Discrimination of Free Space and Subsurface Canonical Metallic Targets Using Hybrid E-Pulse Method

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Abstract—Radar scattered time domain response can be modeled by natural poles using singularity expansion method (SEM) in resonance region. In this paper, limitation of the conventional Extinction pulse method is brought out, and a hybrid of conventional Extinction pulse and auto-regressive (AR) method is proposed for robust discrimination of radar targets. A new target discrimination number (TDN) is suggested, which gives very good discrimination margin for enhanced decision process. The Hybrid Extinction pulse technique is applied on the free space targets as well as subsurface canonical metallic targets and the result obtained shows good discrimination margin. The free space target response was obtained using FDTD simulation and the subsurface target response was obtained using frequency domain measurement done for the targets buried under dry sand.

1. INTRODUCTION

The resonance based radar target discrimination schemes using time domain target response has generated considerable interest in the past [1–5]. Targets can be identified by their natural frequencies, which are extracted from the late time scattered responses when irradiated with transient electromagnetic signals. One of the most popular methods used in recent time for resonance based target discrimination is the Extinction pulse (E-pulse) discrimination technique introduced by Rothwell et al. [3]. The E-pulse discrimination number (EDN) was introduced to quantify the discrimination process for automated E-pulse technique [4]. The pulse basis function is conventionally used for construction of E-pulse, but recently other basis functions were also used and shown [12] that exponential basis function gives improved discrimination capacity. The late time response of a low SNR signal may give an erroneous result. To overcome this limitation, E-pulse method was used for early-time signal [6]. The early-time E-pulse technique requires separate waveforms for each target aspect angles and thus needs significantly more storage and processing time. The combination of early time/late time was employed to overcome this limitation [7]. The possibility that some of resonant modes may not be well excited at particular incident aspect and polarization state, Lui and Shuley [14] suggested the use of full polarimetric target signature for more reliable results. Detection and imaging of radar targets buried below an interface has been of interest to microwave and radar engineers over many years. Lui et al. [13] monitored the small physical changes of hip prosthesis target below an interface using E-pulse technique. Amidst the significant development observed in the area of resonance based target discrimination, one aspect which was not seriously addressed is finding the onset of late time, which depends on the dimension of the targets.

The autoregressive (AR) approach suggested by Primak el al. [11] to obtain the E-pulse directly in time domain solves the key issue of finding the late time in conventional E-pulse method [3]. The AR method is very sensitive to noise because E-pulse is directly constructed from target response data. The

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discrimination process in both cases is accomplished by the convolution of E-pulse with the late-time scattered field response of the target. This paper suggest hybrid of conventional E-pulse and AR method for robust target discrimination with improved discrimination margin. The new method was applied to target response of various canonical metallic shapes like cylinders of different lengths, rectangular plates, etc. The conventional E-pulse is briefly discussed in Section 2. The limitation of discrimination using convolved energy in late time is shown in Section 3. The hybrid E-pulse method and new target discrimination number for automated discrimination process is discussed in Section 4. Validation of this method for subsurface canonical scatterers is discussed in Section 5. Conclusion is presented in Section 6.

2. E-PULSE CONCEPT

An E-pulse is a finite duration waveform which when convolved with the anticipated target response annihilates the contribution of a selected numbers of natural resonances to the late time target response. The time domain scattered field response of a conducting target has been observed to be composed of an early-time forced period, when the excitation field is interacting with the scatterer, followed by immediately by a late-time period during which the target oscillates freely [3]. The time domain response of a target is written as early-time and late-time response [3]. The late-time response $r_l(t)$ is expressed by a finite sum of damped sinusoids [1–4]:

$$r_l\left(t\right) = \sum\nolimits_{n=1}^{N} a_n e^{\sigma_n t} \cos\left(\omega_n t + \varphi_n\right) \tag{1}$$

where, $t > T_l$, where, T_l is the onset of the late time, and (σ_n, ω_n) is the pole of the nth resonance mode of the target described by a damping coefficient and a resonance pulsation. $(a_n$ and $\varphi_n)$ are the amplitude and the phase of the nth resonance mode, and N is the number of modes assumed to be excited by the incident field waveform.

The convolution of an E-pulse waveform $e(t)$ with the late time measured response waveform is:

$$c\left(t\right) = e\left(t\right) * r_l(t), \quad t > T_L$$
$$c\left(t\right) = \int_0^{T_e} e\left(t'\right) r_l\left(t - t'\right) dt' \tag{2}$$

Now the condition for the convolved response of the anticipated target in late time has to be zero enforces:

$$c\left(t\right) = 0$$

3. LIMITATIONS OF CONVENTIONAL E-PULSE

The time domain response of cylinder of different dimensions used for discrimination process is obtained using finite difference time domain (FDTD) solver XFDTD (v7.1). A Gaussian plane wave pulse of 50 ps full width half maximum (FWHM), 50 ps rise time as shown in Fig. 1 is used as excitation for simulation. The target cylinders are individually placed vertically and the incoming plane wave with the dominant polarization matching to the axial dimension of the cylinder interacts with the target and backscattered target response is obtained. An absorbing boundary condition with seven perfectly matched layers (PML) and cell size 4 mm is used for this simulation. In conventional method the discrimination is automated by E-pulse discrimination number (EDN) [3, 4] given by the ratio of convolved energy in late time to energy of E-pulse.

$$EDN = \frac{\int_{T_l}^{T_l+W} |c(t)|^2 \, dt}{\int_0^{T_e} |E - pulse(t)|^2 \, dt} \tag{3}$$

where, W represents the time window, corresponds to 95% of the power of the convolution product for $t > T_L$ [9].

The E-pulse is constructed for cylinders of radius 10 cm and length 25 cm. The incoming response of the cylinders of same radius 10 cm and different lengths (1 m, 75 cm, 50 cm and 25 cm) are shown in
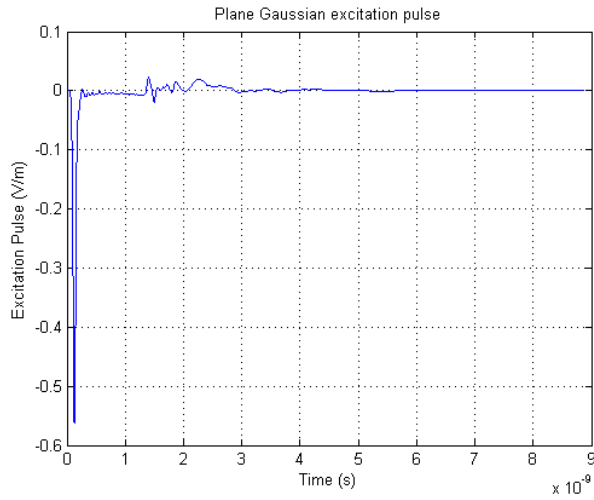
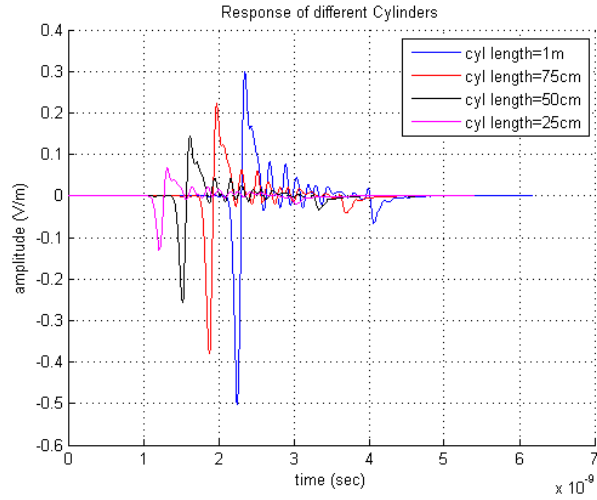**Figure 1.** Excitation Gaussian plane wave.

Figure 2. Time domain responses of cylinders of varying lengths.

Fig. 2. When the time domain response of these four cylinders is convolved with the E-pulse of cylinder-4 sequentially, the minimum EDN is found for the cylinder-4, hence the discrimination is proper in this case as shown in the second column of Table 1. The convolved waveform of the different responses with E-pulse of cylinder-4 of length 25 cm is shown in Fig. 3. The EDN is calculated from the convolved energy after time $T_l + T_e$, where, T_l is the onset of late time of the unknown target and T_e is the duration of E-pulse. The onset of time $T_l + T_e$ in case of convolution with cylinder-4 is 2.11 ns as shown in Fig. 3. The convolved energy of cylinder-4 after 2.11 ns has been reduced to minimal value, while the convolved energy of other cylinders is more, giving rise to a higher EDN (dB), hence cylinder-4 is discriminated as the expected target. The EDN (dB) values obtained from convolved responses of different cylinders, with the E-pulse of cylinder-4, are given in Table 1.

Table 1. EDN (dB) values.

Target Responses	EDN (dB) with convolution with E-pulse of cylinder 25 cm	EDN (dB) with convolution with E-pulse of cylinder 1 m
Cylinder-1 (Length = 1 m)	-21.6731	-4.717
Cylinder-2 (Length = 0.75 m)	-24.1565	-9.027
Cylinder-3 (Length = 0.5 m)	-27.8478	-12.500
Cylinder-4 (Length = 0.25 m)	**-44.0497**	**-15.7139**

The convolved output of different cylinders responses with E-pulse of cylinder-1 as the reference target is shown in Fig. 4. The onset of $T_l + T_e$ for the reference target cylinder-1 is 3.22 ns as shown in Fig. 4. The convolved energy of cylinder-1 has been reduced to minimal value after 3.22 ns, but the convolved energy of other cylinders have been reduced to much lower value cylinder-1 because of early onset of late time (T_l). The EDN (dB) values obtained from the convolved responses of different cylinders with the E-pulse of cylinder-1 are given in the third column of Table 1. The EDN (dB) value in Table 1 shows cylinder-4 as discriminated target as against expected cylinder-1. The wrong identification occurs in this case because the convolved energy of target depends on the onset of late time. The onset of late time, T_l, starts very early as in the case of smaller objects as shown in Fig. 3 and Fig. 4, and it moves farther as the size of the object increases. Therefore, the smallest dimension
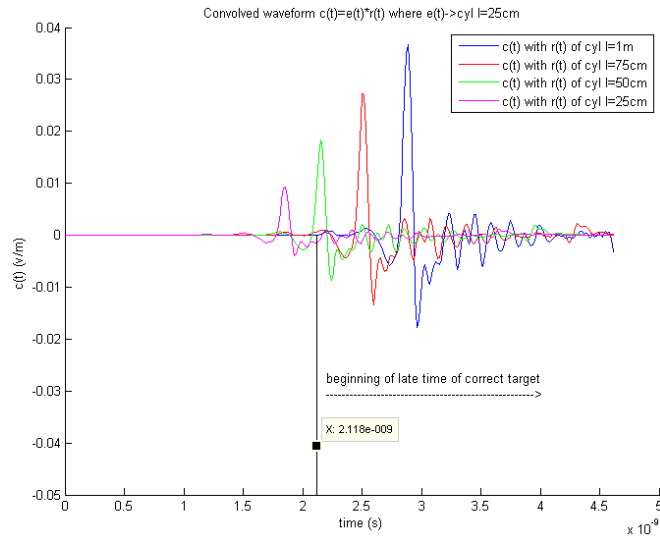
Figure 3. Convolved output of different cylinders with the E-pulse of cylinder-1.
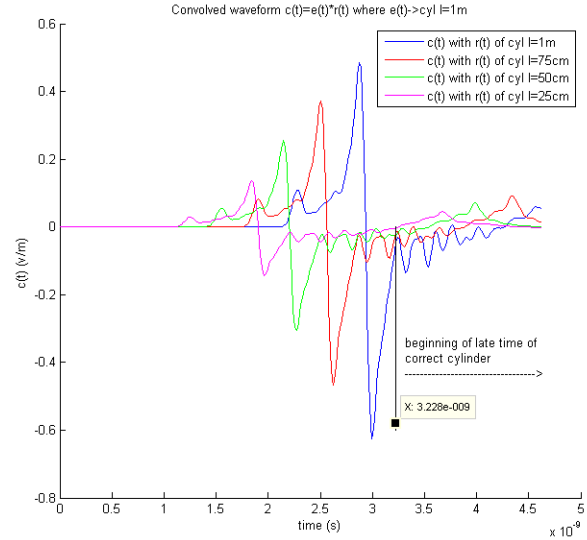
Figure 4. Convolved responses of different cylinders with the E-pulse of cylinder-4.

target will always have minimum convolved energy and minimum EDN when convolved with any E-pulse. Hence, the application of EDN value for discrimination process is not robust and leads to wrong classification in some circumstances.

4. HYBRID E-PULSE METHOD

The above-mentioned limitation of conventional E-pulse method is overcome by using a hybrid method. The hybrid approach used here is combination of conventional E-pulse method [3] and the AR method [11] along with new discrimination module for ensuring robust discrimination process and improved margin of discrimination. The steps followed in the hybrid method are as follows: determination of start of late time using AR method, extraction of poles from late-time response using matrix pencil method, selection of dominant poles using stability criteria, formation of E-pulse by solving systems of linear equations as done in conventional E-pulse method and a new terminology called target discrimination number (TDN) is introduced in the target discrimination module to overcome the limitation of discrimination process in conventional method. The steps followed are explained below.

4.1. Start of Late Time

The determination of starting of late time from time domain target response is one of the critical requirements for natural resonance based discrimination process. Auto regressive (AR) method is used to determine the start of late time. To get the start of late time, the following algorithm is followed [11].

- For the available target response data set $\{R_k\}$, $i = 1, \ldots, N$, a subset of data $\{r_j\} = \{R_j\}$, $j = k, \ldots, N$ is considered, where $k > l$, l being the index of the data point at which the late-time starts. Let $k = N - 1$ initially.

- The AR coefficients for $\{r_j\}$ are determined using the Yule-Walker algorithm. The Yule-Walker algorithm estimates the parameters of AR model, by replacing the theoretical covariance with the estimated value.

- If the change in the AR parameters with respect to their previous values is lower than a threshold, then whole process is repeated after decreasing k. Otherwise the k is stored. The smallest k gives the index of the data point at which the response time starts.
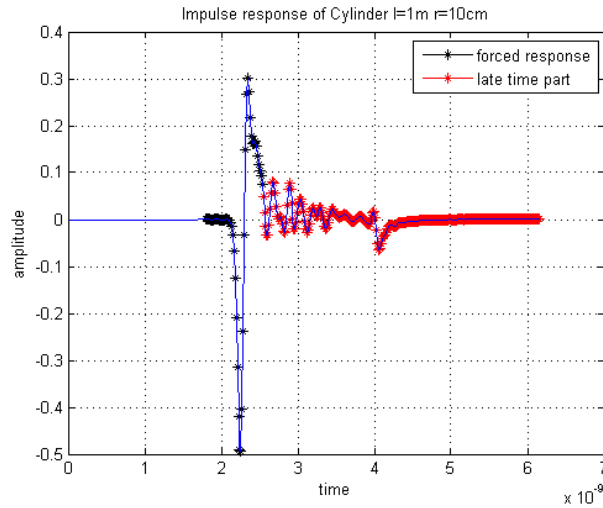
Figure 5. Waveform showing the forced response and the late-time part of response of cylinder-1.

- To this start time, three times the excitation pulse width in terms of data samples is added to avoid the forced impulse response in late time. This gives the required start of late time for Hybrid E-pulse method.

The forced part and late-time portion of the impulse response of 1 m cylinder found using above algorithm are shown in Fig. 5.

4.2. Extraction of Poles

Matrix pencil method is used for extraction of poles from the time domain target response. This method is used for the approximation of a function by a sum of complex exponentials [8]. It is more robust to noise and computationally more efficient. It provides smaller variance of parameters in the presence of noise than polynomial method. It is a one-step process.

4.3. Selection of Dominant Poles

The poles extracted from the target response may have many parasitical poles, less contributing poles and dominant poles. The dominant poles are enough to faithfully reconstruct late-time target response with minimal mean square error. The selection of these dominant poles is done using following two criteria [10]:

(i) The poles lying on the right half of the s-plane ($\sigma_m > 0$) representing the instability of the system, and the purely real poles ($\omega_m = 0$), which contribute only to the damping, are eliminated.

(ii) Neglecting the pairs of poles with weight $\frac{|R_m|}{|\sigma_m|}$ much lower than the weight $\frac{|R_d|}{|\sigma_d|}$ of the dominant pair of poles. $\{(\frac{|R_m|}{|\sigma_m|})_{norm} > threshold\}$, where $\frac{|R_m|}{|\sigma_m|}$ is normalized by $\frac{|R_d|}{|\sigma_d|}$.

All the poles extracted from late-time response of cylinder-1 of length 1 m and the dominant poles are shown in Fig. 6. The reconstructed response from the dominant poles closely follows the original late-time response as shown in Fig. 7.

4.4. E-Pulse Waveforms

An extinction pulse (E-pulse) is defined as a finite duration waveform which, upon interaction with a particular target, eliminates a preselected portion of the target's natural mode [3]. The Equation (2)
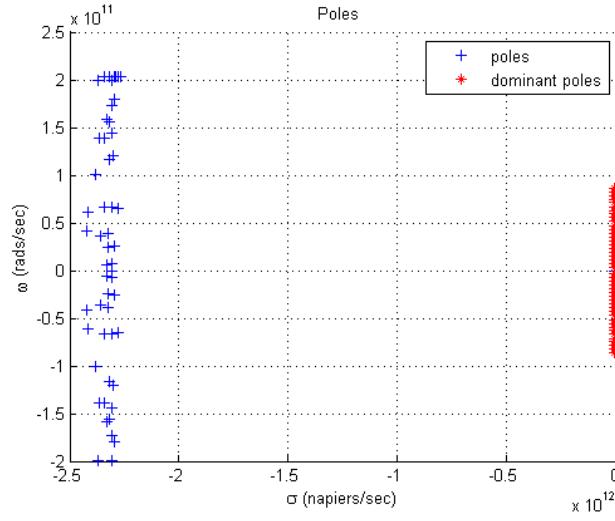
Figure 6. Poles extracted using matrix pencil method and dominant poles.
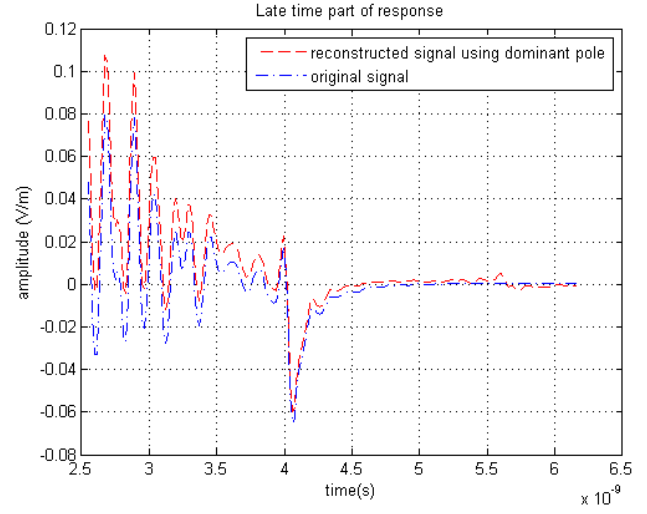
Figure 7. Original response and reconstructed response.

showing the convolution of an E-pulse waveform $e(t)$ with the late-time response waveform can be rewritten as:

$$\sum_{n=1}^{N} a_n e^{\sigma_n t} \left[A_n \cos\left(\omega_n t + \varphi_n\right) + B_n \sin\left(\omega_n t + \varphi_n\right) \right] \tag{4}$$

where, $t > T_L = T_l + T_e$, T_e is the finite duration of $e(t)$ and:

$$A_n = \int_0^{T_e} e\left(t'\right) e^{-\sigma_n t'} \cos\left(\omega_n t'\right) dt' \tag{5a}$$

$$B_n = \int_0^{T_e} e\left(t'\right) e^{-\sigma_n t'} \sin\left(\omega_n t'\right) dt' \tag{5b}$$

E-pulse waveform is designed to eliminate the entire finite expected natural mode spectrum of the target, know as natural E-pulse, which nullify late-time response of the anticipated target. The derivation of natural E-pulse [3] is as follows:

Laplace transform of $e(t)$ is given by:

$$E\left(s\right) = L\left(e\left(t\right)\right) = \int_0^{T_e} e(t)e^{-s_n t} dt = \int_0^{T_e} e\left(t\right) e^{-\sigma_n t} \left(\cos\left(\omega_n t\right) - j \sin\left(\omega_n t\right)\right) dt$$

$$= \int_0^{T_e} e(t)e^{-\sigma_n t} \cos\left(\omega_n t\right) dt - j \int_0^{T_e} e(t)e^{-\sigma_n t} \sin\left(\omega_n t\right) dt \tag{6}$$

From (5a),

$$A_n = R_e\left\{L(e\left(t\right))\right\} = R_e[E(s)] \tag{7a}$$

From (5b),

$$B_n = -I_m\left\{L\left(e\left(t\right)\right)\right\} = -I_m[E\left(s\right)] \tag{7b}$$

The condition that convolved response of anticipated target in late time is zero:

$$c\left(t\right) = 0 \tag{8}$$

requires from (2)

$$A_n = B_n = 0, \quad 1 \leq n \leq N \tag{9a}$$
$$R_e\left[E\left(s\right)\right] = 0; \quad I_m\left[E\left(s\right)\right] = 0 \tag{9b}$$
$$[E(s)] = 0 \tag{9c}$$

The natural E-pulse can be expressed as set of basis functions:

$$e(t) = \sum_{m=1}^{2N} \alpha_m f_m(t) \tag{10}$$

where, $f_m(t)$ is the pulse basis function defined as:

$$f_m(t) = \begin{cases} g(t-(m-1)\Delta), & \text{if } (m-1)\Delta \le t \le m\Delta \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

where, Δ is the pulse width. The Laplace transform of the pulse basis function is given as:

$$F_m(s) = \int_0^{T_e} g(t-(m-1)\Delta)e^{-st}dt = F_1(s)e^{s\Delta}e^{-sm\Delta}$$

$$F_m(s) = F_1(s)e^{-(m-1)s\Delta} \tag{12}$$

$$\begin{bmatrix} 1 & z_1 & z_1^2 & : & : & : & : & z_1^{2N-1} \\ : & : & : & : & : & : & : & : \\ : & : & : & : & : & : & : & : \\ 1 & z_N & z_N^2 & : & : & : & : & z_N^{2N-1} \\ 1 & z_1^* & (z_1^*)^2 & : & : & : & : & (z_1^*)^{2N-1} \\ : & : & : & : & : & : & : & : \\ : & : & : & : & : & : & : & : \\ 1 & z_N^* & (z_N^*)^2 & : & : & : & : & (z_N^*)^{2N-1} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ : \\ \alpha_N \\ \alpha_{N+1} \\ : \\ : \\ \alpha_{2N} \end{bmatrix} = 0 \tag{13}$$

where, $z = e^{-s_n\Delta}$.

The above Z matrix is a Vandermonde matrix, and Equation (13) is a homogeneous, thus has solution only when the determinant of the matrix is zero. The determinant of a Vandermonde matrix is given by:

$$\prod (z_i - z_j)$$

where, $i = 1,\ldots,2N$, $j = 1,\ldots,(N-1)$ and $i > j$.

The homogeneous equation of the form $AX = 0$, either $X = 0$ or $A = 0$.

For nontrivial solution, i.e., $X \ne 0 \therefore \det A = 0$

$$\prod (z_i - z_j) = 0 \tag{14}$$

The only possible solution is $z_i = z_i^*$

$$\Delta = \frac{p\pi}{\omega_i} \tag{15}$$

where, $p = 1, 2, 3, \ldots$.

The E-pulse signal duration depends only on the imaginary part of one of the natural frequencies. The minimal T_e value is calculated by choosing the greatest value of ω_i

$$(T_e)_{\min} = \frac{2N\pi}{\omega_i} \tag{16}$$

Once Δ determined, the amplitudes of the basis functions are calculated using the theory of determinants. The orthonormal basis of the Vandermonde matrix in Equation (16) gives the E-pulse amplitudes. The E-pulse constructed for the various cylinders is shown in Fig. 8.

4.5. Target Discrimination Module

Unlike E-pulse Discrimination number (EDN in dB), which is the ratio of convolved energy in late time to energy of E-pulse, convolution energy depends on starting of late time and in turn on the target dimensions, hence smaller targets will always give minimum EDN value irrespective of the incoming target response. Therefore, a more fundamental methodology is applied wherein solution of the simultaneous linear equation $Ax = 0$ is used for the discrimination process. The products of A

and x are compared for the different targets. The E-pulse of the matching target will always satisfy the linear equation.

$$\begin{bmatrix} 1 & z_1 & z_1^2 & : & : & : & : & z_1^{2N-1} \\ : & : & : & : & : & : & : & : \\ : & : & : & : & : & : & : & : \\ 1 & z_N & z_N^2 & : & : & : & : & z_N^{2N-1} \\ 1 & z_1^* & (z_1^*)^2 & : & : & : & : & (z_N^*)^{2N-1} \\ : & : & : & : & : & : & : & : \\ : & : & : & : & : & : & : & : \\ 1 & z_N^* & (z_N^*)^2 & : & : & : & : & (z_N^*)^{2N-1} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ : \\ \alpha_N \\ \alpha_{N+1} \\ : \\ : \\ \alpha_{2N} \end{bmatrix} = 0 \tag{17}$$

where, $2N$ is the total number of samples in E-pulse, α the amplitude of E-pulse, N the number of dominant poles in unknown target response and:

$$z_i = e^{(-s_i \Delta)}, \text{ where } s_i = \sigma_i + j\omega_i.$$

Let $R = Z\alpha$ where Z is $2N \times 2N$ matrix of poles and α the E-pulse vector as given in the above equation.

A new terminology namely Target Discrimination number (TDN) is defined as:

$$\text{TDN} = \frac{\|R'\|_1}{\|\alpha'\|_1} \tag{18}$$

where, $\| \|_1$ is the 1-norm; $R' = [|r_i|^2]$ and $\alpha' = [|\alpha_i|^2]$ are the elements of the vectors with the square of the modulii of the elements R and α, respectively. The 2-norm and ∞-norm were also used for finding the TDN value, but there was no significant difference in the value, hence 1-norm is implemented for calculation of TDN.

$$\text{TDN}_{\text{dB}} = 10\log_{10}(\text{TDN}) \tag{19}$$

The hybrid E-pulse discrimination process can be depicted by the block diagram shown in Fig. 9. The robustness of the method was verified by using many target response data. The TDN (dB) values obtained for all the different cylinders is shown in Table 2. The lowest value of the TDN is found to be of the expected target in each row of Table 2. The margin of the discrimination is much larger than other methods [3, 11], which enhances the decision making capability of automated target discrimination process.

Table 2. TDN (dB) values obtained for different combinations of target and E-pulse.

Target→	Cylinder $l = 1\,\text{m}$ $r = 10\,\text{cm}$	Cylinder $l = 75\,\text{cm}$ $r = 10\,\text{cm}$	Cylinder $l = 50\,\text{cm}$ $r = 10\,\text{cm}$	Cylinder $l = 25\,\text{cm}$ $r = 10\,\text{cm}$
E-Pulse ↓	1	2	3	4
1	**−98.0122**	4.7113	−0.0231	−2.9028
2	4.6226	**−91.0573**	3.5902	0.9556
3	2.6345	−4.6167	**−93.2396**	−2.3345
4	3.1486	−2.3107	0.5229	**−101.0636**

5. DISCRIMINATION OF SUBSURFACE SCATTERERS

The improved discrimination margin is demonstrated using hybrid E-pulse method in the earlier section on the set of time domain response data of different cylinders obtained from FDTD simulation. The method was further validated using measured response of canonical metallic scatterers buried under dry sand. The metal pieces of different shapes and sizes were buried 8 cm below the surface of dry
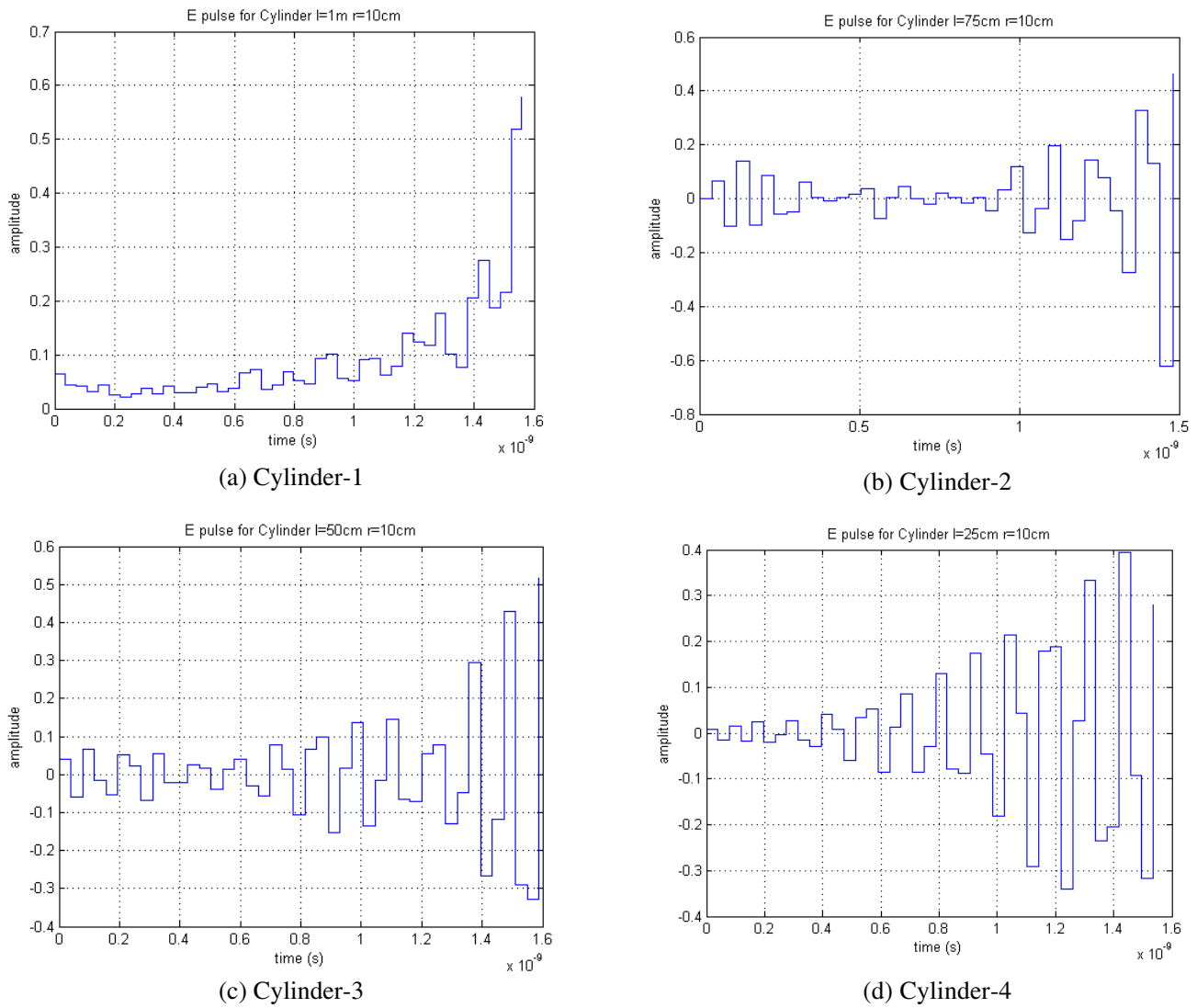
(a) Cylinder-1

(b) Cylinder-2

(c) Cylinder-3

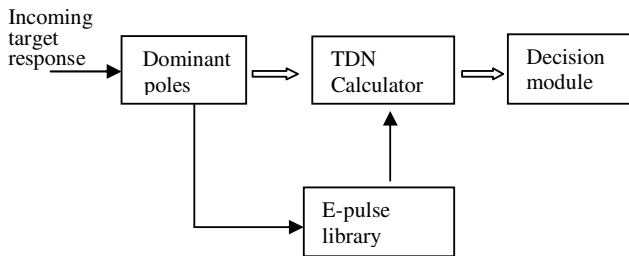(d) Cylinder-4

Figure 8. E-pulse for different cylinders.

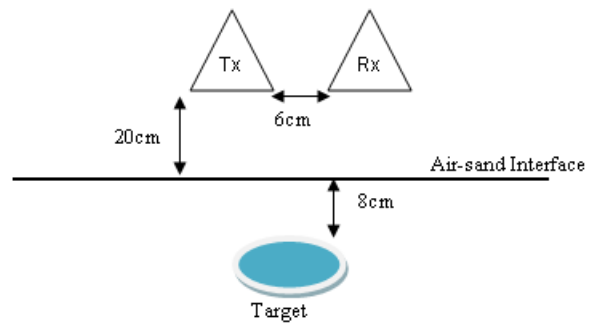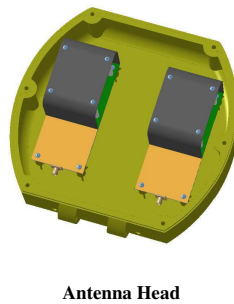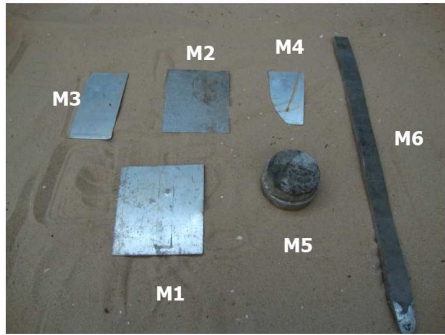**Figure 9.** Block diagram of target discrimination process.

Figure 10. Schematics of subsurface target response measurement setup.

Figure 11. Subsurface canonical scatterers used for target discrimination.

Figure 12. Sand test bed used for scattering experiment.

sand, and a bistatic antenna head was placed 20 cm above the sand surface as depicted in Fig. 10. The metallic targets used for the experimentation are made of aluminium of different shapes such as rectangles (M1, M2, M3, and M4), cylinder (M5) and rectangular strip (M6) as shown in Fig. 11. The measurement was done using stepped frequency method in the frequency range of 500 MHz to 3.2 GHz using network analyser and bistatic ultra wide band (UWB) antennas. The UWB antenna used for the experiments is reflector-backed printed hybrid monopole. The ground penetrating radar (GPR) test bed filled with dry sand used for the experimentation and antenna search head of handheld GPR is shown in Fig. 12. The frequency domain response of various canonical scatterers obtained from the measurement was converted to time domain response using inverse Fourier transform. The response of the different targets was obtained individually, and their E-pulse library was created. The hybrid E-pulse method was applied to the time domain response of these subsurface scatterers, and the TDN value obtained is shown in Table 3.

Table 3. TDN (dB) values for different combinations of scatterers and E-pulses.

Target→ E-Pulse ↓	M1 (17 cm×15 cm)	M2 (16 cm×13 cm)	M3 (17 cm×9 cm)	M4 (13 cm×7 cm)	M5 (dia 10 cm)	M6 (62 cm×4 cm)
1	**−82.007**	−3.1175	−43.6205	−44.0608	−21.7801	−30.7502
2	−16.1058	**−80.5446**	−13.5786	−12.4182	−15.2949	−14.6258
3	−23.9218	0.4475	**−82.6404**	−52.8881	−13.5298	−22.052
4	−17.9856	2.7547	−35.2914	**−83.2996**	−9.4673	−15.4639
5	−31.7526	−7.9731	−45.9107	−48.3525	**−81.4082**	−26.6325
6	−35.2889	−6.9524	−36.6731	−44.2656	−29.017	**−83.7832**

6. CONCLUSION

Target discrimination with E-pulse has been used for discriminating various radar targets. The conventional E-pulses method is studied, and their limitations are brought out. A Hybrid E-pulse method, which is a combination of conventional E-pulse and AR method, is proposed here to overcome these limitations. This method has been tested using time domain response data of various cylinders placed in free space obtained using FDTD simulation. The hybrid E-pulse method was also validated using measured response of canonical scatterers buried under dry sand. The large discrimination margin offered by this method improves the decision process and reduces the fall alarm of the automated discrimination process. This method can be used as additional target classification routine to remove clutter for GPR used for imaging targets as IEDs and UXOs.

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