

ESTIMATION OF PROMINENT GLOBAL POSITIONING SYSTEM MEASUREMENT ERRORS FOR GAGAN APPLICATIONS

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Abstract

The Global Positioning System (GPS) has been in use for providing positioning, navigation and timing (PNT) services in many parts of the world. There are several errors that affect the positional accuracy of GPS. Prominent among them are ephemeris errors, satellite and receiver clock errors, multipath errors, signal propagation errors such as ionospheric delay, tropospheric delay, and instrumental biases of the satellite and receiver. In this paper, prominent estimation techniques to characterize various GPS measurement errors are reviewed. The GPS data in the Receiver INdependent EXchange (RINEX) format obtained from a Dual frequency GPS receiver is used in this analysis. Among all the errors, ionospheric delay is found to be the most dominant. However, these delay measurements are affected by the satellite and receiver instrumental biases. The instrumental biases exist as the signals at the two GPS frequencies experience different delays inside the satellite and receiver hardware. For estimation of the instrumental biases Kalman filter technique is adopted. The user equivalent range error (UERE) obtained due to all the error sources is of the order of few metres. After accounting for various errors, the estimation accuracy is significantly improved.

Keywords: GPS, GAGAN, Error sources, Data processing, Ionospheric delay, UERE

Introduction

Global Positioning System (GPS) is a satellite based navigation system developed by the U.S. Department of Defense (DoD), U.S.A. GPS transmits two pseudorandom noise (PRN) codes, viz. C/A and P-code, on two L-band frequencies ($f_1 = 1575.42$ MHz, $f_2 = 1227.60$ MHz) that are processed in a GPS receiver to provide position, velocity and time information (Hofmann et al, 2001). GPS does not meet the more stringent air navigation

requirements, particularly during the critical phases of flight like non-precision and precision approaches. The horizontal and vertical accuracy required for Category-I Precision Approach (PA) applications of civil aviation is 16 m and 6 to 4 m respectively (with 95 % probability). To overcome these deficiencies of GPS and to use it for all phases of flight, augmentation systems have been proposed (A35-WP/229, 2004). In recent years, there has been a widespread growth in the development of Satellite Based Augmentation Systems (SBAS) around the world. It uses data collected by a number of widely separated ground reference stations to compute error corrections that are broadcasted to users via geostationary (GEO) satellites.

GPS Services and Accuracy standards

In an effort to make GPS service available to commercial, national and international civil users while maintaining the original U.S. military function, two GPS services are provided.

Standard Positioning Service (SPS): Available to all users on a continuous and worldwide basis. It uses the C/A code and is provided on the L1 signal only.

Precise Positioning Service (PPS): Restricted to U.S. armed forces, and some selected allied military organizations and agencies. It uses the P(Y) code on the L1 and the L2 signal.

The GPS performance is dynamic, changing both with time and place as the satellite geometry and measurement errors change. A global characterization of the performance is based on various parameters such as satellite constellation strength, signal propagation anomalies, and receiver capabilities. The performance specifications are given in statistical terms, for e.g. as rms error or 95th percentile of the error distribution. The SPS and PPS positioning and timing accuracy based on a 95% probability level are given in Table 1 (U.S. Dept. of Defense, 2008; Kaplan, 2006). The performance levels shown are for the signal in space (SIS) and contributions of ionosphere, troposphere, receiver, multipath error, etc. are not included. The PPS performance is actually better than that for SPS (SA off), even though the actual specifications show lower accuracy (Misra and Enge, 2001).

Table 1 SPS and PPS positioning and timing accuracy based on a 95% probability level

Accuracy standards			
S.No.	Description	SPS (SA off)	PPS
a.) Global average positioning domain accuracy			
1.	Horizontal error	≤ 9 m	22 m (98.2%)
2.	Vertical error	≤ 15 m	27.7 m

b.) Time transfer accuracy			
3.	Time transfer error	≤ 40 ns	200 ns

Satellite Based Augmentation System

A Satellite Based Augmentation System (SBAS) consists of a number of dual frequency GPS receivers placed at precisely known reference locations that are spread over a wide geographic area. These receivers continuously monitor all the GPS satellites, and are called wide area reference stations (WRS). The raw GPS measurements collected by the WRS are transmitted to the central processing facilities at wide area master stations (WMS) (Enge and Van Dierendonck, 1996). The master stations use the measurements to generate wide area differential (WAD) corrections for each satellite (Kee and Parkinson, 1996). These include satellite clock corrections, a correction for the three-dimensional position of the satellite, and a set of corrections for the ionospheric delay. Additionally, the WMS performs several integrity checks to validate the satellite signals. The differential corrections along with the integrity information are transmitted using C-band signals to the geostationary satellite (GEO), which relay the information using L-band signals to the users. SBAS provides three major components of information for performance enhancement: (i) the differential corrections improve the accuracy of the position solution, (ii) the GPS-like signals transmitted by the geostationary satellite provide an additional ranging signal, which improves the availability and continuity, (iii) the integrity information of the SBAS signals enhance the safety by alerting users within 6 seconds of any malfunction in the GNSS / SBAS system (Hofmann et al, 2008). The SBAS system will enable GPS to be used as the primary navigational aid in civil aviation for all phases of the flight from takeoff through Category-I precision approach. In addition, SBAS can also provide benefits beyond aviation to all modes of transportation including maritime, highways, and railroads (Website 1). Currently, a number of SBAS systems all over the world are at various stages of development. These include the Wide Area Augmentation System (WAAS) in USA, European Geostationary Navigation Overlay Service (EGNOS) in Europe, Multi-functional transport Satellite-based Augmentation System (MSAS) in Japan, and the GPS Aided Geo Augmented Navigation (GAGAN) in India (Enge et al, 1996; Berenguer et al, 2005; Suryanarayana Rao, 2007). The Indian SBAS is being jointly implemented by the Airports Authority of India (AAI) and Indian Space Research Organisation (ISRO) to meet civil aviation requirements for various phases of a flight, over the Indian airspace. All the SBAS systems must comply with Standards And Recommended Practices (SARPs) specified by the

International Civil Aviation Organisation (ICAO), for providing seamless navigation of civilian aircrafts across the globe (Kibe, 2003).

Errors in GAGAN / SBAS and Methods of Error Correction

The objectives of the SBAS are to provide integrity, accuracy, availability and continuity for GPS, GLONASS, and Galileo Standard Positioning Service (SPS). An SBAS system should provide necessary corrections for majority of the GNSS errors. The leftover data errors (referred to as residual errors) are mitigated by the transmission of residual error bounding information. The SBAS corrections improve the accuracy of satellite signals. The integrity data ensure that the residual errors are bounded (Grewal et al, 2007). The GNSS measurements are affected by several types of random errors and biases. Some of the errors can be removed and some can be reduced. These can be broadly categorized into three categories namely satellite based errors, propagation medium-related errors and receiver based errors.

Satellite based errors

The errors originating at the satellite include ephemeris error, satellite clock error, relativistic effects due to different gravitational potential experienced by satellites, and the satellite instrumental bias error.

- i) *Ephemeris error:* The GPS Master Control Station (MCS) collects the code and carrier phase data from the monitor stations, and predicts the ephemeris of the satellites using sophisticated software models. The ephemeris parameters are uploaded to the satellites and subsequently broadcasted to the users as part of the navigation message. A small residual error exists due to difference between the actual satellite position and the position predicted by the MCS (Langley, 1997). The ephemeris error is of the order of 1.5 m. The ephemeris error can be avoided by having a network of reference stations that transmit the three-dimensional error in the reported ephemeris or predicted ephemeris determined based on the reference stations own measurements (Kaplan, 1996). In component form, these ephemeris data do not decorrelate spatially and decorrelate very slowly in time (Enge and Dierendonck, 1996).
- ii) *Satellite clock error:* The GPS satellite clocks although highly stable are correct to about 1 to 2 parts in 10^{13} over a one-day period. The satellite clock offset does not decorrelate spatially but can decorrelate temporally. The drift in the satellite clock can cause an error of about 8.64 to 17.28 ns per day (El-Rabbany, 2002). The corresponding range error is about 2.59 m to 5.18 m. The master control station determines the clock error of each satellite and transmits clock correction parameters to the satellites for rebroadcast of these

in the navigation message. The satellite clock error for the C/A code pseudorange observation is modeled as a second-degree polynomial (Navstar GPS Joint Program Office, 2004),

$$\delta t^s = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + \nabla t_r \quad (1)$$

where a_{f0} =clock bias (s); a_{f1} =clock drift (s/s); a_{f2} =frequency drift (s/s²); t_{oc} =clock data reference time (s); t =current time epoch (s); ∇t_r =correction due to relativistic effects (s).

iii) *Relativistic effects:* The satellite clock is affected by the general and special theories of relativity. According to the general theory of relativity, the satellite clock would run faster than the receiver clock, due to the difference in gravitational potential experienced by the clocks of the satellite and receiver. According to the special theory of relativity, a clock aboard the satellite traveling at a constant speed would appear to run slowly relative to a clock on the ground. General relativity predicts that the GPS satellite clocks should get ahead of receiver clocks by 43 μ s per day. Special relativity predicts that the satellite clocks fall behind receiver clocks by about 9 μ s per day. The total of these two relativistic effects for the satellite clock is 34 μ s faster per day. This leads to a clock rate offset of 4.45×10^{-10} faster for the satellite clock (Samama, 2008). In order to compensate for the above mentioned relativistic effects, the satellite clock frequency is adjusted to 10.2299999543 MHz prior to launch (Navstar GPS Joint Program Office, 2004). Then, the frequency observed by the user at sea level would be 10.23 MHz. A user receiver has to make correction for another periodic effect that arises due to the assumption of a circular orbit. In an elliptical orbit, both the speed of the satellite and the gravitational potential change with the position of the satellite in its orbit. This relativistic correction (in seconds) to the satellite clock time is given by (Navstar GPS Joint Program Office, 2004),

$$\nabla t_r = Fe\sqrt{A} \sin E_k \quad (2)$$

where F =constant ($-4.442807633 \times 10^{-10}$ s / \sqrt{m}); e =satellite orbit eccentricity; A =semi-major axis of the satellite orbit; E_k =eccentric anomaly.

iv) *Satellite instrumental bias:* Within the GPS satellite hardware, the L1 and L2 signals propagate through different analog circuitry, before digitization. That is, L1 and L2 signals undergo different propagation delays within the satellite causing instrumental bias (Warnant and Pottiaux, 2000). There exists an instrumental bias (delay) in the signals of each of the two GPS frequencies. The difference of the instrumental bias of the individual frequencies is known as the differential instrumental bias, also known as interfrequency

bias (Sardon et al, 1994). The ionospheric delay measurements obtained from a dual frequency receiver are corrupted by the differential instrumental biases of the satellites. The instrumental biases must be estimated and removed to obtain accurate estimates of the ionospheric delay. The satellite differential instrumental bias can be as large as 1.5 m.

Errors due to Propagation medium

The signal propagation errors include the delay of the GPS signal as it passes through the ionospheric and tropospheric layers of the atmosphere.

- i) *Ionospheric delay:* As stated earlier, ionosphere is a region of ionized gases consisting of free electrons and ions, and extends from about 50 km to more than 1000 km. As the GPS signal travels from the satellite to the receiver, the presence of free electrons in the ionosphere changes the velocity (speed and direction) of propagation of the signals. The ionosphere affects the GPS signal propagation by delaying the code phase measurements and advancing the carrier phase measurements (Misra and Enge, 2001). The ionospheric delay for a satellite at zenith, typically varies from about 1 m at nighttime to about 5-15 m during midday (Wells et al, 1987). As India comes under equatorial and low latitude region, the spatial and temporal variability of the ionospheric delay is much greater. In an SBAS system, dual frequency GPS data from various reference stations is used for estimating the ionospheric delay corrections for user receivers. The ionospheric group delay (in metres) at GPS L1 frequency can be obtained using dual frequency code measurements as,

$$I_{L1} = \frac{f_2^2}{(f_1^2 - f_2^2)} (P_2 - P_1) \quad (3)$$

where f_1 =GPS L1 frequency (Hz); f_2 =GPS L2 frequency (Hz); P_1 =pseudorange measurement on L1 frequency (m); P_2 =pseudorange measurement on L2 frequency (m).

- ii) *Tropospheric delay:* The troposphere is the lower part of the earth's atmosphere where temperature decreases with an increase in altitude. The height of the troposphere extends to about 9 km over the poles and upto about 16 km near the equator. The GPS signals are affected by the presence of neutral atoms and molecules in the troposphere (Langley, 1997). The troposphere causes a delay in both the code phase and carrier phase observations. Unlike the ionosphere, the troposphere is non-dispersive at GPS frequencies. Since the tropospheric delay is not frequency dependent, it cannot be canceled out by using dual frequency measurements. However, it can be modeled accurately using several mathematical models including Hopfield (1969), Black (1978) and Saastamoinen (1973). The total tropospheric path delay can be split into two parts:

the dry component, which constitutes about 90% of the total refraction, and the wet component, which constitutes the remaining 10%. These models use meteorological data including local temperature, pressure and relative humidity, and satellite elevation angle to compute the tropospheric delay. The tropospheric delay in the zenith direction is about 2 m (Guochang Xu, 2003). The total tropospheric path delay (in metres) using the Hopfield model is given by (Hofmann et al, 2001),

$$\Delta^{Trop}(E) = \Delta_d^{Trop}(E) + \Delta_w^{Trop}(E) \quad (4)$$

where $\Delta_d^{Trop}(E)$ is the dry component given by,

$$\Delta_d^{Trop}(E) = \frac{10^{-6}}{5} \frac{77.64 \frac{P}{T}}{\sin \sqrt{E^2 + 6.25}} [40136 + 148.72(T - 273.16)] \quad (5)$$

and $\Delta_w^{Trop}(E)$ is the wet component given by,

$$\Delta_w^{Trop}(E) = \frac{10^{-6}}{5} \frac{-12.96T + 3.718 \cdot 10^5}{\sin \sqrt{E^2 + 2.25}} \frac{e}{T^2} 11000 \quad (6)$$

$$\text{and } e = 6.1 \frac{RH}{100} 10^{7.4475T_c / (234.7 + T_c)} \quad (7)$$

where P =total pressure (millibars); T =absolute temperature ($^{\circ}$ K); E =elevation angle (degrees); e =partial pressure of water vapour (millibars); RH =relative humidity (%); T_c =temperature ($^{\circ}$ C).

Receiver based errors

The errors originating in the receiver include receiver clock error, receiver measurement noise, multipath error, and the receiver instrumental bias error.

- i) *Receiver clock error:* GPS receivers use relatively inexpensive crystal clocks which are less accurate than the satellite clocks. Due to this, the receiver clock error is much higher than that of the satellite clock. The receiver clock error is estimated by considering it as an additional unknown parameter in the user position estimation algorithm (Langley, 1991b). The pseudorange measurements and satellite positions from four satellites can be solved to determine the user position in three dimensions (x_u, y_u, z_u) and the receiver clock error (δt_u). The receiver clock error estimated at an epoch of time using the Bancroft algorithm (Bancroft, 1985) is 98.13 m for a NovAtel make DL-4 *plus* dual frequency GPS receiver.
- ii) *Receiver measurement noise:* The GPS measurements are affected by random measurement noise which include thermal noise introduced by the antenna, amplifiers,

cables, and the receiver; multi-access noise due to interference from other GPS like signals; and signal quantization noise. With modern microprocessor and chip technology, a GPS receiver introduces less than 0.5 m code measurement error and about 1-2 mm carrier phase measurement error due to receiver noise (Misra and Enge, 2001).

- iii) *Multipath error:* Occurs when the signal arrives at the receiver via multiple paths due to reflections from the Earth and objects in the vicinity of a receiving antenna (Kaplan, 1996). The reflected signals get superimposed on the desired direct-path signal, and distort the amplitude and phase of the direct-path signal. Multipath affects both code and carrier phase measurements, but the magnitude of the error differ significantly. The multipath mitigation techniques employed in the GPS field are broadly classified into three categories: 1. Pre-receiver techniques, 2. Receiver signal processing techniques, 3. Post-receiver signal processing techniques. Pre-receiver techniques include good antenna design and use of choke-ring or pinwheel antennas to reduce the multipath error (Kubo et al, 2005). The receiver signal processing techniques mostly rely on modifying the tracking loop discriminator so as to resist multipath signals (Van Dierendonck et al, 1993).The multipath error causes about 1-5 m error in code phase measurements and about 1-5 cm in the carrier phase measurements. The smoothing of the code phase measurements using the carrier phase measurements also reduces the effect of code measurement noise and multipath.
- iv) *Receiver instrumental bias:* There exists an instrumental bias error due to the frequency dependent transmission delays caused by the analog hardware within the receiver. The instrumental bias error is specific to dual frequency receivers. This receiver interfrequency bias or differential instrumental bias of the receiver also affects the ionospheric delay measurements. The receiver differential instrumental delay can be as large as 5.0 m (Wilson et al, 1993). Various methods based on Kalman filtering, Self Calibration Of pseudoRange Error (SCORE) algorithm, and least squares adjustment technique are reported in literature for estimation the instrumental biases of the satellite and receiver along with the TEC using dual frequency GPS data for mid latitude regions (Sardon et al, 1994; Bishop et al, 1996; Ma and Maruyama, 2003).

User Equivalent Range Error

The significance of various errors and biases that affect the accuracy of GPS/ SBAS system are discussed. However, one needs a parameter that signifies the effect of all errors. User Equivalent Range Error (UERE) is one such parameter. It is a statistical ranging error that represents the total effect of all the contributing error sources. UERE is defined as the

root-sum-square of the various error sources affecting the pseudorange measurement, all expressed in units of length (Misra and Enge, 2001),

$$\sigma_{UERE} = \sqrt{\sigma_{R1}^2 + \sigma_{R2}^2 + \dots + \sigma_{Rn}^2} \quad (8)$$

where $\sigma_{R1}, \sigma_{R2}, \dots, \sigma_{Rn}$ are the rms range errors due to various error sources.

Dilution of Precision

The geometry of the visible satellites plays a very important role in the total positioning accuracy (Wells et al, 1987). Good satellite geometry is obtained when the satellites considered in the position solution are more spread out in the sky (Langley, 1999). Figure 1 illustrates the satellite geometry effect using two satellites assuming a two-dimensional case. The satellites are assumed to be at the centre of the circles having radius equal to the satellite-receiver distance. Due to various measurement errors, the measured range will not be exact. This uncertainty in the range measurement is shown by shaded grey region on either sides of each circle. For the two-dimensional case shown, the receiver may be located anywhere on the intersection area (A) of the two circles.



Fig. 1 (a) Good satellite geometry, (b) Bad satellite geometry

In Fig. 1 (a), as the satellites are more spread out, the receiver position uncertainty area (A) will be small. In Fig. 1 (b), as the satellites are close to each other, bad satellite geometry results, therefore, the receiver position uncertainty area (A) will be large.

To indicate the quality of the satellite geometry, a dimensionless quantity known as the dilution of precision (DOP) is used. If the satellite geometry is good, the corresponding DOP value will be lower, and error in the receiver position estimate will be small. There are various forms of DOP that relate various parameters of the user position and time bias errors to those of pseudorange errors (Kaplan, 1996). For examining the three-dimensional positioning accuracy, the position dilution of precision (PDOP) parameter is used. The effect of satellite geometry on the horizontal component of position accuracy is represented by the

horizontal DOP (HDOP) parameter, and that on the vertical component of position accuracy is measured by the vertical DOP (VDOP) parameter (El-Rabbany, 2002). With an elevation mask angle of 5°, HDOP value typically ranges between one and 1.5. VDOP value is larger than the HDOP, and typically ranges between two and three (Langley, 1999). The UERE is multiplied by the appropriate DOP value to determine the expected error in the GPS position at one-sigma (1σ) level (68.3% confidence level). To determine the expected position error at two-sigma (2σ) level (95.4% confidence level), UERE is multiplied by twice the appropriate DOP value (Hofmann et al, 2001).

Estimation of Error Corrections using Dual Frequency GPS Receiver Data

Under the GAGAN project, about 20 reference stations are located at various places covering the entire Indian subcontinent. Limited data provided by Space Applications Centre, ISRO, Ahmedabad, India is used for estimation of various errors. The data is acquired at a sampling rate of 60 Hz. The raw data is converted into the Receiver Independent Exchange (RINEX) format using “Convert” software. Three different file types are defined in RINEX, viz. navigation, observation and meteorological data. From the navigation data, 16 ephemeris parameters and 3 clock coefficients are extracted along with time of epoch (t_{oc}) and the space vehicle time (t_{sv}). These parameters are used to compute the GPS time, satellite position and SV clock correction. The dual frequency carrier phase and pseudoranges are extracted from the observation data. The data processing involves estimation of the various errors, and correction of the measured pseudoranges to provide more accurate pseudorange information. Table 2 shows a typical GPS error budget calculation assuming an HDOP of 1.5 and VDOP of 2.0. The Hyderabad GAGAN station data is used in the estimation of ionospheric delay (Eq. 3), tropospheric delay (Eq. 4), multipath error and instrumental biases. The satellite position of various satellites and the corresponding pseudoranges are used to compute the receiver position using Bancroft algorithm (Bancroft, 1985). The slant ionospheric delay is computed from the precise carrier phase observables after removal of integer ambiguities. Further, the elevation angle, slant factor, and ionospheric pierce point coordinates are computed. These parameters along with the slant ionospheric delay are fed to a five state Kalman filter for estimating the receiver instrumental bias (Sunehra et al, 2010). The satellite instrumental bias (PRN 26) provided by Centre for Orbit Determination (CODE), Europe is used in the estimation. The tropospheric delay is estimated by using the pressure, temperature and relative humidity parameters obtained from the meteorological data. The multipath error is estimated using the TEQC software available in public domain. For ephemeris error, satellite clock error and receiver noise, typical values reported in literature are used (Misra

and Enge, 2001). After estimating the range error due to various sources, UERE is computed. Multiplying the UERE by twice the appropriate Dilution of Precision (DOP) value produces the expected precision of the GPS positioning at the two-sigma (2σ) level.

Table 2 GPS Error Budget Computation
(Hyderabad GAGAN station, 4 March 2005, PRN 26, El: 45.91° , 01:30 hrs LT)

S.No.	Error Source	RMS range error (m)
1.	Ephemeris error	1.5
2.	Satellite clock error	1.5
3.	Tropospheric delay	2.78
4.	Ionospheric delay	6.15
5.	Receiver noise	0.5
6.	Multipath error	0.18
7.	Satellite instrumental bias	0.55
8.	Receiver instrumental bias	1.49
System UERE, rms		7.27
Horizontal position error (2σ), m (= $2 \times \text{HDOP} \times \text{UERE}$)		21.81
Vertical position error (2σ), m (= $2 \times \text{VDOP} \times \text{UERE}$)		29.08

Table 2 presents the rms range error obtained due to various error sources, UERE and the horizontal and vertical position errors in metres. UERE due to all the error sources is of the order of 7.27 m.

Conclusion

The dual frequency GPS code and carrier phase observations are affected by various biases and errors. In order to obtain better position estimates, it is necessary to correct the GPS observations for various errors. In this investigation, prominent existing methods are used for estimation of various errors for improving the position accuracy. It is obvious from the GPS error budget (Table 2) that ionospheric delay is the most predominant error and is typically of the order of 5-15 m during midday. The dual frequency GPS receiver can be used to estimate the ionospheric delay accurately. However, the instrumental biases of the satellite and receiver affect the ionospheric delay measurements obtained from a dual frequency receiver. The instrumental delay due to the satellite can cause an error as large as 1.5 m in the ionospheric delay estimate, whereas the instrumental delay due to the receiver can be as large as 5 m. In order to estimate the ionospheric delay accurately, these instrumental biases have to be estimated and removed. The data processing methods suggested here can be extended to other GAGAN stations.

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