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Chapter 7

Solving Ordinary Differential Equations by theDormand Prince Method

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Abstract. In general, differential equations in mathematics can be defined as an equation that comprises of one or more functions and its derivatives. Meanwhile ordinary differential equation in mathematics is declared as differential equations that contains one or more functions of one independent variable and its ordinary derivatives. Unlike partial differential equations, ordinary differential equations involve only the ordinary derivatives with respect to one independent variable. This research was conducted to solve ordinary differential equations by a numerical method called the Dormand Prince method. Consequently the solutions obtained are compared with the other numerical method in terms of accuracy. Dormand Prince method is one of the similar methods as RungeKutta method. It is used to solve an ordinary differential equation explicitly by six function evaluations. Throughout this research, the accuracy of the Dormand Prince method in solving ordinary differential equations was examined by comparing it with the other numerical method, which is Runge Kutta Fehlberg method.

Keywords. Ordinarydifferential equations, Dormand Prince method, RungeKutta method, Accuracy

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1 Introduction

In real life, mathematics plays a very important role to solve the problem of mankind by transforming the problem encountered into its own language so that it can be solved and consequently makes the life simple [1]. Mathematics is a very broad study and one of the branches of mathematics is differential equations. Differential equations study the relationship between a function and its derivatives. Generally, simple differential equations can be solved directly by using formula.

Unfortunately, problems arise when there are many complex differential equations that involve many derivatives with higher order. They cannot be solved easily by explicit formulas [2]. Therefore, numerical methods are used to get the numerical approximation for complex differential equations.Runge Kutta method is one of the famous numerical methods that use the idea of multiplicity of evaluations of the function f within each step [3].

Dormand Prince methodis a family of Runge Kutta methods in solving ordinary differential equations [4]. It uses both 4th and 5th order of Runge Kutta method to gain the numerical solution while the absolute difference between them is known as the error in the solution. Besides, Dormand Prince method exhibits a special property known as the ‘First Same As Last’ property (FSAL). This property allows six function evaluations in Dormand Prince method to get the solution even though it comprises of seven stages.

The main purpose of this research is to utilize the algorithm of Dormand Prince method in solving a differential equations. Then, the solution obtained is compared with the other numerical methods in term of accuracy. However, this research is limited to the scope of linear ordinary differential equations of first and second order. Initial value problem is emphasized in this research [5, 6].

2 Problem formulation

In order to solve ordinary differential equations by the Dormand Prince method, there are some procedures that need to be bear in mind in order to get a satisfactory solution.

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2.1 Dormand Prince Method

The function evaluation of each stage is calculated based on the Butcher tableau of the Dormand Prince method:

$$k_1 = hf\left(x_i \ , y_i\right)$$
$$k_2 = hf\left(x_i + \frac{1}{5}h \ , y_i + \frac{1}{5}k_1\right)$$
$$k_3 = hf\left(x_i + \frac{3}{10}h \ , y_i + \frac{3}{40}k_1 + \frac{9}{40}k_2\right)$$
$$k_4 = hf\left(x_i + \frac{4}{5}h \ , y_i + \frac{44}{45}k_1 - \frac{56}{15}k_2 + \frac{32}{9}k_3\right)$$
$$k_5 = hf\left(x_i + \frac{8}{9}h \ , y_i + \frac{19372}{6561}k_1 - \frac{25360}{2187}k_2 + \frac{64448}{6561}k_3 - \frac{212}{729}k_4\right)$$
$$k_6 = hf\left(x_i + h \ , y_i + \frac{9017}{3168}k_1 - \frac{355}{33}k_2 + \frac{46732}{5247}k_3 + \frac{49}{176}k_4 - \frac{5103}{18656}k_5\right)$$
$$k_7 = hf\left(x_i + h \ , y_i + \frac{35}{384}k_1 + \frac{500}{1113}k_3 + \frac{125}{192}k_4 - \frac{2187}{6784}k_5 + \frac{11}{84}k_6\right)$$
$$y_{k+1} = y_k + \frac{35}{384}k_1 + \frac{500}{1113}k_3 + \frac{125}{192}k_4 - \frac{2187}{6784}k_5 + \frac{11}{84}k_6$$

2.2 Runge Kutta Fehlberg Method

The function evaluation of each stage is calculated based on the Butcher tableau of RungeKuttaFehlberg method:

$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + \frac{1}{4}h, y_i + \frac{1}{4}k_1)$$
$$k_3 = hf(x_i + \frac{3}{8}h, y_i + \frac{3}{32}k_1 + \frac{9}{32}k_2)$$
$$k_4 = hf(x_i + \frac{12}{13}h, y_i + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3)$$
$$k_5 = hf(x_i + h, y_i + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4)$$
$$k_6 = hf(x_i + \frac{1}{2}h, y_i - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5)$$
$$y_{i+1} = y_i + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6$$

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3 Problem solution

There were two examples solved in this research where the Example 1 focused on the first order ordinary differential equations while the Example 2 emphasized on second order ordinary differential equations. The solutions obtained from the analytical method, Fifth Order Dormand Prince method (A), Fourth Order Dormand Prince method (B) and Runge Kutta Fehlberg method (C) were recorded for further comparison and discussion.

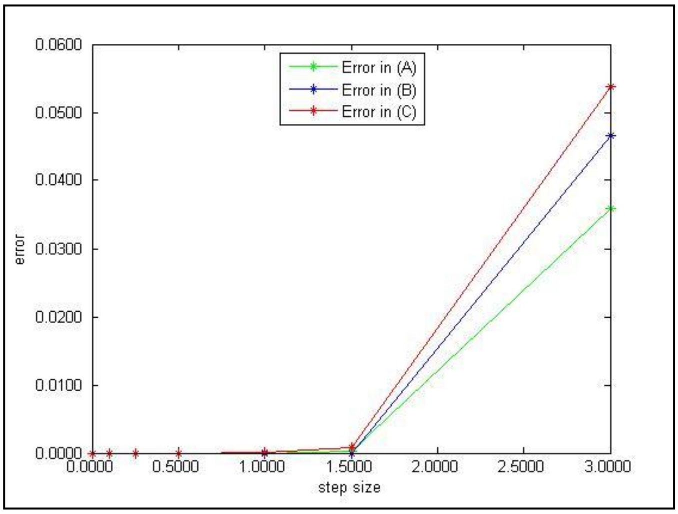

Fig. 1: Errors in the Dormand Prince and the Runge Kutta Fehlberg with different step size for Example 1

From Fig. 1, it can be clearly seen that the solutions obtained from the fifth order of Dormand Prince method (A) are more accurate than the fourth order of Dormand Prince method (B) and Runge Kutta Fehlberg method (C) in different step size. The errors in Dormand Prince Method is smaller if compared to Runge Kutta Fehlberg method.

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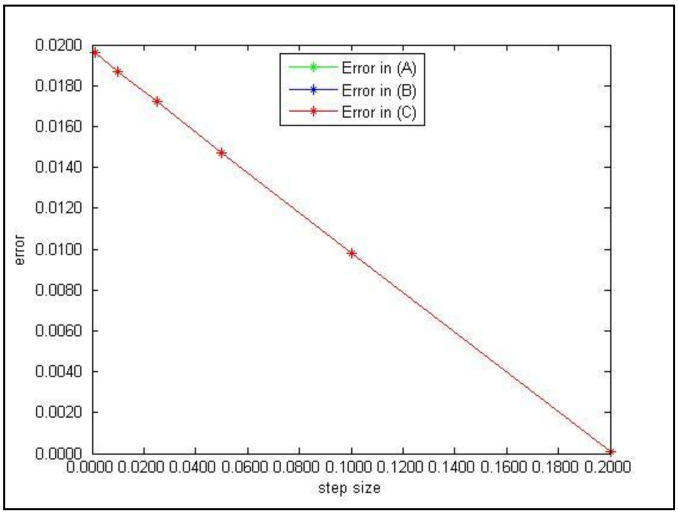

Fig. 2: Errors in the Dormand Prince and the Runge Kutta Fehlberg with different step size for Example 2

From Fig. 2, it can be clearly seen that the solutions obtained from the Dormand Prince method labelled with (B) and (C) and Runge Kutta Fehlberg (C) method are not accurate in different step sizes. The errors in both methods are the same. This means that the solutions obtained from both methods are not accurate. However, the accuracy of both methods is the same.

4 Conclusions

The results obtained showed that Dormand Prince method is more accurate than Runge Kutta Fehlberg method in solving first order ordinary differential equations. Besides, the accuracy increases with the decreasing step size. This is because the smaller step size will calculate the particular solution in a lower range and more accurate solution can be obtained. The error proposed by Runge Kutta Fehlberg method is larger than the Dormand Prince method. However, in second order ordinary differential equations, both Dormand Prince method and RungeKutta method are not accurate and deviates from the analytical solution. The accuracy of Dormand Prince method is the same as the Runge Kutta Fehlberg method because both of the solutions proposed the same value of error.

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The main purpose of this research was met where the algorithm of Dormand Prince method was utilized to solve both first order and second order ordinary differential equation. The solution obtained was compared with the Runge Kutta Fehlberg method. There are many numerical methods that can be used. However, the suitability of a method depends on the step size, number of iterations and order of the differential equations.

Lastly, the algorithm of Dormand Prince method and Runge Kutta Fehlberg method can be utilized in some application problems in science and engineering fields [7, 8]. Some real life problems can be solved by using the algorithms [9, 10]. The convergence and stability properties in numerical methods can be studied and improved in order to compare the accuracies and suitability among the numerical methods.

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