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On Some Structural Properties of $G_{m,n}$ Graphs

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Abstract

This is the continuation of the study on an undirected graph $G_{m,n}$ where vertex set $V = I_n = \{1, 2, 3, \cdots, n\}$ and $a, b \in V$ are adjacent if and only if $a \neq b$ and $a + b$ is not divisible by m, where $m(> 1) \in \mathbb{N}$. In the present paper we computed the diameter, Weiner index, degree distance, independence number of the graph $G_{m,n}$. We also studied the complement of the graph $G_{m,n}$.

Keywords: Divisibility graph, power graph, annihilator graph, connected graph, Weiner index, degree distance, independence number of graphs

Mathematics Subject Classification (2010): 05C10

1. Wiener index and degree distance of $G_{m,n}$

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The distance $d(u, v)$ between any two vertices $u, v \in V(G)$ is the minimum number of edges on a path in G between u and v. The diameter of a graph G is the maximum of distances between the every pairs of vertices in $V(G)$.

The Wiener index of the vertex v in G is defined as $W(v, G) = \sum_{u \in V(G)} d(u, v)$. The Wiener index of a graph G is defined as $W(G) = \sum_{\{u,v\} \subseteq V} d(u, v)$. The degree distance of a graph G is defined as $DD(G) = \sum_{\{u,v\} \subseteq V} (\deg(u) + \deg d(v)) d(u, v)$.

Lemma 1.1. *For any m, n, the diameter of the graph $G_{m,n}$ is less than or equal to 2.*

Proof. Let $G = G_{m,n}$. Let $a, b \in V$, where $a \neq b$ and a, b are adjacent, then $d(a, b) = 1$. Let $m \geq 2n$, then the graph $G_{m,n}$ is complete[3] so $d(i, j) = 1$ for all $i, j \in V$. Let $m < 2n$.

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Case I. Let $m < n$, then the vertex set $V = \{1, 2, \ldots, m-i, \ldots, 2m-i, \ldots, km-i, \ldots, n\}$. Consider the set $P(x) = \{m-x, 2m-x, \ldots, km-x\}$. Let $x, y \in V$, the vertex x is adjacent to the vertex y if $y \notin P(x)$. Let $y \in P(x)$, then the vertex x is not adjacent to the vertex y. Again for each $y \in V$, the vertex y is not adjacent to any other vertex $y_1 \in P(y)$ where $P(y) = \{m-y, 2m-y, \ldots, km-y\}$. Consider the set $V - \{P(x) \bigcup P(y)\}$. Let $x, y \in V$, where $x, y \neq km$ for some $k \in \mathbb{N}$. Then the vertex $v = km \in V - \{P(x) \bigcup P(y)\}$. Again let $x \neq km, y = km$ for some $k \in \mathbb{N}$ then the vertex $v = y \in V - \{P(x) \bigcup P(y)\}$. Lastly consider $x = k_1 m, y = k_2 m$, where $k_1, k_2 \in \mathbb{N}$ then the vertex $v = 1 \in V - \{P(x) \bigcup P(y)\}$. Thus the set $V - \{P(x) \bigcup P(y)\}$ is nonempty. Let $z \in V - \{P(x) \bigcup P(y)\}$. Then the vertex z is adjacent to the vertex x as well as to the vertex y. Thus the vertices x, y are connected via the vertex z. Hence, $d(x, y) = 2$.

Case II. Let $2 < n < m < 2n$. Let $x, y \in V$. By definition, the vertex x and y are not adjacent if and only if m divides $(x+y)$. But in this case $2n < 2m$ which implies m divides $(x+y) \Leftrightarrow m = x+y$. Then the only non adjacent pairs of vertices are $P = \{(n, m-n), (n-1, m-n+1), \ldots, (\frac{m}{2}, \frac{m}{2})\}$ if m is even and $P = \{(n, m-n), (n-1, m-n+1), \ldots, (\frac{m+1}{2}, \frac{m-1}{2})\}$ if m is odd. Let $(x, y) \in P$, then the vertices x, y are not adjacent. Then for any $z \in V$, where $z \notin \{x, y\}$, the vertices x, y are adjacent to the vertex z. Thus the vertices x, y are connected via the vertex z, hence $d(x, y) = 2$. Again for any $(x, y) \ni P$, then the vertices x, y are adjacent.

Case III. Let $m = n$ and $a, b \in V$.

Case A. Let the vertices a and b are adjacent, then $d(a, b) = 1$.

Case B. Let the vertices a, b are not adjacent. Since $m = n$, m does not divide $(n + a)$ for all $a \in V$ which implies the vertices n and a are adjacent. Similarly we can say the vertices n and b are adjacent. So the vertices a, b are connected via the vertex n, which gives $d(a, b) = 2$. Thus we can conclude that the distance between any two distinct vertices in $G_{m,n}$ is 1 or 2. □

Theorem 1.2. *Let $m \geq 2n$, then the diameter of the graph $G_{m,n}$ is one.*

Proof. For $m \geq 2n$, $G_{m,n}$ is complete, hence the diameter of the graph $G_{m,n}$ is one. □

Theorem 1.3. *Let $m < 2n$, then the diameter of the graph $G_{m,n}$ is two.*

Proof. For $m < 2n$, the diameter of $G_{m,n}$ is two, which follows from the lemma 1.1. □

Theorem 1.4. *Let $G = G_{m,n}$ be a graph. Then the Wiener index of any vertex $i \in V$, where V is the vertex set of the graph G is $W(i, G) = 2n - deg\, i - 2$.*

Proof. Let $i \in V(G)$. Then the Wiener index of i is $W(i, G) = \sum_{j \in V(G)} d(i, j)$. The distance between any two distinct vertices in G is 1 or 2. The total number of vertices of distance 1 from i is equal to the number of vertices adjacent to i which is nothing but the degree of the vertex i. Again the vertices which are not adjacent to the vertex i are at the distance two and the number of such vertices are $n - deg\, i - 1$. Thus $W(i,G) = \sum_{j\in V(G)} d(i,j) = deg\, i + (n - deg\, i - 1)\cdot 2 = 2n - deg\, i - 2$. □

Theorem 1.5. *Let $m = 2$ and n be even. The the Wiener index of the graph $G_{2,n}$ is $\frac{3}{4}n^2 - n$.*

Proof. Let $m = 2$ and n be even. Let $V_1 = \{1,3,\cdots,n-1\}, V_2 = \{2,4,\cdots,n\}$ where $V = V_1 \bigcup V_2$. Then the degree of each vertex $i \in V_1$ and $j \in V_2$ is $\frac{n}{2}$. Let i,j be adjacent, then $d(i,j) = 1$ and $\sum_{(i,j)\subseteq V} d(i,j) = \frac{1}{2}(n\cdot\frac{n}{2}\cdot 1)$. Again consider $i,j \in V$ where i,j are not adjacent, then $d(i,j) = 2$ and each vertex is not adjacent to $n - \frac{n}{2}$ number of vertices, thus $\sum_{(i,j)\subseteq V} d(i,j) = \frac{1}{2}n\cdot(n-\frac{n}{2}-1)\cdot 2$. Thus the Wiener index of the graph $G_{2,n}$ is

$$W(G_{2,n}) = \sum_{(i,j)\subseteq V} d(i,j) = \frac{1}{2}(n\cdot\frac{n}{2}\cdot 1) + \frac{1}{2}n\cdot(n-\frac{n}{2}-1)\cdot 2 = \frac{3}{4}n^2 - n.$$

□

Theorem 1.6. *Let $m = 2$ and n be even. The degree distance of $G_{m,n}$ is $\frac{1}{4}[n^2(3n-4)]$.*

Proof. Consider the graph $G = G_{2,n}$ where n be even. Let $V_1 = \{1,3,\cdots,n-1\}, V_2 = \{2,4,\cdots,n\}$ where $V = V_1 \bigcup V_2$. The order of the sets $V_1 = V_2 = \frac{n}{2}$. Again the degree of each vertex in V_1 and V_2 is $\frac{n}{2}$. Again no two vertices in V_1 or in V_2 are adjacent, hence the distance between any two vertices either in V_1 or in V_2 are 2. But the distance between any two vertices where $v_i \in V_1, v_j \in V_2$ is 1 as they are adjacent. Let $i \in V_1$ and $j \in V_2$ then $\sum_{(i,j)\subseteq V}(\deg(i)+\deg(j))d(i,j) = (\frac{n}{2}+\frac{n}{2})\cdot 1\cdot\frac{n}{2}\cdot\frac{n}{2}$.

Again let $i,j \in V_1$ or $i,j \in V_2$ then $\sum_{(i,j)\subseteq V}(\deg(i)+\deg(j))d(i,j) = (\frac{n}{2}+\frac{n}{2})\cdot 2\cdot\binom{\frac{n}{2}}{2}$. Thus $\sum_{(i,j)\subseteq V}(\deg(i)+\deg(j))d(i,j) = (\frac{n}{2}+\frac{n}{2})\cdot 1\cdot\frac{n}{2}\cdot\frac{n}{2} + (\frac{n}{2}+\frac{n}{2})\cdot 2\cdot\frac{(\frac{n}{2})(\frac{n}{2}-1)}{2} + (\frac{n}{2}+\frac{n}{2})\cdot 2\cdot\frac{(\frac{n}{2})(\frac{n}{2}-1)}{2} = \frac{n^3}{4} + \frac{n^2(n-2)}{4} + \frac{n^2(n-2)}{4} = \frac{n^2}{4}(3n-4)$. □

Theorem 1.7. *Let $m = 2$ and n be odd. Then the Wiener index of the graph $G_{2,n}$ is $\frac{1}{4}(n-1)(3n-1)$.*

Proof. Let $m = 2$ and n be odd. Let $V_1 = \{1,3,\cdots,n\}, V_2 = \{2,4,\cdots,n-1\}$ where $V = V_1 \bigcup V_2$. Then the degree of each vertex $i \in V_1$ is $\frac{n-1}{2}$ and the degree of each vertex $j \in V_2$ is $\frac{n+1}{2}$. Thus $\frac{n-1}{2}$ vertices of V_1 are at distance one from $\frac{n+1}{2}$ vertices of V_2 and vice versa. Again $(\frac{n+1}{2}-1)$ vertices of V_2 are at distance two from $\frac{n+1}{2}$ vertices of V_2. Similarly $(\frac{n-1}{2}-1)$ vertices of V_1 are at distance two from $\frac{n-1}{2}$ vertices of V_1. Thus the Wiener index of $G_{2,n}$ is $W(G_{2,n}) = \sum_{(i,j)\subseteq V} d(i,j) = \frac{1}{2}[(\frac{n-1}{2}\cdot 1\cdot\frac{n+1}{2}) + ((\frac{n+1}{2}-1)\cdot 2\cdot\frac{n+1}{2}) + (\frac{n+1}{2}\cdot 1\cdot\frac{n-1}{2}) + ((\frac{n-1}{2}-1)\cdot 2\cdot\frac{n-1}{2})] = \frac{1}{4}(n-1)(3n-1)$. □

Theorem 1.8. *Let $m = 2$ and n be odd. Then the degree distance of the graph $G_{2,n}$ is $\frac{n^2-1}{4}(3n-4)$.*

Proof. Let $m = 2$ and n be odd. Let $V_1 = \{1, 3, \cdots, n\}$, $V_2 = \{2, 4, \cdots, n-1\}$ where $V = V_1 \bigcup V_2$. Then the degree of each vertex $i \in V_1$ is $\frac{n-1}{2}$ and the degree of each vertex $j \in V_2$ is $\frac{n+1}{2}$. Let $i \in V_1, j \in V_2$, $\sum_{(i,j)\subseteq V}(\deg(i) + \deg(j))d(i,j) = (\frac{n-1}{2} + \frac{n+1}{2}) \cdot 1 \cdot \frac{n+1}{2} \cdot \frac{n-1}{2}$. Again let $i, j \in V_1$, then $\sum_{(i,j)\subseteq V}(\deg(i) + \deg(j))d(i,j) = (\frac{n-1}{2} + \frac{n-1}{2}) \cdot 2 \cdot \binom{\frac{n+1}{2}}{2}$. Similarly let $i, j \in V_2$, then $\sum_{(i,j)\subseteq V}(d(i) + d(j))d(i,j) = (\frac{n+1}{2} + \frac{n+1}{2}) \cdot 2 \cdot \binom{\frac{n-1}{2}}{2}$. Thus the degree distance of $G_{2,n}$ is $\sum_{(i,j)\subseteq V}(d(i) + d(j))d(i,j) = \frac{n(n^2-1)}{4} + \frac{(n-1)(n^2-1)}{4} + \frac{(n-3)(n^2-1)}{4} = \frac{(n^2-1)(3n-4)}{4}$. □

Theorem 1.9. *Let $m(\neq 2)$ be a prime and n be multiple of m. Then the graph $G_{m,n}$ contains k vertices of degree $n-k$ and $n-k$ vertices of degree $n-k-1$ where $n = km$, $k \in \mathbb{N}$.*

Proof. Let $m(\neq 2)$ be a prime and $n = km$, $k \in \mathbb{N}$. Let

$$V = \{1, 2, \ldots, n\} = \{1, 2, \ldots, m-1, m, \ldots, 2m-1, 2m, \ldots, km-1, km\}.$$

The vertices k_1m where $k_1 = 1, 2, \ldots, k$ are adjacent to all other vertices except k_2m where $k_1 \neq k_2$ and $k_2 = 1, 2, \ldots, k$. Thus the degree of the vertices of the form k_1m is $n-(k-1)-1 = n-k$. Again the vertex k_1m-i is not adjacent to the vertex k_2m-j where $k_1 \neq k_2$, $k_1, k_2 = 1, 2, \ldots, k$, $i, j = 1, 2, \ldots, m$ and $i + j = m$. Thus the degree of the vertices of the form $k_1m - i$ is $n-k-1$ (subtracting 1 because $k_1m - i$ is not adjacent to itself). □

Theorem 1.10. *Let $m(\neq 2)$ be a prime and $n = km$ where $k \in \mathbb{N}$. Then the Wiener index of the graph $G_{m,n}$ is $\frac{1}{2m}(n-1)(n+nm)$.*

Proof. Let $G = G_{m,n}$, $m(\neq 2)$ be a prime and $n = km$ where $k \in \mathbb{N}$. Let $i \in V$ such that $i \neq k_1m$, where $k_1 = 1, 2, \cdots, k(= \frac{n}{m})$. Then the vertex i is not adjacent to any other vertex of the form $k_1m - i$, thus $d(i, k_1m - i) = 2$. And there are $n - k = n - \frac{n}{m}$ number of vertices of the form $i \neq k_1m$. So for each vertex of the form $i \neq k_1m$ there are $k = \frac{n}{m}$ vertices which are at distance two and $(n - \frac{n}{m} - 1)$ vertices which are at distance one. Again there are $\frac{n}{m}$ number of vertices of the form $i = k_1m$ where $k_1 = 1, 2, \ldots, k$. Consider $i \in V$ such that $i = k_1m$, then the vertex i is not adjacent to the vertex $j \in V$ where $i \neq j$ and $j = k_1m$. Thus in that case also $d(i,j) = 2$. And for each vertex of the form $i = k_1m$, there are $n - \frac{n}{m}$ number of vertices which are at distance one. Thus the Wiener index of $G_{m,n}$ is

$$\begin{aligned} W(G_{m,n}) &= \sum_{(i,j)\subseteq V} d(i,j) \\ &= \frac{1}{2}[(n - \tfrac{n}{m})\{\tfrac{n}{m}2 + (n - \tfrac{n}{m} - 1)1\} + \tfrac{n}{m}\{(\tfrac{n}{m} - 1)2 + (n - \tfrac{n}{m}).1\}] \\ &= \frac{(n-1)(n+mn)}{2m}. \end{aligned}$$ □

Theorem 1.11. *Let $m(\neq 2)$ be a prime and $n = km$ where $k \in \mathbb{N}$. The degree distance of $G_{m,n}$ is $(n-k)((n+k)(n-2)+1)$.*

Proof. Let $G = G_{m,n}$ where $m(\neq 2)$ be a prime and $n = km$ for $k \in \mathbb{N}$. The degree distance of G is $DD(G) = \sum_{\{i,j\}\in V}(d(i)+d(j))d(i,j)$. Let $V = \{1, 2, \ldots, m-1, m, \ldots, 2m-1, 2m, \ldots, km-1, km\}$. The degree of the vertices k_1m is $n-k$ and the degree of the vertices $k_1m - i$ is $n-k-1$. There are k vertices of degree $n-k$ and $n-k$ vertices of degree $n-k-1$. Again for each $n-k$ vertices there are k vertices of degree $n-k$ which are at distance one, k vertices of degree $n-k-1$ at distance two and $(n-k-k-1)$ vertices of degree $n-k-1$ at distance one. Similarly for each vertex k of the form k_1m, there are k vertices of degree $n-k$ at distance two and $(n-k)$ vertices of degree $n-k-1$ at distance one. Thus the degree distance of G is

$$\begin{aligned} DD(G) &= \sum_{\{i,j\}\in V}(d(i)+d(j))d(i,j) \\ &= \tfrac{1}{2}(n-k)(n-k-1+n-k-1)2k + (n-k-1+n-k).1.k \\ &\quad +(n-k-1+n-k-1).1.(n-k-k-1) \\ &\quad +k(n-k+n-k)2k + (n-k+n-k-1)1.(n-k) \\ &= (n-k)((n+k)(n-2)+1). \end{aligned}$$

□

Theorem 1.12. *Let m, n be primes and $m = n = p$. Then $G_{m,n}$ has one vertex of degree $p-1$ and $(p-1)$ vertices of degree $p-2$.*

Proof. Let $m = n = p$. Let $V = \{1, 2, \ldots, p\}$. Let $i(\neq p) \in V$. Then the vertex $v_i = i$ is not adjacent to the vertex $v_j = j = p - i$. Thus the degree of the vertex $v_i = i$ is $p-2$ (as it is not adjacent to itself too). Again the vertex $m = n = p$ is adjacent to all other vertices other than itself as $p \nmid i + p$ where $i(< p) \in V$. Thus the degree of the vertex $n = p$ is $p-1$. Hence the result follows. □

Theorem 1.13. *Let $G = G_{m,n}$ where $m = n = p$, p be a prime. Then the Wiener index of the graph G is $\frac{p^2-1}{2}$.*

Proof. Let $m = n = p$, where p be a prime. The Weiner index of a graph G is $W(G) = \sum_{\{i,j\}\subseteq V} d(i,j)$. From the above theorem, it follows that $d(p,i) = 1$ for all $i(\neq p) \in V$. Thus $\sum_{\{p,i\}\subseteq V} d(p,i) = \deg(p)$, where $\deg(p)$ represents the degree of the vertex p. Again the vertex $i(\neq p)$ is not adjacent to two vertices one is itself and the other one is $p-i$. Thus $\sum_{\{i,j\}\subseteq V} d(i,j) = \frac{(p-2)\cdot 1\cdot (p-1)}{2}$ where $i, j \neq p$ and i, j are adjacent. Again for each vertex $i \neq p$ there is only one vertex $p-i$ which is at distance 2, so $\sum_{\{i,j\}\subseteq V} d(i,j) = \frac{2\cdot(p-1)}{2}$ where $i, j \neq p$ and i, j are not adjacent. Hence $W(G) = \sum_{\{i,j\}\subseteq V} d(i,j) = (p-1) + \frac{(p-1)\cdot(p-2)\cdot 1}{2} + (p-1) = \frac{p^2-1}{2}$. □

Theorem 1.14. *Let $m = n = p$, where p be a prime. Then the degree distance of $G_{m,n}$ is $(p-1)(p^2-p-1)$.*

Proof. Let $G = G_{m,n}$ and $m = n = p$ where p be a prime. Then the degree distance of G is $DD(G) = \sum_{\{i,j\}\subseteq V}(d(i)+d(j))d(i,j) = d(p)\cdot(d(p)+d(i))\cdot d(p,i) + (d(i)+d(j))\cdot 1\cdot(p-3)\cdot\frac{p-1}{2} + (d(i)+d(j))\cdot 2\cdot\frac{(p-1)}{2} = (p-1)((p-1)+(p-2))\cdot 1+(p-2+p-2)\cdot 1\cdot(p-3)\cdot\frac{(p-1)}{2}+(p-2+p-2)\cdot 2\cdot\frac{(p-1)}{2} = (p-1)(p^2-p-1)$, where p represents the vertex $n = p$ and i, j represents the vertices $i, j(\neq p) \in V$. □

2. Complement of the graph $G_{m,n}$, independence number and independence sets of the graph $G_{m,n}$

Let the graph $\bar{G}_{m,n}$ be the complement of the graph $G_{m,n}$. Then the two distinct vertices $a, b \in \bar{G}_{m,n}$ are adjacent if m divides $(a+b)$ where the vertex set $V = \{1, 2, \ldots, n\}$.

Theorem 2.1. *Let $n = m$, then the independence number of $G_{m,n}$ is 2.*

Proof. Consider the graph $\bar{G}_{m,n}$. Let $V = \{1, 2, \ldots, n\}$. The pair of vertices $(i, n-i)$ where $i \in V$, $i \neq n$ and $i \neq \frac{n}{2}$ forms cliques in $\bar{G}_{m,n}$. Thus the independent sets of $G_{m,n}$ are $\{i, n-i\}$. Hence the independence number of $G_{m,n}$ is 2 which is the cardinality of the set $\{i, n-i\}$. □

Theorem 2.2. *Let $m = n$ where m is odd. Then the number of independent sets of $G_{m,n}$ is $\left\lfloor\frac{n}{2}\right\rfloor$.*

Proof. Consider the graph $\bar{G} = \bar{G}_{m,n}$, where $m = n$ and m is odd. Let the vertex set $V = \{1, 2, \ldots, n\}$. Then the vertex $v = n$ is isolated vertex in the graph $\bar{G}$ as $m \nmid i+n$ for $i(i \neq n) \in V$ since $i < n$. And for any vertex $i \in V$ where $i = 1, 2, \ldots, n-1$ is adjacent to the vertex $n-i$ and the number of such pairs is $\left\lfloor\frac{n}{2}\right\rfloor$. Thus the number of cliques in $\bar{G}$ is $\left\lfloor\frac{n}{2}\right\rfloor$. Hence the number of independence sets of $G_{m,n}$ is $\left\lfloor\frac{n}{2}\right\rfloor$. □

Theorem 2.3. *Let $m = n$ where m is even. Then the number of independent sets of $G_{m,n}$ is $\frac{n}{2}-1$.*

Proof. Consider the graph $\bar{G}_{m,n}$ where $m = n$ and m be even. Let the vertex set $V = \{1, 2, \ldots, n\}$. The vertices n and $\frac{n}{2}$ are not adjacent as $m \nmid n + \frac{n}{2}$. Thus the vertices n and $\frac{n}{2}$ will not form a clique in the graph $\bar{G}_{m,n}$. Again let $j \in V$ where $j \neq n, \frac{n}{2}$. Consider the vertex $i = n - j$ where $j = 1, 2, \ldots, \frac{n}{2}-1$. Then the vertices $j, n-j$ forms cliques in $\bar{G}_{m,n}$ for all j as m divides $j + (n-j)$. Thus the number of cliques in $\bar{G}_{m,n}$ is $\frac{n}{2}-1$. Hence the number of independent sets of $G_{m,n}$ is $\frac{n}{2}-1$. □

Theorem 2.4. *Let $m > n$. Then the independence number of $G_{m,n}$ is 2.*

Proof. Consider the graph $\bar{G}_{m,n}$ where $m > n$ and the vertex set $V = \{1, 2, \ldots, n\}$. Then the vertices n and $m - n$ form a clique in $\bar{G}_{m,n}$. Thus the independence number of $G_{m,n}$ is 2 for $m > n$. □

Theorem 2.5. *Let $m < n$ where $m \neq 2$. Then the independence number of $G_{m,n}$ is $\left\lfloor \frac{n}{m} \right\rfloor$.*

Proof. Let $m < n$ where $m \neq 2$. The vertices $\{m, 2m, \ldots, km\}$ where $km \leq n$ forms an independent set in $G_{m,n}$ as they form a clique in $\bar{G}_{m,n}$. And the cardinality of the set $\{m, 2m, \ldots, km\}$ is $\left\lfloor \frac{n}{m} \right\rfloor$. Hence the results follows. □

Theorem 2.6. *Let $m = 2$ and $n \in \mathbb{N}$. Then the independence number of $G_{m,n}$ is $\left\lfloor \frac{n}{2} \right\rfloor$. The number of independent set is 2.*

Proof. Let $G = G_{m,n}$ where $m = 2$ and $n \in \mathbb{N}$. Let $V = \{1, 2, \ldots, n\}$. The set $E_1 = \{2, 4, \ldots\} \subseteq V$ form the independent set of G as no two vertices of E_1 are adjacent in G. This set is maximal. Since for a given n there are $\left\lfloor \frac{n}{2} \right\rfloor$ number of even numbers in the set $\{1, 2, \ldots, n\}$. Thus the independence number of G is $\left\lfloor \frac{n}{2} \right\rfloor$. Thus the sets $O_1 = \{1, 3, 5, \ldots\}$ and $E_1 = \{2, 4, 6, \ldots\}$ are the independent sets of G as no two vertices in E_1 are adjacent as well as no two vertices in O_1 are adjacent. Thus the number of independent set is two. □

3. Conclusion

In this article, we computed the diameter, Weiner index of a vertex, and Weiner index and degree distance of the graphs $G_{2,n}$, $G_{m,n}$, where $m \neq 2$ is a prime, n is a multiple of m and $G_{p,p}$, where p is a prime. In future one can study various energies, domination, planarity etc. of the graph $G_{m,n}$.

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