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# TORQUE PULSATIONS IN A THREE PHASE INDUCTION MOTOR SUBJECT TO SINUSOIDAL VOLTAGE FLUCTUATIONS ASSOCIATED TO LIGHT FLICKER EFFECT

## PULSACIONES DE PAR EN UN MOTOR DE INDUCCIÓN TRIFÁSICO SUJETO A FLUCTUACIONES DE TENSIÓN SINUSOIDALES ASOCIADAS AL EFECTO LUMÍNICO FLICKER

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### ABSTRACT

A statistical indicator used to measure human visual perception of light flicker is the short term flicker severity index ($P_{st}$). A value equal to one indicates visual annoyance, thus becoming a limit to the flicker phenomenon of lamps and, indirectly, to the voltage fluctuation that causes it. Limits of voltage fluctuations are only based on its effects on lamps; consequently, it is important to investigate the effects on other electrical equipment. With this purpose, torque response of a three phase induction motor subject to sinusoidal voltage fluctuations was evaluated in this study. For each voltage fluctuation, the associated flicker index $P_{st}$ was determined. A wide range in the index, including flicker limits, was considered. Through simulations, pulsating torques in steady state were observed which could give rise to vibration problems. Given that $P_{st}$ is not suitable to establish limits of voltage fluctuations in the case of induction motor, other parameters were proposed in this respect. Results are supported by means of analytical methods.

**Key words**: Power quality, induction machine, pulsating torques.

### RESUMEN

Un indicador estadístico utilizado para medir la percepción visual humana del efecto lumínico flicker es el índice de severidad de flicker de corta duración ($P_{st}$). Un valor igual a uno implica molestia visual, convirtiéndose así en el valor límite para el fenómeno flicker de las lámparas y, de manera indirecta, para la fluctuación de tensión que lo origina. Los límites a las fluctuaciones de tensión se basan únicamente en sus efectos sobre las lámparas; en consecuencia, es importante investigar los efectos sobre otros equipos eléctricos. Con este propósito, en este artículo se estudia la respuesta del par de un motor de inducción trifásico sujeto a fluctuaciones de tensión sinusoidales. Para cada fluctuación de tensión, se determinó el índice de flicker $P_{st}$ asociado. Se consideró un amplio rango del índice, incluyendo límites de flicker. A través de simulaciones, se observaron pares pulsantes en estado estacionario que podrían dar lugar a problemas de vibraciones. Dado que el índice $P_{st}$ no es apropiado para establecer límites a las fluctuaciones de tensión en el caso del motor de inducción, se propusieron otros parámetros en este sentido. Los resultados son respaldados por medio de métodos analíticos.

**Palabras clave**: Calidad del servicio eléctrico, máquina de inducción, pares pulsantes.

### INTRODUCTION

An index used to establish limits to the flicker phenomenon of lamps and, indirectly, to the associated voltage fluctuation, is the short term flicker severity index ($P_{st}$), as indicated in IEEE Standard 1453-2004 (2004) and IEC International Standard 61000-4-15:1997+A1:2003 (2003). Specifically, a $P_{st}$ index equal to one corresponds to the limit. Limits of voltage fluctuations do not take into account the possible effects on electrical equipment apart from lamps; therefore, it is important to investigate these effects.

Due to the importance of three phase induction motor in industry, its response to voltage fluctuations must be studied. This is the scope of this paper using simulation and analytical methods. Specifically, torque response in steady state is analyzed when sinusoidal voltage fluctuations exist in the supply.

Although $P_{st}$ index is not appropriate to establish limits of voltage fluctuations in the case of induction motor as it is associated to a visual phenomenon, the index will be used as a first approach to characterize the fluctuations. A wide range in the index, including flicker limits ($P_{st}$ = 1), will be taken into account. First, previous works on the subject are reviewed.

In the work by Medeiros and de Oliveira (2002), authors determined by means of simulations time response of an induction motor subject to three phase sinusoidal voltage fluctuations related to flicker limits. Modulation frequency and magnitude of the fluctuations were obtained using an approximate equation. Small

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variations in torque and speed were observed under starting and steady state conditions. Authors suggested the possibility of a flicker limits relaxation in the case of electrical loads different from lamps.

In the work by Bucci *et al.* (2005), it was studied by means of simulations and laboratory measurements, the effects of three phase rectangular voltage fluctuations on induction motor transient response. Modulation frequency was varied from 1 Hz to 10 Hz and magnitude from 1% to 5%. It was obtained a maximum torque variation of 63.9% with respect to the mean torque. Also, small speed fluctuations were observed. Authors pointed out that torque variations can involve an unacceptable torsion stress as well as vibrations.

In the work by Tennakoon *et al.* (2008), it was simulated the response of four three phase induction motors subject to balanced three phase sinusoidal voltage fluctuations. Speed variations and sideband components in the stator current were observed. Authors concluded that speed variations can produce extra frequency components of significance which appear as sidebands in relation to the mains frequency. Specifically, stator current exhibited sideband components of which the major ones were identified.

In the work by Zhao *et al.* (2012), induction motor response to three phase sinusoidal voltage fluctuations was studied by means of the dynamic model of the machine and the per-phase equivalent circuit. Also, experimental verification work was carried out. Stator and rotor RMS currents were determined considering magnitudes of voltage fluctuations from 1% to 10% and modulation frequencies from 1 Hz to 35 Hz. It was concluded that currents in induction motor increase considerably when the machine is subjected to high values of voltage change and modulation frequencies. Also, it was pointed out that a new voltage fluctuation index different from $P_{st}$ needed to be proposed, in order to indicate the risk to electrical equipment.

In other works, the three phase induction motor was subject to sinusoidal voltage fluctuations associated to flicker limits (Cruz and Molina 2010, Cruz and Molina 2012b), and the effects on torque and speed in steady state investigated through simulation methods. A frequency domain analysis was done in which alternating components of negligible magnitudes were observed. However, only two case studies were considered. Finally, Pst index was increased (Cruz and Molina 2012a) and the effects on torque and currents studied. Additional alternating components apart from the fundamental components arose in the currents. Also, alternating components were again observed in motor torque. However, in this paper an analytical support for the torque response was not provided and the magnitudes of the alternating components in the torque were not presented. Also, only three phase voltage fluctuations were taken into account.

Torque pulsations are important because they produce vibrations in the machine which may affect motor operation and life. Therefore, one main contribution of this paper is the calculation of the torque pulsations for various case studies, thus helping establish basis to further research that may inquire about limits of voltage fluctuations in the case of induction motor. It is important to remark that the current limit given by $P_{st}$ index cannot be changed, nor the index is suitable to be applied to the motor, since $P_{st}$ is related to the human visual annoyance given by the flicker effect of light sources. This fact was also pointed out in the work by Zhao *et al*. (2012). Nevertheless, new limits for electrical equipment different from lamps can be investigated. Additional contributions are: two parameters are proposed in order to establish limits of voltage fluctuations for the induction motor, both single phase and three phase sinusoidal voltage fluctuations are taken into account and the analytical support provided gives insight into the operation of induction motor under the imposed conditions.

## MATERIALS AND METHODS

### Induction motor model

Induction motor response will be obtained using computer simulations. The dynamic model of the machine will be employed. This model is presented next, considering the reference frame theory (Krause *et al*. 1995a) as a basis.

Because the transformation used to refer variables from *abc* domain to *dq0* domain will be alluded to in the paper, the transformation is shown in Equation (1).

$$\mathbf{T}_s = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (1)$$

Voltage equations are given by Equation (2)

$$\mathbf{v}_{qd0} = \mathbf{A}\,\mathbf{i}_{qd0} \qquad (2)$$

Where

$$(\mathbf{v}_{qd0})^T = (v_{qs} \quad v_{ds} \quad v_{0s} \quad v'_{qr} \quad v'_{dr} \quad v'_{0r}) \; (3)$$

$$(\mathbf{i}_{qd0})^T = (i_{qs} \quad i_{ds} \quad i_{0s} \quad i'_{qr} \quad i'_{dr} \quad i'_{0r}), \; (4)$$

and matrix **A** is given by Equation (5), where $p$ is the derivative operator, $r_s$ is the stator resistance, $r'_r$ is the rotor resistance referred to the stator, $X_{ls}$ is the stator leakage reactance, $X'_{lr}$ is the rotor leakage reactance referred to the stator, $X_M$ is the mutual reactance, $\omega$ is the angular velocity of the arbitrary reference frame, $\omega_r$ is the electrical angular velocity associated to the rotor and $\omega_b$ is the base angular velocity. Reactances $X_{ss}$ and $X'_{rr}$ are given by Equations (6) and (7), respectively.

$$\mathbf{A} = \begin{bmatrix} r_s + \frac{p}{\omega_b}X_{ss} & \frac{\omega}{\omega_b}X_{ss} & 0 & \frac{p}{\omega_b}X_M & \frac{\omega}{\omega_b}X_M & 0 \\ -\frac{\omega}{\omega_b}X_{ss} & r_s + \frac{p}{\omega_b}X_{ss} & 0 & -\frac{\omega}{\omega_b}X_M & \frac{p}{\omega_b}X_M & 0 \\ 0 & 0 & r_s + \frac{p}{\omega_b}X_{ls} & 0 & 0 & 0 \\ \frac{p}{\omega_b}X_M & \left(\frac{\omega - \omega_r}{\omega_b}\right)X_M & 0 & r'_r + \frac{p}{\omega_b}X'_{rr} & \left(\frac{\omega - \omega_r}{\omega_b}\right)X'_{rr} & 0 \\ -\left(\frac{\omega - \omega_r}{\omega_b}\right)X_M & \frac{p}{\omega_b}X_M & 0 & -\left(\frac{\omega - \omega_r}{\omega_b}\right)X'_{rr} & r'_r + \frac{p}{\omega_b}X'_{rr} & 0 \\ 0 & 0 & 0 & 0 & 0 & r'_r + \frac{p}{\omega_b}X'_{lr} \end{bmatrix} \qquad (5)$$

$$X_{ss} = X_{ls} + X_M \qquad (6)$$

$$X'_{rr} = X'_{lr} + X_M \,. \qquad (7)$$

To perform simulations, a stationary reference frame fixed in the stator was considered.

For a $P$-pole machine, the mechanical equation is given by Equation (8)

$$T_e - T_L = J\left(\frac{2}{P}\right)p\omega_r \qquad (8)$$

Where $T_e$ is the electromagnetic torque and $T_L$ is the load torque. Besides, the electromagnetic torque can be written as

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)M\left(i_{qs}i'_{dr} - i_{ds}i'_{qr}\right) \qquad (9)$$

Where $M$ is the mutual inductance.

**Characteristics of the motor**

The machine is a 220 V, 3 Hp, 60 Hz, 1710 RPM induction motor whose parameters are shown in Table 1. The rotor moment of inertia is $J = 0{,}089$ kg.m$^2$ and includes inertia of load. These characteristics were taken from the work by Cathey *et al.* (1973). A constant load torque equal to the torque delivered by the motor at rated speed was assumed.

Prior to showing the simulation results, the mathematical representation of the voltage fluctuations is presented.

Table 1. Induction motor parameters.

| $r_s$ (Ω) | $r'_r$ (Ω) | $X_{ls}$ (Ω) | $X'_{lr}$ (Ω) | $X_M$ (Ω) |
|---|---|---|---|---|
| 0,435 | 0,816 | 0,754 | 0,754 | 26,13 |

## VOLTAGE FLUCTUATIONS

### Definition

As indicated in IEEE Standard 1159-2009 (2009), voltage fluctuations that cause lamp flicker generally appear as an amplitude modulation of the fundamental frequency waveform. Therefore, a sinusoidal voltage fluctuation may be expressed by Equation (10) as it was done in previous investigations (Medeiros and de Oliveira 2002, Tennakoon *et al.* 2008, Zhao *et al*. 2012).

$$v(t) = V_p \sin(2\pi f_p t)[1 + m\sin(2\pi f_m t)]\,. \qquad (10)$$

In Equation (10), $V_p$ is the amplitude of the fundamental voltage waveform, $f_p$ is the fundamental frequency, $f_m$ is the modulation frequency and $m$ is the modulation index which can be written as

$$m = \frac{V_m}{V_p} \qquad (11)$$

Where $V_m$ is the amplitude of the modulating waveform.

If $\Delta V$ is the peak-to-peak amplitude of the modulating waveform, the modulation index is

$$m = \frac{\Delta V}{2V_p} \qquad (12)$$

Where the parameter $\Delta V$ also corresponds to the magnitude of the voltage fluctuation.

Substituting Equation (12) into Equation (10) and considering $\Delta V$ as a percentage of $V_p$, Equation (13) is obtained:

$$v(t) = V_p \sin(2\pi f_p t)\left[1 + \frac{1}{2}\Delta V(\%)\sin(2\pi f_m t)\right] \qquad (13)$$

From Equation (13), the voltage fluctuation may be expressed by Equation (14), as it was done in the work by Cruz and Molina (2012b).

The form given by Equation (14) is convenient, because it explicitly shows the frequency components of the voltage waveform.

**Single phase and three phase sinusoidal voltage fluctuations**

In the case of single phase sinusoidal voltage fluctuations, it will be supposed that the fluctuation is located in phase $a$. Thus, Equations (15), (16), and (17) are obtained.

$$v(t) = V_p \sin(2\pi f_p t) + \frac{1}{4}\Delta V(\%)V_p \cos[2\pi(f_p - f_m)t] - \frac{1}{4}\Delta V(\%)V_p \cos[2\pi(f_p + f_m)t]. \qquad (14)$$

$$v_a(t) = V_p \sin(2\pi f_p t) + \frac{1}{4}\Delta V(\%)V_p \cos[2\pi(f_p - f_m)t] - \frac{1}{4}\Delta V(\%)V_p \cos[2\pi(f_p + f_m)t] \qquad (15)$$

$$v_b(t) = V_p \sin\left(2\pi f_p t - \frac{2\pi}{3}\right) \qquad (16)$$

$$v_c(t) = V_p \sin\left(2\pi f_p t + \frac{2\pi}{3}\right) \qquad (17)$$

In the case of three phase sinusoidal voltage fluctuations, it will be supposed that the voltage waveforms are the same but shifted in phase by 120º:

$$v_a(t) = V_p \sin(2\pi f_p t)\left[1 + \frac{1}{2}\Delta V(\%)\sin(2\pi f_m t)\right] \qquad (18)$$

$$v_b(t) = V_p \sin\left(2\pi f_p t - \frac{2\pi}{3}\right)\left[1 + \frac{1}{2}\Delta V(\%)\sin\left(2\pi f_m t - \frac{2\pi}{3}\frac{f_m}{f_p}\right)\right] \qquad (19)$$

$$v_c(t) = V_p \sin\left(2\pi f_p t + \frac{2\pi}{3}\right)\left[1 + \frac{1}{2}\Delta V(\%)\sin\left(2\pi f_m t + \frac{2\pi}{3}\frac{f_m}{f_p}\right)\right] \qquad (20)$$

**Parameters**

Modulation frequency and magnitude are two parameters of the voltage fluctuations. In Table 2, these parameters associated to six different cases related to flicker limits ($P_{st}$ = 1) and sinusoidal voltage fluctuations are shown. The parameters were obtained using a computer based flickermeter (Montoya 2008).

In Table 3, the parameters for a wide range in the $P_{st}$ index are presented.

Table 2. Parameters of the voltage fluctuations for $P_{st}$ = 1.

| $f_m$ (Hz) | $\Delta V$ (%) |
|---|---|
| 1 | 2,10 |
| 5 | 0,67 |
| 10 | 0,48 |
| 15 | 0,84 |
| 20 | 1,40 |
| 25 | 2,09 |

Table 3. Parameters of the voltage fluctuations for a wide range in the $P_{st}$ index.

| $f_m$ (Hz) | ΔV (%) | $P_{st}$ | $f_m$ (Hz) | ΔV (%) | $P_{st}$ | $f_m$ (Hz) | ΔV (%) | $P_{st}$ |
|---|---|---|---|---|---|---|---|---|
| | 1 | 0,47 | | 1 | 1,52 | | 1 | 2,09 |
| | 2 | 0,95 | | 2 | 3,04 | | 2 | 4,17 |
| 1 | 3 | 1,42 | 5 | 3 | 4,58 | 10 | 3 | 6,22 |
| | 4 | 1,90 | | 4 | 5,96 | | 4 | 8,21 |
| | 5 | 2,39 | | 5 | 7,60 | | 5 | 10,43 |
| | 1 | 1,20 | | 1 | 0,72 | | 1 | 0,48 |
| | 2 | 2,40 | | 2 | 1,43 | | 2 | 0,96 |
| 15 | 3 | 3,58 | 20 | 3 | 2,18 | 25 | 3 | 1,43 |
| | 4 | 4,80 | | 4 | 2,87 | | 4 | 1,93 |
| | 5 | 5,96 | | 5 | 3,66 | | 5 | 2,43 |

## RESULTS

In Figure 1, torque frequency response corresponding to the six cases presented in Table 2 and three phase sinusoidal voltage fluctuations is shown. Figure 1 is symmetric about the plane Frequency = 0. Only constant components are observed because they are much larger than alternating components. In Figure 2, only alternating components are shown.

In a similar manner, Figures 3 and 4 show torque frequency response in the case of single phase sinusoidal voltage fluctuations and $P_{st}$ = 1. Frequencies and magnitudes of the components are summarized in Table 4.

In Tables 5 and 6, results corresponding to a wide range in the $P_{st}$ index are shown. Only alternating components are presented. The magnitudes are in *percent* of the constant component.

Frequency response indicates that apart from the constant component, sinusoidal components at different frequencies arise which correspond to pulsating torques. The origin of these torques will be treated next, using theoretical reasoning and analytical methods. Also, magnitudes of the pulsating torques will be discussed.

Table 4. Torque components for $P_{st}$ = 1.

| $f_m$ (Hz) | $f$ (Hz) | Three phase voltage fluctuations | | Single phase voltage fluctuations | |
|---|---|---|---|---|---|
| | | Torque (N.m) | Torque (%) | Torque (N.m) | Torque (%) |
| 1 | 0 | 14,0269 | 100 | 14,0269 | 100 |
| | 1 | 0,1104 | 0,79 | 0,0368 | 0,26 |
| | 119 | 0,0144 | 0,10 | 0,2396 | 1,71 |
| | 121 | 0,0142 | 0,10 | 0,2332 | 1,66 |
| 5 | 0 | 14,0269 | 100 | 14,0269 | 100 |
| | 5 | 0,0788 | 0,56 | 0,0266 | 0,19 |
| | 115 | 0,0230 | 0,16 | 0,0808 | 0,58 |
| | 125 | 0,0224 | 0,16 | 0,0706 | 0,50 |
| 10 | 0 | 14,0269 | 100 | 14,0269 | 100 |
| | 10 | 0,0390 | 0,28 | 0,0136 | 0,10 |
| | 110 | 0,0332 | 0,24 | 0,0624 | 0,44 |
| | 130 | 0,0310 | 0,22 | 0,0476 | 0,34 |
| 15 | 0 | 14,0269 | 100 | 14,0269 | 100 |
| | 15 | 0,0176 | 0,13 | 0,0064 | 0,05 |
| | 105 | 0,0866 | 0,62 | 0,1182 | 0,84 |
| | 135 | 0,0786 | 0,56 | 0,0786 | 0,56 |
| 20 | 0 | 14,0269 | 100 | 14,0269 | 100 |
| | 20 | 0,1306 | 0,93 | 0,0516 | 0,37 |
| | 100 | 0,1888 | 1,35 | 0,2146 | 1,53 |
| | 140 | 0,1670 | 1,19 | 0,1240 | 0,88 |
| 25 | 0 | 14,0269 | 100 | 14,0269 | 100 |
| | 25 | 0,4086 | 2,91 | 0,1788 | 1,27 |
| | 95 | 0,3408 | 2,43 | 0,3516 | 2,51 |
| | 145 | 0,2958 | 2,11 | 0,1758 | 1,25 |

Table 5. Alternating torque components for a wide range in the $P_{st}$ index and three phase voltage fluctuations.

| $f_m$ (Hz) | $f$ (Hz) | ΔV 1 % | 2 % | 3 % | 4 % | 5 % |
|---|---|---|---|---|---|---|
| 1 | 1 | 0,37 | 0,75 | 1,12 | 1,50 | 1,87 |
| | 119 | 0,05 | 0,10 | 0,15 | 0,19 | 0,24 |
| | 121 | 0,05 | 0,10 | 0,15 | 0,19 | 0,24 |
| 5 | 5 | 0,84 | 1,68 | 2,52 | 3,36 | 4,20 |
| | 115 | 0,25 | 0,49 | 0,74 | 0,98 | 1,23 |
| | 125 | 0,24 | 0,48 | 0,71 | 0,95 | 1,19 |
| 10 | 10 | 0,58 | 1,16 | 1,74 | 2,32 | 2,90 |
| | 110 | 0,49 | 0,99 | 1,48 | 1,97 | 2,46 |
| | 130 | 0,46 | 0,92 | 1,38 | 1,85 | 2,31 |
| 15 | 15 | 0,15 | 0,30 | 0,45 | 0,60 | 0,74 |
| | 105 | 0,73 | 1,47 | 2,20 | 2,94 | 3,67 |
| | 135 | 0,67 | 1,33 | 2,00 | 2,67 | 3,34 |
| 20 | 20 | 0,66 | 1,33 | 1,99 | 2,66 | 3,32 |
| | 100 | 0,96 | 1,92 | 2,88 | 3,84 | 4,80 |
| | 140 | 0,85 | 1,70 | 2,55 | 3,40 | 4,25 |
| 25 | 25 | 1,39 | 2,79 | 4,18 | 5,57 | 6,96 |
| | 95 | 1,16 | 2,33 | 3,49 | 4,65 | 5,81 |
| | 145 | 1,01 | 2,02 | 3,03 | 4,04 | 5,05 |

Table 6. Alternating torque components for a wide range in the $P_{st}$ index and single phase voltage fluctuations.

| $f_m$ (Hz) | $f$ (Hz) | ΔV 1 % | 2 % | 3 % | 4 % | 5 % |
|---|---|---|---|---|---|---|
| 1 | 1 | 0,12 | 0,25 | 0,37 | 0,50 | 0,62 |
| | 119 | 0,81 | 1,63 | 2,44 | 3,25 | 4,07 |
| | 121 | 0,79 | 1,58 | 2,38 | 3,17 | 3,96 |
| 5 | 5 | 0,28 | 0,57 | 0,85 | 1,13 | 1,41 |
| | 115 | 0,86 | 1,72 | 2,58 | 3,44 | 4,30 |
| | 125 | 0,75 | 1,50 | 2,26 | 3,01 | 3,76 |
| 10 | 10 | 0,20 | 0,40 | 0,60 | 0,80 | 1,01 |
| | 110 | 0,93 | 1,85 | 2,78 | 3,70 | 4,63 |
| | 130 | 0,71 | 1,41 | 2,12 | 2,83 | 3,54 |
| 15 | 15 | 0,05 | 0,11 | 0,16 | 0,22 | 0,27 |
| | 105 | 1,00 | 2,01 | 3,01 | 4,01 | 5,01 |
| | 135 | 0,67 | 1,33 | 2,00 | 2,67 | 3,34 |
| 20 | 20 | 0,26 | 0,53 | 0,79 | 1,05 | 1,31 |
| | 100 | 1,09 | 2,19 | 3,28 | 4,37 | 5,46 |
| | 140 | 0,63 | 1,26 | 1,89 | 2,53 | 3,16 |
| 25 | 25 | 0,61 | 1,22 | 1,83 | 2,44 | 3,05 |
| | 95 | 1,20 | 2,40 | 3,60 | 4,80 | 6,00 |
| | 145 | 0,60 | 1,20 | 1,80 | 2,40 | 3,00 |

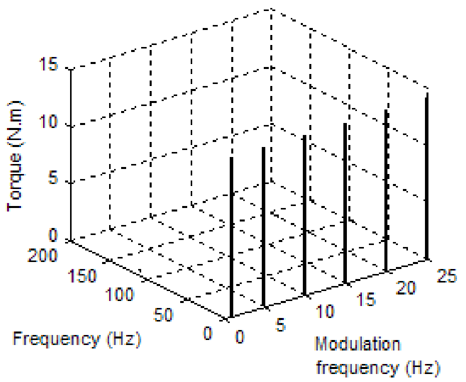


Figure 1. Torque frequency response for $P_{st}$ = 1 and three phase voltage fluctuations.

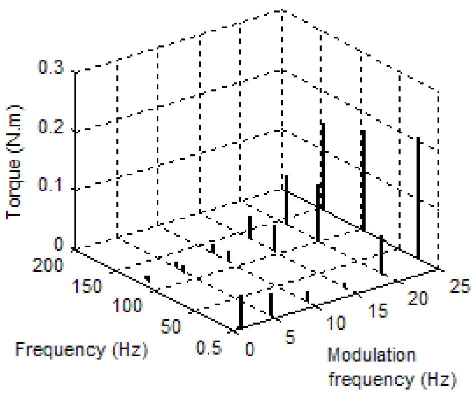


Figure 2. Alternating torque components for $P_{st}$ = 1 and three phase voltage fluctuations.

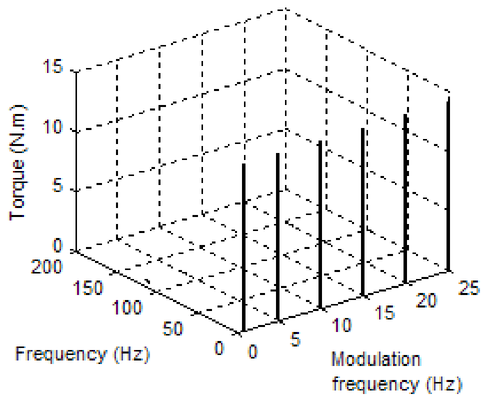


Figure 3. Torque frequency response for $P_{st}$ = 1 and single phase voltage fluctuations.

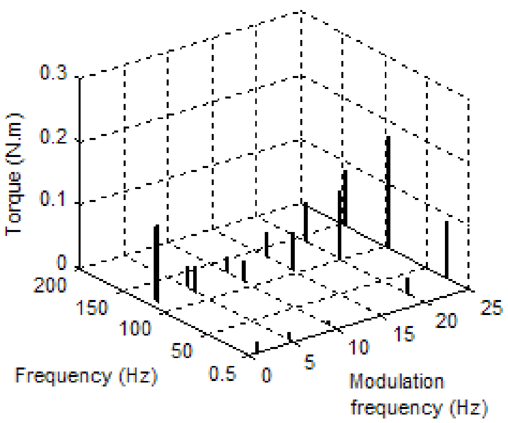


Figure 4. Alternating torque components for $P_{st}$ = 1 and single phase voltage fluctuation.

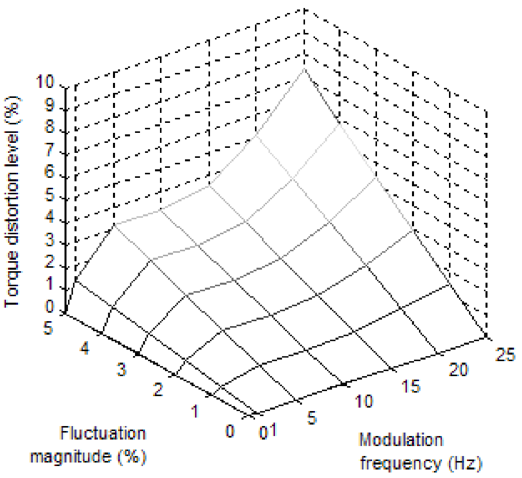


Figure 5. TDL for a wide range in the $P_{st}$ index and three phase voltage fluctuations.

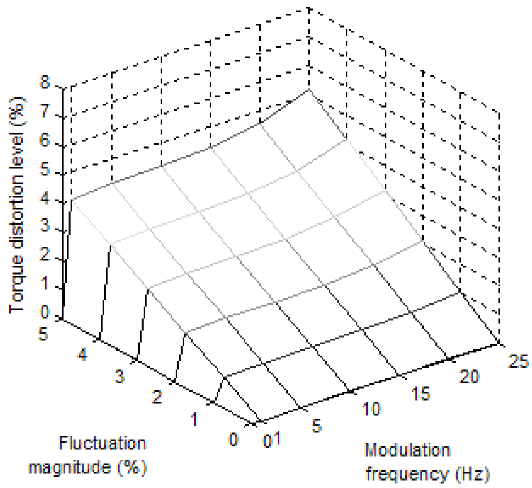


Figure 6. TDL for a wide range in the $P_{st}$ index and single phase voltage fluctuations.

## DISCUSSION

The origin of the pulsating torques will be explained considering the case of single phase sinusoidal voltage fluctuations. A similar reasoning is valid in the case of three phase sinusoidal voltage fluctuations.

In Equations (15), (16) and (17) we may observe various sets of voltages. A balanced three phase set at a frequency $f_p$, an unbalanced three phase set at a frequency $f_p$-$f_m$ and an unbalanced three phase set at a frequency $f_p$+$f_m$.

According to the theory of symmetrical components, each unbalanced set can be decomposed into a positive sequence set, a negative sequence set and a zero sequence set. Thus, there exist balanced sets of voltages, and therefore of currents, at different sequences. In Table 7, frequency and sequence of the balanced sets in the stator are shown. Extra frequency components that might appear in the stator different from those at $f_p \pm f_m$ will be ignored due to their relatively negligible magnitudes (Tennakoon *et al*. 2008).

Table 7. Frequency and sequence of the balanced sets in the stator.

| Frequency of the balanced set | Sequence of the balanced set |
|---|---|
| $f_p$ | Positive |
| $f_p$-$f_m$ | Positive |
| $f_p$-$f_m$ | Negative |
| $f_p$+$f_m$ | Positive |
| $f_p$+$f_m$ | Negative |

Based on induction motor principle of operation, torque is produced by the interaction of two rotating magnetomotive forces, one originated in the stator and the other in the rotor.

Magnetomotive forces are established by the respective balanced set of currents. Constant torque is produced when the two interacting magnetomotive forces have the same angular velocity. Meanwhile, the interaction between a stator magnetomotive force and a rotor magnetomotive force, both revolving with different angular velocities, gives rise to a pulsating torque in steady state where the frequency of the torque is given by the difference in the angular velocity of the magnetomotive forces (Krause *et al*. 1995b).

In Table 8, rotational frequencies of the magnetomotive forces are shown. These frequencies are considered relative to the stator reference frame.

Table 8. Rotational frequencies of the magnetomotive forces.

| $f_m$ (Hz) | Rotational frequency (Hz) | | | |
|---|---|---|---|---|
| | Stator | | Rotor | |
| | Seq (+) | Seq (-) | Seq (+) | Seq (-) |
| 1 | 60 | - | 60 | - |
| | 59 | 59 | 59 | 59 |
| | 61 | 61 | 61 | 61 |
| 5 | 60 | - | 60 | - |
| | 55 | 55 | 55 | 55 |
| | 65 | 65 | 65 | 65 |
| 10 | 60 | - | 60 | - |
| | 50 | 50 | 50 | 50 |
| | 70 | 70 | 70 | 70 |
| 15 | 60 | - | 60 | - |
| | 45 | 45 | 45 | 45 |
| | 75 | 75 | 75 | 75 |
| 20 | 60 | - | 60 | - |
| | 40 | 40 | 40 | 40 |
| | 80 | 80 | 80 | 80 |
| 25 | 60 | - | 60 | - |
| | 35 | 35 | 35 | 35 |
| | 85 | 85 | 85 | 85 |

In Table 9, frequencies of the pulsating torques obtained from the difference in the rotational frequency of the magnetomotive forces are presented.

Table 9. Frequencies of the pulsating torques.

| $f_m$ (Hz) | $f$ (Hz) |
|---|---|
| 1 | 0, 1, 2, 118, 119, 120, 121, 122 |
| 5 | 0, 5, 10, 110, 115, 120, 125, 130 |
| 10 | 0, 10, 20, 100, 110, 120, 130, 140 |
| 15 | 0, 15, 30, 90, 105, 120, 135, 150 |
| 20 | 0, 20, 40, 80, 100, 120, 140, 160 |
| 25 | 0, 25, 50, 70, 95, 120, 145, 170 |

It is seen that frequencies of the pulsating torques given in Table 4 appear in Table 9; however, some frequencies in Table 9 do not appear in Table 4. These frequencies correspond to torque components much smaller than those observed in Figures 2 and 4 (Cruz and Molina 2012b) and will not be taken into account.

An analytical foundation to the origin of the pulsating torques will be presented next.

Let

$$2\pi f_p = \omega_1 \qquad (21)$$

$$2\pi\left(f_p - f_m\right) = \omega_2 \qquad (22)$$

$$2\pi\left(f_p + f_m\right) = \omega_3 . \qquad (23)$$

Applying the transformation given by Equation (1) to Equations (15), (16) and (17), yields Equations (24), (25) and (26).

$$v_{qs} = V_p \sin(\omega_1 t - \theta) + \frac{mV_p}{6}\cos(\omega_2 t - \theta) + \frac{mV_p}{6}\cos(\omega_2 t + \theta) - \frac{mV_p}{6}\cos(\omega_3 t - \theta) - \frac{mV_p}{6}\cos(\omega_3 t + \theta) \qquad (24)$$

$$v_{ds} = V_p \cos(\omega_1 t - \theta) - \frac{mV_p}{6}\sin(\omega_2 t - \theta) + \frac{mV_p}{6}\sin(\omega_2 t + \theta) + \frac{mV_p}{6}\sin(\omega_3 t - \theta) - \frac{mV_p}{6}\sin(\omega_3 t + \theta) \qquad (25)$$

$$v_{0s} = \frac{mV_p}{6}\cos(\omega_2 t) - \frac{mV_p}{6}\cos(\omega_3 t). \qquad (26)$$

In Equations (24) and (25) we may observe, as expected from reference frame theory and its relation to the method of symmetrical components (Krause 1985), five two-phase balanced sets whose frequencies and sequences are in accordance with those previously shown in Table 7.

Voltages given by Equations (24) and (25) yield the following currents

$$\begin{aligned} i_{qs} &= a_{qs}\sin(\omega_1 t-\theta+\phi_1)+b_{qs}\cos(\omega_2 t-\theta+\phi_2)+c_{qs}\cos(\omega_2 t+\theta+\phi_3) \\ &\quad +d_{qs}\cos(\omega_3 t-\theta+\phi_4)+e_{qs}\cos(\omega_3 t+\theta+\phi_5) \end{aligned} \tag{27}$$

$$\begin{aligned} i_{ds} &= a_{ds}\cos(\omega_1 t-\theta+\phi_1)+b_{ds}\sin(\omega_2 t-\theta+\phi_2)+c_{ds}\sin(\omega_2 t+\theta+\phi_3) \\ &\quad +d_{ds}\sin(\omega_3 t-\theta+\phi_4)+e_{ds}\sin(\omega_3 t+\theta+\phi_5) \end{aligned} \tag{28}$$

Where $a_{qs}$, $b_{qs}$, $c_{qs}$, $d_{qs}$, $e_{qs}$ and $a_{ds}$, $b_{ds}$, $c_{ds}$, $d_{ds}$, $e_{ds}$ denote current magnitudes in $q$ and $d$ axes, respectively. Similar equations arise in the rotor circuit:

$$\begin{aligned} i'_{qr} &= a'_{qr}\sin(\omega_1 t-\theta+\phi_6)+b'_{qr}\cos(\omega_2 t-\theta+\phi_7)+c'_{qr}\cos(\omega_2 t+\theta+\phi_8) \\ &\quad +d'_{qr}\cos(\omega_3 t-\theta+\phi_9)+e'_{qr}\cos(\omega_3 t+\theta+\phi_{10}) \end{aligned} \tag{29}$$

$$\begin{aligned} i'_{dr} &= a'_{dr}\cos(\omega_1 t-\theta+\phi_6)+b'_{dr}\sin(\omega_2 t-\theta+\phi_7)+c'_{dr}\sin(\omega_2 t+\theta+\phi_8) \\ &\quad +d'_{dr}\sin(\omega_3 t-\theta+\phi_9)+e'_{dr}\sin(\omega_3 t+\theta+\phi_{10}). \end{aligned} \tag{30}$$

Substituting Equations (27), (28), (29) and (30) into Equation (9), produces Equation (31). Frequencies of the pulsating torques given by the use of Equation (31), match the values shown in Table 9, thus confirming the theoretical reasoning.

Considering the magnitudes obtained, it is seen from Table 4 that in the case of $P_{st}$ = 1 and three phase sinusoidal voltage fluctuations, the maximum alternating component reaches a value of 2,91% relative to the constant component; in the case of single phase sinusoidal voltage fluctuations, the maximum alternating component is 2,51%. In the case of a wide range in the $P_{st}$ index and three phase sinusoidal voltage fluctuations, the maximum alternating component has a value of 6.96% as it is seen from Table 5, and in the case of single phase sinusoidal voltage fluctuations, the largest magnitude is 6%.

$$\begin{aligned} T_e &= \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)M\{a_{ds}a'_{dr}\sin[(\omega_1-\omega_1)t+\phi_1-\phi_6] \\ &\quad +a_{ds}b'_{dr}\cos[(\omega_1-\omega_2)t+\phi_1-\phi_7]-a_{ds}c'_{dr}\cos[(\omega_1+\omega_2)t+\phi_1+\phi_8] \\ &\quad +a_{ds}d'_{dr}\cos[(\omega_1-\omega_3)t+\phi_1-\phi_9]-a_{ds}e'_{dr}\cos[(\omega_1+\omega_3)t+\phi_1+\phi_{10}] \\ &\quad -b_{ds}a'_{dr}\cos[(\omega_2-\omega_1)t+\phi_2-\phi_6]+b_{ds}b'_{dr}\sin[(\omega_2-\omega_2)t+\phi_2-\phi_7] \\ &\quad -b_{ds}c'_{dr}\sin[(\omega_2+\omega_2)t+\phi_2+\phi_8]+b_{ds}d'_{dr}\sin[(\omega_2-\omega_3)t+\phi_2-\phi_9] \\ &\quad -b_{ds}e'_{dr}\sin[(\omega_2+\omega_3)t+\phi_2+\phi_{10}]+c_{ds}a'_{dr}\cos[(\omega_2+\omega_1)t+\phi_3+\phi_6] \\ &\quad +c_{ds}b'_{dr}\sin[(\omega_2+\omega_2)t+\phi_3+\phi_7]-c_{ds}c'_{dr}\sin[(\omega_2-\omega_2)t+\phi_3-\phi_8] \\ &\quad +c_{ds}d'_{dr}\sin[(\omega_2+\omega_3)t+\phi_3+\phi_9]-c_{ds}e'_{dr}\sin[(\omega_2-\omega_3)t+\phi_3-\phi_{10}] \\ &\quad -d_{ds}a'_{dr}\cos[(\omega_3-\omega_1)t+\phi_4-\phi_6]+d_{ds}b'_{dr}\sin[(\omega_3-\omega_2)t+\phi_4-\phi_7] \\ &\quad -d_{ds}c'_{dr}\sin[(\omega_3+\omega_2)t+\phi_4+\phi_8]+d_{ds}d'_{dr}\sin[(\omega_3-\omega_3)t+\phi_4-\phi_9] \\ &\quad -d_{ds}e'_{dr}\sin[(\omega_3+\omega_3)t+\phi_4+\phi_{10}]+e_{ds}a'_{dr}\cos[(\omega_3+\omega_1)t+\phi_5+\phi_6] \\ &\quad +e_{ds}b'_{dr}\sin[(\omega_3+\omega_2)t+\phi_5+\phi_7]-e_{ds}c'_{dr}\sin[(\omega_3-\omega_2)t+\phi_5-\phi_8] \\ &\quad +e_{ds}d'_{dr}\sin[(\omega_3+\omega_3)t+\phi_5+\phi_9]-e_{ds}e'_{dr}\sin[(\omega_3-\omega_3)t+\phi_5-\phi_{10}]\}. \end{aligned} \tag{31}$$

Torque pulsations tend to produce vibrations in the machine which may affect the mechanical properties of the conductor insulation apart from the impact on motor shaft, couplings and bearings. Magnitudes of 6% are not negligible; however, values to use as limits are difficult to find in literature. In the work by Fuchs and Masoum (2008), it is suggested in an application example, a magnitude of 4% of the average torque as an acceptable level for a pulsating torque. Nevertheless, this value cannot be considered conclusive and further studies need to be done in this respect.

In order to make a comparison between the three phase case and the single phase case, and to perform a sensitivity analysis, an expression similar to that used to determine total harmonic distortion is employed (Cruz and Molina 2012a). This magnitude will be named Torque Distortion Level (TDL) and is given by Equation (32).

$$TDL = \frac{\sqrt{\sum T_n^2}}{T_{DC}} \times 100\% \qquad (32)$$

Where $T_n$ is the RMS value of each sinusoidal varying torque and $T_{DC}$ is the constant torque component.

Applying Equation (32) to the case of a wide range in the $P_{st}$ index, Figures 5 and 6 are obtained. In these figures, it is observed that torque distortion increases with both modulation frequency and magnitude of the voltage fluctuation. Also, it is noted that for modulation frequencies below 15 Hz approximately, distortion is greater in the case of single phase voltage fluctuations.

From the values shown in Table 3, it is seen that for modulation frequencies up to 10 Hz approximately, $P_{st}$ increases as modulation frequency increases; but then, $P_{st}$ decreases. Meanwhile, as observed from Figures 5 and 6, torque distortion increases with modulation frequency. A similar fact was observed in the work by Zhao *et al.* (2012) for the stator current. This indicates that a clear relation does not exist between $P_{st}$ index and variables related to the motor. This was expected since $P_{st}$ is associated to a luminous phenomenon. Therefore, $P_{st}$ index is not appropriate to establish limits of voltage fluctuations in the case of induction motor.

Finally, TDL might be useful to evaluate the total severity of the torque pulsations.

## CONCLUSIONS

Sinusoidal voltage fluctuations associated to flicker limits give rise to small alternating torque components with respect to the constant component. Meanwhile, sinusoidal voltage fluctuations associated to a wide range in the $P_{st}$ index give rise to larger alternating components, reaching a maximum of 6,96%. The highest magnitudes obtained are likely to produce vibration problems; however, they need to be mechanically analysed in further studies to assess their impact on motor components, operation and life. This will also help determine limit values for the torque pulsations and, consequently, limits of voltage fluctuations in the case of induction motor.

Frequencies of the pulsating torques depend on the modulating waveform characteristics, specifically modulation frequency. Using the analytical method presented, frequencies of the pulsating torques can be easily calculated in the case of sinusoidal voltage fluctuations.

By means of a parameter named Torque Distortion Level (TDL), it was observed that torque distortion increases with both modulation frequency ($f_m$) and magnitude of the voltage fluctuation ($\Delta V$). Thus, the utilization of parameters $f_m$ and $\Delta V$ is proposed to establish limits of voltage fluctuations in the case of induction motor.

The results obtained help confirm that $P_{st}$ is not suitable to establish limits of voltage fluctuations when electrical equipment different from lamps are involved.

Further investigation should include experimental results and the evaluation of other types of motor load.

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