著者 | 多賀甲 多和
学位授与機関 | 東北大学
学位授与番号 | 東北大学第2621号

博士論文

Metallicity distribution of disk stars and the formation history of the Milky Way

（円盤星の金属量分布と銀河系の形成史）

豊内 大輔

平成２８年
Abstract

We investigate the formation history of the stellar disk component in the Milky Way based on our new chemo-dynamical evolution model. Our model considers several fundamental baryonic processes, including gas inflow and outflow as well as radial migration of disk stars, which is recently receiving a lot of attention. Each of these baryonic processes in the disk evolution is characterized by model parameters, which are determined by fitting to various observational data of the stellar disk in the Milky Way, including the radial dependence of the metallicity distribution function (MDF) of the disk stars, which has recently been derived in the APOGEE survey for the first time. We succeeded to obtain the best set of model parameters, which well reproduces the observed radial dependences of the mean, standard deviation, skewness, and kurtosis of the MDFs for the disk stars. We analyze the basic properties of our model results in detail to get new insights into gas inflow and outflow processes, as well as radial migration of stars in the formation history of the Milky Way. One of the remarkable findings is that the metallicity of inflowing gas rapidly increases up to $[\text{Fe/H}] \sim -0.5$ at the early disk formation phase, probably resulting from re-accretion of metal-enriched gas ejected from the inner disk region. The combination of this process with the inside-out galactic disk evolution is found to be essential to reproduce the observed narrower metallicity distribution functions at more outer disk region. Moreover, important implications for the driving mechanisms of gas outflow and the influence of radial migration on the MDFs are also inferred from our model calculation. We also find that our model is able to reproduce other various observational properties of the Galactic stellar disk. Therefore, our chemo-dynamical evolution model is successfully applied to the evolutionary histories of the stellar disk in the Milky Way.
Contents

1 INTRODUCTION ............................... 7
  1.1 Basic Baryonic Processes in the Evolution of Disk Galaxies ......... 7
  1.2 Constraints on Galaxy Evolution from Metallicity Distribution of Stars .. 9

2 CHEMO-DYNAMICAL EVOLUTION MODEL ................. 15
  2.1 Gas Inflow Rate ........................................ 15
  2.2 Chemical Abundance of Inflowing Gas ........................ 17
  2.3 Star Formation Rate .................................... 17
  2.4 Gas Outflow Rate ..................................... 18
  2.5 Radial Migration ..................................... 19
  2.6 Basic Equations ..................................... 20
  2.7 Determination of Model Parameters ......................... 21

3 FITTING RESULTS ................................ 27

4 FORMATION HISTORY OF THE MILKY WAY REPRODUCED BY
   MODEL CALCULATION ................................. 35
  4.1 Star Formation History ............................... 35
  4.2 Gas Inflow History .................................. 39
  4.3 Time Evolution of Metallicity of Inflowing Gas ............... 43
  4.4 Gas Outflow History .................................. 47
  4.5 Radial Migration History ............................ 53
5 VALIDITY OF OUR MODEL
  5.1 Is Our Model Setting Valid? ........................................... 57
  5.2 Other Observational Properties in the Galactic Stellar Disk ........ 59

6 SUMMARY & CONCLUSION .................................................. 73

A FORMULA FOR SEVERAL BARYONIC PROCESSES IN CHEMICAL EVOLUTION
  A.1 Radial Profile of Gas Inflow ........................................ 83
  A.2 Relations between $\Sigma_{\text{out}}$ and $\Sigma_{\text{SFR}}$ ............... 85

B ADDITIONAL MODEL CALCULATIONS .................................. 89
  B.1 Model Without Break in Radial Profile of Gas Inflow .......... 89
  B.2 Model Without Time Evolution of Metallicity of Inflowing Gas .... 95
  B.3 Model Without Gas Outflow ....................................... 100
  B.4 Model Without Radial Migration .................................. 105
Chapter 1

INTRODUCTION

1.1 Basic Baryonic Processes in the Evolution of Disk Galaxies

Disk galaxies are a dominant galaxy system in the present universe (Delgado-Serrano et al. 2010), and unraveling their formation and evolution histories is one of the most important subjects in galactic astronomy. Formation of disk galaxies involves various baryonic processes, including gas inflow, gas outflow and radial migration of disk stars, which are essential in understanding the present properties of disk galaxies. Past studies for each of these basic processes are summarized as follows.

1.1.1 Gas inflow

Gas inflow provides a significant influence on the budget of baryon and star formation activity in galaxies, and therefore plays an important role in galaxy evolution. In the grand picture of galaxy formation and evolution based on LCDM cosmology, gas inflow onto a galactic disk occurs when gas accretes from the circumgalactic region into the dark matter halo and subsequently collapses toward its central region. However, it is difficult to predict how gas is actually funneled into star-forming regions in disk galaxies, because the gas cooling and collapsing processes in disk galaxies are significantly affected by
various feedback processes associated with, e.g., UV radiation, supernovae explosions, and active galactic nuclei. These feedback processes are essential to reproduce the observed structural properties of disk galaxies. Indeed, earlier semi-analytical models and numerical simulations with only modest feedback effects produce much more massive and compact disk galaxies than the observed ones (e.g., White & Frenk 1991; Navarro & Steinmetz 2000). Recent high-resolution hydrodynamical simulations taking into account stronger feedback effects based on a more realistic physical background have partially resolved such problems in galaxy formation (e.g., Sawala et al. 2014; Vogelsberger et al. 2014), but have not been completely successful yet (e.g., Sparre et al. 2015; Oman et al. 2015).

1.1.2 Gas outflow

Feedback processes in galaxies are regarded as a principal driver of galactic outflow, which has been observed ubiquitously in starburst galaxies at both high and low redshifts (e.g., Shapley et al. 2003; Rupke et al. 2005; Tremonti et al. 2007; Weiner et al. 2009). It is especially remarkable from the observations of such galaxies that a large amount of gas and heavy elements is expelled from their galactic disks (e.g., Bouché et al. 2005, 2006, 2007; Zahid et al. 2012; Peeples et al. 2014), thereby implying the importance of galactic outflow in the chemical and structural evolutions of galaxies.

The possible mechanisms driving such large scale galactic gas outflows are the injection of kinetic energy (e.g., Springel & Hernquist 2003; Okamoto et al. 2010) or momentum (e.g., Murray et al. 2005; Oppenheimer & Davé 2006) from stellar feedback into the interstellar medium (ISM). The former and latter outflow mechanisms are called the energy-driven wind (EDW) and momentum-driven wind (MDW), respectively. Both mechanisms are expected to be essential in galaxy formation. Indeed, Okamoto et al. (2014) showed that a baryonic feedback model including both EDW and MDW successfully reproduces the various properties of galaxies, for example stellar mass function, stellar mass–halo mass relation, and their redshift evolutions from $z = 4$ to 0. According to the hydrodynamical simulation with a much detailed description of feedback processes by Hopkins et al. (2012), the importances of EDW and MDW mechanisms significantly depend on the gas density in the galaxy; for a galaxy, in which the gas density is not so high as to efficiently dissipate kinetic energy in the ISM by radiative cooling, the EDW tends to
1.2. CONSTRAINTS ON GALAXY EVOLUTION FROM METALLICITY DISTRIBUTION OF STARS

be the main driver of the galactic wind, whereas for a galaxy with high gas density, the MDW preferentially occurs. However, how these mechanisms actually work in the galaxy formation and evolution is not still clearly understood.

1.1.3 Radial migration of disk stars

Radial migration of stars along a galactic disk affects the structure and dynamics of the disk galaxy significantly. Actually, various theoretical and observational studies have suggested the importance of radial redistributions of disk stars (e.g., Schönrich & Binney 2009; Hayden et al. 2015; Kordopatis et al. 2015; Morishita et al. 2015). Such radial migration processes are generally triggered by gravitational interactions between disk stars and bar/spiral structures and giant molecular clouds (GMCs) (e.g., Sellwood & Binney 2002; Roškar et al. 2008; Loebman et al. 2011), and minor mergers of satellite galaxies (e.g., Quinn et al. 1993; Velázquez & White 1999; Villalobos & Helmi 2008). The basic properties of radial migration depend significantly on its triggering process. Therefore, the detailed investigation of radial migration history in a disk galaxy is expected to provide important implications for the internal and external gravitational perturbations working in the galaxy. Deducing the effect of radial migration in external galaxies is generally impossible, whereas only the Milky Way (MW) enables us to constrain its characteristic properties from various observational information as described below.

1.2 Constraints on Galaxy Evolution from Metallicity Distribution of Stars

The MW galaxy is the best site for studying the formation and evolution processes of disk galaxies, because we can observe individual stars composing the stellar disk in great detail. Many observations of the MW disk stars have shown the various detailed properties of the present stellar disk, such as the spatial structure (e.g., Yoshii 1982; Gilmore & Reid 1983; Jurić et al. 2008; Bovy et al. 2012, 2016), [α/Fe]—metallicity relation (e.g., Bensby et al. 2003, 2014; Lee et al. 2011; Adibekyan et al. 2012), and radial metallicity gradient (e.g., Nordström et al. 2004; Allende Prieto et al. 2006; Cheng
et al. 2012; Toyouchi & Chiba 2014). These observed properties of the Galactic stellar disk are expected to be useful constraints on its formation history. In particular the metallicity distribution function (MDF) of the Galactic disk stars is a very useful tool for studying the formation history of the MW. The MDF of disk stars has been actively investigated in various spectroscopic observations since 1990s (e.g., Wyse & Gilmore 1995; Lee et al. 2011). Recently, the observation of SDSS-III/APOGEE, which is one of the latest large surveys for the MW stellar disk, revealed the radial dependence of the MDFs in the wide radial range, \( R = 3 - 16 \) kpc (Anders et al. 2014; Hayden et al. 2015).

To show how the MDF reflects formation history of a galaxy we here introduce a simplified model for demonstration. In this model, a galaxy is described with the one-zone approximation, in which only the total mass of star, gas, and metals are considered by marginalizing any spatial dependence. The chemical evolution in the galaxy can be provided by numerically solving the mass conservation equations of gas and metals with the effects of star formation, gas inflow, and gas outflow, for which the detail descriptions are presented in equations (2.10) and (2.12) in Section 2.6. In this model, the star formation rate, \( \Psi \), gas inflow rate, \( \dot{M}_{\text{in}} \), and gas outflow rate, \( \dot{M}_{\text{out}} \) are described as follows, respectively,

\[
\Psi = \frac{M_{\text{gas}}}{5 \ \text{Gyr}},
\]

\[
\dot{M}_{\text{in}} = \left( \frac{10^{10} \ \text{M}_\odot}{\tau_{\text{in}}} \right) \exp \left( -\frac{t}{\tau_{\text{in}}} \right),
\]

\[
\dot{M}_{\text{out}} = \Lambda \Psi.
\]

Here, \( \tau_{\text{in}} \) and \( \Lambda \) are basic parameters in this model, corresponding to the time scale of gas inflow and the efficiency of gas outflow to star formation, called mass loading factor, respectively. In addition to these two parameters, in this model the chemical abundance of inflowing gas, \( [X/H]_{\text{in}} \), is another parameter. These three parameters provide essential influences on the shapes of MDFs.

Figure 1.1, 1.2, and 1.3 represent the dependence of the MDF on \( \tau_{\text{in}} \), \( Z_{\text{in}} \), and \( \Lambda \), respectively. Figure 1.4 demonstrates that a shorter time scale of gas inflow produces
1.2. CONSTRAINTS ON GALAXY EVOLUTION FROM METALLICITY DISTRIBUTION OF STARS

broader MDF because many stars can rapidly form while the ISM is still metal-poor. On the other hand, it is evident from Figure 1.2 that the value of \( Z_{\text{in}} \) can set the metal-poor edge of the distribution function due to the lack of more metal-poor stars than inflowing gas. Additionally, Figure 1.3 shows that the higher value of \( \Lambda \) suppresses the chemical enrichment efficiency, consequently making the metallicity at the peak of the MDF more metal-poor. Thus, the shape of the MDF essentially reflects the properties of gas inflow and outflow histories, and therefore the comparison between model results and the actual radial dependence of the MDF observed in the APOGEE survey will provide important constraints on the formation histories of the MW stellar disk.

In addition to gas inflow and outflow, radial migration of disk stars is an important process affecting the observed MDF. If the radial redistribution of disk stars occurs in a galactic disk, the stellar population at any radius is regarded as a mixture of disk stars born at various radii, implying no longer the shape of the MDF is determined only from the gas inflow and outflow histories in the radius. Thus, examining chemo-dynamical evolution models, which include the effect of radial migration, unlike the above one-zone model, is necessary in understanding the origin of the observed MDFs.

Such chemo-dynamical evolution models have been studied in many of previous papers (e.g., Schörich & Binney 2009; Kubryk et al. 2015). In this work, by combining our new chemo-dynamical model with the parameter surveys based on the Markov Chain Monte Carlo (MCMC) method, we attempt to derive the best solution for galaxy formation and evolution, which reproduces the observed radial dependence of the MDF of the Galactic disk stars. Moreover, based on our model experiments, we discuss the gas inflow, outflow and radial migration histories of the MW.

This thesis is organized as follows. In Chapter 2, we introduce our chemo-dynamical evolution model. In Chapter 3, we show the fitting results to the various properties of the present Galactic stellar disk based on the MCMC method. In Chapter 4, we discuss our model calculation results in detail and present new implications for the formation history of the MW. In Chapter 5, we show the validity of our model calculation. Finally, our conclusions are drawn in Chapter 6.
Figure 1.1: The dependence of the distribution function of stars in [O/H] on $\tau_{\text{in}}$ obtained from the one-zone chemical evolution model. The red, green, and blue lines represent the result for $\tau_{\text{in}} = 1$ Gyr, 3 Gyr, and 5 Gyr, respectively. Here, we set $[O/H]_{\text{in}} = -1$, and $\Lambda = 0.5$. The detailed description of this one-zone chemical evolution model is given in Appendix A.
1.2. CONSTRAINTS ON GALAXY EVOLUTION FROM METALLICITY DISTRIBUTION OF STARS

Figure 1.2: The same figure as Figure 1.1, but for the dependence on [O/H]_in. The red, green, and blue lines represent the result for [O/H]_in = −2, −1, and −0.5, respectively. Here, we set τ_in = 3 Gyr, and Λ = 0.5.
Figure 1.3: The same figure as Figure [1.1](#), but for the dependence on $\Lambda$. The red, green, and blue lines represent the result for $\Lambda = 0.5$, 1.5, and 2.5, respectively. Here, we set $\tau_m = 3$ Gyr, and $[\text{O/H}]_m = -1$. 
Chapter 2

CHEMO-DYNAMICAL EVOLUTION MODEL

In this thesis, we adopt a standard one-dimensional chemo-dynamical model, as studied in many previous works (e.g., Schönrich & Binney 2009; Kubryk et al. 2015). In this model, we consider a galactic disk consisting of many co-center rings, and by calculating baryonic mass evolution for each ring at each time step, we obtain the surface density of gas, $\Sigma_{\text{gas}}$, that of stars, $\Sigma_{\text{star}}$, and a mass fraction of heavy element $i$, $Z_i$, respectively, at any time, $t$, and at any radius, $R$. These calculations are carried out in a radial range from $R = 0$ to $R_{\text{out}}$ ($= 16 \text{ kpc}$) with a grid of $\Delta R = 1 \text{ kpc}$ over $t = 0$ to $t_p$ ($= 12 \text{ Gyr}$) with a grid of $\Delta t = 50 \text{ Myr}$. We are particularly interested in both the $[\text{Fe/H}]$ and $[\alpha/\text{Fe}]$ distributions of disk stars and their evolution, where $[\alpha/\text{Fe}]$ is defined as the average for the typical $\alpha$ elements of O, Mg, Si, Ca, and Ti relative to the Fe abundance.

In Section 2.1-2.5, we introduce each important process in our chemo-dynamical evolution model. Basic equations to calculate baryonic mass evolution in a galactic disk are described in Section 2.6. Finally, in Section 2.7 we discuss how we select the values of the important free parameters in our model.

2.1 Gas Inflow Rate

Inflow of gas from the circumgalactic region into a dark matter halo is an essential
process in the course of galaxy formation. In our model, we assume that the surface density of gas inflow rate, $\Sigma_{\text{in}}$, as functions of $t$ and $R$ is given as,

$$
\Sigma_{\text{in}}(t, R) = \begin{cases} 
\Sigma_{\text{in},0} \exp\left(-R/h_{\text{R,in1}} - t/\tau_{\text{in}}\right) & (R \leq R_b) \\
\Sigma_{\text{in},0} \exp\left\{-(R - R_b)/h_{\text{R,in2}} - R_b/h_{\text{R,in1}} - t/\tau_{\text{in}}\right\} & (R > R_b)
\end{cases},
$$

(2.1)

where $M_{\text{tot,in}}$ is the total mass of inflowing gas on the disk plane. $R_b$ is the break radius where the scale length of the radial profile of gas inflow changes from $h_{\text{R,in1}}$ at inner radii to $h_{\text{R,in2}}$ at outer radii. In our previous model shown in Toyouchi & Chiba (2016), we adopted a single exponential profile of gas inflow without any break. However, van den Bosch et al. (2001) based on a theoretical consideration suggest that the radial profile of gas inflow can be more complex, as noted in more detail in Section 4.2, and therefore in this work we consider $R_b$, $h_{\text{R,in1}}$, and $h_{\text{R,in2}}$ to take into account a broken radial profile of gas inflow. The necessity of a break in the radial profile of gas inflow for reproducing the observational property of the Milky Way will be discussed in Section 5.1.

$\tau_{\text{in}}$ is a time scale of gas inflow rate at $R$, described with $\tau_{\text{in},0}$, $\tau_{\text{in},8}$ and $\alpha$ in our model as,

$$
\tau_{\text{in}}(R) = \tau_{\text{in},0} + (\tau_{\text{in},8} - \tau_{\text{in},0}) \left(\frac{R}{8 \text{ kpc}}\right)^{\alpha}.
$$

(2.3)

Such an assumption of radially dependent time scale of gas inflow has been adopted in many previous studies (e.g., Chiappini et al. 1997, 2001), all of which assumed that the time scale increases with increasing $R$ based on the inside-out galaxy formation scenario. In this model, we don’t assume such an inside-out scenario explicitly. The gas inflow history is characterized by the seven free-parameters ($M_{\text{tot,in}}$, $R_b$, $h_{\text{R,in1}}$, $h_{\text{R,in2}}$, $\tau_{\text{in},0}$, $\tau_{\text{in},8}$, $\alpha$).
2.2 Chemical Abundance of Inflowing Gas

Inflowing gas on a galactic disk consists of not only primordial gas, but also metal-enriched gas, stripped from accreting satellite galaxies and re-accreting after ejected from the disk plane as outflow. The fraction of these metal-enriched gas in inflowing gas is very low at the early phase of galaxy formation, but increases with increasing time, leading to the growth of the metallicity of the inflowing gas (e.g., Tsujimoto et al. 2010).

In this model, we assume the time evolution of iron and $\alpha$ abundance of inflowing gas, $[\text{Fe}/H]_{\text{in}}$ and $[\alpha/H]_{\text{in}}$, respectively, as follows,

$$
([\text{Fe}/H]_{\text{in}}, [\alpha/H]_{\text{in}}) = \begin{cases} 
-1.0 + ([\text{Fe}/H]_{\text{in},0} + 1.0) \frac{t}{t_{\text{in}}}, & \text{if } t \leq t_{\text{in}} \\
-0.6 + ([\alpha/H]_{\text{in},0} + 0.6) \frac{t}{t_{\text{in}}}, & \text{if } t > t_{\text{in}}.
\end{cases}
$$

(2.4)

We here set the initial abundance of inflowing gas of $[\text{Fe}/H]_{\text{in}} = -1$ and $[\alpha/H]_{\text{in}} = -0.6$, which are in rough agreement with the low-metal edge of the distribution function of $[\text{Fe}/H]$ and $[\alpha/H]$ observed in APOGEE survey (Hayden et al. 2015). $[\text{Fe}/H]_{\text{in}}$ and $[\alpha/H]_{\text{in}}$ increase with increasing time up to $[\text{Fe}/H]_{\text{in},0}$ and $[\alpha/H]_{\text{in},0}$, respectively, by $t = t_{\text{in}}$, and after this time, $[\text{Fe}/H]_{\text{in}}$ and $[\alpha/H]_{\text{in}}$ are constant.

In this model, $t_{\text{in}}$ and $[\text{Fe}/H]_{\text{in},0}$ are parameters characterizing the time evolution of chemical abundance of inflowing gas. We here set $[\alpha/H]_{\text{in},0} = [\text{Fe}/H]_{\text{in},0}$, corresponding to $[\alpha/\text{Fe}]_{\text{in}} = 0$, because the observational facts adopted in our MCMC procedure introduced in Section 2.7 cannot constrain the value of $[\alpha/H]_{\text{in},0}$ so well.

2.3 Star Formation Rate

The surface density of star formation rate, $\Sigma_{\text{SFR}}$, in our model follows the observation-
ally motivated star formation law by Bigiel et al. (2008), in which \( \Sigma_{\text{SFR}} \) is proportional to the surface density of \( \text{H}_2 \) gas. The star formation rate can be written using the mass ratio of \( \text{H}_2 \) to HI gas, \( R_{\text{mol}} \), as follows,

\[
\Sigma_{\text{SFR}} = 1.6 \frac{R_{\text{mol}}}{R_{\text{mol}} + 1} \left( \frac{\Sigma_{\text{gas}}}{M_\odot \text{pc}^{-2}} \right) [M_\odot \text{ pc}^{-2} \text{ Gyr}^{-1}]. \tag{2.5}
\]

To calculate \( \Sigma_{\text{SFR}} \) from this equation, we additionally make use of the semi-empirical law of \( R_{\text{mol}} \) provided by Blitz & Rosolowsky (2006), which depends on \( \Sigma_{\text{gas}} \) and \( \Sigma_{\text{star}} \) as follows,

\[
R_{\text{mol}} = 0.23 \left[ \left( \frac{\Sigma_{\text{gas}}}{10 M_\odot \text{pc}^{-2}} \right) \left( \frac{\Sigma_{\text{star}}}{35 M_\odot \text{pc}^{-2}} \right)^{0.5} \right]^{0.92}. \tag{2.6}
\]

These empirical laws of equations (2.5) and (2.6) have been known to reproduce the present spatial distribution of HI, \( \text{H}_2 \) and star formation in the Milky Way (Blitz & Rosolowsky 2006; Kubryk et al. 2015).

### 2.4 Gas Outflow Rate

Feedback processes associated with star formation activity, such as radiation pressure from massive stars and supernova explosions, may drive a strong galactic gas outflow, which gives a significant impact on the chemical evolution of a galactic disk. In this model, we assume that the surface density of gas outflow rate, \( \Sigma_{\text{out}} \), is proportional to \( \Sigma_{\text{SFR}} \) with an assumed outflow-mass loading factor, \( \Lambda \), which is described as a following function,

\[
\Lambda(R) = \Lambda_0 + (\Lambda_8 - \Lambda_0) \left( \frac{R}{8 \text{ kpc}} \right)^{\beta}, \tag{2.7}
\]

where \( \Lambda_0 \), \( \Lambda_8 \), and \( \beta \) are parameters characterizing the radial dependence of \( \Lambda \).

It is worth noting that this expression of gas outflow tightly linking with star formation approximately includes the effect of gas radial flow along a galactic disk, because the radial gas flow is expected to be driven by gravitational torques related to the formation of spiral/bar or giant molecular clouds, which are also important drivers of star formation, so that radial gas flow may occur with a time scales similar to that of star formation.
2.5. RADIAL MIGRATION

(Yoshii & Sommer-Larsen 1989). Therefore, in our model $\Lambda$ can be negative, when the amount of gas expelled from the disk region as outflow is less than that of gas supplied into the region by radial gas flow, although $\Lambda$ calculated in Chapter 3 is found to be positive at all radii.

We also note that while for simplicity we assume that $\Lambda$ does not evolve with time, this assumption may not be appropriate. According to previous works on chemical evolution models applied to several observational results of extra-galactic star-forming galaxies, the outflow-mass loading factor increases with increasing redshift (e.g. Yabe et al. 2015; Toyouchi & Chiba 2015). Therefore, our time-independent mass loading factor may be regarded as an time-averaged one, although it actually changes with time.

2.5 Radial Migration

To take into account the effect of radial migration of disk stars in our model, we make use of the method adopted in Sellwood & Binney (2002), which can reproduce the basic properties of radial migration obtained in N-body simulations well. This method provides the probability, in which a star born at radius $R_f$ and with age $\tau$ is found in radius $R$, $P(\tau, R_f, R)$, and this is expressed in terms of the following Gaussian function

$$P(\tau, R_f, R) = \frac{1}{\sqrt{2\pi}\sigma_{RM}^2} \exp\left(-\frac{(R - R_f)^2}{2\sigma_{RM}^2}\right),$$  \hspace{1cm} (2.8)

where $\sigma_{RM}$ corresponds to the diffusion length of stars by radial migration, which is generally a function of $\tau$ and $R_f$. Thus, in this method the radial migration history can be characterized by the time dependence of $\sigma_{RM}$.

In our model, we assume that $\sigma_{RM}$ monotonically increases with increasing $\tau$ as follows,

$$\sigma_{RM}(\tau, R_f) = (a_{rm} R_f + b_{rm}) \left(\frac{\tau}{5 \text{ Gyr}}\right)^\gamma \text{ [kpc]},$$  \hspace{1cm} (2.9)

where, $a_{rm}$, $b_{rm}$, and $\gamma$ are parameters characterizing the radial migration history. This description of $\sigma_{RM}$ is the same as that adopted in Kubryk et al. (2013), who analyzed the radial migration history in a bar-dominated disk galaxy in the high-resolution N-body+smoothed particle hydrodynamics simulation, and determined $(a_{rm}, b_{rm}, \gamma) = (-0.0667, 2.75 \text{ kpc}, 0.5)$. We note here that their choices of the values of $a_{rm}$, $b_{rm}$, and $\gamma$ was
based on the numerical experiment for only one example of a disk galaxy. Therefore, it is worth searching for another set of \((a_{rm}, b_{rm}, \gamma)\) to represent the radial migration history of the Galactic stellar disk based on the analysis of the observed MDFs of the disk stars.

We also note here that such a continuous evolution of \(\sigma_{RM}\) assumed in equation (2.9) supposes the radial migration events driven by internal gravitational processes, such as interactions between bar/spiral structures and disk stars. However, our recent work in Toyouchi & Chiba (2016) suggests that not only such a continuous radial migration event, but also discontinuous one driven by external disk heating event, such as minor merging of satellite galaxies, are important in the chemical evolution in the galactic disk. Therefore while in this study we focus on the effect of continuous radial migration events for simplicity, more extensive modelings, including such discontinuous events, is one of the important subjects in our future study.

### 2.6 Basic Equations

To obtain the time evolution of \(\Sigma_{\text{gas}}, \Sigma_{\text{star}}\) and \(Z_i\), we solve the following equations, including the effects of star formation, gas inflow, gas outflow and radial migration of stars,

\[
\frac{\partial \Sigma_{\text{gas}}}{\partial t} = -(1 - R)\Sigma_{\text{SFR}} + \Sigma_{\text{in}} - \Sigma_{\text{out}},
\]

\[
\Sigma_{\text{star}}(t, R) = (1 - R) \int_0^{R_{\text{out}}} \int_0^t \frac{R_t}{R} \Sigma_{\text{SFR}}(t, R_t)P(t - t_t, R, R_t) dt_t dR_t,
\]

\[
\frac{\partial (Z_i \Sigma_{\text{gas}})}{\partial t} = (Y_{\text{II},i} + Y_{\text{Ia},i})\Sigma_{\text{SFR}} - Z_i(1 - R)\Sigma_{\text{SFR}} + Z_{\text{in},i}\Sigma_{\text{in}} - Z_{\text{out},i}\Sigma_{\text{out}}.
\]

In equation (2.10) the first term on the right hand side represents the net gas consumption by star formation, where \(R\) is the mass fraction returned back to interstellar medium (ISM) via stellar mass loss, and the description of this term is based on an instantaneous recycling approximation. In this paper we set \(R = 0.45\) corresponding to the Chabrier initial mass function (Leitner & Kravtsov 2011).
2.7. DETERMINATION OF MODEL PARAMETERS

Equation (2.11) implies that the time evolution of $\Sigma_{\text{star}}$ at any radius depends on the past star formation and radial migration history over the galactic disk. Here, $t_f$ corresponds to the formation time of stars. $R_{\text{out}}$ in equation (2.11) is the outer limit of $R$ in our calculation, where we set $R_{\text{out}} = 16$ kpc, i.e., much larger than the disk size of the Milky Way to avoid its effect on the calculation.

In equation (2.12) the first term on the right hand side describes the supply of heavy element $i$ newly synthesized in stars, where $Y_{\text{II},i}$ and $Y_{\text{Ia},i}$ are the nucleosynthetic yields from Type II and Ia SNe (hereafter SN II and SN Ia), respectively. We assume that $Y_{\text{II},i}$ is constant and adopt the SN II yield from François et al. (2004). On the other hand, $Y_{\text{Ia},i}$ changes with time and radius, reflecting the past star formation and radial migration history as follows,

$$Y_{\text{Ia},i} = (1-R) f_{\text{Ia}} \frac{m_{\text{Ia},i}}{\Sigma_{\text{SFR}}(t, R)} \int_0^{R_{\text{out}}} \int_0^{t-\Delta t_{\text{Ia}}} \frac{R_{\text{Ia}}}{R} \frac{\Sigma_{\text{SFR}}(t_{\text{Ia}}, R_{\text{Ia}})P(t-t_{\text{Ia}}, R_{\text{Ia}})}{t-t_{\text{Ia}}} dt_{\text{Ia}} dR_{\text{Ia}},$$

(2.13)

where $f_{\text{Ia}}$ is a free parameter controlling the SN Ia rate in the galactic disk, and $\Delta t_{\text{Ia}}$ is a minimum delayed time of SN Ia. Here we set $\Delta t_{\text{Ia}} = 0.5$ Gyr suggested by Homma et al. (2015), which reproduced, with this value of $\Delta t_{\text{Ia}}$, star formation histories and chemical evolutions of the Galactic dwarf galaxies self-consistently. Another SN Ia parameter, $m_{\text{Ia},i}$, in equation (2.13) is the released mass of heavy element $i$ per a Type Ia supernova, for which we adopt the SN Ia yield of W7 model in Iwamoto et al. (1999).

The second term on the right hand side of equation (2.12) is the mass of heavy elements finally locked up in stars. The third and fourth terms on the right hand side of equation (2.12) denote the mass injection and ejection of heavy elements associated with inflow and outflow, respectively, where $Z_{\text{in},i}$ and $Z_{\text{out},i}$ are mass fractions of heavy element $i$ in inflowing and outflowing gas, respectively. $Z_{\text{in},i}$ follows the description of the time evolution of $[\text{Fe/H}]_{\text{in}}$ and $[\alpha/\text{H}]_{\text{in}}$ in Section 2.2, and the metallicity of outflowing gas always corresponds to that of the ISM, implying $Z_{\text{out},i} = Z_i$.

2.7 Determination of Model Parameters

Our chemical evolution model described above contains 16 free parameters, which are
summarized in Table 2.1. In this study, we adopt the MCMC method (Metropolis et al. 1953; Hastings 1970) to obtain the best set of these free parameters, which reproduce the radial profile of star, gas, [O/H] and [Fe/H] of the ISM. In this model, following the results of several previous works, we adopt the Galactic stellar disk with the scale length of 2.3 kpc and the stellar density at \( R = 8 \) kpc of 35 \( M_\odot \) pc\(^{-2}\) (Flynn et al. 2006). For the total gas density profile, consisting of HI and H\(_2\), in the Galactic disk, we adopt the work of Kubryk et al. (2015) shown in their Figure A.2, and the radial profile of [O/H] and [Fe/H] in the disk is taken from the observation of Cepheids in Luck & Lambert (2011).

In addition to these observational constraints, we also adopt the radial dependence of the MDF of disk stars observed in APOGEE survey (Hayden et al. 2015) as the new observational constraints to determine the model parameters in our MCMC procedure. Figure 2.1 shows the radial dependence of the MDF for the APOGEE red-giant star sample locating at \( |Z| < 2 \) kpc. From this figure, it is clear that the properties of the MDFs, such as their peak and high and low-metallicity tales, significantly depend on the radii. Here, to quantify this radial change of the observed MDFs, we explore the mean, standard deviation, skewness, and kurtosis of the MDF at each radius in Figure 2.2. We find that the mean and standard deviation decrease with increasing \( R \), whereas the skewness increases from \( \approx -1 \) at \( R = 4 \) kpc to \( \geq 0 \) at \( R = 12 \) kpc, which accompanies the growth of the high-metallicity tail in the MDF with increasing \( R \). The kurtosis is mostly constant with \( R \), but rapidly increases in the outer disk region, \( R > 12 \) kpc. In our MCMC procedure, we use these radial profile of the moments of the MDF as the observational constraints. Thus we attempt to obtain the evolution history of the Milky Way, which reproduces the observed MDF for the Galactic disk stars.
2.7. DETERMINATION OF MODEL PARAMETERS

Table 2.1: The list of free parameters in our chemo-dynamical model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\text{Tot, in}}$</td>
<td>Total mass of inflowing gas</td>
</tr>
<tr>
<td>$R_b$</td>
<td>Break radius in the radial profile of gas inflow</td>
</tr>
<tr>
<td>$h_{R,\text{in}1}$</td>
<td>Scale length of inflowing gas at the inside of $R_b$</td>
</tr>
<tr>
<td>$h_{R,\text{in}2}$</td>
<td>Scale length of inflowing gas at the outside of $R_b$</td>
</tr>
<tr>
<td>$\tau_{\text{in},0}$</td>
<td>Time scale of gas inflow at $R = 0 \text{ kpc}$</td>
</tr>
<tr>
<td>$\tau_{\text{in},8}$</td>
<td>Time scale of gas inflow at $R = 8 \text{ kpc}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Power-law index characterizing the radial dependence of the time scale of gas inflow</td>
</tr>
<tr>
<td>$\Lambda_0$</td>
<td>Mass loading factor at $R = 0 \text{ kpc}$</td>
</tr>
<tr>
<td>$\Lambda_8$</td>
<td>Mass loading factor at $R = 8 \text{ kpc}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Power-law index characterizing the radial dependence of mass loading factor</td>
</tr>
<tr>
<td>$a_{\text{rm}}$</td>
<td>Radial gradient of diffusion length of disk stars by radial migration</td>
</tr>
<tr>
<td>$b_{\text{rm}}$</td>
<td>Diffusion length of disk stars by radial migration at $R = 0 \text{ kpc}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Power-law index characterizing the dependence of radial diffusion length on stellar age</td>
</tr>
<tr>
<td>$t_{\text{in}}$</td>
<td>Characteristic time in evolution of metallicity of inflowing gas</td>
</tr>
<tr>
<td>$[\text{Fe/H}]_{\text{in},0}$</td>
<td>Metallicity of inflowing gas at $t = t_{\text{in}}$</td>
</tr>
<tr>
<td>$f_{\text{Ia}}$</td>
<td>Parameter controlling the rate of Type Ia supernova</td>
</tr>
</tbody>
</table>
Figure 2.1: The MDFs of the Galactic disk stars observed by the APOGEE survey (Hayden et al. 2015). The observed radial range of each MDF is denoted on the top of each panel.
2.7. DETERMINATION OF MODEL PARAMETERS

Figure 2.2: The mean, standard deviation, skewness, and kurtosis of the observed MDFs as a function of $R$ are shown in the top-left to bottom-right panels, respectively.
Chapter 3

FITTING RESULTS

In our MCMC procedure, we carry out 5 MCMC chains starting from different parameter sets. Each chain consists of 100,000 iterations, and by compiling the later 50,000 iterations in all chains, we get the posterior probability distribution for each parameter. Figure 3.1 shows the posterior probability distributions of the 16 parameters. We find from this figure that the MCMC chains for all parameters converge successfully. The results of this estimation of the 16 parameters are summarized in Table 3.1.

We here present the result of the model calculation based on the set of best-fit parameters. Figure 3.2 shows the results of the time evolution of the radial profiles of gas, star, [O/H] and [Fe/H] of interstellar medium. The green, blue, yellow and red lines in each panel represent the results at $t = 2, 4, 8,$ and 12 Gyr, respectively. These results imply that the structural and chemical evolutions have proceeded faster at inner disk region, corresponding to the inside-out formation scenario. Additionally, in each panel of Figure 3.2 we show the observed profiles with the black lines. From comparison between the red and black lines in these figures, we find that our model can give the moderately good reproduction for the present chemo-structural properties of the Galactic disk.

The reproduced MDFs at the four radial ranges and the radial profiles of their mean, standard deviation, skewness, and kurtosis are shown in Figure 3.3 and 3.4, respectively. In these figures, the red and black lines represent the result from the model calculation and the observations as shown in Figure 2.1 and 2.2, respectively. These results show that our model can reproduce the observed MDF of the Galactic disk stars successfully.
Thus our chemical evolution models are appropriate in comparison of the Galactic stellar disk. In the next chapter, we show the detailed formation history of the MW predicted by the best fit model and present the implications for the gas inflow, gas outflow and radial migration processes in the evolution of the MW stellar disk.
Table 3.1: The results of the estimation of free parameters

<table>
<thead>
<tr>
<th></th>
<th>best(^\text{a})</th>
<th>16 %(^\text{b})</th>
<th>50 %(^\text{c})</th>
<th>84 %(^\text{d})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{\text{tot,in}}) [10(^{10}) (M_\odot)]</td>
<td>18.1</td>
<td>15.3</td>
<td>21.6</td>
<td>32.4</td>
</tr>
<tr>
<td>(R_b) [kpc]</td>
<td>4.44</td>
<td>4.86</td>
<td>5.67</td>
<td>6.55</td>
</tr>
<tr>
<td>(h_{R, in1}) [kpc]</td>
<td>1.02</td>
<td>1.13</td>
<td>1.42</td>
<td>1.83</td>
</tr>
<tr>
<td>(h_{R, in2}) [kpc]</td>
<td>9.97</td>
<td>7.27</td>
<td>11.4</td>
<td>16.5</td>
</tr>
<tr>
<td>(\tau_{in,0}) [Gyr]</td>
<td>0.98</td>
<td>1.37</td>
<td>2.32</td>
<td>2.69</td>
</tr>
<tr>
<td>(\tau_{in,8}) [Gyr]</td>
<td>21.9</td>
<td>9.29</td>
<td>15.0</td>
<td>22.9</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>1.61</td>
<td>1.32</td>
<td>2.48</td>
<td>3.79</td>
</tr>
<tr>
<td>(\Lambda_0)</td>
<td>0.52</td>
<td>0.51</td>
<td>0.67</td>
<td>0.86</td>
</tr>
<tr>
<td>(\Lambda_8)</td>
<td>1.77</td>
<td>1.45</td>
<td>1.77</td>
<td>2.19</td>
</tr>
<tr>
<td>(\beta)</td>
<td>2.87</td>
<td>2.56</td>
<td>3.33</td>
<td>4.17</td>
</tr>
<tr>
<td>(a_{rm}) [10(^{-2})]</td>
<td>-9.07</td>
<td>-5.45</td>
<td>0.99</td>
<td>6.44</td>
</tr>
<tr>
<td>(b_{rm}) [kpc]</td>
<td>2.77</td>
<td>1.95</td>
<td>2.27</td>
<td>2.56</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.24</td>
<td>0.06</td>
<td>0.19</td>
<td>0.37</td>
</tr>
<tr>
<td>(t_{in}) [Gyr]</td>
<td>1.47</td>
<td>0.83</td>
<td>1.12</td>
<td>1.48</td>
</tr>
<tr>
<td>([\text{Fe/H}]_{in,0})</td>
<td>-0.41</td>
<td>-0.50</td>
<td>-0.43</td>
<td>-0.38</td>
</tr>
<tr>
<td>(f_{1a}) [10(^{-4})]</td>
<td>4.39</td>
<td>3.87</td>
<td>5.14</td>
<td>6.59</td>
</tr>
</tbody>
</table>

\(^{a}\) The best value obtained from the MCMC estimation.

\(^{b}\) The 16 percentile value obtained from the MCMC estimation.

\(^{c}\) The 50 percentile value obtained from the MCMC estimation.

\(^{d}\) The 84 percentile value obtained from the MCMC estimation.
Figure 3.1: The diagonal and off-diagonal panels show the 1D and 2D posterior probability distributions of the 16 parameters, respectively. The red, green and blue contours in each off-diagonal panel represent the 1, 2 and 3 $\sigma$ regions of the posterior probability distributions, respectively.
Figure 3.2: The radial profiles of surface densities for [O/H], [Fe/H], gas, and stars obtained in the best fit model. The green, blue, yellow, and red lines show the results at \( t = 2, 4, 8, \) and 12 Gyr, respectively. The black lines are the observed radial profiles.
Figure 3.3: The MDFs at the four radial ranges obtained in the best fit model are represented with the red lines. The black lines are the observed MDFs at the radial ranges.
Figure 3.4: The radial dependences of the mean, standard deviation, skewness, and kurtosis of the MDFs obtained in the best fit model are described with the red lines. The black lines are the observations shown in Figure 2.2.
Chapter 4

FORMATION HISTORY OF THE MILKY WAY REPRODUCED BY MODEL CALCULATION

4.1 Star Formation History

The red lines in upper and lower panels in Figure 4.1 represent the model results of the time evolution of total star formation rate and stellar mass, respectively. For comparison, the star formation history of the MW like galaxies provided by van Dokkum et al. (2013) based on abundance matching method are also shown with the black lines in these figures. While their present stellar mass is higher than that of our model by $\sim 10^{10} \, M_\odot$, this difference may be because the mass from bulge component is not included in our model. According to our model calculation, the star formation rate became highest around at look back time of 11 Gyr and more than half of the present stellar mass has already formed by look back time of 8 Gyr, generally corresponding to the results from van Dokkum et al. (2013).

The time evolution of the radial profile of $\Sigma_{SFR}$ is depicted in Figure 4.2. This figure shows that at the inner disk region the star formation activity is significantly high at the early disk formation phase, and rapidly declines with increasing time. In contrast,
the star formation rate in the outer disk region is lower than that in the inner one at all epochs, but mostly independent of time. This star formation history corresponds to the inside-out disk evolution as shown in Figure 3.2. In the following subchapters, we investigate the detailed properties of important processes, which can significantly affect the star formation history.
Figure 4.1: The time evolution of the total star formation rate and the stellar mass in the galaxy are shown in the upper and lower panels, respectively. The red and black lines represent the results of our model calculation and the observation for the MW like galaxies by van Dokkum et al. (2013), respectively.
Figure 4.2: The radial profiles of $\Sigma_{\text{SFR}}$ as a function of $t$ obtained in the best fit model. The green, blue, yellow, and red lines show the results at $t = 2$, 4, 8, and 12 Gyr, respectively.
4.2 Gas Inflow History

The red line in Figure 4.3 presents the radial profiles of time scale of gas inflow based on the best fit parameters. It follows that the time scales of gas inflow in the central and outer most disk regions are shorter than 1 Gyr and much longer than 10 Gyr, respectively. This radial dependence of \( \tau_{\text{in}} \) is simply understood with a shorter cooling and collapsing time at inner gas halo, in which the gas density is higher (Molla & Diaz 2005; Molla et al. 2016).

Additionally, we describe the surface density of the total mass of gas inflow as a function of \( R \) with red line in Figure 4.4, and find that the radial profile of gas inflow is clearly up-bending. To understand the origin of such an up-bending profile of gas inflow, we present a theoretical prediction by van den Bosch et al. (2001) with the black line in Figure 4.4. The prediction is based on an assumption that gas in a halo follows the same specific angular momentum distribution as the dark matter, produced in N-body simulations of structure formation, and all the gas cool and form the disk as conserving the specific angular momentum distribution completely (see Appendix A for the detailed model description). The radial profile of gas inflow from this simple prediction is a double power law rather than a single exponential, and it is generally similar to our up-bending profile. This fact suggests that such an up-bending profile of gas inflow results from the original distribution of angular momentum of gas accreting onto the galactic disk plane. However, we note here that the above assumption about the angular momentum distribution of gas is somewhat simplistic because gas can cool and form fine structures through energy dissipation process, and consequently the gravitational torque to gas component is much different from that to the dark matter component (Danovich et al. 2015). Therefore the origin of the up-bending profile of gas inflow have to be investigated more strictly based on hydrodynamical simulations.

The resulting gas inflow history is shown as a function of \( R \) and \( t \) in Figure 4.5, and we find that at early disk formation phase a large amount of gas rapidly accretes onto the inside of \( R \approx 4 \) kpc, whereas at the outside of the radius gas stationary accretes over the whole disk formation phase. This inflow history is the key ingredient leading to the inside-out star formation history described in the previous chapter.
Figure 4.3: The radial profile of $\tau_m$ in the best fit model.
4.2. GAS INFLOW HISTORY

Figure 4.4: The radial profile for the surface density of the total mass of gas inflow in the best fit model is shown with the red line. The black line represents the theoretical prediction by van den Bosch et al. (2001), described in equation (A.4) with $\lambda = 0.05$, $\mu = 1.5$, and $r_{\text{vir}} = 250$ kpc.
Figure 4.5: The radial profile of $\Sigma_{in}$ as a function of $t$ in the best fit model. The green, blue, yellow, and red lines show the results at $t = 2$, $4$, $8$, and $12$ Gyr, respectively.
4.3 Time Evolution of Metallicity of Inflowing Gas

As denoted in Table 3.1, our MCMC procedure gives the best fit solution of $(t_{\text{in}}, [\text{Fe/H}]_{\text{in},0}) = (1.47 \text{ Gyr}, -0.41)$, implying rapid and significant increase of $[\text{Fe/H}]_{\text{in}}$ at the early disk formation phase as shown in Figure 4.6. This rapid time evolution of $[\text{Fe/H}]_{\text{in}}$ may be possible in the realistic galactic evolution context, if such an evolution is driven by re-accretion of metal-enriched galactic wind. According to the previous theoretical studies based on semi-analytical models (e.g., Spitoni et al. 2009) and hydrodynamical simulations (e.g., Bekki et al. 2009), the typical time scale of falling back of gas ejected from the disk plane is $\sim$ a few 100 Myr. Additionally, as shown below in detail, our model predicts the large amount of metal-enriched outflowing gas with $[\text{Fe/H}] \gtrsim -0.5$ from the inner disk region, $R < 4 \text{ kpc}$, at the early phase, $t \leq 1 \text{ Gyr}$. These metal-enriched gas ejected from the inner disk region is expected to acquire additional angular momentum via feedback processes and preferentially re-accrete into disk regions further out than the radii where they originally resided (Bekki et al. 2009). Therefore, if such a recycling process of galactic wind occurs in the formation history of the MW, the time evolution of $[\text{Fe/H}]_{\text{in}}$ in our model is well understood, although we need further investigation with detailed numerical simulations.

To understand the importance of such a rapid increase of $[\text{Fe/H}]_{\text{in}}$ in the formation history of the MW, we here present an additional MCMC experiment for the case with a fixed set of parameters of $([\text{Fe/H}]_{\text{in}}, [\alpha/\text{H}]_{\text{in}}) = (-1.0, -0.6)$. Figure 4.7 and 4.8 are the same figures as Figure 3.3 and 3.4, respectively, but for the best fit result without the effect of time evolution of $[\text{Fe/H}]_{\text{in}}$, for which the more detailed information of the fitting is presented in Appendix B.2. It is evident from Figure 4.7 and 4.8 that the accretion of such a metal-poor gas forms more significant metal-poor tales in the MDFs and consequently makes their standard deviations much larger than the observation especially at the outer disk regions. This result implies that the evolution of $[\text{Fe/H}]_{\text{in}}$ is a key ingredient to reproduce the observed radial dependence of the MDF of disk stars. We conclude based on the rapid increase of $[\text{Fe/H}]_{\text{in}}$ and the inside-out star formation history obtained from our model calculation that the inner disk region of the Milky Way formed very rapidly from metal-poor gas, whereas the outer one has been gradually constructed from metal-
Figure 4.6: The time evolution of $[\text{Fe/H}]_{\text{in}}$ in the best fit model.

rich gas supplied from re-accretion of galactic wind, and such a formation history of the Galactic stellar disk is recorded into the radial dependence of the low metallicity side of the MDF of the disk stars.
4.3. TIME EVOLUTION OF METALLICITY OF INFLOWING GAS

Figure 4.7: The same figure as Figure 3.3, but for the model calculation without the time evolution of $[\text{Fe}/\text{H}]_\text{in}$. 
Figure 4.8: The same figure as Figure 3.4, but for the model calculation without the time evolution of [Fe/H]_{in}.
4.4 Gas Outflow History

In our best fit model, $\Lambda$ is mostly constant of $\sim 0.5$ at the inner disk region, and drastically increases at the outer disk region, as shown in Figure 4.9. Since the radius, $R \sim 3$ kpc, at which $\Lambda$ begins to increase is roughly consistent with the scale length of the present Galactic stellar disk, the rapid increase of $\Lambda$ at the outer disk region is associated with the rapid decline of the local gravitational binding energy of the stellar disk. The resulting outflow history is presented in Figure 4.10, and we find that while $\Lambda$ increases with increasing $R$, $\Sigma_{\text{out}}$ is higher at inner disk region, in particular at the early intense disk formation phase, when the star formation activity is significantly centrally concentrated, as shown in Figure 4.2.

The gas outflow history obtained in this study are very similar to that in our previous study shown in Toyouchi & Chiba (2015), which analyzed the observed chemo-structural evolution of distant star-forming galaxies based on their chemical evolution model. In this previous study, we suggested that when the relation between $\Sigma_{\text{out}}$ and $\Sigma_{\text{SFR}}$ is fitted by a functional form, $\Sigma_{\text{out}} \propto \Sigma_{\text{SFR}}^{k_{\text{out}}}$, the value of the index, $k_{\text{out}}$, becomes much different between the EDW and MDW models. In Toyouchi & Chiba (2015), we described $\Sigma_{\text{out}}$ as a function of $\Sigma_{\text{SFR}}$ for both cases of EDW and MDW models, as follows,

$$\Sigma_{\text{out}} \propto \begin{cases} 
\Sigma_{\text{SFR}}^{0.29} & (\text{EDW}) \\
\Sigma_{\text{SFR}}^{0.64} & (\text{MDW in optically thin limit}) \\
\Sigma_{\text{SFR}}^{1.36} & (\text{MDW in optically thick limit})
\end{cases} \quad (4.1)$$

For the MDW model we considered the optically thin and thick limit to the infrared radiation, because the contribution of the re-radiation from the dust grains to the injection of the momentum into the ISM significantly depends on the optical depth of the ambient gas. The detailed derivation of Equation (4.1) are described in Appendix A. Thus, investigating the dependence of $\Sigma_{\text{out}}$ on $\Sigma_{\text{SFR}}$ can provide an important implication for the driving mechanism of galactic wind.

Figure 4.11 shows the relation between $\Sigma_{\text{out}}$ and $\Sigma_{\text{SFR}}$ for our best fit model. The green, blue, yellow and red lines represent the $\Sigma_{\text{out}} - \Sigma_{\text{SFR}}$ relations at $t = 2, 4, 8,$ and 12 Gyr, respectively. We find the break points on the relations at all epochs, which
corresponds to the breaks in the radial profiles of $\Sigma_{\text{out}}$ in Figure 4.10 at around $R = 4.5$ kpc, originating from the up-bending profile of gas inflow as in Figure 4.3. Therefore, these breaks in the $\Sigma_{\text{out}} - \Sigma_{\text{SFR}}$ relations imply that $\Sigma_{\text{out}}$ is roughly proportional and unproportional to $\Sigma_{\text{SFR}}$ at the inner and outer disk regions, respectively.

To quantify the difference of the $\Sigma_{\text{out}} - \Sigma_{\text{SFR}}$ relations between the inner and outer disk regions, for each of the regions we investigate the time evolution of the value of $k_{\text{out}}$ by fitting the $\Sigma_{\text{out}} - \Sigma_{\text{SFR}}$ relation by a function, $\Sigma_{\text{out}} \propto \Sigma_{\text{SFR}}^{k_{\text{out}}}$, at each time step. Figure 4.12 shows the time evolution of $k_{\text{out}}$, where the red and blue lines correspond to the value of $k_{\text{out}}$ measured below and beyond $R = 4.5$ kpc, respectively, and for comparison the expected values of $k_{\text{out}}$ for the EDW, and MDW in optically thin and thick limits described in Equation (4.11) are drawn with the dashed, dash-dot, and dotted lines, respectively. We find from this figure that while at the inner disk region $k_{\text{out}}$ is always confined in the range between optically thin and thick limits for the MDW model, at the outer one it roughly agrees with the EDW model, except for the early several Gyrs, when the $\Sigma_{\text{out}} - \Sigma_{\text{SFR}}$ relation may not be applied because the activities of star formation and gas outflow at the disk region are still negligibly low. These behaviors of $k_{\text{out}}$ at the inner and outer disk regions are expected to reflect the difference of the gas densities in between these regions. Namely, the high gas density in the inner disk region enables the efficient dissipation of kinetic energy of the ISM due to radiative cooling, and consequently momentum injection into the ISM can be a main mechanism to drive wind at this region, whereas the low gas density in the outer disk region allows the kinetic energy of the ISM to be mostly conserved because of the less efficient radiative cooling, and therefore the galactic wind from this region can be described in terms of the EDW model. Thus, our model calculation provides new important implications for the gas outflow processes from the star forming disk and their driving mechanisms.
Figure 4.9: The radial profile of $\Lambda$ in the best fit model.
Figure 4.10: The radial profile of $\Sigma_{\text{out}}$ as a function of $t$ in the best fit model. The green, blue, yellow, and red lines show the results at $t = 2$, 4, 8, and 12 Gyr, respectively.
Figure 4.11: The relation between $\Sigma_{out}$ and $\Sigma_{SFR}$ as a function of $t$ in the best fit model. The green, blue, yellow, and red lines show the results at $t = 2$, $4$, $8$, and $12$ Gyr, respectively. The black dashed line corresponds to $\Sigma_{out} = \Sigma_{SFR}$.
Figure 4.12: The time evolution of $k_{\text{out}}$ measured below and beyond $R = 4.5$ kpc are shown with the red and blue lines, respectively. The black dashed, dash-dot, and dotted lines represent the theoretical expectations of $k_{\text{out}}$ in the case of the EDW, and the MDW in optically thin and thick limits, respectively, described in Equation (4.1).
4.5 Radial Migration History

Our MCMC procedure determines the best fit parameters, characterizing the radial migration history, as \((a_{rm}, b_{rm}, \gamma) = (-0.0907, 2.77 \text{ kpc}, 0.24)\), which is very similar to the results from the high-resolution N-body+smoothed particle hydrodynamics simulation by Kubryk et al. (2013). The evolution of \(\sigma_{RM}\) based on the best fit parameters is demonstrated in Figure 4.13. Here, we show the cases for \(R_f = 4, 8\) and 12 kpc with red, blue, and green lines, respectively, and find that the diffusion length of radial migration is larger for disk stars born at the inner disk region. This radial dependence of \(\sigma_{RM}\) is simply understood from the fact that an inner disk part is more gravitationally unstable and consequently has more bar/spiral structures and GMCs, triggering the radial migration of disk stars. Additionally, this figure shows that for age \(< 2 \text{ Gyr}\) \(\sigma_{RM}\) drastically evolves although for age \(\geq 2 \text{ Gyr}\) the evolution of \(\sigma_{RM}\) slows down. Such a evolution of \(\sigma_{RM}\) implies that although young disk stars can effectively move radially due to interactions with bar/spiral structures and GMCs, the efficiency of radial migration of disk stars quickly declines with increasing stellar age because such interactions simultaneously disturb the dynamics of disk stars and weaken the gravitational connections with bar/spirals and GMCs (e.g., Hämminen & Flynn 2002; De Simone et al. 2004; Aumer et al. 2016).

To clarify the impact of this radial migration history on the MDF of the disk stars, we show the MDF as a function of \(R\) and \(R_f\) in the top and bottom panels, respectively, in Figure 4.14. From these panels it is found that the MDFs observed in any \(R\), especially outer disk region, have more significant high-metallicity tails than the MDFs of disk stars with the same \(R_f\). This implies that the observed high metallicity tails in MDFs at outer disk regions reveal due to the net radial transfer of metal-rich disk stars born at the inner disk region toward the outer one. Similar suggestion was recently provided in Loebman et al. (2016) based on the numerical simulation for the formation of an isolated disk galaxy. Thus, the radial migration effects are necessary to interpret the observed radial change of the shape of MDFs.
Figure 4.13: The dependence of $\sigma_{RM}$ on stellar age in the best fit model. The red, blue, and green lines are $\sigma_{RM}$ for $R_f = 4$, 8, and 12 kpc, respectively.
Figure 4.14: The upper and lower panels show the MDF as a function of $R$ and $R_f$, respectively, in the best fit model.
Chapter 5

VALIDITY OF OUR MODEL

5.1 Is Our Model Setting Valid?

In our model, the four effects, namely the presence of break in the radial profile of gas inflow, time evolution of $[\text{Fe/H}]_{\text{in}}$, gas outflow, and radial migration, are taken into account, and these were not necessarily included in previous chemical evolution models. In this section, we assess quantitatively how important these four effects are in galaxy evolution. For this purpose, we carry out MCMC fittings when either of these effects is turned off, and examine the resulting fitting result in terms of the value of the so-called bayesian information criterion (BIC), as defined by the following equation,

$$ BIC = -2 \ln(L_{\text{max}}) + N_{\text{para}} \ln(n) \ . $$

Here, $L_{\text{max}}$ and $N_{\text{para}}$ are the maximum value of likelihood and the number of free parameters, respectively. $n$ is the number of data points to measure likelihood, where $n = 40$ in our MCMC procedures. BIC can evaluate the goodness of the fitting with different $N_{\text{para}}$ for each model, and its value becomes lower for a better model. If the value of BIC for the model without including either of the above four effects is lower than that for the fiducial model including all effects, the effect in concern is regarded unimportant to understand the present properties of the MW stellar disk.

Table 5.1 summarizes the maximum value of likelihood and BIC for the model experiment when either of the four effects is not included. In brief, the value of BIC is the
Table 5.1: The informations of estimation for the model without each important process.

<table>
<thead>
<tr>
<th>Model</th>
<th>$N_{\text{para}}^a$</th>
<th>$(\ln L)_{\text{max}}^b$</th>
<th>BIC$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiducial</td>
<td>16</td>
<td>$-6.18$</td>
<td>71.38</td>
</tr>
<tr>
<td>w/o break in radial profile of gas inflow</td>
<td>14</td>
<td>$-16.56$</td>
<td>84.75</td>
</tr>
<tr>
<td>w/o time evolution of $[\text{Fe/H}]_{\text{in}}$</td>
<td>14</td>
<td>$-26.26$</td>
<td>104.17</td>
</tr>
<tr>
<td>w/o gas outflow</td>
<td>13</td>
<td>$-51.04$</td>
<td>150.04</td>
</tr>
<tr>
<td>w/o radial migration</td>
<td>13</td>
<td>$-27.54$</td>
<td>103.04</td>
</tr>
</tbody>
</table>

$^a$ The number of free parameters  
$^b$ The value of maximum logarithmic likelihood  
$^c$ The value of bayesian information criterion

smallest for the fiducial model, implying that all of the four effects are important in the formation and evolution history of the MW. More details on these results are given in Appendix B. Therefore, we conclude that our model setting, including these four effects, is reasonably valid for investigating galaxy formation and evolution.
5.2. OTHER OBSERVATIONAL PROPERTIES IN THE GALACTIC STELLAR DISK

5.2 Other Observational Properties in the Galactic Stellar Disk

Here, to test the importance of our model results, we discuss the other important observational properties in the Galactic stellar disk based on our best fit model.

5.2.1 Stellar distribution on the \([\alpha/Fe]-[Fe/H]\) plane

Many of previous observations have reported that the distribution of the Galactic disk stars on the \([\alpha/Fe]-[Fe/H]\) plane is bimodal: there are the high-\([\alpha/Fe]\) and low-\([\alpha/Fe]\) peaks in the stellar density at the \([\alpha/Fe]\) ratio of \(\sim 0.2-0.3\) and \(\sim 0\), respectively (e.g., Bensby et al. 2003; Lee et al. 2011; Adibekyan et al. 2012; Anders et al. 2014; Mikolaitis et al. 2014). Figure 5.1 shows one of the latest observational results for the bimodal distribution of disk stars on the \([\alpha/Fe]-[Fe/H]\) plane by the APOGEE survey (Hayden et al. 2015), indicating the dependence of such a bimodality on the distances from the Galactic center, \(R\), and the disk plane, \(|Z|\). In this figure, the grey solid lines in these panels are common and describe the rough ridge line of the observed high-\([\alpha/Fe]\) sequence. It is clear from this figure that the high-\([\alpha/Fe]\) peak is more significant than the low-\([\alpha/Fe]\) one at smaller \(R\) and larger \(|Z|\), implying that the high-\([\alpha/Fe]\) disk population is geometrically thicker and more concentrate than the low-\([\alpha/Fe]\) one. Additionally, the comparison between the observed stellar distributions and the grey lines shows that the the high-\([\alpha/Fe]\) sequence does not strongly change along the disk plane, whereas the metallicity of the low-\([\alpha/Fe]\) peak decreases with increasing \(R\). These observed properties of the stellar distribution on the \([\alpha/Fe]-[Fe/H]\) plane are closely associated with the formation history of the Galactic stellar disk. However, the origin of such a bimodal distribution is not still clearly understood, in spite of many previous studies (e.g., Chiappini et al. 1997, 2001; Schönrich & Binney 2009; Haywood et al. 2016; Toyouchi & Chiba 2016).

Here, we present the stellar distributions on the \([\alpha/Fe]-[Fe/H]\) plane produced by our best fit model with color maps and contours in Figure 5.2. The panels from top left to bottom right represent the stellar distribution observed at the inner \((R = 5-7\ kpc)\), solar neighborhood \((R = 7-9\ kpc)\), the outer \((R = 9-11\ kpc)\), and the outermost region...
(R = 11-13 kpc), respectively. From this figure, we find that the clear bimodal stellar
distributions on the [\(\alpha/Fe\)]-[Fe/H] plane emerge at the inner radii, R < 9 kpc, while
the high-[\(\alpha/Fe\)] sequence rapidly fades out at the outer radii. Moreover, the high-[\(\alpha/Fe\)]
sequence is generally independent of radius, whereas the metallicity of the low-[\(\alpha/Fe\)]
peak becomes lower at outer disk region. Thus, our model can successfully reproduce the
observed properties in the stellar distributions on the [\(\alpha/Fe\)]-[Fe/H] plane, as shown in
Figure 5.1.

To understand how such a bimodal stellar distribution is formed in our model, in
Figure 5.3 we plot the [\(\alpha/Fe\)] vs. [Fe/H] for gas in each radial ring at each time step in
the model calculation. The color of each plot in the top and bottom panels shows the
look back time and the radius, respectively. From the comparison between Figure 5.2
and 5.3, we find that the high-[\(\alpha/Fe\)] and low-[\(\alpha/Fe\)] sequences obtained in our model
generally consist of the old disk stars born in the inner disk regions of R \(\lesssim\) 3 kpc at \(\gtrsim\)
9 Gyr ago, and the young ones born in the outer disk regions of \(\gtrsim\) 5 kpc at \(\lesssim\) 7 Gyr
ago, respectively. Here, it is worth noting that the decrease of [\(\alpha/Fe\)] at around R \(\sim\) 4-5
kpc is much faster than those in the other regions, thereby the evolutionally paths on the
[\(\alpha/Fe\)]-[Fe/H] diagram in the inner and outer disk regions are made distinctly different,
roughly corresponding to the high and low-[\(\alpha/Fe\)] sequence. The rapid decrease of [\(\alpha/Fe\)]
is due to the significant increase of the number ratio of SNe Ia to SNe II, N_{Ia}/N_{II}, caused
by radial migration in which a large amount of intermediate and old disk stars, which
eventually explode as SNe Ia, migrate from the inner disk region, R \(\lesssim\) 3 kpc, to the outer
one, R \(\gtrsim\) 5 kpc.

In fact, such an effect of radial migration on the stellar distribution on the [\(\alpha/Fe\)]-
[Fe/H] plane was already found in our previous model calculation by Toyouchi & Chiba
(2016). The previous model calculation suggested that to reproduce a bimodal stellar
distribution on the [\(\alpha/Fe\)]-[Fe/H] plane, a discontinuous radial migration event, as driven
by minor merger with a massive satellite, can be an essential mechanism. However our new
model taking into account only a continuous radial migration implies that such a special
dynamical event is not necessarily a unique solution. The difference of results between
our new and previous models suggests that the presence of a discontinuous density profile
in inflowing gas, namely its up-bending profile newly implied in this study gives rise to
a bimodal distribution in the concerned abundance-ratio diagram, because this density profile leads to a more massive inner disk and subsequently more significant transfer of disk stars from inner to outer disk regions than our previous model. Thus the observed bimodal distribution of the Galactic disk stars on the [$\alpha$/Fe]-[Fe/H] plane is an important record for the presence of discontinuity in the gas inflow history and dynamical evolution in the Milky Way stellar disk.
Figure 5.1: The density distributions of the APOGEE red-giant stars on the $\alpha/Fe$-[Fe/H] plane as a function of $R$ and $|Z|$. The top, middle, and bottom panels show the results at $|Z| = 0.0-0.5$ kpc, 0.5-1.0 kpc, and 1.0-2.0 kpc, respectively. The grey solid lines in these panels are common and describe the rough ridge line of the observed high-$\alpha/Fe$ sequence. This figure is taken from Hayden et al. (2015), Figure 4.
5.2. OTHER OBSERVATIONAL PROPERTIES IN THE GALACTIC STELLAR DISK

Figure 5.2: The color maps represent the stellar distributions on the $[\alpha/\text{Fe}] - [\text{Fe/H}]$ plane at $t = 12$ Gyr, reproduced in the best fit model. The stellar distribution observed at the inner ($R = 5-7$ kpc), solar neighborhood ($R = 7-9$ kpc), the outer ($R = 9-11$ kpc), and the outer-most disk region ($R = 11-13$ kpc) are shown from the top-left to bottom-right panels, respectively.
Figure 5.3: Each plot shows the [$\alpha$/Fe] and [Fe/H] of gas in each radial ring at each time step, respectively. The color of each plot in the top and bottom panels corresponds to the look back time and the radius, respectively.
5.2. OTHER OBSERVATIONAL PROPERTIES IN THE GALACTIC STELLAR DISK

5.2.2 Dependence of radial metallicity gradient of disk stars on their age

The radial metallicity gradient of the Galactic disk stars has been measured using various tracers including HII regions, star clusters, gaseous nebulae and Cepheid variable stars (e.g., Maciel & Costa 2009 and references therein). In particular, the field main-sequence turn-off stars are useful tracers, which can provide the time evolution of the radial metallicity gradient (e.g., Nordström et al. 2004). Figure 5.4 shows the radial metallicity gradients, $\Delta[\text{Fe/H}]/\Delta R$, of the disk stars as a function of stellar age, recently revealed by the LAMOST survey, which is one of the latest optical spectroscopic survey for the Galactic stars (Xiang et al. 2015). Each panel presents the result for the different bin of $|Z|$. The black and red squares correspond to the value of $\Delta[\text{Fe/H}]/\Delta R$ without and with correction of color magnitude diagram weights, respectively (see Xiang et al. 2015 for more details). This figure shows that the radial metallicity gradient is mostly flat for the oldest disk stars with age $> 10$ Gyr, whereas the younger disk stars with age $< 9$ Gyr show the negative radial metallicity gradients with the steepest value of $\sim -0.01$ dex/kpc at age $\sim 7$ Gyr. Such a dependence of radial metallicity gradient of disk stars on their age is generally consistent with the results provided by the previous studies (Casagrande et al. 2011; Toyouchi & Chiba 2014).

We here present the radial metallicity gradient of stars as a function of age obtained in our best fit model in Figure 5.5. The red and black lines are our model result and the observed one at $|Z| = 0.5-0.7$ kpc shown in Figure 5.4, respectively. We find that our model can roughly reproduce the observed trends, while at age $\sim 7$ Gyr the radial metallicity gradient in our model is somewhat flatter than that in the observation. In our model calculation, the flat radial metallicity gradient for disk stars with age $> 10$ Gyr is naturally realized, because such old disk stars were mostly born in the limited region of $R < 3$ kpc and subsequently have widely spread out over the disk due to stellar radial migrations. After the earliest phase, as star formation occurs even at the outer disk region, $R > 5$ kpc, the radial metallicity gradients for the younger stars with age $< 9$ Gyr become negative. Thus our model calculation implies that the observed dependence of radial metallicity gradient of disk stars on their age can be understood as evidence of
both the inside-out disk evolution and the stellar radial migration in the Galactic disk.
5.2. OTHER OBSERVATIONAL PROPERTIES IN THE GALACTIC STELLAR DISK

Figure 5.4: The radial metallicity gradients as a function of age for disk stars in different $|Z|$ bins, as marked in the plot. The black and red squares represent results without and with the correction of the color magnitude diagram weights (see Xiang et al. 2015 for more details). The horizontal and vertical error bars represent the age ranges of stars adopted to derive the radial gradients, and the fitting errors of radial gradients, respectively. They grey crosses in the $|Z| < 0.1$ kpc bin are measurements from Nordström et al. (2004). Xiang et al., RAA [2015], 1209, Figure 16. Reprinted with permission from Research in Astronomy and Astrophysics.
Figure 5.5: The radial metallicity gradient of disk stars as a function of age. The red and black lines correspond to our model result and the observation for disk stars, locating at $|Z| = 0.5$-0.7 kpc, shown in Figure 5.4, respectively.
5.2. OTHER OBSERVATIONAL PROPERTIES IN THE GALACTIC STELLAR DISK

5.2.3 Radial stellar density profiles as a function of chemical abundance

The spatial distribution of the Galactic disk stars as a function of stellar age or chemical abundances is a key piece of information to elucidate the formation history of the Milky Way. To obtain this information, Bovy et al. (2016) divided the APOGEE red-clump stars into several subsamples by their [$\alpha$/Fe] and [Fe/H], and estimated the spatial density profiles for these abundance-selected subsamples with a careful collection of the observational selection biases in the APOGEE survey. Figure 5.6 shows the radial density profiles for several subsamples estimated by Bovy et al. (2016). The top and bottom panels present the results for the subsamples, generally categorized into the high and low-[\alpha/Fe] sequences on the [$\alpha$/Fe]-[Fe/H] plane, respectively. From this figure it is clear that the properties of the density profiles are much different between the high and low-[\alpha/Fe] subsamples; the radial profiles for the high-[\alpha/Fe] subsamples are well fitted by single exponentials with a common scale length, whereas those for the low-[\alpha/Fe] ones are described by broken profiles, for which the radial density gradients are positive and negative at the inner and outer disk regions, and the break radii in the radial profiles move outward with decreasing [Fe/H] of subsamples.

The difference of density structures between the high and low-[\alpha/Fe] disk populations is expected to reflect the difference of their formation mechanisms. Here, we present the radial density profiles for several abundance-selected subsamples produced by our best fit model in Figure 5.7. In this figure, the left and right panels show the radial density profiles for the high and low-[\alpha/Fe] subsamples, respectively, and the values of [$\alpha$/Fe] and [Fe/H] of each subsample are denoted at the left end of the radial density profile. We find from this figure that our model can remarkably reproduce the properties of the observed radial density profiles for both the high and low-[\alpha/Fe] subsamples. This result is not so surprising because our model can also reproduce the stellar distribution on the [$\alpha$/Fe]-[Fe/H] plane and its spatial dependence. As noted in Section 5.2.1, the high and low-[\alpha/Fe] disk stars formed at $R < 3$ kpc and $> 5$ kpc, respectively, and they have radially migrated inward and outward from their birth radii. Therefore, the present stellar density profiles of the high-[\alpha/Fe] subsamples monotonically decreases with increasing radii at $R$
> 4 kpc, whereas those of the low-[α/Fe] ones have the peaks around at the radii, roughly corresponding to the average birth radii for the member stars of the subsample, thereby the peak radii becomes larger for more metal-poor subsamples.

Thus, our chemo-dynamical evolution model enables us to understand not only the radial dependence of the MDF of disk stars, but also the other several important observational properties of the Galactic stellar disk. Therefore we conclude that the formation history of the Milky Way is clarified in this thesis.
5.2. OTHER OBSERVATIONAL PROPERTIES IN THE GALACTIC STELLAR DISK

Figure 5.6: The radial density profiles of several abundance-selected subsamples, estimated from the spatial distribution of the APOGEE red-clump stars. The top and bottom panels present the radial profiles of the subsamples, generally categorized into the high and low-[$\alpha$/Fe] sequences on the [$\alpha$/Fe]-[Fe/H] plane, respectively. For the high-[$\alpha$/Fe] sequence, the density profiles from top to bottom are presented for the subsamples with [$\alpha$/Fe] = 0.20 for [Fe/H] = (−0.5, −0.4), [$\alpha$/Fe] = 0.15 for [Fe/H] = (−0.3, −0.2), and [$\alpha$/Fe] = 0.10 for [Fe/H] = −0.1. For the low-[$\alpha$/Fe] sequence, the density profiles from top to bottom are presented for the subsamples with [$\alpha$/Fe] = 0.05 up to [Fe/H] = −0.4 and [$\alpha$/Fe] = 0.0 at higher [Fe/H]. In this figure, arbitrary offsets in the vertical direction are applied to clearly display the density profile of each subsample. This figure is taken from Bovy et al. (2016), Figure 11.
Figure 5.7: The radial density profiles for several abundance-selected subsamples produced by our best fit model. The left and right panels show the radial density profiles for the high and low-[α/Fe] subsamples, respectively, and the values of [α/Fe] and [Fe/H] of each subsample are denoted at the left end of the radial density profile. In this figure, arbitrary offsets in the vertical direction are applied to clearly display the density profile of each subsample.
Chapter 6

SUMMARY & CONCLUSION

In this thesis, to get new insights into the formation history of the stellar disk in the Milky Way, we have attempted to reproduce the radial dependence of the metallicity distribution function of the Galactic disk stars with the chemo-dynamical evolution model. Our new chemo-dynamical evolution model includes the effects of gas inflow, gas outflow, and radial migration processes in galaxy evolution, each of which is characterized by model parameters. We have determined the best set of model parameters, which fit to the present structural and chemical properties of the Milky Way stellar disk based on the Markov Chain Monte Carlo method. We succeeded to find the solution to this galaxy formation model, which reproduces the observed radial dependences of the mean, standard deviation, skewness, and kurtosis of the metallicity distribution functions. We have also derived various fundamental results, and obtained the following new implications for gas inflow, gas outflow, and radial migration processes in the formation history of the Milky Way.

- The time scale of gas inflow is shorter at inner disk regions, which is associated with a shorter gas cooling and collapsing time at inner halo. The total surface mass density of gas inflow follows an up-bending profile with the break radius of \( \sim 4 \) kpc, and such a profile is expected to reflect the angular momentum distribution of inflowing gas on the galactic disk plane. As a result, gas accretion onto the disk region below \( R \sim 4 \) kpc occurs very rapidly at the early disk formation phase, whereas gas accretion onto outer radii proceeds slowly over the whole disk formation
phase. This gas inflow history leads to the inside-out galactic disk evolution.

- The metallicity of inflowing gas rapidly increases up to $[\text{Fe/H}] \sim -0.5$ at the early disk formation phase, probably resulting from re-accretion of metal-enriched gas ejected from the inner disk region. The combination of this process with the inside-out galactic disk evolution is a key ingredient to reproduce the observed narrower metallicity distribution functions at more outer disk regions.

- Gas outflow rate is higher at inner disk regions, especially at the earlier disk formation phase. The relation between star formation rate and gas outflow rate is much different between radii below and beyond $R \sim 4$ kpc, and the difference implies that the galactic winds from the inner and outer disk regions, where the gas densities are relatively high and low, are described by the momentum and energy driven wind models, respectively.

- The net radial transport of metal-rich disk stars from the inner to outer disk regions produces significant high-metallicity tails in the metallicity distribution functions at the outer disk regions, as observed in the Galactic stellar disk. Therefore, the radial migration process is essential to reproduce the radial dependence of the skewness of the metallicity distribution functions of the disk stars.

Moreover, based on the further analysis of our best fit model, we have found that our model can simultaneously reproduce other various important observational properties of the Galactic stellar disk, e.g., the $[\alpha/\text{Fe}] - [\text{Fe/H}]$ relation and age–radial metallicity gradient relation. Therefore, our chemo-dynamical evolution model is successfully applied to the description of galaxy evolution. In future, we will improve our model to include more realistic description of stellar dynamics for comparing with the detail kinematic information given by the forthcoming data release of the Gaia survey. Additionally, we will further apply this model to other relevant subjects, including the detail analysis of the SEDs of local star-forming disk galaxies and the subject of the galactic habitable regions in the Milky Way. Thus, our chemo-dynamical evolution model has a great potential as a tool for studying various subjects relating to the evolutions of disk galaxies.
Acknowledgement

Here, I would like to express my gratitude to people who have supported my graduate research.

First of all, I would like to express the deepest appreciation to my supervisor, Professor Masashi Chiba, who taught me how interesting astronomy is. Conversations with him about our common interests greatly enhances my research motivation, and shows a right direction for my research. I will never forget his kind and constant support for my student life.

I am greatly indebted to members in Galactic Archaeology seminar, Mikito Tanaka, Miho Ishigaki, Kohei Hayashi, Hidetomo Honma, Yusuke Komuro, Yuta Suzuki, Jun Uchi Nishizaki, Mihoko Kato, Daisuke Honma, and Hana Sasaki. This seminar provided me an invaluable opportunity to share latest exciting topic on studies of galaxy, and greatly deepened my research.

I also thank to all the members of our institute, especially my great colleagues, Satoshi Yamanaka, Takuya Otsuka, Toshiyuki Mizuki, Takahiro Morishita, and Masayo Morioka. When I faced some difficulties, they listened to my worries and encouraged me to relieve my anxiety.

Finally, I express my deep gratitude to my parents, sisters and brother for their invaluable love and support.
Bibliography

Appendix A

FORMULA FOR SEVERAL BARYONIC PROCESSES IN CHEMICAL EVOLUTION

A.1 Radial Profile of Gas Inflow

Here, we give the theoretical prediction of the radial profile of gas inflow by van den Bosch et al. (2001). This prediction is based on the assumption that the angular momentum distribution of gas accreting on the galactic disk plane is completely consistent with that of dark matters in the galaxy. Using N-body simulations of structure formation, Bullock et al. (2001) investigated the specific angular momentum distributions of dark matter halos, and found that they can be expressed with the following distribution function,

\[
P(\mathcal{J}) = \begin{cases} 
\frac{\mu \mathcal{J}_0}{(\mathcal{J}_0 + \mathcal{J})^2} & (\mathcal{J} \geq 0) \\
0 & (\mathcal{J} < 0) 
\end{cases}
\]  

(A.1)

Here, \(\mathcal{J}\) is the specific angular momentum in the direction of the total angular momentum vector. \(\mathcal{J}_0\) is the characteristic value of \(\mathcal{J}\) and \(\mu\) is the free-shape parameter. With this distribution function, the halo mass with specific angular momentum less than \(\mathcal{J}\),
M(< \mathcal{J}) can be written as follows,

\begin{equation}
M(< \mathcal{J}) = \begin{cases} 
M_{\text{vir}} \frac{\mu \mathcal{J}}{\mu + \mathcal{J}} & (\mathcal{J} \geq 0) \\
0 & (\mathcal{J} < 0)
\end{cases},
\end{equation}

where, $M_{\text{vir}}$ is the virial mass of the halo, defined as $M(< \mathcal{J}) = M_{\text{vir}}$ for the maximum specific angular momentum in the distribution, $\mathcal{J}_{\text{max}}$, which gives $\mathcal{J}_{\text{max}} = \mathcal{J}_0/(\mu - 1)$. The total specific angular momentum of a halo, $\mathcal{J}_{\text{tot}}$, is expressed by the following function of $\mathcal{J}_0$ and $\mu$,

\begin{align}
\mathcal{J}_{\text{tot}} &= \int_{0}^{\mathcal{J}_{\text{max}}} \mathcal{J} P(\mathcal{J}) d\mathcal{J} \\
&= \mathcal{J}_0 \left\{ \mu \ln \left( \frac{\mu}{\mu - 1} \right) - 1 \right\}.
\end{align}

$\mathcal{J}_{\text{tot}}$ can be rewritten as the dimensionless spin parameter,

\begin{equation}
\lambda = \frac{\mathcal{J}_{\text{tot}}}{\sqrt{2} V_{\text{vir}} r_{\text{vir}}},
\end{equation}

where, $V_{\text{vir}}$ and $r_{\text{vir}}$ are the virial velocity and virial radius of the halo, respectively. Thus, the angular momentum distribution of a halo can be specified by $\lambda$ and $\mu$. According to Bullock et al. (2001), $\lambda$ and $\mu$ for a halo follow the log-normal distributions with mean $\bar{\lambda} \simeq 0.035$ and $\bar{\mu} \simeq 1.25$, and scatter $\sigma_{\ln \lambda} \simeq 0.5$ and $\sigma_{\ln \mu} \simeq 0.4$, respectively.

If gas in a halo follows the same specific angular momentum distribution as the dark matter, produced in N-body simulations of structure formation, and all the gas cool and form the disk as conserving the specific angular momentum distribution completely, the mass distribution of gas disk obeys

\begin{equation}
\frac{M_d(R)}{M_{d,\text{tot}}} = \frac{M_h(< \mathcal{J})}{M_{\text{vir}}},
\end{equation}

where, $M_{d,\text{tot}}$ and $M_d(R)$ are the total mass and the enclosed mass within the radius $R$ of the gas disk, respectively, and through the circular velocity, $V_c$, $\mathcal{J}$ and $R$ are related according to $\mathcal{J} = RV_c(R)$. As the density profile of gas disk is written as

\begin{equation}
\Sigma_d(R) = \frac{1}{2\pi R} \frac{dM_d(R)}{dR},
\end{equation}
A.2 RELATIONS BETWEEN $\Sigma_{\text{OUT}}$ AND $\Sigma_{\text{SFR}}$

one can be

$$\Sigma_d(R) = \frac{M_{d,\text{tot}}}{2 \pi R^2} P(\mathcal{I}) RV_c(R) \left[ 1 + \frac{\partial \ln V_c}{\partial \ln R} \right].$$  \hspace{1cm} (A.8)

For an assumption of a flat rotation curve, $V_c(R) = V_c$, equation (A.8) reduces to

$$\Sigma_d(R) = \mu \frac{M_{d,\text{tot}}}{2 \pi R_d^2} \left( \frac{R}{R_d} \right)^{-1} \left( 1 + \frac{R}{R_d} \right)^{-2},$$  \hspace{1cm} (A.9)

where,

$$R_d \equiv \mathcal{I}_0/V_c = \sqrt{2} \lambda r_{\text{vir}} \left\{ \mu \ln \left( \frac{\mu}{\mu - 1} \right) - 1 \right\}^{-1}.$$  \hspace{1cm} (A.10)

The radial profile in equation (A.9) is a double power law rather than an single exponential. The radial profile of equation (A.9) with $\lambda = 0.05$, $\mu = 1.5$, and $r_{\text{vir}} = 250$ kpc is drawn with the black line in Figure 4.4.

A.2 Relations between $\Sigma_{\text{out}}$ and $\Sigma_{\text{SFR}}$

In this section, we present the derivations of the relations between $\Sigma_{\text{out}}$ and $\Sigma_{\text{SFR}}$ shown in equation (4.1), which were given by Toyouchi & Chiba (2015).

A.2.1 Energy-driven wind

First, we attempt to describe $\Sigma_{\text{out}}$ as a function of $\Sigma_{\text{SFR}}$ in the framework of the EDW model. Here we consider a star cluster with stellar mass of $M_{\text{cl}}$. Because the energy injected from the star cluster into ISM is proportional to $M_{\text{cl}}$ in the EDW model, the mass of outflow ejected from the star cluster, $m_w$, is described by using a typical velocity of outflowing gas when leaving the disk, $v_w$, as follows,

$$m_w \propto \frac{M_{\text{cl}}}{v_w^2}.$$  \hspace{1cm} (A.11)

We assume that $v_w$ corresponds to the escape velocity of the disk defined as,

$$v_w \sim \sqrt{\frac{\pi G H (\Sigma_{\text{star}} + \Sigma_{\text{gas}})}{\Sigma_{\text{gas}} \propto \sqrt{(1 + \mu^{-1})\Sigma_{\text{gas}}}}},$$  \hspace{1cm} (A.12)
where $\mu \equiv \Sigma_{\text{gas}}/\Sigma_{\text{star}}$, and $H$ is the scale height of the disk and is assumed to be constant along the radius. As the number of star clusters with $M_{\text{cl}}$ in the region where stars are formed in the rate of $\Sigma_{\text{SFR}}$ is estimated as $\Sigma_{\text{SFR}}/M_{\text{cl}}$, we can describe $\Sigma_{\text{out}}$ in terms of $\Sigma_{\text{SFR}}$ as follows,

$$\Sigma_{\text{out}} \sim m_{\text{w}} \frac{\Sigma_{\text{SFR}}}{M_{\text{cl}}}.$$  \hspace{1cm} (A.13)

By substituting equations (A.11) and (A.12) into (A.13) and adopting the Kennicutt and Schmidt (KS) relation (Schmidt 1959; Kennicutt 1998), $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^a$, we express $\Sigma_{\text{out}}$ as functions of $\mu$ and $\Sigma_{\text{SFR}}$ as follows,

$$\Sigma_{\text{out}} \propto \frac{\mu}{1 + \mu} \frac{\Sigma_{\text{SFR}}^{a-1}}{\Sigma_{\text{SFR}}}.$$  \hspace{1cm} (A.14)

Here we regard the factor $\mu/(1 + \mu)$ in this equation as a mostly constant value over the disk, because the variation of $\mu$ along the radius is much smaller than that of $\Sigma_{\text{SFR}}$. Therefore $(a - 1)/a$ in this equation correspond to $k_{\text{out}}$ introduced in Section 4.4. This formulation reveals that $k_{\text{out}}$ is independent of time and that by adopting $a = 1.4$ as observed in local star-forming galaxies we find $k_{\text{out}} = (a - 1)/a \sim 0.29$ as shown in equation (4.1).

### A.2.2 Momentum-driven wind

We now derive the relation between $\Sigma_{\text{out}}$ and $\Sigma_{\text{SFR}}$ based on the MDW model. We consider that the radiation emitted from a star cluster drives the ambient gas as outflow. The momentum injected into the ambient gas from a star cluster via the radiation during the time scale of $\tau_{\text{inj}}$, which should roughly correspond to the lifetime of massive stars, i.e., $\sim 10$ Myr, is described as follows,

$$P \sim (1 + \tau_{\text{IR}}) \frac{L}{c} \tau_{\text{inj}},$$  \hspace{1cm} (A.15)

where $L$ and $\tau_{\text{IR}}$ are the luminosity of the star cluster and the optical depth of the ambient gas to the infrared emission by dust grains, respectively.

We note that $\tau_{\text{IR}}$ is proportional to the surface gas density of giant molecular clouds (GMCs) surrounding the star cluster, $\Sigma_{\text{GMC}}$, rather than $\Sigma_{\text{gas}}$ as in equations (2.11) and (2.12). Therefore, in order to explicitly describe the relation between $\Sigma_{\text{GMC}}$ and $\Sigma_{\text{gas}}$, we
consider here that each of a GMC is originally a fragment of the galactic gas disk with $\Sigma_{\text{gas}}$ and the scale of fragmentation is roughly consistent with the scale height of the gas disk. Then the mass of a GMC can be described as $M_{\text{GMC}} = \pi \Sigma_{\text{gas}} H^2 = \pi \Sigma_{\text{GMC}} r_{\text{GMC}}^2$, where $r_{\text{GMC}}$ is the radius of a GMC. Assuming that the ratio of $H$ to $r_{\text{GMC}}$ is nearly constant, as in Murray et al. (2011), we obtain $\tau_{\text{IR}} \propto \Sigma_{\text{gas}}$. In this manner, we connect the physics in a galactic disk scale with that in a GMC scale in this discussion.

Here we consider the optically thin ($\tau_{\text{IR}} \ll 1$) and thick ($\tau_{\text{IR}} \gg 1$) limits, and the mass of outflow ejected from the star cluster for each limit is described as follows,

$$m_w \sim \begin{cases} 
\tau_{\text{inj}} (L/c) / v_w \propto M_{\text{cl}} / v_w & \text{(optically thin limit)} \\
\tau_{\text{inj}} (\tau_{\text{IR}} L/c) / v_w \propto \Sigma_{\text{gas}} M_{\text{cl}} / v_w & \text{(optically thick limit)}
\end{cases}$$  \hspace{1cm} (A.16)

where we assume that $L$ is proportional to $M_{\text{cl}}$. From equations (A.12), (A.13) and (A.16), $\Sigma_{\text{out}}$ for the MDW model can be presented as follows;

$$\Sigma_{\text{out}} \propto \begin{cases} 
\sqrt{\frac{\mu}{1+\mu}} \Sigma_{\text{SFR}}^{(2a-1)/2} & \text{(optically thin limit)} \\
\sqrt{\frac{\mu}{1+\mu}} \Sigma_{\text{SFR}}^{(2a+1)/2} & \text{(optically thick limit)}
\end{cases}$$  \hspace{1cm} (A.17)

Thus, as in equation (A.14), the value of $k_{\text{out}}$ can be expressed as a function of $a$.

The remarkable difference from the case of the EDW model is that $k_{\text{out}}$ depends on the optical depth of the ambient gas and thus the surface gas density. Adopting $a = 1.4$ leads to $k_{\text{out}} \sim 0.64$ and $1.36$ for the optically thin and thick limit, respectively, as shown in equation (A.14).
Appendix B

ADDITIONAL MODEL CALCULATIONS

In this section, we show the detail of the fitting results of the additional model experiments introduced in Section 5.1.

B.1 Model Without Break in Radial Profile of Gas Inflow
Table B.1: The results of the estimation of free parameters in the model calculation without break in radial profile of gas inflow

<table>
<thead>
<tr>
<th>Parameter</th>
<th>best(^a)</th>
<th>16 (^b)</th>
<th>50 (^c)</th>
<th>84 (^d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{\text{tot,in}} \times 10^{10} M_\odot)</td>
<td>13.3</td>
<td>9.89</td>
<td>12.2</td>
<td>15.4</td>
</tr>
<tr>
<td>(h_{R,\text{in1}} \text{[kpc]})</td>
<td>3.63</td>
<td>3.33</td>
<td>3.98</td>
<td>4.93</td>
</tr>
<tr>
<td>(\tau_{\text{in,0}} \text{[Gyr]})</td>
<td>3.20</td>
<td>2.75</td>
<td>3.07</td>
<td>3.43</td>
</tr>
<tr>
<td>(\tau_{\text{in,8}} \text{[Gyr]})</td>
<td>12.0</td>
<td>9.73</td>
<td>14.2</td>
<td>21.8</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>4.99</td>
<td>2.94</td>
<td>3.96</td>
<td>4.65</td>
</tr>
<tr>
<td>(\Lambda_0)</td>
<td>0.45</td>
<td>0.28</td>
<td>0.39</td>
<td>0.52</td>
</tr>
<tr>
<td>(\Lambda_8)</td>
<td>2.10</td>
<td>1.89</td>
<td>2.34</td>
<td>2.86</td>
</tr>
<tr>
<td>(\beta)</td>
<td>3.02</td>
<td>2.35</td>
<td>3.02</td>
<td>3.76</td>
</tr>
<tr>
<td>(a_{\text{rm}} \times 10^{-2})</td>
<td>-7.95</td>
<td>-8.40</td>
<td>-5.22</td>
<td>0.49</td>
</tr>
<tr>
<td>(b_{\text{rm}} \text{[kpc]})</td>
<td>2.40</td>
<td>1.71</td>
<td>2.12</td>
<td>2.46</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.07</td>
<td>0.2</td>
<td>0.59</td>
<td>1.12</td>
</tr>
<tr>
<td>(t_{\text{in}} \text{[Gyr]})</td>
<td>0.72</td>
<td>0.60</td>
<td>0.78</td>
<td>0.97</td>
</tr>
<tr>
<td>([\text{Fe}/\text{H}]_{\text{in,0}})</td>
<td>-0.41</td>
<td>-0.46</td>
<td>-0.41</td>
<td>-0.36</td>
</tr>
<tr>
<td>(f_{\text{in}} \times 10^{-4})</td>
<td>4.15</td>
<td>3.46</td>
<td>4.62</td>
<td>6.02</td>
</tr>
</tbody>
</table>

\(^a\) The best value obtained from the MCMC estimation.
\(^b\) The 16 percentile value obtained from the MCMC estimation.
\(^c\) The 50 percentile value obtained from the MCMC estimation.
\(^d\) The 84 percentile value obtained from the MCMC estimation.
Figure B.1: The same figure as Figure 3.1, but for the model calculation without break in radial profile of gas inflow.
Figure B.2: The same figure as Figure 3.2, but for the model calculation without break in radial profile of gas inflow.
B.1. MODEL WITHOUT BREAK IN RADIAL PROFILE OF GAS INFLOW

Figure B.3: The same figure as Figure 3.3, but for the model calculation without break in radial profile of gas inflow.
Figure B.4: The same figure as Figure 3.4, but for the model calculation without break in radial profile of gas inflow.
### B.2 Model Without Time Evolution of Metallicity of Inflowing Gas

Table B.2: The results of the estimation of free parameters in the model calculation without time evolution of metallicity of inflowing gas

<table>
<thead>
<tr>
<th>Parameter</th>
<th>best(^a)</th>
<th>16 %(^b)</th>
<th>50 %(^c)</th>
<th>84 %(^d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\text{tot,in}} [10^{10} M_\odot]$</td>
<td>11.9</td>
<td>10.0</td>
<td>12.5</td>
<td>17.2</td>
</tr>
<tr>
<td>$R_b$ [kpc]</td>
<td>4.50</td>
<td>4.17</td>
<td>5.06</td>
<td>5.69</td>
</tr>
<tr>
<td>$h_{R,\text{in1}}$ [kpc]</td>
<td>1.24</td>
<td>1.03</td>
<td>1.21</td>
<td>1.41</td>
</tr>
<tr>
<td>$h_{R,\text{in2}}$ [kpc]</td>
<td>4.81</td>
<td>5.04</td>
<td>7.52</td>
<td>11.6</td>
</tr>
<tr>
<td>$\tau_{\text{in,0}}$ [Gyr]</td>
<td>1.55</td>
<td>1.14</td>
<td>2.23</td>
<td>2.68</td>
</tr>
<tr>
<td>$\tau_{\text{in,8}}$ [Gyr]</td>
<td>17.4</td>
<td>19.8</td>
<td>25.1</td>
<td>28.4</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.47</td>
<td>1.62</td>
<td>2.43</td>
<td>3.35</td>
</tr>
<tr>
<td>$\Lambda_0$</td>
<td>0.67</td>
<td>0.29</td>
<td>0.45</td>
<td>0.60</td>
</tr>
<tr>
<td>$\Lambda_8$</td>
<td>1.35</td>
<td>1.17</td>
<td>1.39</td>
<td>1.65</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.84</td>
<td>1.45</td>
<td>2.18</td>
<td>2.95</td>
</tr>
<tr>
<td>$a_{\text{rm}} [10^{-2}]$</td>
<td>7.44</td>
<td>-8.1</td>
<td>-3.88</td>
<td>3.79</td>
</tr>
<tr>
<td>$b_{\text{rm}}$ [kpc]</td>
<td>2.22</td>
<td>2.66</td>
<td>3.00</td>
<td>3.30</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.002</td>
<td>0.01</td>
<td>0.04</td>
<td>0.10</td>
</tr>
<tr>
<td>$f_{\text{In}} [10^{-4}]$</td>
<td>7.38</td>
<td>4.40</td>
<td>5.76</td>
<td>7.31</td>
</tr>
</tbody>
</table>

\(^a\) The best value obtained from the MCMC estimation.
\(^b\) The 16 percentile value obtained from the MCMC estimation.
\(^c\) The 50 percentile value obtained from the MCMC estimation.
\(^d\) The 84 percentile value obtained from the MCMC estimation.
Figure B.5: The same figure as Figure 3.1, but for the model calculation without time evolution of metallicity of inflowing gas.
B.2. MODEL WITHOUT TIME EVOLUTION OF METALLICITY OF INFLOWING GAS

Figure B.6: The same figure as Figure 3.2, but for the model calculation without time evolution of metallicity of inflowing gas.
Figure B.7: The same figure as Figure 3.3, but for the model calculation without time evolution of metallicity of inflowing gas.
Figure B.8: The same figure as Figure 3.4, but for the model calculation without time evolution of metallicity of inflowing gas.
## B.3 Model Without Gas Outflow

Table B.3: The results of the estimation of free parameters in the model calculation without gas outflow.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>best</th>
<th>16 %</th>
<th>50 %</th>
<th>84 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\text{tot, in}}$ [$10^{10} M_\odot$]</td>
<td>7.62</td>
<td>6.15</td>
<td>6.97</td>
<td>7.64</td>
</tr>
<tr>
<td>$R_b$ [kpc]</td>
<td>2.52</td>
<td>2.51</td>
<td>2.56</td>
<td>2.64</td>
</tr>
<tr>
<td>$h_{R,\text{in1}}$ [kpc]</td>
<td>0.32</td>
<td>0.32</td>
<td>0.33</td>
<td>0.35</td>
</tr>
<tr>
<td>$h_{R,\text{in2}}$ [kpc]</td>
<td>19.0</td>
<td>16.3</td>
<td>18.1</td>
<td>19.3</td>
</tr>
<tr>
<td>$\tau_{\text{in,0}}$ [Gyr]</td>
<td>2.02</td>
<td>2.02</td>
<td>2.58</td>
<td>3.18</td>
</tr>
<tr>
<td>$\tau_{\text{in,8}}$ [Gyr]</td>
<td>27.6</td>
<td>15.8</td>
<td>23.0</td>
<td>27.3</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.61</td>
<td>1.36</td>
<td>1.68</td>
<td>2.15</td>
</tr>
<tr>
<td>$a_{\text{rm}}$ [$10^{-2}$]</td>
<td>-3.11</td>
<td>-4.90</td>
<td>1.33</td>
<td>6.75</td>
</tr>
<tr>
<td>$b_{\text{rm}}$ [kpc]</td>
<td>1.88</td>
<td>1.70</td>
<td>1.86</td>
<td>2.15</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.38</td>
<td>1.14</td>
<td>1.34</td>
<td>1.45</td>
</tr>
<tr>
<td>$t_{\text{in}}$ [Gyr]</td>
<td>3.51</td>
<td>3.33</td>
<td>3.85</td>
<td>5.15</td>
</tr>
<tr>
<td>$[\text{Fe}/\text{H}]_{\text{in,0}}$</td>
<td>-0.37</td>
<td>-0.37</td>
<td>-0.33</td>
<td>-0.27</td>
</tr>
<tr>
<td>$f_{\text{Ia}}$ [$10^{-4}$]</td>
<td>0.45</td>
<td>0.44</td>
<td>0.80</td>
<td>1.23</td>
</tr>
</tbody>
</table>

- a: The best value obtained from the MCMC estimation.
- b: The 16 percentile value obtained from the MCMC estimation.
- c: The 50 percentile value obtained from the MCMC estimation.
- d: The 84 percentile value obtained from the MCMC estimation.
Figure B.9: The same figure as Figure 3.1, but for the model calculation without gas outflow.
Figure B.10: The same figure as Figure 3.2, but for the model calculation without gas outflow.
B.3. MODEL WITHOUT GAS OUTFLOW

Figure B.11: The same figure as Figure 3.3, but for the model calculation without gas outflow.
Figure B.12: The same figure as Figure 3.4, but for the model calculation without gas outflow.
### B.4. Model Without Radial Migration

#### Table B.4: The results of the estimation of free parameters in the model calculation without radial migration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>best(^a)</th>
<th>16 %(^b)</th>
<th>50 %(^c)</th>
<th>84 %(^d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{\text{tot,in}} [10^{10} , M_\odot])</td>
<td>13.0</td>
<td>12.0</td>
<td>14.1</td>
<td>16.6</td>
</tr>
<tr>
<td>(R_b [\text{kpc}])</td>
<td>6.66</td>
<td>3.11</td>
<td>4.70</td>
<td>6.90</td>
</tr>
<tr>
<td>(h_{R,\text{in1}} [\text{kpc}])</td>
<td>3.25</td>
<td>2.70</td>
<td>4.90</td>
<td>8.00</td>
</tr>
<tr>
<td>(h_{R,\text{in2}} [\text{kpc}])</td>
<td>2.32</td>
<td>2.19</td>
<td>2.44</td>
<td>2.74</td>
</tr>
<tr>
<td>(\tau_{\text{in,0}} [\text{Gyr}])</td>
<td>2.80</td>
<td>1.63</td>
<td>2.72</td>
<td>3.31</td>
</tr>
<tr>
<td>(\tau_{\text{in,8}} [\text{Gyr}])</td>
<td>4.91</td>
<td>4.66</td>
<td>5.30</td>
<td>6.12</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>1.31</td>
<td>0.49</td>
<td>0.99</td>
<td>1.93</td>
</tr>
<tr>
<td>(\Lambda_0)</td>
<td>0.80</td>
<td>0.48</td>
<td>0.96</td>
<td>1.38</td>
</tr>
<tr>
<td>(\Lambda_3)</td>
<td>1.81</td>
<td>1.68</td>
<td>1.94</td>
<td>2.29</td>
</tr>
<tr>
<td>(\beta)</td>
<td>1.71</td>
<td>0.82</td>
<td>1.94</td>
<td>3.11</td>
</tr>
<tr>
<td>(t_{\text{in}} [\text{Gyr}])</td>
<td>0.73</td>
<td>0.66</td>
<td>0.79</td>
<td>0.95</td>
</tr>
<tr>
<td>([\text{Fe/H}]_{\text{in,0}})</td>
<td>-0.39</td>
<td>-0.46</td>
<td>-0.41</td>
<td>-0.37</td>
</tr>
<tr>
<td>(f_{\text{IA}} [10^{-4}])</td>
<td>6.22</td>
<td>6.92</td>
<td>8.46</td>
<td>10.3</td>
</tr>
</tbody>
</table>

\(^a\) The best value obtained from the MCMC estimation.
\(^b\) The 16 percentile value obtained from the MCMC estimation.
\(^c\) The 50 percentile value obtained from the MCMC estimation.
\(^d\) The 84 percentile value obtained from the MCMC estimation.
Figure B.13: The same figure as Figure 3.1, but for the model calculation without radial migration.
Figure B.14: The same figure as Figure 3.2, but for the model calculation without radial migration.
Figure B.15: The same figure as Figure 3.3, but for the model calculation without radial migration.
Figure B.16: The same figure as Figure 3.4, but for the model calculation without radial migration.