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UNSTEADY MHD THREE DIMENSIONAL FLOW OF MAXWELL FLUID THROUGH POROUS MEDIUM IN A PARALLEL PLATE CHANNEL UNDER THE INFLUENCE OF INCLINED MAGNETIC FIELD

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ABSTRACT

In this paper, we discuss the unsteady hydro magnetic flow of an electrically conducting Maxwell fluid in a parallel plate channel bounded by porous medium under the influence of a uniform magnetic field of strength Ho inclined at an angle of inclination with the normal to the boundaries. The perturbations are created by a constant pressure gradient along the plates. The time required for the transient state to decay and the ultimate steady state solution are discussed in detail. The exact solutions for the velocity of the Maxwell fluid consists of steady state are analytically derived, its behaviour computationally discussed with reference to the various governing parameters with the help of graphs. The shear stresses on the boundaries are also obtained analytically and their behaviour is computationally discussed in detail.

KEYWORDS: *Maxwell fluids, unsteady flows, porous medium, parallel plate channels, MHD flows*

I. INTRODUCTION

Several fluids including butter, cosmetics and toiletries, paints, lubricants, certain oils, blood, mud, jams, jellies, shampoo, soaps, soups, and marmalades have rheological characteristics and are referred to as the non-Newtonian fluids. The rheological properties of all these fluids cannot be explained by using a single constitutive relationship between stress and shear rate which is quite different than the viscous fluids [1, 2]. Such understanding of the non-Newtonian fluids forced researchers to propose more models of non-Newtonian fluids. In general, the classification of the non-Newtonian fluid models is given under three categories which are called the differential, the rate, and the integral types [3]. Out of these, the differential and rate types have been studied in more detail. In the present analysis we discuss the Maxwell fluid which is the subclass of rate-type fluids which take the relaxation phenomenon into consideration. It was employed to study various problems due to its relatively simple structure. Moreover, one can reasonably hope to obtain exact solutions from Maxwell fluid. This motivates us to choose the Maxwell model in this study. The exact solutions are important as these provide standard reference for checking the accuracy of many approximate solutions which can be numerical or empirical in nature. They can also be used as tests for verifying numerical schemes that are being developed for studying more complex flow problems [4–9]. On the other hand, these equations in the non-Newtonian fluids offer exciting challenges to mathematical physicists for their exact solutions. The equations become more problematic, when a non-Newtonian fluid is discussed in the presence of MHD and porous medium. Despite this fact, various researchers are still making their interesting contributions in the field (e.g., see some recent studies [1–15]). Few investigations which provide the examination of non-Newtonian fluids in a rotating frame are also presented [1–19]. Recently Faisal Salah [20] discussed two explicit examples of acceleration subject to a rigid plate are taken into account. Constitutive equations of a Maxwell fluid are used and modified Darcy's law has been utilized. The exact solutions to the resulting problem are developed by Fourier sine transform. With respect to physical applications, the graphs are plotted in order to illustrate the variations of embedded flow parameters. The mathematical results of many existing situations are shown as the special cases of that study. Such studies have special relevance in meteorology, geophysics, and astrophysics. Hayat *et.al* [21] investigated to analyze the MHD rotating flow of a Maxwell fluid through a porous medium in parallel plate channel. M.V. Krishna [22] discussed analytical solution for the unsteady MHD flow is constructed in a rotating non-Newtonian fluid through a porous medium taking hall current into account. In this paper, we examine the MHD flow of Maxwell fluid through a porous medium in a parallel plate channel with inclined magnetic field, the perturbations in the flow are created by a constant pressure gradient along the plates. The time required for the transient effects to decay and the ultimate steady state solution are discussed in detail. The exact solutions of the velocity in the Maxwell fluid consists of steady state are analytically derived, its behaviour computationally discussed with reference to the various governing parameters with the help of graphs. The shear stresses on the boundaries are also obtained analytically and their behaviour is computationally discussed.

II. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the unsteady flow of an electrically conducting Maxwell fluid through porous medium in a parallel plate channel subjected to a uniform transverse magnetic field of strength H_o inclined at an angle of inclination α normal to the channel walls. The boundary plates are assumed to be parallel to *xy*-plane and the magnetic field to the *z*-axis in the transverse *xz*-plane. The component along *z*-direction induces a secondary flow in that direction while its *x*-components changes perturbation to the axial flow. At $t > 0$ the fluid is driven by a prescribed pressure gradient parallel to the channel walls. We choose a Cartesian system O(*x, y, z*) such that the boundary walls are at $z = 0$ and $z = l$, since the plates extends to infinity along *x* and *y* directions, all the physical quantities except the pressure depend on *z* and *t* alone. The unsteady hydro magnetic equations governing the electrically conducting Maxwell fluid under the influence of transverse magnetic field with reference to a frame are

$$\rho\left[\frac{\partial V}{\partial t} + (V.\nabla)V\right] = -\nabla p + div\,S + J \times B + R \tag{2.1}$$

$$\nabla.V = 0 \tag{2.2}$$

$$\nabla.B = 0 \tag{2.3}$$

$$\nabla \times B = \mu_m J \tag{2.4}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2.5}$$

Where, *J* is the current density, *B* is the total magnetic field, *E* is the total electric field, μ_m is the magnetic permeability, *V* = (*u, v, w*) is the velocity field, *T* is the Cauchy stress tensor, *B* is the total magnetic field so that $B = B_0 Sin\alpha + b$, where B_0 is the applied magnetic field parallel to the *z*-axis and *b* is the induced magnetic field. The induced magnetic field is negligible so that the total magnetic field $B = (0, 0, B_0 Sin\alpha)$, the Lorentz force $J \times B = -\sigma B_0^2 Sin^2\alpha V$, σ is the electrical conductivity of the fluid, ρ is the density of the fluid, and $\frac{D}{Dt}$ is the material derivative and *R* is the Darcy resistance. The extra tensor ***S*** for a Maxwell fluid is

$$T = -\,p\,I + S \tag{2.6}$$

$$S + \lambda\left(\frac{DS}{Dt} - LS - SL^T\right) = \mu A \tag{2.7}$$

where $-pI$ is the stress due to constraint of the impermeability, here p is the static fluid pressure, I is the identity tensor, μ is the viscosity of the fluid, λ is the material time constants referred to as relaxation time, it is assumed that $\lambda \geq 0$. The first Rivlin-Ericksen tensor A_1 is defined as

$$A_1 = (\text{grad } V) + (\text{grad } V)^T \tag{2.8}$$

It should be noted that this model includes the viscous Navier-Stokes fluid as a special case for $\lambda = 0$. Let us indicate the stress tensor and the velocity component as

$$V(z, t) = (u, 0, w) \tag{2.9}$$

According to Tan and Masuoka [4] Darcy's resistance in an Oldroyd-B fluid satisfies the following expression:

$$\left(1+\lambda\frac{\partial}{\partial t}\right)R = -\frac{\mu\phi}{k}\left(1+\lambda_r\frac{\partial}{\partial t}\right)V \tag{2.10}$$

where λ_r is the retardation time, ϕ is the porosity (0<ϕ<1), and k is the permeability of the porous medium. For Maxwell fluid λ_r= 0, and hence,

$$\left(1+\lambda\frac{\partial}{\partial t}\right)R = -\frac{\mu\phi}{k}V \tag{2.11}$$

Making use of the equations (2.6), (2.7) and (2.8), the equation (2.1) reduces to

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}+\frac{\partial S_{xz}}{\partial z}-\sigma B_0^2 Sin^2\alpha\, u + R_x \tag{2.12}$$

$$\rho\frac{\partial w}{\partial t} = \frac{\partial S_{yz}}{\partial z}-\sigma B_0^2 Sin^2\alpha\; w + R_z \tag{2.13}$$

Where R_x and R_z are x and z-components of Darcy's resistance R;

$$\left(1+\lambda\frac{\partial}{\partial t}\right)S_{xz} = \mu\frac{\partial u}{\partial z} \text{ and } \left(1+\lambda\frac{\partial}{\partial t}\right)S_{yz} = \mu\frac{\partial w}{\partial z} \tag{2.14}$$

The equations (2.12) and (2.13) reduces to

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}+\frac{\partial S_{xz}}{\partial z}-\sigma B_0^2 Sin^2\alpha\; u-\frac{\mu\varphi}{k}\; u \tag{2.15}$$

$$\rho\frac{\partial w}{\partial t} = \frac{\partial S_{yz}}{\partial z}-\sigma B_0^2 Sin^2\alpha\; v-\frac{\mu\varphi}{k}\; w \tag{2.16}$$

Let $q = u + iw$ Combining equations (2.15) and (2.16), we obtain

$$\rho\frac{\partial q}{\partial t} = -\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}(S_{xz}+iS_{yz})-\sigma B_0^2 Sin^2\alpha\, q-\frac{\mu\varphi}{k}q \tag{2.17}$$

Since

$$\left(1+\lambda\frac{\partial}{\partial t}\right)(S_{xz}+iS_{yz}) = \mu\frac{\partial q}{\partial z} \tag{2.18}$$

Substituting the equation (2.18) in the equation (2.17), we obtain the equation for the governing the flow through a porous medium with respect to the rotating frame is given by

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial q}{\partial t}+\left(\frac{\sigma B_0^2 Sin^2\alpha}{\rho}+\frac{v\phi}{k}\right)\left(1+\lambda\frac{\partial}{\partial t}\right)q = -\frac{1}{\rho}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x}+v\frac{\partial^2 q}{\partial z^2} \tag{2.19}$$

The boundary and initial conditions are

$$q = 0 \quad t > 0, \quad z = 0 \tag{2.20}$$

$$q = 0, \quad t \neq 0, \quad z = l \tag{2.21}$$

$$q(z,t) = 0, \quad \frac{dq(z,t)}{dt} = 0, \quad t \leq 0, \quad for\ all\ \ z \tag{2.22}$$

We introduce the following non dimensional variables are

$$z^* = \frac{z}{l},\ q^* = \frac{q\,l}{v},\ t^* = \frac{tv}{l^2}, \omega^* = \frac{\omega l^2}{v},\ \xi^* = \frac{\xi}{l},\ P^* = \frac{Pl^2}{\rho v^2}$$

Using non dimensional variables the governing equations are (dropping asterisks in all forms)

$$\left(1+\beta_1\frac{\partial}{\partial t}\right)\frac{\partial q}{\partial t}+(M^2Sin^2\alpha+D^{-1}\phi)\left(1+\beta_1\frac{\partial}{\partial t}\right)q=\frac{1}{\rho}\left(1+\beta_1\frac{\partial}{\partial t}\right)P+\frac{\partial^2 q}{\partial z^2} \tag{2.23}$$

where, $M^2=\frac{\sigma\mu_e{}^2H_0{}^2l^2}{\rho\nu}$ is the Hartmann number, $D^{-1}=\frac{l^2}{k}$ is the inverse Darcy Parameter, $\beta_1=\frac{\lambda\nu}{l^2}$ is the material parameter related to relaxation time and $P=-\frac{\partial p}{\partial x}$ is the pressure gradient.

The corresponding initial and boundary conditions are

$$q=0 \quad t>0, \quad z=0 \tag{2.24}$$

$$q=0, \quad t\neq 0, \quad z=1 \tag{2.25}$$

$$q(z,t)=0, \quad \frac{dq(z,t)}{dt}=0, \quad t\leq 0, \quad for\ all\ \ z \tag{2.26}$$

supposing the pressure is given by

$$P=\begin{cases}P_0+P_1e^{i\omega_1 t}, & t>0 \quad \forall\ z,\\ 0\ , & t<0 \quad \forall\ z\end{cases} \tag{2.27}$$

Taking Laplace transforms of equations (2.23) and (2.27) using initial conditions (2.26) the governing equations in terms of the transformed variable reduces to

$$\frac{d^2\bar{q}}{dz^2}-\left[\beta_1 s^2+(1+\beta_1(M^2Sin^2\alpha+D^{-1}\phi))s+(M^2Sin^2\alpha+D^{-1}\phi)\right]\bar{q}=$$

$$-(1+i\beta_1\omega_1)\frac{P_1}{s-i\omega_1}-\frac{P_0}{s} \tag{2.28}$$

Solving equation (2.28) subjected to the conditions (2.24) and (2.25), we obtain

$$\begin{aligned}\bar{q}=&-\frac{P_1(1+i\beta_1\omega_1)Cosh(\lambda_1 z)}{\lambda_1{}^2(s-i\omega_1)}-\frac{P_0\,Cosh(\lambda_1 z)}{\lambda_1{}^2 s}\\&+\frac{P_1(1+i\beta_1\omega_1)Cosh(\lambda_1)Sinh(\lambda_1 z)}{\lambda_1{}^2(s-i\omega_1).Sinh(\lambda_1)}+\frac{P_0\,Cosh(\lambda_1).Sinh(\lambda_1 z)}{\lambda_1{}^2 s.\,Sinh(\lambda_1)}-\\&-\frac{P_1(1+i\beta_1\omega_1).Sinh(\lambda_1 z)}{\lambda_1{}^2(s-i\omega_1).\,Sinh(\lambda_1)}-\frac{P_0.Sinh(\lambda_1 z)}{\lambda_1{}^2 s.Sinh(\lambda_1)}+\frac{P_1(1+i\beta_1\omega_1)}{\lambda_1{}^2(s-i\omega_1)}+\frac{P_0}{\lambda_1{}^2 s(1+s\alpha)}\end{aligned} \tag{2.29}$$

Where $\lambda_1^2=\beta_1 s^2+(1+\beta_1(M^2Sin^2\alpha+D^{-1}\varphi))s+(M^2\,Sin^2\alpha+D^{-1}\varphi)$

Taking the inverse Laplace transforms to the equations (2.29) on both sides, We obtain

$$\begin{aligned}q=&-\frac{P_0\ Cosh(b_0 z)}{b_0^2}+\frac{P_0\ Cosh(b_0).Sinh(b_0 z)}{b_0^2\ Sinh(b_0)}-\frac{P_0\ \ Sinh(b_0 z)}{b_0^2\ Sinh(b_0)}+\frac{P_0}{b_0^2}+\\&+\frac{P_1(1+i\beta_1\omega_1)}{(i\omega_1-s_1)(i\omega_1-s_2)}\left\{-Cosh(b_4 z)+\frac{Cosh(b_4).Sinh(b_4 z)}{Sinh(b_4)}-\right.\\&\left.-\frac{Sinh(b_4 z)}{Sinh(b_4)}+1\right\}e^{i\omega_1 t}+\left\{-\frac{P_1(1+i\beta_1\omega_1).Cosh(b_3 z)}{(s_1-s_2)(s_1-i\omega_1)}+\right.\\&+\frac{P_1(1+i\beta_1\omega_1).Cosh(b_3).Sinh(b_3 z)}{(s_1-s_2)(s_1-i\omega_1).Sinh(b_3)}-\frac{P_1(1+i\beta_1\omega_1).Sinh(b_3 z)}{(s_1-s_2)(s_1-i\omega_1)Sinh(b_3)}\\&+\frac{P_1(1+i\beta_1\omega_1)}{(s_1-s_2)(s_1-i\omega_1)}-\frac{P_0\,Cosh(b_3 z)}{(s_1-s_2)s_1}+\frac{P_0\,Cosh(b_3).Sinh(b_3 z)}{(s_1-s_2)(s_1).Sinh(b_3)}-\\&\left.-\frac{P_0\,.Sinh(b_3 z)}{(s_1-s_2)(s_1).Sinh(b_3)}+\frac{P_0}{(s_1-s_2)(s_1)}\right\}e^{s_1 t}+\end{aligned}$$

$$+\left\{-\frac{P_1(1+i\beta_1\omega_1).Cosh(b_7 z)}{(s_2-s_1)(s_2-i\omega_1)}+\right.$$
$$+\frac{P_1(1+i\beta_1\omega_1).Cosh(b_7).Sinh(b_7 z)}{(s_2-s_1)(s_2-i\omega_1).Sinh(b_7)}-\frac{P_1(1+i\beta_1\omega_1).Sinh(b_7 z)}{(s_2-s_1)(s_2-i\omega_1)Sinh(b_7)}$$
$$+\frac{P_1(1+i\beta_1\omega_1)}{(s_2-s_1)(s_2-i\omega_1)}-\frac{P_0\,Cosh(b_7 z)}{(s_2-s_1)s_2}+\frac{P_0\,Cosh(b_7).Sinh(b_7 z)}{(s_2-s_1)s_2.Sinh(b_7)}-$$
$$\left.-\frac{P_0.Sinh(b_7 z)}{(s_2-s_1)s_2.Sinh(b_7)}+\frac{P_0}{(s_2-s_1)s_2}\right\}e^{s_2 t}+$$
$$\sum_{n=0}^{\infty}\left\{\frac{P_1(1+i\beta_1\omega_1).Cosh(b_6).Sinh(b_6 z)}{b_6^2(s_3-s_4)(s_3-i\omega_1)}-\frac{P_1(1+i\beta_1\omega_1).\,Sinh(b_6 z)}{b_6^2(s_3-s_4)(s_3-i\omega_1)}+\right.$$
$$\left.+\frac{P_0\ Cosh(b_6).Sinh(b_6 z)}{b_6^2(s_3-s_4)(s_3)}-\frac{P_0\ Sinh(b_6 z)}{b_6^2(s_3-s_4)(s_3)}\right\}e^{s_3 t}$$
$$+\sum_{n=0}^{\infty}\left\{\frac{P_1(1+i\beta_1\omega_1).Cosh(b_5).Sinh(b_5 z)}{b_5^2(s_4-s_3)(s_4-i\omega_1)}-\frac{P_1(1+i\beta_1\omega_1).\,Sinh(b_5 z)}{b_5^2(s_4-s_3)(s_4-i\omega_1)}+\right.$$
$$\left.+\frac{P_0\ Cosh(b_5).Sinh(b_5 z)}{b_5^2(s_4-s_3)(s_4)}-\frac{P_0\ Sinh(b_5 z)}{b_5^2(s_4-s_3)(s_4)}\right\}e^{s_4 t} \quad (2.30)$$

(Where the constants are mentioned in the appendix)

The shear stresses on the upper and lower plate are given by

$$\tau_U=\left(\frac{dq}{dz}\right)_{z=1} \text{ and } \tau_L=\left(\frac{dq}{dz}\right)_{z=0} \quad (2.31)$$

III. RESULTS AND DISCUSSION

We discuss the unsteady flow of an electrically conducting Maxwell fluid through a porous medium in parallel plate channel subjected to uniform magnetic field. In unperturbed state the perturbation are created by performing to imposition of constant pressure gradient along the axis (*OX*) of the channel walls the velocity component along the imposed pressure gradient and normal to it. Under the boundary layer assumptions these velocity components are to functions of *z* and *t* alone, where *z* corresponds to the direction of axis of the channel. The transverse magnetic field once arising give rise to Lorentz forces resisting the flow along normal to the channel wall.

The constitutive equations relating the stress and rate of strain are chosen to depict the Maxwell fluid. The Brinkman's model has been chosen to analyses the flow through a porous medium. The equation governing the velocity components and with reference to frame ultimately can be combined into a single equation by defining the complex velocity $q=u+iw$ The expression for the components of the stresses are manipulated from the stress and strain relationships. Under these assumptions the ultimate governing equations for the unsteady flow through a porous medium with reference to frame is formulated the corresponding boundary and initial conditions. This boundary value problem has been solved using non-dimensional variables making use of Laplace transform technique.

The solution for the combined velocity *q* consists of two kinds of terms 1. Steady state 2. The transient terms involving exponentially varying time dependence. The analysis of transient terms indicates that this transient velocity decay exponentially in dimensionless time to of order i.e., $t>\max\left\{\beta_1,\frac{1}{|s_3|},\frac{1}{|s_4|}\right\}$. This decay in the transient term depends on the non-dimensional parameters β_1, M and D^{-1} . When these transient terms decay the ultimate velocity consists of steady and oscillatory components.

$$(q)_{steady} == -\frac{P_0\ Cosh(b_0 z)}{b_0^2} + \frac{P_0\ Cosh(b_0).Sinh(b_0 z)}{b_0^2\ Sinh(b_0)} - \frac{P_0\ Sinh(b_0 z)}{b_0^2\ Sinh(b_0)} + \frac{P_0}{b_0^2} \quad (3.1)$$

The flow governed by the non-dimensional parameters namely viz. M the magnetic field parameter (the Hartmann number), D^{-1} the inverse Darcy parameter, β_1 is the material time parameter referred as relaxation time. The computational analysis has been carried out to discuss the behaviour of velocity components u and w on the flow in the rotating parallel plate channel and the lower plate executes non-torsional oscillations in its own plane with reference to variations in the governing parameters may be analyzed from figures (1-3) and (4-6) respectively (P_0=P_1=10, t =0.1, $\omega_1 = \pi/4, \alpha = \pi/3$).

We may note that the effect of the magnetic field on the flow from figures (1 and 4). The magnitude of the velocity component u reduces and the velocity component w increases with increase in the Hartmann number M. However, the resultant velocity reduces throughout the fluid region with increase in the intensity of the magnetic field (the Hartmann number M). The figures (2 and 5) represent the velocity profiles with different variation in the inverse Darcy parameter D^{-1}. We find that the magnitude of u reduces with decrease in the permeability of the porous medium, while the magnitude of w experiences a slight enhancement with increase in the inverse Darcy parameter D^{-1}. It is interesting to note that lesser the permeability of the porous medium lower the magnitude of the resultant velocity. i.e., the resultant velocity reduces throughout the fluid region with increase in the inverse Darcy parameter D^{-1}. Both the velocity components u and w enhances with increase in the relaxation time entire fluid region. These displayed in the figures (3 and 6). The resultant velocity enhances throughout the fluid region with increase in the relaxation time. The shear stresses on the upper and lower plates have been calculated with reference to variations in the governing parameters and are tabulated in the tables (I-IV). On the upper plate the magnitude of the stresses τ_x enhances with increase in M and β_1 , while it reduces with increase in the inverse Darcy parameter D^{-1}. The magnitude of the stresses τ_y enhances with increase in for all governing parameters M, $D^{-1}$and β_1 (tables. I-II). On the lower plate the magnitude of the stresses τ_x and τ_y enhances with increase in M and β_1 , while these reduces with increase in the inverse Darcy parameter D^{-1} (tables. III-IV).

IV. CONCLUSIONS

1. The resultant velocity reduces throughout the fluid region with increase in the intensity of the magnetic field (the Hartmann number M).
2. Lesser the permeability of the porous medium lower the magnitude of the resultant velocity. *i.e.*, the resultant velocity reduces throughout the fluid region with increase in the inverse Darcy parameter D^{-1}.
3. Both the velocity components u and w and the resultant velocity enhances with increase in the relaxation time in the entire fluid region.
4. On the upper plate the magnitude of the stresses τ_x enhances with increase in M and β_1, while it reduces with increase in the inverse Darcy parameter D^{-1}.
5. The magnitude of the stresses τ_y enhances with increase in for all governing parameters M, D^{-1} and β_1 . On the lower plate the magnitude of the stresses τ_x and τ_y enhances with increase in M, and β_1 , while these reduces with increase in the inverse Darcy parameter D^{-1}.

V. GRAPHS AND TABLES

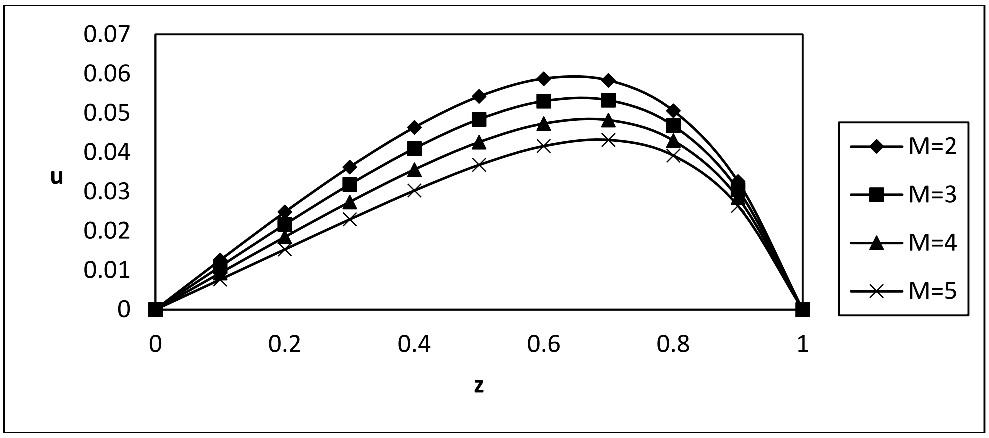

Fig. 1: The velocity profile for *u* with *M*.

$\beta_1 = 1$, D^{-1} =2000,

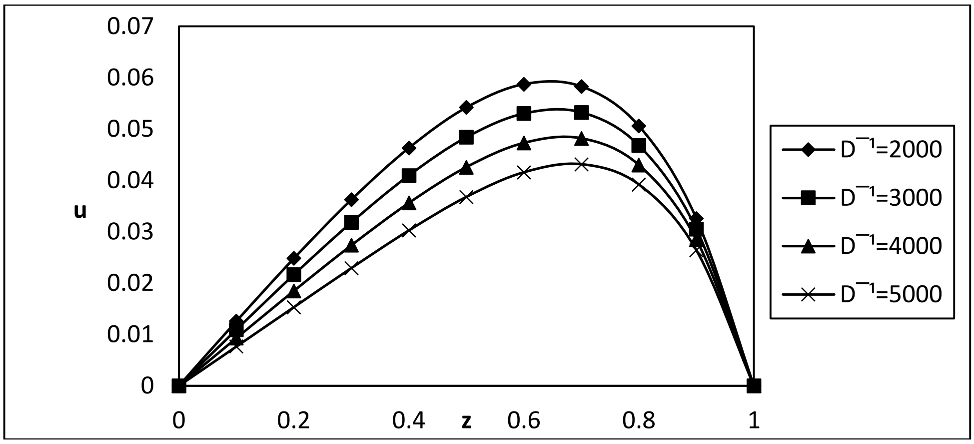

Fig. 2: The velocity profile for *u* with D^{-1}.

$\beta_1 = 1$, *M=2*

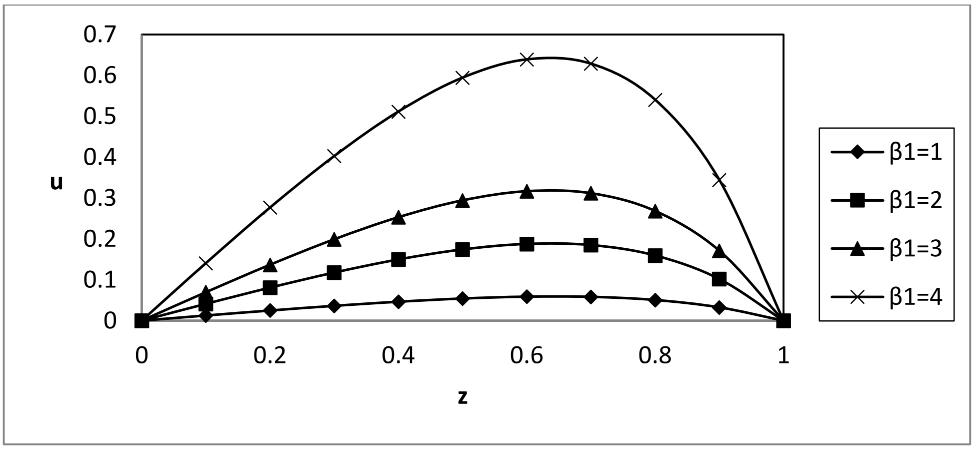

Fig. 3: The velocity profile for *u* with β_1 .

D^{-1} =2000, *M=2*

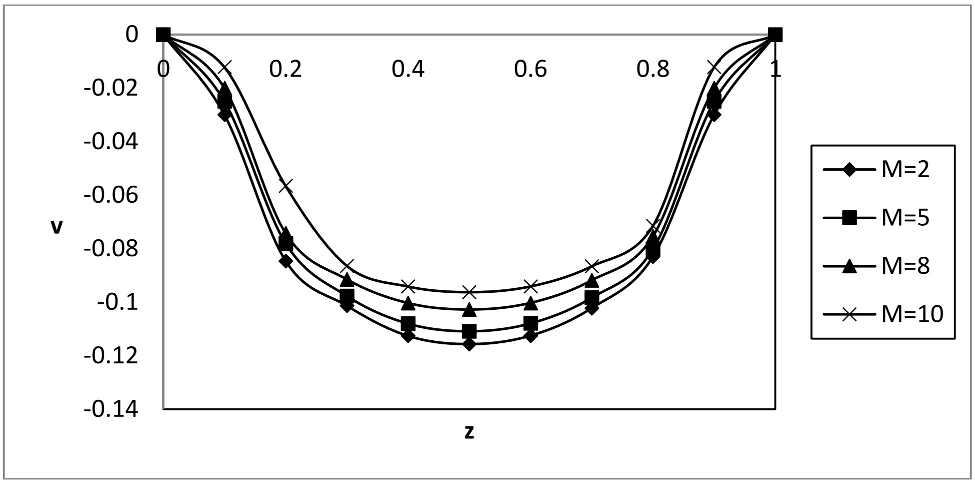

Fig. 4: The velocity profile for *w* with *M*.

$\beta_1 = 1, \ D^{-1} = 2000,$

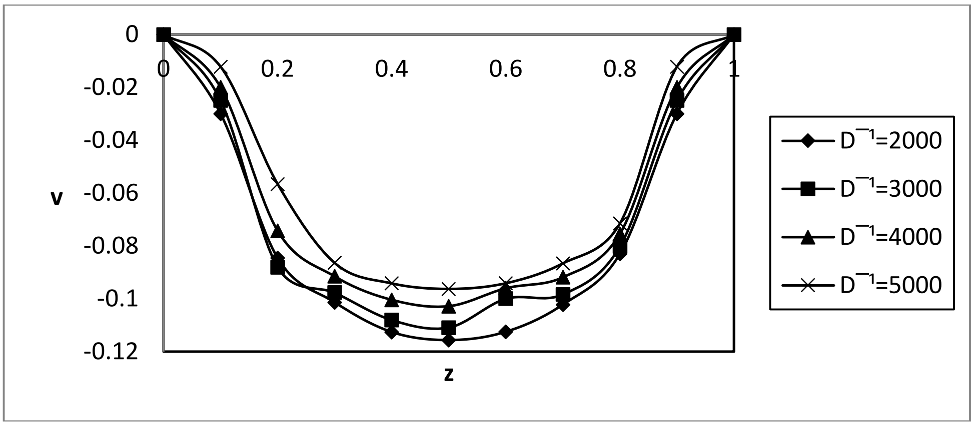

Fig. 5: The velocity profile for *w* with D^{-1}.

$\beta_1 = 1, \ M=2$

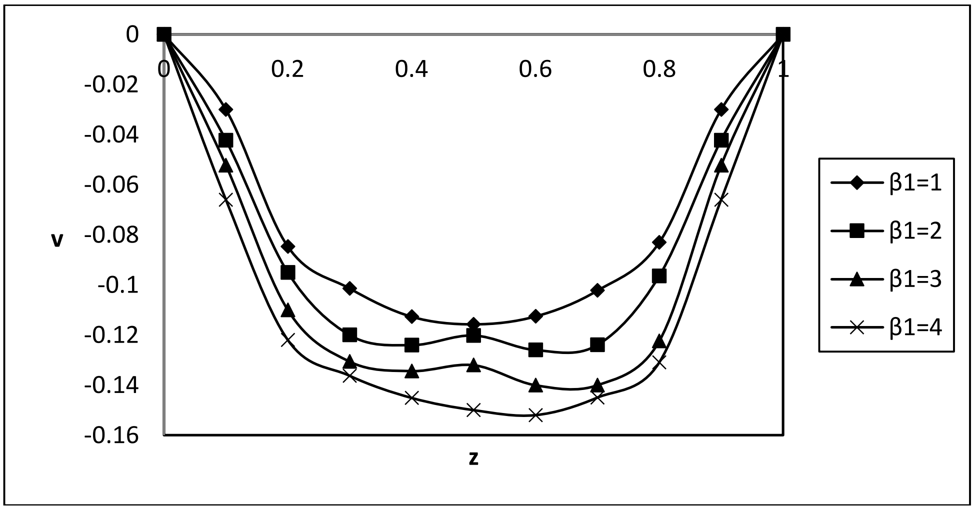

Fig. 6: The velocity profile for *w* with β_1.

$D^{-1} = 2000, M=2$

Table I: The shear stresses (τ_x) on the upper plate

$P_0 = P_1$	I	II	III	IV	V	VI	VII
2	0.084673	0.156783	0.246352	0.062501	0.046782	0.107466	0.145336
4	0.121453	0.186299	0.268751	0.116002	0.083146	0.144236	0.181673
6	0.146755	0.208888	0.278752	0.118208	0.121482	0.180083	0.256335
10	0.163752	0.408755	0.544799	0.127436	0.118442	0.207853	0.501652

	I	II	III	IV	V	VI	VII
M	2	5	8	2	2	2	2
D^{-1}	2000	2000	2000	3000	4000	2000	2000
β_1	5	5	5	5	5	6	8

Table II: The shear stresses (τ_y) on the upper plate

$P_0 = P_1$	I	II	III	IV	V	VI	VII
2	-0.01467	-0.02561	-0.03216	-0.01565	-0.01682	-0.01512	-0.01811
4	-0.01814	-0.02848	-0.04821	-0.01255	-0.02845	-0.02147	-0.02533
6	-0.02107	-0.03245	-0.04552	-0.02856	-0.03215	-0.02658	-0.03275
10	-0.04251	-0.06837	-0.07550	-0.05478	-0.06253	-0.05865	-0.08314

	I	II	III	IV	V	VI	VII
M	2	5	8	2	2	2	2
D^{-1}	2000	2000	2000	3000	4000	2000	2000
β_1	5	5	5	5	5	6	8

Table III: The shear stresses (τ_x) on the lower plate

$P_0 = P_1$	I	II	III	IV	V	VI	VII
2	0.000048	0.000054	0.000064	0.000041	0.000032	0.000052	0.000084
4	0.000066	0.000072	0.000084	0.000042	0.000035	0.000062	0.000098
6	0.000072	0.000078	0.000089	0.000052	0.000042	0.000082	0.000099
10	0.000084	0.000094	0.000132	0.000062	0.000048	0.000092	0.000147

	I	II	III	IV	V	VI	VII
M	2	5	8	2	2	2	2
D^{-1}	2000	2000	2000	3000	4000	2000	2000
β_1	5	5	5	5	5	6	8

Table IV: The shear stresses (τ_y) on the lower plate

$P_0 = P_1$	I	II	III	IV	V	VI	VII
2	-0.00467	-0.00599	-0.00653	-0.00321	-0.00301	-0.00546	-0.00675
4	-0.00521	-0.00684	-0.00744	-0.00427	-0.00357	-0.00584	-0.00748
6	-0.00633	-0.00744	-0.00831	-0.00524	-0.00427	-0.00752	-0.00846
10	-0.00801	-0.00856	-0.00946	-0.00622	-0.00582	-0.00942	-0.00999

	I	II	III	IV	V	VI	VII
M	2	5	8	2	2	2	2
D^{-1}	2000	2000	2000	3000	4000	2000	2000
β_1	5	5	5	5	5	6	8

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REFERENCES

[1]. P. Puri, "Rotary flowof an elastico-viscous fluid on an oscillating plate," *Journal of Applied Mathematics and Mechanics*, Vol. 54, No. 11, pp. 743–745, 1974.
[2]. M. Hussain, T. Hayat, S. Asghar, and C. Fetecau, "Oscillatory flows of second grade fluid in a porous space," *Nonlinear Analysis: Real World Applications*, Vol. 11, No. 4, pp. 2403–2414, 2010.
[3]. C. Fetecau, S. C. Prasad, and K. R. Rajagopal, "A note on the flow induced by a constantly accelerating plate in an Oldroyd-B fluid," *Applied Mathematical Modelling*, Vol. 31, No. 4, pp. 647– 654, 2007.
[4]. W. Tan and T. Masuoka, "Stokes' first problem for a second grade fluid in a porous half-space with heated boundary," *International Journal of Non-Linear Mechanics*, Vol. 40, No. 4, pp. 515–522, 2005.
[5]. C. Fetecau, M. Athar, and C. Fetecau, "Unsteady flow of a generalized Maxwell fluid with fractional derivative due to a constantly accelerating plate," *Computers andMathematics with Applications*, Vol. 57, No. 4, pp. 596–603, 2009.
[6]. M. Husain, T. Hayat, C. Fetecau, and S. Asghar, "On accelerated flows of an Oldroyd—B fluid in a porous medium," *Nonlinear Analysis: Real World Applications*, Vol. 9, No. 4, pp. 1394–1408, 2008.
[7]. M. Khan, E. Naheed, C. Fetecau, and T. Hayat, "Exact solutions of starting flows for second grade fluid in a porous medium," *International Journal of Non-LinearMechanics*, Vol. 43, No. 9, pp. 868–879, 2008.
[8]. F. Salah, Z. A. Aziz, and D. L. C. Ching, "New exact solution for Rayleigh-Stokes problem ofMaxwell fluid in a porous medium and rotating frame," *Results in Physics*, Vol. 1, No. 1,pp. 9–12, 2011.
[9]. M. Khan, M. Saleem, C. Fetecau, and T. Hayat, "Transient oscillatory and constantly accelerated non-Newtonian flow in a porous medium," *International Journal of Non-Linear Mechanics*, Vol. 42, No. 10, pp. 1224–1239, 2007.
[10]. F. Salah, Z.Abdul Aziz, andD. L.C.Ching, "New exact solutions for MHD transient rotating flow of a second-grade fluid in a porous medium," *Journal of Applied Mathematics*, Vol. 2011, Article ID 823034, 8 pages, 2011.
[11]. C. Fetecau, T. Hayat, M. Khan, and C. Fetecau, "Erratum: Unsteady flow of an Oldroyd-B fluid induced by the impulsive motion of a plate between two side walls perpendicular to the plate," *Acta Mechanica*, Vol. 216, No. 1–4, pp. 359–361, 2011.
[12]. C. Fetecau, T. Hayat, J. Zierep, and M. Sajid, "Energetic balance for the Rayleigh-Stokes problem of an Oldroyd-B fluid," *Nonlinear Analysis: Real World Applications*, Vol. 12, No. 1, pp. 1–13, 2011.
[13]. K. R. Rajagopal and A. S.Gupta, "On a class of exact solutions to the equations of motion of a second grade fluid," *International Journal of Engineering Science*, Vol. 19, No. 7, pp. 1009–1014, 1981.
[14]. M. E. Erdo˘gan and C. E. Imrak, "On unsteady unidirectional flows of a second grade fluid," *International Journal of Non-Linear Mechanics*, Vol. 40, No. 10, pp. 1238–1251, 2005.
[15]. F. Salah, Z. A. Aziz, and D. L. C. Ching, "Accelerated flows of a magnetohydrodynamic (MHD) second grade fluid over an oscillating plate in a porous medium and rotating frame," *International Journal of Physical Sciences*, Vol. 6, No. 36, pp. 8027–8035, 2011.
[16]. C. Fetecau andC. Fetecau, "Starting solutions for some unsteady unidirectional flows of a second grade fluid," *International Journal of Engineering Science*, Vol. 43, No. 10, pp. 781–789, 2005.
[17]. T. Hayat, K. Hutter, S. Asghar, and A.M. Siddiqui, "MHD flows of an Oldroyd-B fluid," *Mathematical and Computer Modelling*, Vol. 36, No. 9-10, pp. 987–995, 2002.
[18]. T.Hayat, S.Nadeem, S.Asghar, andA.M. Siddiqui, "Fluctuating flow of a third-grade fluid on a porous plate in a rotating medium," *International Journal of Non-Linear Mechanics*, Vol. 36, No. 6, pp. 901–916, 2001.
[19]. S. Abelman, E. Momoniat, and T. Hayat, "Steady MHD flow of a third grade fluid in a rotating frame and porous space," *Nonlinear Analysis: Real World Applications*, Vol. 10, No. 6, pp. 3322–3328, 2009.
[20]. Faisal Salah, Zainal Abdul Aziz, Mahad Ayem, and Dennis Ling Chuan Ching, " MHD Accelerated Flow of Maxwell Fluid in a Porous Medium and Rotating Frame," *ISRN Mathematical Physics*, Vol. 2013, Article ID 485805, 10 pages, 2013, http://dx.doi.org/ 10.1155/2013/485805.
[21]. Hayat.T, C.Fetecau, M.Sajid., *Physics Letters A*, Vol. 372, pp. 1639-1644, 2008.
[22]. M.VeeraKrishna, S.V.Suneetha and R.SivaPrasad, "Hall current effects on unsteady MHD flow of rotating Maxwell fluid through a porous medium," *Ultra Scientist of Physical Sciences*, Vol. 21(1)M, pp. 133-144, 2010.

Appendix

$$z_1 = M^2 + D^{-1}\varphi, \qquad z_2 = 1 + \beta_1(M^2 + D^{-1}\varphi),\ b_0 = \sqrt{\frac{z_1}{\beta_1}},\ b_1 = \sqrt{\frac{-\beta_1\omega^2 + iz_2\omega + z_1}{1 + i\beta_1\omega}},$$

$$b_2 = \sqrt{\frac{-\beta_1\omega^2 - iz_2\omega + z_1}{1 - i\beta_1\omega}},\ b_3 = \sqrt{\frac{\beta_1 s_1^2 + z_2 s_1 + z_1}{1 + \beta_2 s_1}},\ b_4 = \sqrt{\frac{-\beta_1\omega_1^2 + iz_2\omega_1 + z_1}{1 + i\beta_1\omega_1}},$$

$$b_5 = \beta_1 s_4^2 + z_2 s_4 + (z_1 + n^2\pi^2), \qquad b_6 = \beta_1 s_3^2 + z_2 s_3 + (z_1 + n^2\pi^2)$$

$$b_7 = \sqrt{\beta_1 s_2^2 + z_2 s_2 + z_1},\ s_1 = \frac{-z_2 + \sqrt{z_2^2 - 4\beta_1 z_1}}{2\beta_1},\ s_2 = \frac{-z_2 - \sqrt{z_2^2 - 4\beta_1 z_1}}{2\beta_1},$$

$$s_3 = \frac{-z_2 + \sqrt{{z_2}^2 - 4\beta_1(z_1 + n^2\pi^2)}}{2\beta_1},\ s_4 = \frac{-z_2 - \sqrt{{z_2}^2 - 4\beta_1(z_1 + n^2\pi^2)}}{2\beta_1}$$

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