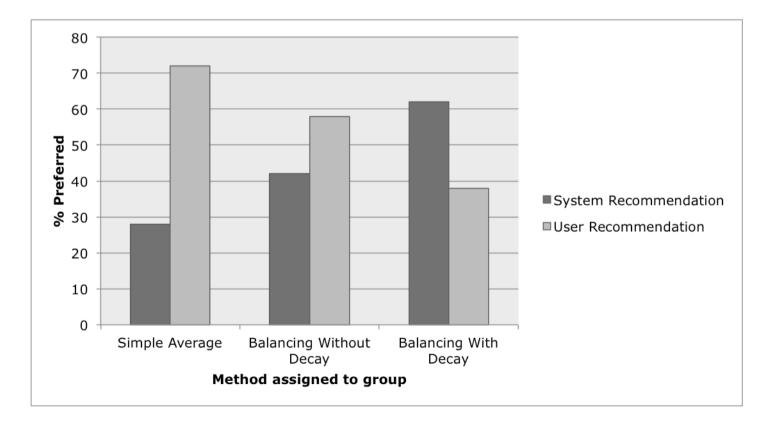
- Large scale live user study
- Fully functional sequential group recommender
- We compared:
  - Balancing' without Decay
  - Balancing' with Decay
  - Average
  - User generated
- Participant tasks included:
  - Rate music tracks
  - Get assigned into groups
  - Compile a sequence suggestion to one's group
  - Evaluate other track sequences

## **Experimental setup II**

- Music track corpus of 1068 tracks
- 77 users have left 5160 ratings with the average of 67 ratings per user and 5 ratings per track
- Out of 38 groups created 32 have finished the experiment at least partly
- Each group was assigned one of the three methods to be tested: 'Average', 'Balancing without Decay' and 'Balancing with Decay'

### **Results: preferred sequence**

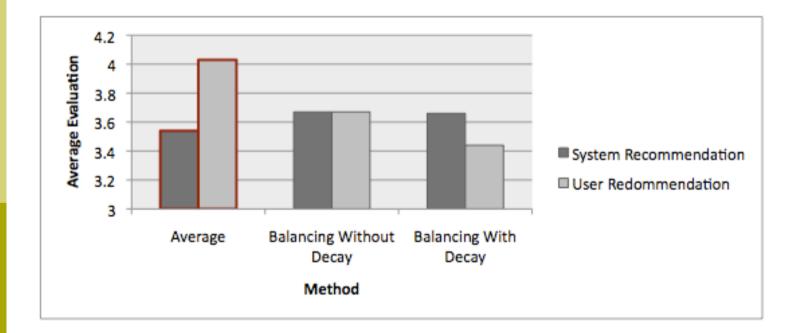
- Choice between system produced and human made recommendations:
  - Q5: Which recommended sequence would you select for your group? 
    Sequence 1
    Sequence 2



#### **Results: goodness for group**

Q1: How good is this recommended sequence for your group?





#of users per condition: Average 39; Balancing 26, Balancing with decay 24.

#### **Results: novelty**

Q3: To what extend does this recommended sequence contain interesting and unexpected tracks that you think you group would like?

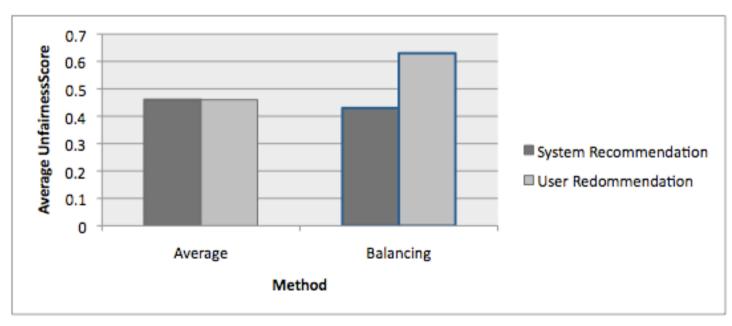




### **Results: fairness**

Group recommendation is fair if the following two are close:

- Goodness for group (Q1) Q1: How good is this recommended sequence for your group?
- Personal satisfaction (Q2)<sup>Q2:</sup> How good is this recommended sequence for you personally?
- For each group member calculate the absolute difference:
   |Q1 Q2|
- Take an average of those differences as an unfairness score for the group (the smaller the score, the better results)



# Human Rec. Strategies

More than 10 strategies were found analysing the user comments about how they built music track sequences

Strategy type	Comment
Intersection of everyone's preferences	"Sorted tracks by user evaluations and picked the ones that all group members marked with 5 or 4 stars."
Compromise (a bit for each)	"Chose songs, highly rated by one of the members, each member a few."
Compromise (at least not hated by anybody)	"Not many ratings in common so I chose songs which had minimum 3 stars from minimum 2 users."
Guessing/reasoning from available information	"First, I looked for tracks with high ratings by all members. I then filled up the list with tracks that were rated by one member only but, based on what other members liked, I thought they would have been rated highly by the other members as well, had they listened to them.";
Own preferences first	
Egoistic	"I have chosen the baroque style music, since it is not very popular among people, but I think everyone should be at least 4 familiar to it.";

## **Stable vs. Ephemeral groups**

- The more recommendations are requested (at different times) the more opportunities the system has to balance users' preferences
- Stable groups call for sequential recommendations and balancing approaches
- More shared knowledge of the group members' preferences – change the attitude of the members towards the recommendations
- The social role and the reciprocal influence of group members must be modeled.

# **Group Recs for Individual Users (I)**

 Aggregating recommendations for different users is similar to aggregating multi criteria recommendations (ex. 3 criteria for news recommendations)

	А	В	С	D	Е	F	G	Н	Ι	J
Topic Location	10 1				10 7		6 6	8 9	10 3	8 8
Recency	10	5	2	7	9	8	5	6	7	6

- We must not treat all the criteria in the same way (they are not anymore representing users)
- **Weights** differently the criteria
- Use the criteria to better predict the user final overall satisfaction.

## Group Recs for Individual Users (II)

- Aggregating recommendations for different users can be used to tame the cold-start problem
- User-based collaborative filtering is "aggregating" recommendations of neighbor users
- The two main group recommendation approaches (profile aggregation and recommendation aggregation) can be turned into techniques for individual users recommendations
  - How many fictitious group members?
  - How much to weight the target user (few and unreliable ratings) vs. the other group members' preferences in cold-start situations?
  - How this compare with state of the art techniques (Matrix factorization).

# **Conclusions (I)**

- Rank aggregation techniques provide a viable approach to group recommendation
- Group recommendations may be better than individual recommendations
  - Both for random groups and high similar groups
- Users are more similar among them as one can expect
- It could be used as an individual recommendation technique: search for similar users – make individual predictions to all of them and then aggregate the predictions for the target user
- Groups with high inner similarity (generally) have better group recommendations.

# **Conclusions (II)**

- First online study where users evaluated system generated group recommendations (vs. user generated)
- For generating sequences of recommendation
   `Balancing' outperforms state of the art (averaging)
- Balancing performs well even compared to humanmade recommendations
- Average' method inferior to human recommendations when considering:
  - Overall quality
  - Goodness for the group
  - Novelty

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