

2015

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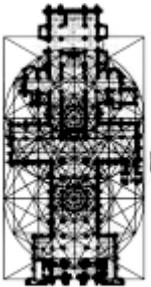


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Ramzy, Nelly Shafik. "The Dual Language of Geometry in Gothic Architecture: The Symbolic Message of Euclidian Geometry versus the Visual Dialogue of Fractal Geometry." *Peregrinations: Journal of Medieval Art and Architecture* 5, 2 (2015): 135-172. <https://digital.kenyon.edu/perejournal/vol5/iss2/7>

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The Dual Language of Geometry in Gothic Architecture: The Symbolic Message of Euclidian Geometry versus the Visual Dialogue of Fractal Geometry

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1. Introduction

When performing geometrical analysis of historical buildings, it is important to keep in mind what were the intentions of the originators, even though these intentions have likely changed many times as the master masons changed. For the medieval builders, geometry in design was a tool used to structure ideas and aesthetic impulses or perhaps to incorporate into this work a meaningful system of symbols; it was the internal logic of the building that mattered more than achieving beauty or following the correct canonical models as was the case in Renaissance era.¹ Geometry was used in Gothic architecture as visual tools for contemplating the mathematical nature of the Universe, which was directly linked to the Divine, the architect of the Universe as illustrated in the famous painting of *God the Geometer* (Austrian National Library, Codex Vindobonensis 2554). To seek these principles would thus be worshipping God.

The 1390s- *Constitutions of Masonry* or *The Regius Poem* opens with: "Here begin the Constitutions of the art of Geometry according to Euclid."² Regardless of the accuracy of this document, the medieval masons clearly understood that the science of Geometry that they used was received from its inventor: *Euclid*.

¹ Robert Bork, *The Geometry of Creation*, 2011: 5.

² William Hughan, *The Old Charges of British Freemasons*, 1872: 2-3.

The medieval geometry of Euclid had nothing to do with the geometry that is taught in schools today; no knowledge of mathematics or theoretical geometry of any kind was required for the construction process of medieval edifices. Using only a compass and a straight-edge,³ Gothic masons created myriad lace-like designs, making stone hang in the air and glass seem to chant. In a similar manner, although they did not know the recently discovered principles of Fractal geometry, Gothic artists created a style that was based on the geometry of Nature, which contains myriad of fractal patterns.

Architectural theorists Nikos Salingaros and Christopher Alexander believe that buildings must convey languages, among them are pattern language and form language; the former is the basic system for the design of any space, while the latter is what makes the former unique and beautiful; patterns are general and consistent throughout certain style, while forms are local and changeable from one building to another.⁴ Applying this to Gothic architecture, one may say that its general features, like pointed arches or high spires, are pattern language, while the specific design of each building and how these patterns came together is the form language. Salingaros also writes that, with the elimination of the ornaments and details within the range of scales -5mm to 2m or thereabouts- which corresponds to human scale, the dialogue between architecture and human beings is also removed.⁵ Based on this, the paper assumes that the Gothic cathedral, with its unlimited scale, yet very detailed structure, was an externalization of a dual language that was meant to address human cognition through its details, while addressing the eye of the Divine through the overall structure, using what was thought to be the divine language of the Universe. It discusses the hypothesis that geometry was used in the Gothic style to reproduce forms and patterns that reflect this dual language. In doing so, master masons distinguished between the abstract overall proportional lines of plans and elevations (form language), and the organic ornamental patterns on walls, ceiling, openings, and pavements (pattern language). It is suggested here that Euclidian geometry was employed for the, usually invisible, proportional or working lines of the

³ Lon Shelby, *Gothic Design Techniques*, 1977: 82–83.

⁴ Nikos Salingaros, *Twelve Lectures on Architecture*, 2010: 26.

⁵ Nikos Salingaros, *Architecture, Patterns, and Mathematics*, 1999: 61-72.

former, which are perceived only by the designer himself or by analytical view of the drawings with a mesh of imaginary lines; while Fractal Geometry, was used for the visible details of the latter, at which the worshipers can gaze and wonder about and may establish a "visual dialogue"; the former is to address the eye of the Divine, while the latter is to address the earthly eye of humans.

This paper will test this hypothesis by finding the actual logic and context (proportional lines, decorating pattern, invisible line, visible details, etc), with which these two types of geometry were used in Gothic buildings, locating the settings (in structural elements, in plans, in decoration elements, ...etc.), in which they functioned in these buildings. It also aims to find the bridge(s) that linked and integrated them into a unified whole. To achieve this, the paper depended on a qualitative methodology, using purposefully selected examples/case studies from different western-European countries that typify the geometrical characteristics of Gothic architecture. With regard to the applications of Euclidian geometry, theoretical reviews and documents of the period, as well as extensive geometrical analyses of several examples of the period (at least three examples for each category of Euclidian geometry), will be introduced in order to seek documented, analytical and illustrative support for the applications of the different notions of Euclidian geometry. As for the applications of fractal geometry, geometrical analysis and illustrative comparisons for the most prominent motifs of Gothic architecture against the different models of fractal geometry is accomplished and supported by some recent theoretical reviews. In both cases, the focus of the analyses includes locating the settings of the applications and their context as well as the aspects in common that linked the two types.

This paper concludes that both kinds of geometry were used to represent the geometry of Nature in its own way. While Euclidian geometry was used to compose an overall form language, using invisible proportional or working lines that were meant to address the eye of the Heaven through its *symbolic expression*, fractal patterns were used to reproduce the patterns of Nature in a more *visual expression* through ornaments and details. It was also found that ***geometrical***

progression, the *language of nature* and *modules* acted as bridges, linking these two kinds of geometry in a unified composition.

2. Theoretical Background

Plato described geometry as “the reference to the very laws of the Divine Will and Harmony of Being.”⁶ The term Sacred Geometry is commonly used to encompass the spiritual beliefs surrounding geometry in various cultures, where symbolic meanings are ascribed to certain geometric shapes and proportions, mostly derived from Nature and the mathematical principles at work therein (**Figure 1**). It is a catch-all term covering Pythagorean geometry and neo-Platonic geometry, as well as the perceived relationships between organic and logarithmic curves.⁷

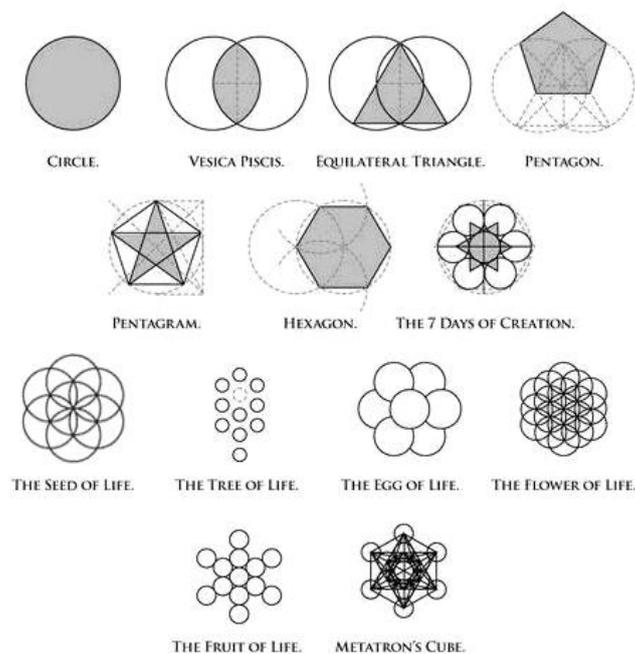


Figure 1 Principle elements of sacred geometry. Diagram: author.

⁶ Loai Dabbour, *Geometric Proportions*, 2012: 380-391.

⁷ Stephen Skinner, *Sacred Geometry: Deciphering the Code*, 2009: 91.

This geometry here was not aimed at beauty, though it often arrived at it, but at harmonization with the divine geometry of God's creation.⁸ In addition to representing cosmological and philosophical structures at the level of form, it was seen as powerful representations of some central concepts of the divine nature. The geometry and proportions of human body, as the culmination of God's creation, was one of the most important sources of this geometry (**Figures 7, 8, 10**).

Colin Dudley writes that: "It is in the light of the ancient cosmology that one needs to envisage the culture that created the great medieval churches, all of which incorporate a geometry that is purposefully created in order to provide, though its supposed supernatural power, divine protection from the destructive powers of the earthly world and the Devil, and to attract the presence of the Almighty, creator of all the geometry in the universe."⁹ That is, the form language in these churches was not addressing the humans' eyes, but rather the eyes of Heaven. To build a house of God without his geometry would be vain; the purpose of geometry here was to "unite the building with the eternal world of Heaven and thus to preserve it from disasters."¹⁰ This did not have anything to do with its static structure; at this time no one, including master-masons, bishops and abbots, was aware of the laws of structural engineering.¹¹

The renowned booklets of *Schmuttermeyer* and *Roriczer*,¹² together with other related documents of the period,¹³ are telling much of the manner in which these great buildings were created geometrically. For example, the circle and the sphere were seen as forms that belonged to the eternal and all-powerful heavens, while the square belonged to the earthly world. In this context, geometry acquired two rules: one is that all constructions must begin with a circle; the other is that symmetry must be maintained. The latter has its origins in the Augustinian belief in that the

⁸ Bork, 2011: 370-372.

⁹ Colin Dudley, *By craft of Ewclyde*, 2001: Chap. 1:16

¹⁰ Dudley, 2001: Chap. 1:20

¹¹ Walter Leedy, *Fan Vaulting: A Study of Form, Technology & Meaning*, 1980: 146-149

¹² Matthäus Roriczer, was a 15th-century German architect, master builder of Regensburg cathedral, and author of several booklets on medieval architectural design. In one of these texts, *Büchlein von der Fialen Gerechtigkeit* (1486), he describes the manner in which medieval masons used a "single dimensional unit" to produce a ground plan, when there was no internationally agreed upon standard of measurement, to give other relative measurements in a process called "constructive geometry". Similar instructions are found in another period publication by Hans Schmuttermeyer of Nuremberg, who was an artist, goldsmith, and master mason, whose booklet (1487) graphically illustrates how to design a pinnacle and gable (small gable)

¹³ Shelby, 1977; Bork, 2011: 29-40

universe owes its stability to the perfect balance of its elements as instituted by the Creator, a stability that will be denied to any building that does not possess symmetry.¹⁴ The former is because architects were aiming in their designs to bring Heaven down to earth, but even more cogent reason is that it was impossible to them to construct a true square, an equilateral triangle, octagon, etc. without a pre-existing circle.¹⁵

The most prominent geometrical patterns produced according to the Euclidean postulates are those developed from square within a circle, (*ad quadratum*) and triangle within a circle (*ad triangulum*), where any geometric design starts with a circle, from which the pattern starts to unfold (**Figure 2**). And for this reason Gothic builders found the *Euclidian geometry* appropriate for their cathedrals.

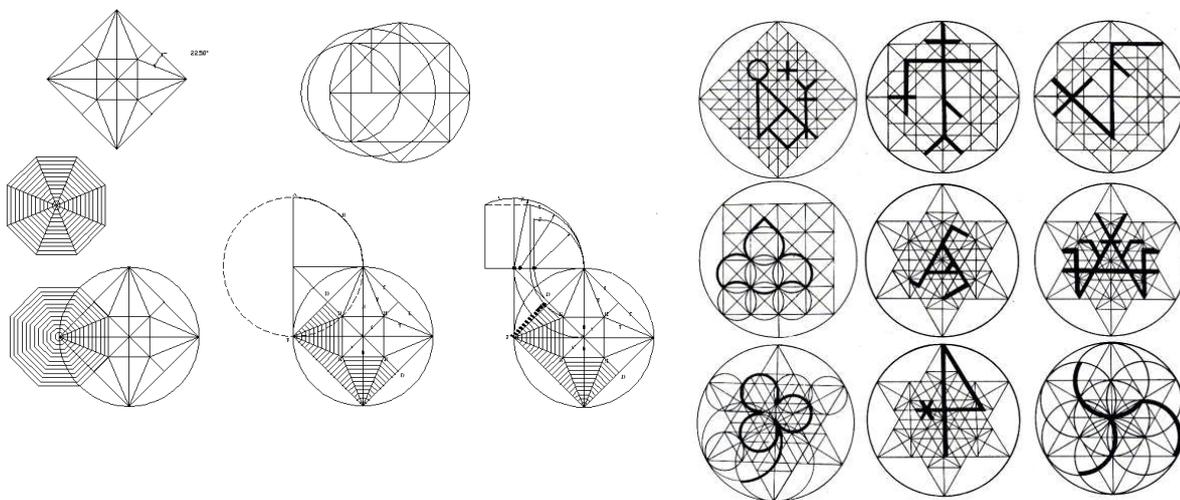


Figure 2 Applications of *ad quadratum* and *ad triangulum* in Gothic architecture: (Left) Methods of constructing Gothic vaults based on *ad quadratum* after Philibert De l'Orme, *Le Premier Tome De L'architecture* (1567), p. 110; (Right) geometrical analyses of mason's marks on different drawings of Gothic cathedrals. Photo: after Franz von Rhiza, *Studien über Steinmertz Zeichen*, 1917, pp. 44-45.

¹⁴ Dudley, 2001, Chap. 1:17-18

¹⁵ Dudley, 2001, Chap. 2: 38

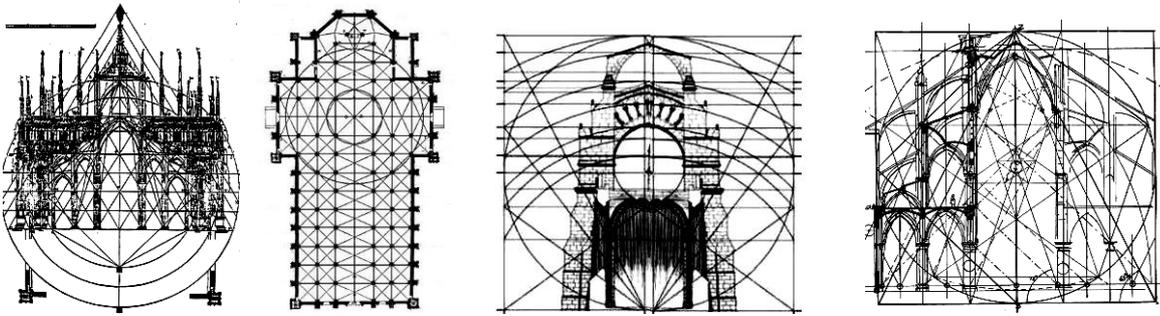


Figure 3: *Ad quadratum* and *ad triangulum* schemes as proportional, imaginary, or working (invisible) lines in the design of medieval churches: (Left) Milan Cathedral plan and section, (middle) partial section at Chartres Cathedral, and (right) Viollet-le-Duc's cross-section of Notre-Dame in Paris. Diagrams: (left) after Paul Frankl, *The Secret of the Medieval Masons* (1945), p. 46-60, (middle) after Rubino, Pintrest, (2001) <http://www.pinterest.com/emmatuzz/south-france-mysteries/>, and (right) after Frederik Lund *Ad Quadratum* (1919), p. 36.

Several architectural historians have long stated that medieval church's geometry of all sorts depended on *ad quadratum* and *ad triangulum* schemes (**Figure 3**). Other authors argued also for the widespread application of other figures, such as the pentagon, which was related to the Golden Section.¹⁶ The drawings in **Figure 3** show also how the lines of the *ad quadratum* and *ad triangulum* schemes were used as proportional working lines that did not appear in the construction itself. The use of geometrical progressions in these schemes is also evident.

Moreover, Euclidian geometry, in addition to being able to create all the simple whole number ratios, also can be used to create those strange ratios that express what are known as *irrational* or *silent* numbers that cannot be accurately expressed numerically. An example of this is $\sqrt{2}$, created by the diagonal of a square, which today one might take to be 1.414, but even this is only an approximation. Similar *surds* also include $\sqrt{3}$ and $\sqrt{5}$, all easily produced as geometric ratios, but comprehended numerically only by the Creator of the cosmos. Euclidian geometry was therefore needed to create micro cosmos with simple regular polygons, which can be easily included into the design process (**Figure 4**). For example, the square includes $\sqrt{2}$, the hexagon

¹⁶ The 45-degree triangle was championed by von Drach in "Das Hüttengeheimnis von gerechten Steinmetzgrund; The Pentagon Figure in Witzel's, Untersuchungen über gotische Proportionengesetze and the Use of Golden Section in Cologne Cathedral by Haase" in *Der Dom zu Köln am Rhein*.

includes $\sqrt{3}$, and the pentagon includes the Golden Section, which was a representation of many natural forms.¹⁷ These figures were, therefore, considered sacramental geometry.

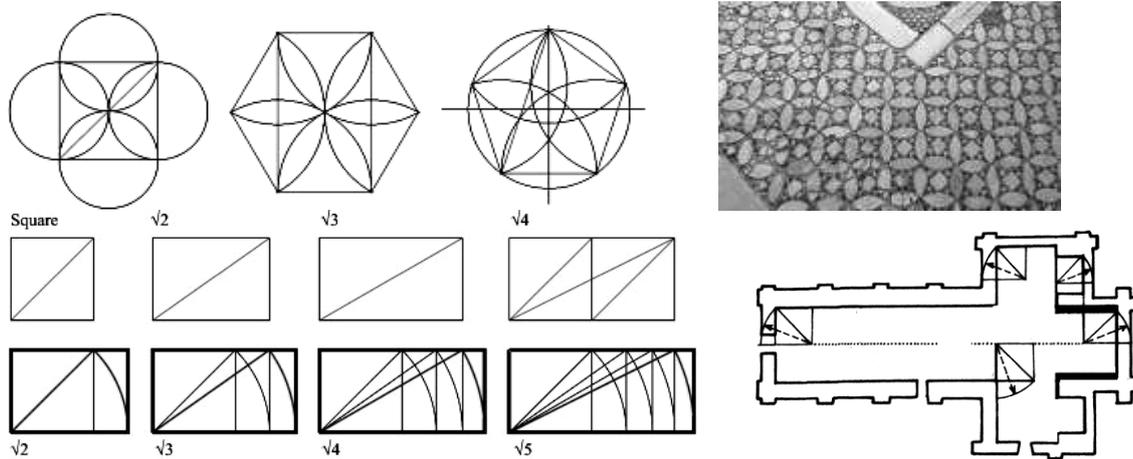


Figure 4: Proportional roots in Gothic architecture: (above left) the geometry of proportional roots based on the circle; (below left) $\sqrt{2}$, $\sqrt{3}$, and Golden Section, roots based on the diagonal of the square; (above right) the geometry of $\sqrt{2}$ in floor tiling at Spoleto's Duomo (below right) proportional roots based on the square. the use of $\sqrt{2}$ geometry to be included inside the wall-thickness in medieval churches, (right). Diagram: after Kinsella, *Thoughts on Medieval Architecture* (2013), <http://soffits.wordpress.com/>

The most mysterious ratio, however, was that between the diameter and the circumference of the circle, known as π . It can not be expressed neither as a root, nor as a ratio between roots, and unlike the square roots, it cannot be created geometrically except by drawing a circle. Such a number is called 'transcendental' and was regarded as truly divine.¹⁸

One further principle controlling the geometry of sacred buildings at this time was that of continuity. When any building was extended or altered it was essential that the geometry of the new work conforms geometrically with the existing work. If it did not, it would not receive divine protection from disasters.¹⁹

It was through *geometrical progression*, which was also originated from *ad quadratum* and the *ad triangulum* principles, that the geometry of a building was thought to receive this protection and be integrated with the geometry of God's universe. A geometrical progression exists when a

¹⁷ Dabbour, 2012, 380-391

¹⁸ Jorge Gracia, *The Transcendentals in the Middle Ages*, 1992: 113-120.

¹⁹ Dudley, 2001, Chap. 2:40.

series of shapes are consecutively related to one another by a common factor, such as square within square within square. It can extend in either direction, larger or smaller, to an infinite degree, to the dome of heaven and beyond. But, unlike arithmetical progression, it can never become a minus quantity or reduce to zero. For this reason, mediaeval theologians believed that it had the power to provide security from destructive forces.²⁰ This means that, on one hand, it has the power of integrating the building into the overall geometry of God's Universe, and on the other hand it has the "divine protection" of continuity, and the immunity from "destructive forces." So, it would have

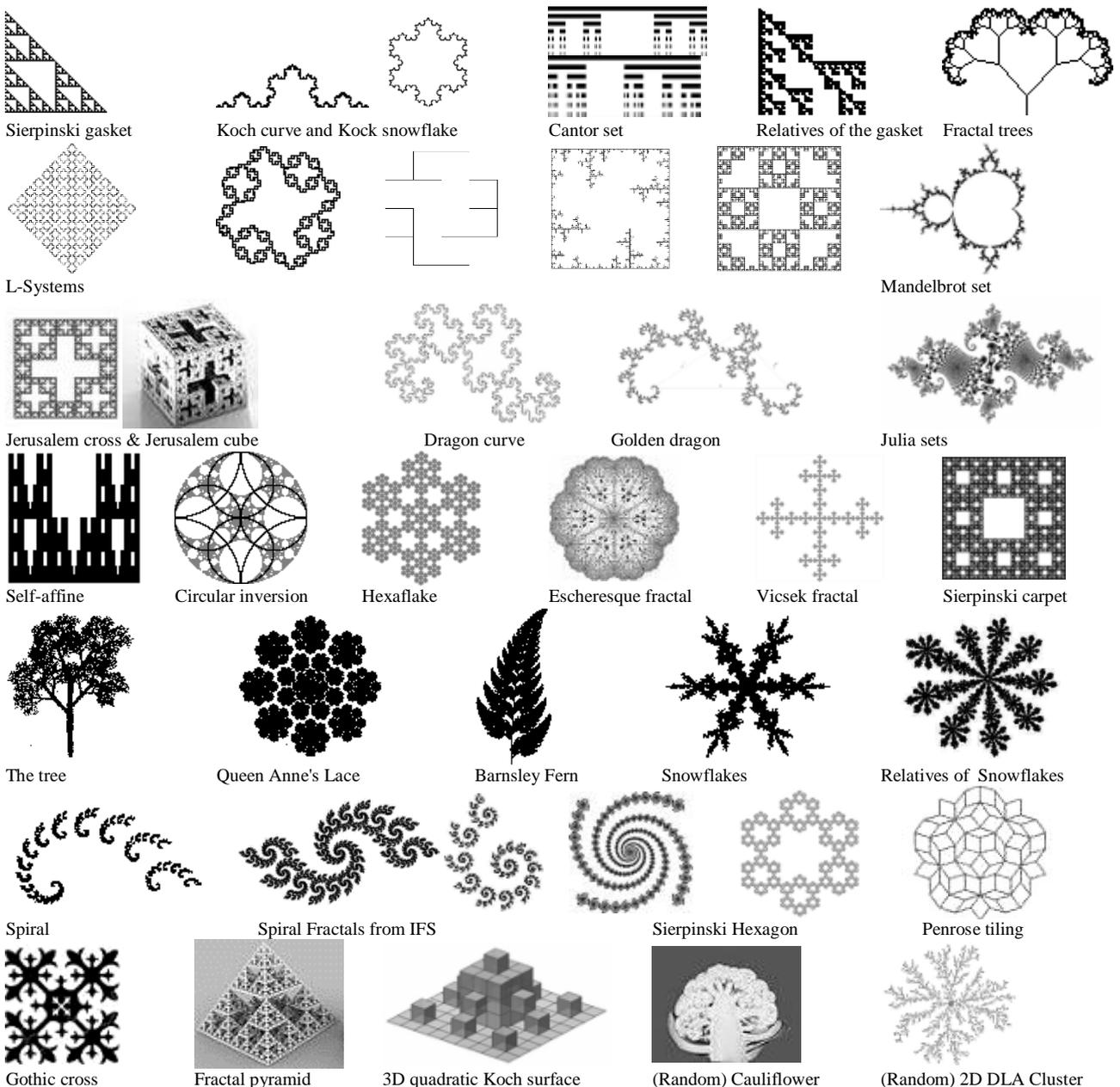


Figure 5: Examples of fractal shapes. Diagram: author.

²⁰ Dudley, 2001, Chap. 2: 41.

been, in the light of such ideas, that theologians, in cooperation with the geometers of Middle Ages, used these progressions as a binding tool to bring its overall geometry in a unified "protected" wholeness and in the same time integrate it into the divine geometry of the universe.

Geometrical progression, together with the tendency to follow the laws of Nature's language, were probably the leading forces behind the development of the other type of geometry, in which these two concepts are essential characteristics, which is *Fractal geometry*. Even though its principles, as they are understood today, were unknown; Gothic cathedrals are considered to be one of the best architectural representation of these principles.²¹

A Fractal is a pattern that repeats itself at different scales to an infinitely small/large scale; this is what mathematicians call *self-similarity* (**Figure 5**). In his book, *Fractals: Form, Chance and Dimension*, Mandelbrot coined the term *fractal* to describe these structures; he derived this term from the Latin *fractus*, defined as broken or shattered glass.²² Fractals are not produced by mere repetition of a shape; they are rather generated by the repetition of a process, which is applied to a shape. This process should be in a way or another related to geometrical progressions (**Figure: 6**), which enables it to extend, larger or smaller, to an infinite degree. In the study of fractals, geometric series often arise as the perimeter, area, or volume of a self-similar figure.²³ In addition to the geometrical progressions built-in in their structure, Fractals are also ideal for modeling Nature as they capture most of its vital qualities, i.e. roughness, self-similarity, intricate detail..., etc.

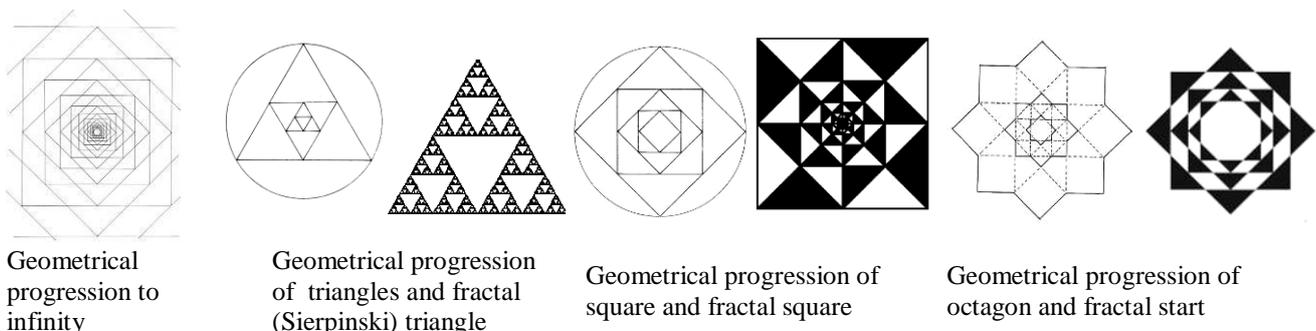


Figure 6: Fractal figures as repetition of geometrical progressions that may extend to infinity and never vanish. Diagram: author.

²¹ Good examples for fractals can also found in Hindu temples, Baroque, and Islamic styles.

²² Benoit Mandelbrot, *Fractals: Form, Chance and Dimension*, 1977: 17.

²³ Luiz Bevilacqua *et al.*, *Geometry, Dynamics and Fractals*, 2008: 11-21.

Fractal Cosmology relates to the usage or appearance of fractals in the study of the cosmos.²⁴ Almost anywhere one looks in the universe; there are fractals or fractal-like structures. Scientists claimed that even the human brain is optimized to process fractals, and in this sense, perception of fractals could be considered as more compatible with human cognitive system and more in tune with its functioning than Euclidian geometry. This is sometimes explained by referring to the fractal characteristics of the brain tissues,²⁵ and therefore it is sometimes claimed that Euclidean shapes are at variance with some of the mathematical preferences of human brains.²⁶ These theories might actually explain how Gothic artists intuitively produced fractal forms, even though they did not have the scientific basis to understand them.

Self-similarity in fractals might refer to: *a) strict self-similarity*, where every detail of the fractal is an exact copy of the whole structure, such as Sierpinski triangle (**Figure 6**); *b) quasi self-similarity*, where the substructure is recognized as being similar to the superstructure, but not in an exact mathematical way, as in Mandelbrot Set (**Figure 5**); and *c) statistical self-similarity*, where some statistical measure or trend is preserved over different scales of magnitude, such as in the random fractals.²⁷

In *The New Paradigm in Architecture*²⁸ and *The Architecture of the Jumping Universe*,²⁹ Charles Jencks argued that fractal architecture can provide an artistic interpretation of physical reality and thereby express the dynamic, creative and self-organizing universe. A strict mathematical definition of a fractal implies that its self-similarity stretches to infinity, which entails that, neither architecture nor anything else in this physical world, can be fractal. A possible alternative is to adopt a more liberal interpretation, where a structure is fractal when it shows a proper degree of self-similarity (5 - 6 hierarchical scales). So, a façade can be given a fractal

²⁴ Jonathan Dickau, *Fractal Cosmology*, 2009: 2103–2105.

²⁵ R.Taylor *et al.*, *Perceptual and Physiological Responses to the Visual Complexity of Fractal Patterns*, 2003, 89-114.

²⁶ Yannick Joye, *Fractal Architecture Could Be Good for You*, 2007, 311- 316.

²⁷ Heinz-Otto Peitgen *et al*, *Chaos and Fractals*, 1992, 55–66.

²⁸ Charles Jencks, *The New Paradigm in Architecture*, 2002.

²⁹ Charles Jencks, *The Architecture of the Jumping Universe*, 1997.

outlook, by repeating architectural details and elements on different scales. Eglash suggests that even a three-fold iteration can be enough to get the concept.³⁰

3. Analysis:

Before starting this analysis, it might be helpful to overview some qualifying comments about the regional differences in Gothic style.

The distinctive characteristic of French cathedrals is their height and their impression of verticality, while in English cathedrals the main internal emphasis was upon their extreme length. French cathedrals tend to be stylistically unified in appearance when compared with English cathedrals, where there is great diversity in almost every building and sometimes every part within the same building as it was not unusual for every part of the building, being built in a different century, to be built in a different style, with no attempt at creating a stylistic unity.

The east ends of French cathedrals are polygonal with ambulatory or chevette of radiating chapels, with slight or no projection of the transepts and subsidiary chapels, while English cathedrals sprawl across their sites, with double, strongly projecting, transepts. The west fronts of French cathedrals are highly consistent, having three portals surmounted by a rose window, and two large towers, where in English cathedrals the west front is usually not as significant, the usual congregational entrance being through a side porch. Their west windows are very large and almost never feature a rose window, which are reserved for the transept gables. The Gothic architecture of Central Europe generally follows the French formula, but the towers are much taller and are surmounted by enormous openwork spires.

The distinctive characteristic of Italian Gothic is the use of polychrome decoration, both externally as marble veneer on the brick façade and internally, where the arches are often made of alternating black and white segments. With the exception of Milan Cathedral which is Germanic in style, Italian cathedrals have few and widely spaced columns. The façades have projecting open

³⁰ Ron Eglash, *African Fractals: Modern Computing and Indigenous Design*, 1999: 3.

porches and ocular or wheel windows rather than roses, and they do not usually have a tower, where the crossing is usually surmounted by a dome.³¹

In the following, an analytical illustrative study is performed on examples of Gothic cathedrals showing the applications of both Euclidian and Fractal geometry in them. The analysis here does not aim at proving the Euclidian or the fractal characteristic of these buildings, which had been explored by many researchers before, such as Bork, Dudley Goldberger and others, but rather aims to find the context or logic, by which they were used, the setting in which they functioned, and the common link(s) or bridges that integrated them with each other.

3.1. Euclidian geometry in Gothic architecture:

In the following, geometry in Gothic architecture, as based on the essential harmonies of Nature, together with various symbolic meanings and theories of perfect proportions, will be reviewed. Man as the core of God's creation, who possesses the most perfect proportions that reflect the divine harmony of being, was the prominent feature of this architecture.

3.1.1. The Golden Mean:

The Golden Mean (or Ratio, or Section) is a proportional system, whereby two elements, or two segments of a line, not equal to each other are related in the formula: $a/b = (a+b)/a = 1.61803$. Some scholars argue that, until Pacioli's 1509 publication, the Golden Ratio was unknown. While others argued that Euclid, in his book *The Elements*, mentions it as: a line AB that is divided in *extreme and mean ratio* by C if $AB:AC = AC:CB$.³² Although he did not use the term, this shall be called the Golden Ratio.³³

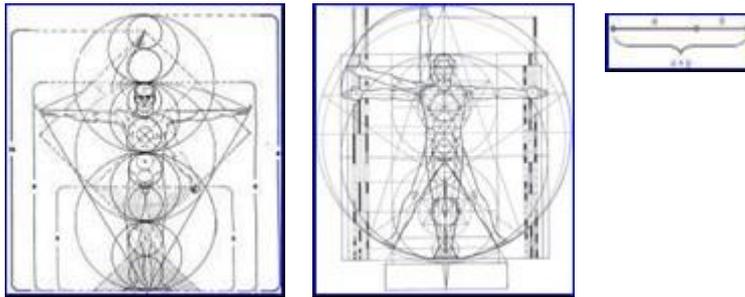
The Golden Mean is the most common proportional system used in architecture throughout history to mirror the divine proportion of the human canon as clearly exhibited in the Vitruvian Man around 1490 (**Figure 7**). These divine proportions were frequently applied to plans, sections or

³¹ Banister Fletcher, *A History of Architecture on the Comparative Method*, 1905: 420-421.

³² The definition appears in Book VI, but the construction is given in Book II, Theorem 11.

³³ Adolf Zeising, *Der goldene Schnitt*, 1884: 4; Max Livio, *Searching for the Golden Ratio*, 2003: 52-58.

Figure 7: Vitruvian man and the Golden Ratio. Diagrams: after Ernst Neufert, *Bauentwurfslehre* (1936), pp. 8-9.



elevations of Gothic cathedrals (**Figure 8-a, b**), to express the spiritual idea of the church as "the body of the Lord".³⁴ In Beauvais Cathedral for example, the height of choir in relation to the overall height of the cathedral is an approximation of the Golden Section as noted by Stephen Murray³⁵ and earlier with regard to a number of Gothic facades by Frederik Macody Lund in his 1919 book *Ad Quadratum*.³⁶

The mathematics of the Golden Ratio is related to the *Fibonacci Series*, which was first published in the 1202 book *Liber Abaci*. The rule governing this sequence is that the next number is the sum of the previous two numbers, as follows: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, etc. If any number in this sequence is divided by the one before it, for example 144/89, or 89/55, the answer is always close to 1.61803.³⁷ The most famous visual expression of this series in Nature are the spiral shapes (**Figure 9-a**), which were frequently seen in Gothic art and architecture in reference to their cosmic significance (**Figure 9-b**).

³⁴ 1 Corinthians 12:12-14.

³⁵ Stephen Murray, *Plotting Gothic*, 2014: 1-14.

³⁶ All the lines that refer to their proportions are imaginary lines that hide behind the actual lines of the building. Walking through the plan of the cathedral or gazing at the actual lines of the facades, a normal beholder would never see any of these lines or any reflection of a human figure as seen in these figures. The domination of modular order is also to be realized in all these illustrations.

³⁷ Parmanand Singh, *The So-called Fibonacci numbers in ancient and medieval India*, 1985: 229-244.

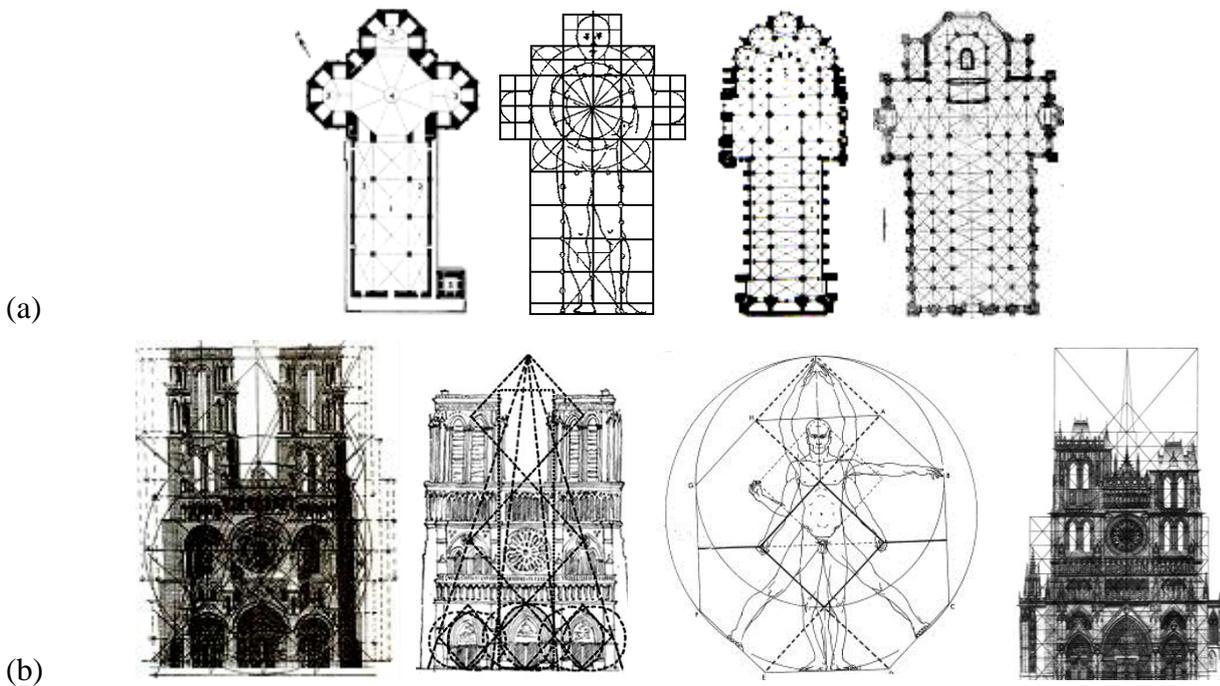


Figure 8: Human proportions in Gothic cathedrals (a): Human proportions and modular system in floor plans of Gothic cathedrals: (from left to right) Florence Cathedral, diagrammatic Latin cross plan based on the proportions of human body, Reims Cathedral, and Milan Cathedral. Figures: after Banister Fletcher, *A History of Architecture on the Comparative Method* (1905), p. 366, 409. (b): Human proportions as imaginary invisible lines with geometric progressions in the façades of Gothic cathedrals: (from left to right) Notre-Dame of Laon, Notre Dame of Paris, and Amiens Cathedrals. Figures: after Lund (1919), p. 48

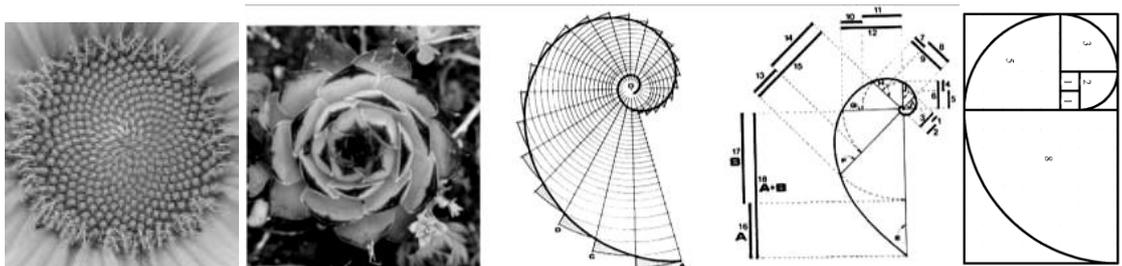


Figure 9-a: 'Fibonacci Series' geometric progression spirals in nature. Diagram: after Adolf Zeising, *Der goldene Schnitt* (1884), p. 220.

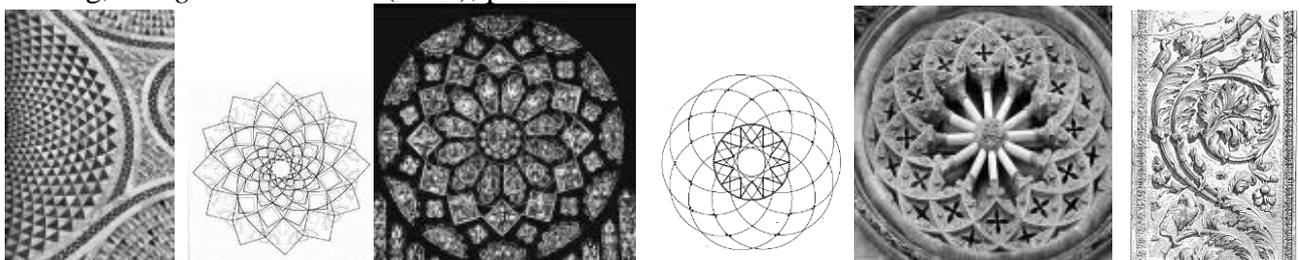


Figure 9-b: *Fibonacci Series* geometric progression and geometry of Nature and in Gothic architecture. (Left to right) *Cosmati* guilloche pattern from San Marco, Venice - Rose windows from Chartres Cathedral and San Francesco d'Assisi, Palermo - Carving on the pulpit in Strasbourg Cathedral. Drawing: after Thomas R. Smith, *Architecture* (1908), p. 172.

3.1.2. The Pentagram Star

In *The Elements*, Euclid presented some applications of the above-mentioned "extreme and mean ratio" such as the regular pentagon. He proved that the diagonals of the regular pentagon cut each other at this "extreme ratio,"³⁸ as illustrated in **Figure 10-a**. During the medieval era, the pentagram star was seen as the symbol of mankind. This idea was related to the ancient Secret Doctrine that "man is a star; an eternal soul that shines deep down beneath the physical, corporeal body."³⁹

Pentagram proportions were present in the designs of Gothic plans, section, and elevations (**Figure 10-b, c**). In Lincoln and Chartres Cathedrals, distances between pillars and the lengths of the nave, transepts, and the choir are all multiples of the Golden Mean.⁴⁰ The overall ground plan in both are based on the intersection of (invisible) two circles containing (also invisible) pentagram in a shape that was called the *Light Matrix*, or, *Ain Sof*, where Spirit (*light*) and Matter (*man*) come together (this is an oculist symbol like those in the book of Agrippa).⁴¹

On another hand, the *Sephiroth* or the *Tree of Life* is another symbolic configuration of ten spiritual principles, arranged in three columns that refer to: the nature of revealed divinity, the human soul, and the spiritual path of ascent by man. In medieval literature this symbol was developed into a depiction of the *Map of Creation*.⁴² This ten-point-symbol, associated with a twin or overlapping pentagrams, was frequently used as geometrical base of Gothic plans (**Figure 10-c**). Again, by looking at the figure, one can see that the lines representing these symbols do not appear as actual elements or walls in the body of the plan, but only as imaginary or working lines and points that enclose the building and bind its components together.

³⁸ Livio, 2003, 52-58.

³⁹ Heinrich Cornelius Agrippa, *De Occulta Philosophia Libri Tres*, (first published 1510) 1949, Book I: 11.

⁴⁰ Francis Bond, *Gothic Architecture in England*, 1906: 216.

⁴¹ Rabbi Krakovsky, *Kabbalah - The Light of Redemption*, 1970: 19.

⁴² Krakovsky, 1970: 19; and Agrippa, 1949, Book III, 10.

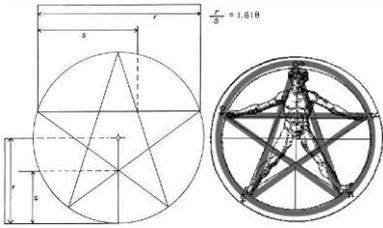


Figure 10-a: Pentagram star as a representation of Euclid's extreme ratio and a symbol of Mankind.

Figure: after Agrippa (1949), p. 11

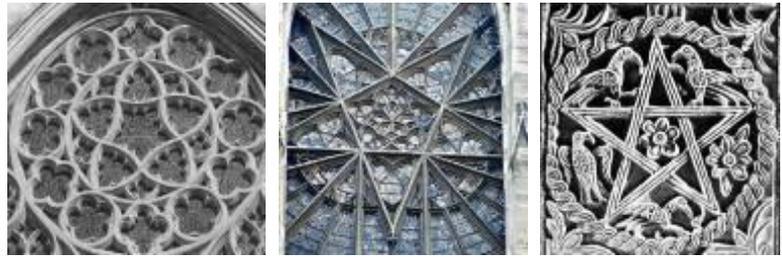


Figure 10-b: Pentagram star in Gothic façades: (left) Rose window in Exeter Cathedral, (middle) side window in Amiens Cathedral, and (right) carving from the baptistery of the St. John Cathedral, Šibenik, Croatia. Photos: author.

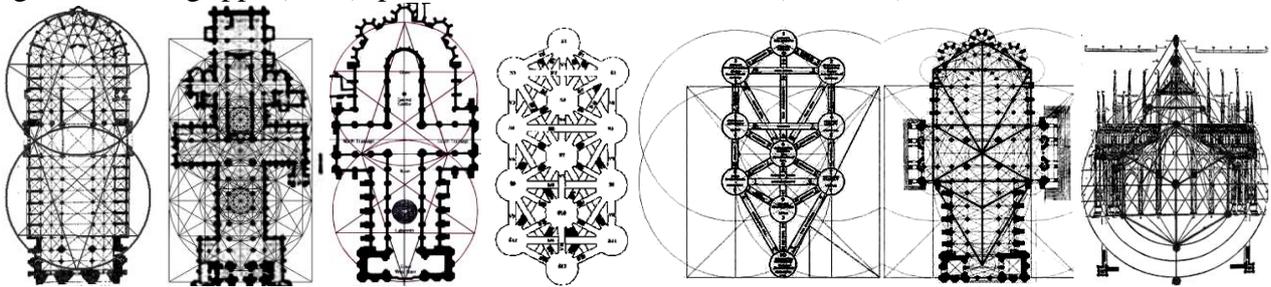


Figure 10- c: Pentagram proportions as invisible working lines in Gothic cathedrals: (left) plans of Notre Dame of Paris and Lincoln Cathedrals, (middle) plan of Chartres Cathedral and its relationship to both the *Light-Matrix* and the *Tree of Life* (both related to the pentagram and the Vesica Pisces), and (right) section of Milan Cathedral showing its relation with both the pentagram and the *Tree of Life*. Diagrams: after (left to right) author, <https://aftermathnews.wordpress.com>, and http://www.bibliotecapleyades.net/cienciareal/cienciareal_ritualcathedral.htm, <http://www.pinterest.com/emmatuzz/south-france-mysteries/>, and Frankl (1945). p. 46, 60.

3.1.3. The Octagon:

Designs based on the octagon are used to create the so-called $\sqrt{2}$ system of proportion. This is because the diagonal of a unit square is $\sqrt{2}$ units in length (**Figure 11**). In Euclidian geometry, it is created by the intersection of two squares in the Sacred Cut or the intersection of nine circles revolving around a fixed center of the Octagon Rosette.⁴³ The relationship between octagons and their circumscribing circles appears frequently in Gothic drawings, especially when "octature" is seen alongside "quadrature" as one of the foremost proportioning strategies of the era (**Figure 12**).⁴⁴

⁴³ Philibert De l'Orme, *Le Premier Tome De L'architecture*, 1567: Pl. 18.

⁴⁴ Bork, 2011, 11.

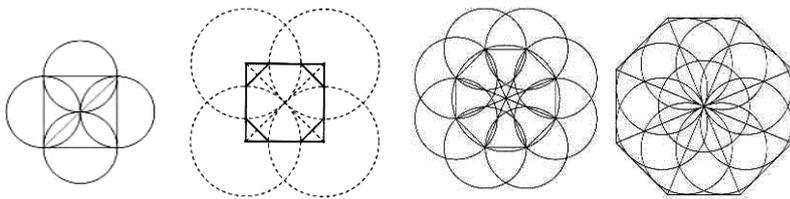


Figure 11: $\sqrt{2}$ system and geometric *rotational* progression in the Sacred Cut and the Octagon Rosette. Diagrams after De l'Orme (1567), pl. 18; and Drunvalo Melchizedek, *The ancient secret of the flower of life* (2000), p. 13.

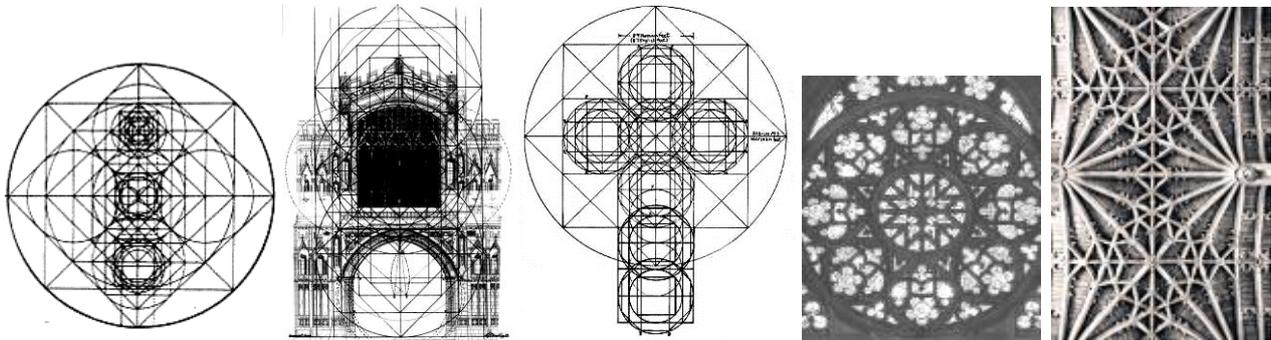


Figure 12: Octagons in Gothic architecture: (a) as invisible working lines with geometric progressions and constant module in the nave, plan and elevation of Peterborough Cathedral. Diagrams after Colin Dudley, *By Craft of Ewclyde* (2001) p. 151, 164, (b), as a center of rose window in St. Vitus Cathedra, Prague, and octature-quadrature proportions in the ribs of the vaulting at Oxford Cathedral. Photos: author.

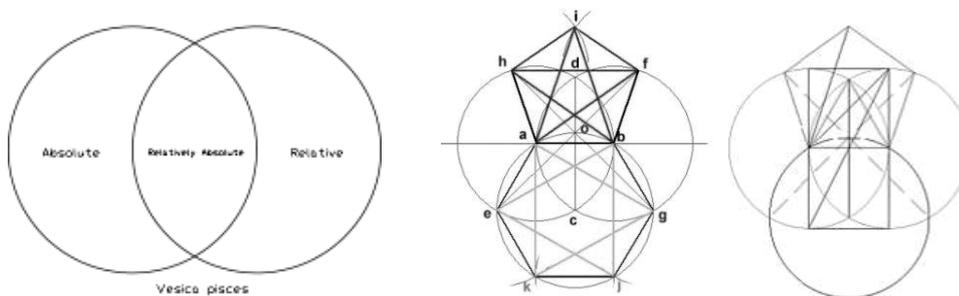


Figure 13-a: Vesica Pisces in Gothic architecture: Relatively Absolute, proportional roots and Golden Mean within the *Vesica Pisces*. Diagrams after De l'Orme (1567), pl. 13.

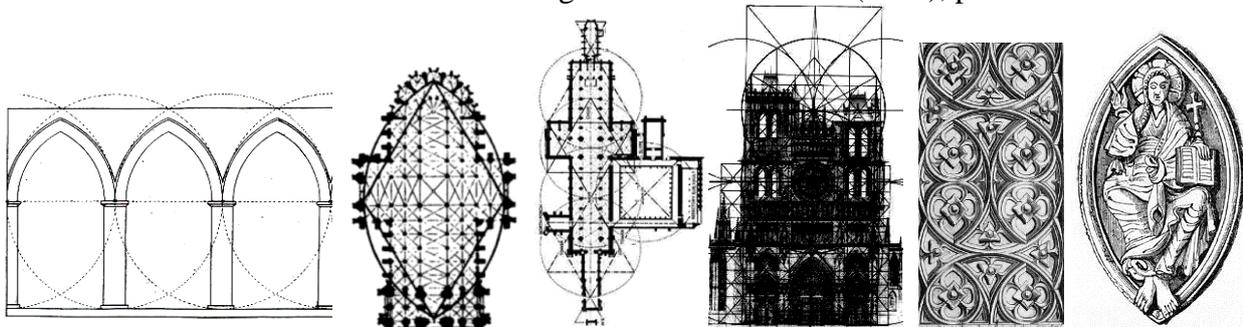


Figure 13-b: Vesica Pisces in the working lines of Gothic architecture: (left to right) the geometry of Gothic pointed arches, plan of Beauvais Cathedral, and Glastonbury Cathedral (<http://www.ancient-wisdom.co.uk/sacredgeometry.htm>), façade of in Amiens Cathedral. Photo: after Lawlor (1979), p. 95; wooden tracery and tympanum icon from Ely Cathedral after Matthew Bloxam, *The Principles of Gothic Ecclesiastical Architecture, Elucidated by Question and Answer* (1882), p. 82.

3.1.4. Vesica Pisces

Vesica Pisces was one of the key starting blocks of sacred geometry throughout the history of the Europe art and architecture. It is the intersection of two circles with the same radius in such a way that the centre of each circle lies on the perimeter of the other and was derived from Euclid's Elements Book 1 Prop 1. The name literally means the 'fish's bladder' in Latin. It represented the symbolic relationship between the absolute and the relative by two overlapped circles. The realm enclosed within this overlap is the '*Relatively Absolute*' and within this realm, the primary proportional roots, i.e., $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$, as well as the Golden Mean, are all present.⁴⁵ By intersecting it with another circle, an equilateral triangle, a hexagon, a pentagon, a square and so on, may be drawn (**Figure 13-a**). As the interlocking/underlying structure between the Pentagram (Man) and the Hexagram (Star of David) it was regarded as a template of the *Demiurgic Science of Creation & Manifestation*.⁴⁶

The Vesica Pisces was frequently used as proportioning system in Gothic architecture as illustrated in Cesare Cesariano 1521 book *The Rule of the German Architects*. This appears in the proportions of the exquisite arches of Gothic architecture that capture the celestial proportions and anchor it into the terrestrial realms. The common rhombus floor in churches and variations of carving and ornaments are also based on the Vesica Pisces proportions. It was also associated with the Christian common symbol of the fish, upon which the geometry of some cathedrals were based (**Figure. 13-b**).

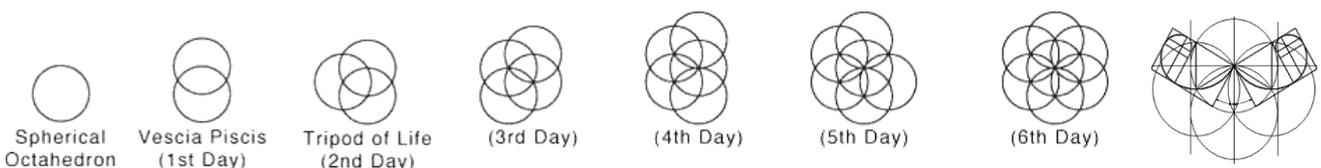


Figure 14-a: The primary circle of the Seed of Life, the six days of Creation, and the geometrical progression of Golden Ratio nested in the figure's evolution (Diagram after Richards, <http://www.goldenspiralresearch.co.uk/geometry.html>), denoting wholeness, completion, unity and infinity.

⁴⁵ Dabbour, 2012, 380-391.

⁴⁶ Gnosticism presents a distinction between the highest, unknowable God and the *demiurgic* creator of the material. In various ancient belief, mainly Platonism and Gnosticism, this is a deity responsible for the creation of the physical universe. The precise nature and character of the Demiurge however varies from a benign architect of matter in some, to the personification of evil in others.

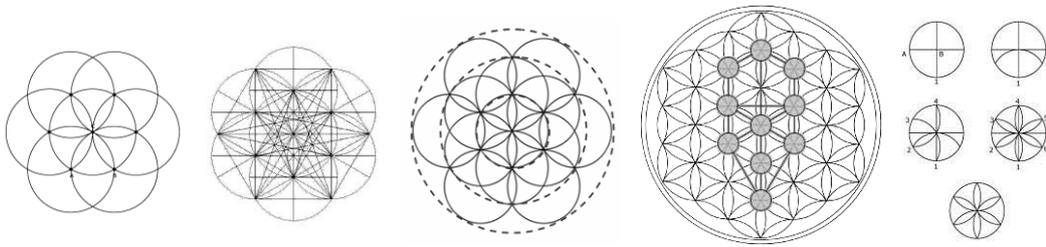
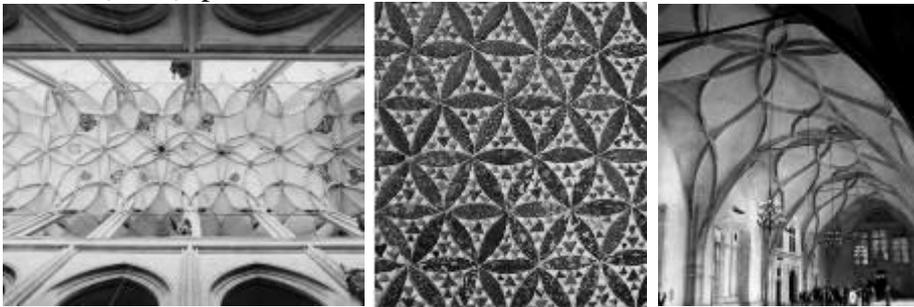
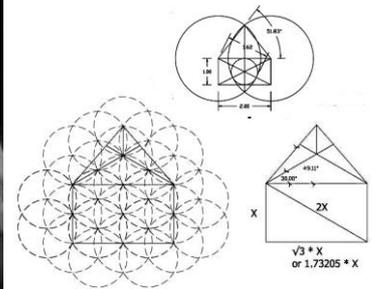


Figure 14-b: The Seed of Life and its associate figures (language of Nature): (left to right) the Seed of Life, the Golden Mean, the Flower of Life (with another geometrical progression in its structure illustrated by imaginary dotted-lines), the Tree of Life and proportional roots. Diagrams after De l'Orme (1567), pl. 13.



Lierne vault in St. Barbara Cathedral, Czech Republic' Westminster Abbey vault at Vladislav hall, Prague



The hidden geometry of medieval framing .Diagrams after De l'Orme (1567), pl. 13.

Figure 14-c: The Seed of Life in Gothic architecture. Photos: author.

3.1.5. Seed of Life or Circle of Unity:

The hexagon is formed in Euclidian geometry from seven circles revolving around a fixed centre in six-fold symmetry, forming a pattern of circles and lenses that is called the Circle of Unity or the Seed of Life (**Figure 14**). In Christian tradition it is a symbol that supposed to depict the six days of Creation (**Figure 14-a**). Further multiplications of the shape produce the symbolic figures of the Flower of Life and Tree of Life (**Figure 14-b**). These grids, in addition to the Fibonacci (geometrical) sequence nested in its structure (**Figure 14-a**), are able to keep repeating themselves to infinity inward and outward in a continuous geometrical progression because a circle around the whole drawing would simply be the central circle of a still larger grid (**Figure 14-b**).⁴⁷

This pattern was used in Gothic vaults, evident in those created by lierne ribs. Robert Willis in *On the Construction of Vaults of the Middle Ages*,⁴⁸ also overviews some Gothic vaulting

⁴⁷ Drunvalo Melchizedek, *The ancient secret of the flower of life*, 2000: 15

⁴⁸ Robert Willis, *On the Construction of Vaults of the Middle Ages*, 1910.

techniques as originally illustrated by Philibert De l'Orme,⁴⁹ Francois Derand⁵⁰ and others. Here, he explains how medieval builders used only such revolving circles (as invisible working lines) to geometrize Gothic vaulting (**Figure 14-c**).

3.1.6. Results:

Looking at **Figures 3, 8, 10, 12, 13**, the following common characteristics are evident:

- (a) The overall design of plans and elevations begins always with one or two **circles** that embrace the building and from which all the other lines start to unfold. Nonetheless, a normal worshiper, who visited the church every week, **would never see this circle**, or any of the other circles and lines unfolding from it, because these are imaginary lines that do not actually appear in the body of the building. The purpose of these circles was not to be seen, but to make the buildings, as discussed in the previous section, belong to Heaven.
- (b) **Figures 8 to 14** illustrate the idea of continuity through an extensive use of **geometrical progressions**, in both visible (in **Figures 9, 14**) and invisible formats (in the rest of the figures).
- (c) The proportional and geometrical lines and ratios that were employed in these buildings were believed to be included in, or representative of, the **geometry of Nature**, while also reflection symbolic spiritual connotations.
- (d) Yet, these lines and ratios are **neither visible nor perceivable** for the medieval layman. Unlike, for example, the façade of the Parthenon, in which the employment of the Golden Ratio is easy to recognize in the actual, continues lines of the columns and the entablature, in these churches, such as in Notre-Dame of Laon (**Figure 8-b**), it is only by drawing a complicated mesh of imaginary lines all over the façade, that one can recognize the role of this ratio in its design. Even the symbolic meaning of these ratios was to be realized only by theologians and clergymen. These lines and ratios were, therefore, probably meant to be a sort of sacramental geometry that addresses the eye

⁴⁹ *Le Premier Tome De L'architecture*, 1567.

⁵⁰ *L'architecture des voutes*, 1643.

that can see the "invisible" working lines and comprehend the "unperceivable" ratios in common between the building and Nature and their connotations; this is the eye of the Creator.

3.2. Fractals in Gothic Architecture

In order to understand the fractal character of Gothic style, it is important to note that architects sometimes use a "module" as main organizational element (**Figure 15**). In this regard, Konrad Hecht noted that Gothic designers worked almost exclusively in modular fashion.⁵¹

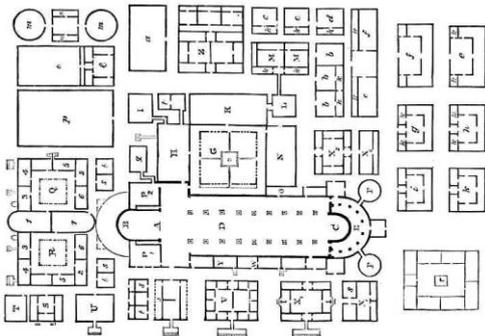


Figure 15: *Module* in the plan of St Gall Switzerland. Figure after Fletcher (1905), p. 261.

The medieval designers aimed to enrich every constructive feature and to embody within the decorative detail the greatest possible amount of allegory and symbolism. Its texture reveals rich set of designs progressing from large to small scale, with ever-increasing intricacy exactly as in fractal elements. An example of this similitude is shown in (**Figure 16**), where the scaling over several levels in both Mandelbrot set⁵² and the exterior of Milan cathedral is illustrated.

3.2.1. Fractals in Gothic Plans:

The most important characteristic of Gothic plans was the apsidal termination of the choir, forming single or double ambulatory or chevet. Looking at the examples in (**Figure 17-a**), the fractal character of chevets and their resemblance with the fractal shape of *Koch Curve*⁵³ are immediately recognized. The moldings found in the cathedrals, too, had similar shapes. This

⁵¹ Konrad Hecht, *Maß und Zahl in der gotischen Baukunst*, 1979, 334–361.

⁵² A set of complex numbers 'c' for which the sequence $(c, c^2 + c, (c^2+c)^2 + c, ((c^2+c)^2+c)^2 + c, (((c^2+c)^2+c)^2+c)^2 + c, \dots)$ does not approach infinity.

⁵³ A mathematical curve that starts with an equilateral triangle, then recursively altering each line segment.

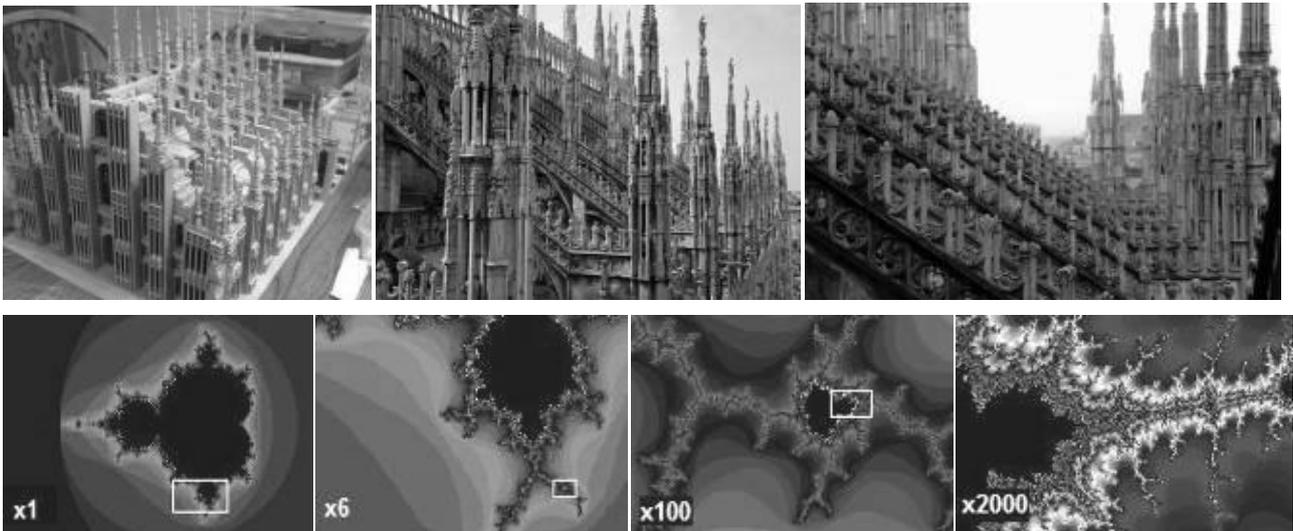


Figure 16: Milan cathedral (above) and Mandelbrot set (below): as one zooms in on the images at finer and finer scales, more patterns appear in an infinitely complex texture. Figures of Mandelbrot set after <http://en.wikipedia.org/wiki/File:Mandelbrot-similar-x6.jpg>. Photos of Milan cathedral: author.

characteristic was further emphasized in the plan of the Church Of Our Lady, Treves, which seems to have been produced by doubling this arrangement on both sides of the transverse axis, in an arrangement that recalls the *Koch Island*.⁵⁴ (Figure. 17-c)

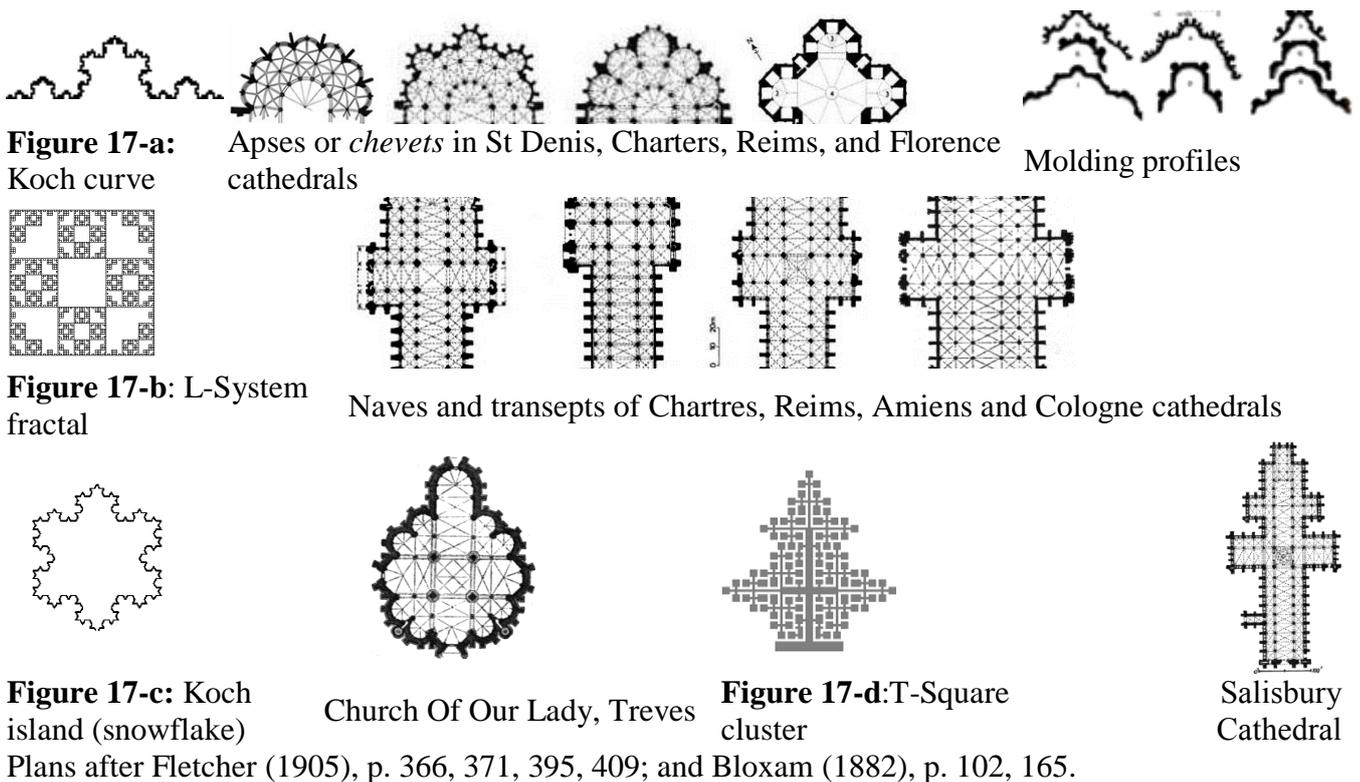


Figure 17-a: Koch curve

Apses or chevets in St Denis, Chartres, Reims, and Florence cathedrals

Molding profiles

Figure 17-b: L-System fractal

Naves and transepts of Chartres, Reims, Amiens and Cologne cathedrals

Figure 17-c: Koch island (snowflake)

Church Of Our Lady, Treves

Figure 17-d: T-Square cluster

Salisbury Cathedral

Plans after Fletcher (1905), p. 366, 371, 395, 409; and Bloxam (1882), p. 102, 165.

⁵⁴ A shape based on the multiplication of the Koch curve.

Another attribute of Gothic models was the multiplication of chapels, where lateral chapels were built at each bay of the side aisles, flanking the nave as well as the choir in fractal-like arrangements (**Figure 17- b**) that follow the L-System model of fractals.⁵⁵ In English cathedrals, the choir had a square termination with secondary transepts with arms of different dimensions symmetrically clustered around the main rectangular. These arms consist of further miniatures, and their corners are filled in with smaller chapels and niches in another fractal-like arrangement (**Figure 17-d**).

An obvious disadvantage of fractal ground plans is that the fractal design is barely visible to the viewer. More conceivable applications may be fractal patterns on tiling. The pattern known as a Cosmatesque or Cosmati (**Figure 18**), which is a style of geometric decorative inlay stonework, was typical of medieval Italy and which later spread throughout Europe.⁵⁶ Interlaced circular patterns known as Guilloche are the focal points of a Cosmati pavement and the polygonal patterns were used to fill up empty spaces inside and between successive interlocking circular shapes (geometrical progressions) with fractal-like patterns. Here, unlike the previously discussed applications of Euclidian geometry in plans, fractal patterns in plans are visibly reflected in the form of: actual walls/components in the body of the building (**Figure 17-a, c, d**), modular rhythm in the space (**Figure 17-b**), and geometrical patterns in tiles (**Figure 18**).

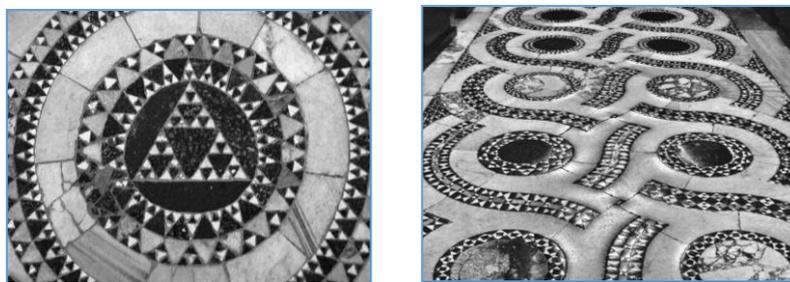


Figure 18 Fractals and geometrical progressions in Cosmatesque pattern: (left) Sierpinski's triangles inside circles: pavement of Santa Maria Cathedral, Trastevere, Italy; (right) double Guilloche filled with fractal details, San Cesareo Cathedral, Terracina, Italy. Photo: author.

⁵⁵ A model that describe the behavior of plant cells and the growth processes of plant development.

⁵⁶ "Cosmati" Russell Sturgis, *A Dictionary of Architecture and Building*, 1901, 691.

3.2.2. Fractals in Gothic Exteriors:

Even more comprehensible are certainly the three-dimensional applications of fractals, or *Arkhitektoiniki*, where the largest component of a building is surrounded with a cascade of smaller and smaller copies.⁵⁷ The same concept is seen in almost all Gothic exteriors (**Figure 19**).

Goldberger states "Fractals capture several key features of Gothic architecture; its carved-out appearance, its wrinkled crenellated surfaces, and its overall self-similarity ... From a distance, the sharp spires are the dominant feature. Closer proximity reveals that these spires are not smooth, but have spiny, outgrowths. Yet closer inspection reveals even more pointed detail superimposed on these ornaments."⁵⁸

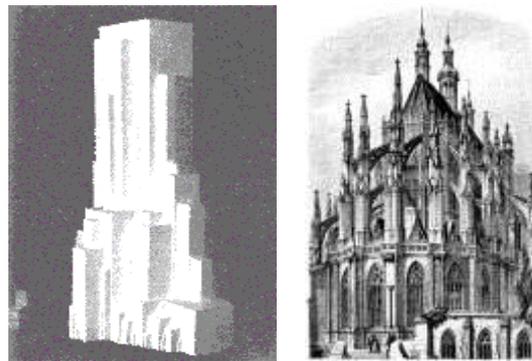


Figure 19: Scaling over several levels: (left) *Arkhitektoiniki* architecture, (right) St. Barbara, Kuttenberg. Photo: Smith, (1908), p. 100.

In the 15th-century booklet of Matthäus Roriczer, the design of Gothic spires or pinnacles is discussed as a process that bears Euclidian order, modular system and geometrical progression. He notes that it starts with a square (Euclidian), rotating a square within it, and then rotating another square within that (geometrical progression). Then these modules are *pulled up* into the third dimension in a process that he calls "Auszug" which means to "extract" it.⁵⁹ These technical processes (all were done in imaginary lines) are then "dressed" in a fractal outfit as in (**Figure 20**), which shows the different logic and context, by which each type of geometry was used.

⁵⁷ Miroslav Novak, *Emergent Nature: Patterns, Growth and Scaling in the Sciences*, 2001: 280.

⁵⁸ Al Goldberger, *Fractals and the Birth of Gothic*, 1996, 99-104.

⁵⁹ Robert Bork, *Gothic Architecture, Geometry, and the Aesthetics of Transcendence*, 2012
(<http://www.uni.edu/universitas/article/gothic-architecture-geometry-and-aesthetics-transcendence>)

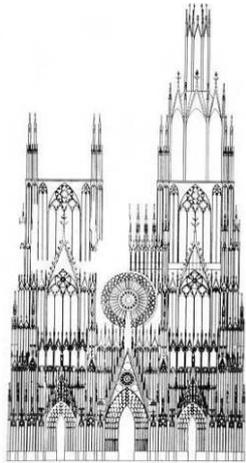


Figure 20-a

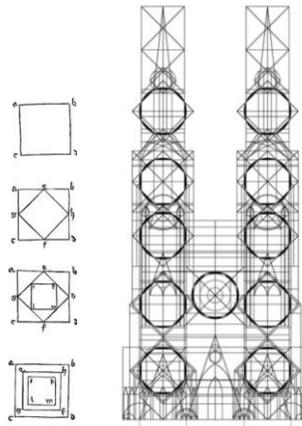
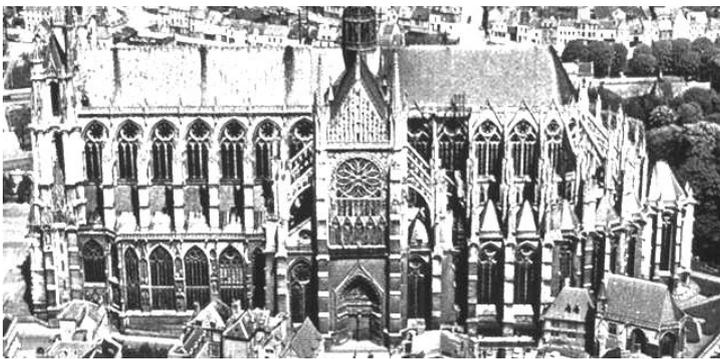
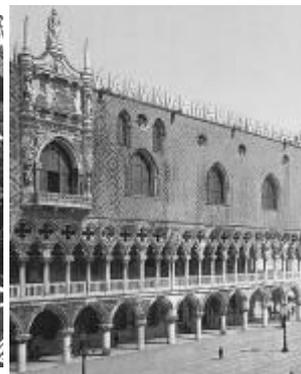


Figure 20-b

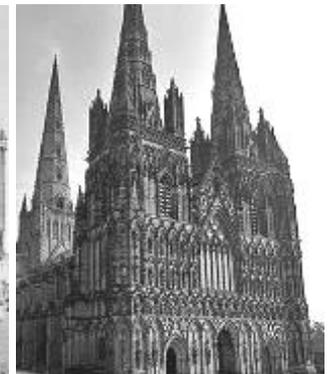
Fractal/Euclidian characteristics in Gothic façades: In (a) fractal order is expressed in the façade of Strasbourg Cathedral in actual visible settings and elements, while in (b) elements of Euclidian geometry are shown in the stages of Gothic spire design according to Roriczer by imaginary lines. It shows also the modular division of the spire's parts (*right*) and the use of geometrical progression (*left*). Figures after Bork, *Gothic Architecture, Geometry, and the Aesthetics of Transcendence* (2012), <http://www.uni.edu/universitas/article/gothic-architecture-geometry-and-aesthetics-transcendence>; and Matthäus Roriczer, *Das Büchlein von der Fialen Gerechtigkeit und die Geometria deutsch* (1490), p. 11.



Amiens Cathedral



Doge's Palace, Venice



Lichfield Cathedral

Figure 21: Fractal repetition, modularity and continuity in Gothic exteriors. Photos: author.

Further visualizations of fractal order in Gothic exteriors are shown in (**Figure 21**). In the Gothic cathedrals of France, and central Europe in general, the shape of the main portal recurs on a smaller scale in the two side portals, or in smaller versions in the different arched windows, which are sometimes divided into smaller parts. The contours of the portals are then repeated inwardly,

surrounded by a wealth of details. The complexity of façades is further increased by the mere repetition of arches, buttresses, spires set in different scales in a combination of complexity and order.

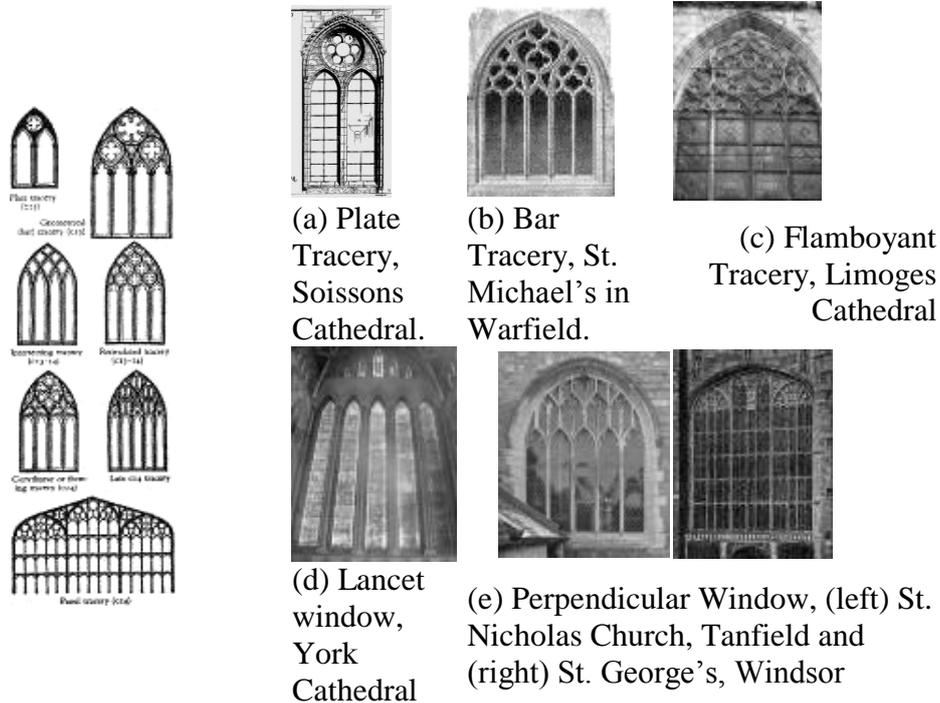


Figure 22: Fractals in Gothic windows and tracery: (left) General patterns of different Gothic panels (right) examples of different styles. Photos: author.

As for windows, a Gothic window is usually an arch sub-divided by two arches, each of which might be made of two still smaller arches and so on in another fractal-like structure (**Figure 22**). Traceries⁶⁰ of interlocked arches were developed in several styles in Gothic architecture to strengthen windows against the pressure of wind. In its earlier forms, tracery consisted merely of decorative openings pierced through slabs of stone (plate-tracery), filling the window-heads over coupled windows (**Figure 22-a**). Later on, the stonework was made lighter and richly molded (bar-tracery) (**Figure 22-b**). These patterns were then abandoned for more flowing and capricious (Flamboyant) tracery (**Figure 22-c**). In England, during the Early English Period (1200– 1300) *lancet windows* were tall, narrow and generally grouped by twos or threes (**Figure 22-d**). In the

⁶⁰ A *tracery* is the ornamental intersecting work in the upper part of a window, screen or panel, or used decoratively in blank arches and vaults.

later Perpendicular Style (**Figure 22-e**) the mullions were carried through to the top of the arch and intersected by horizontal transoms that were developed later into larger and more-detailed forms.



Figure 23: The resemblance between cusping in Gothic architecture and different stages of Koch Curve. Photos: author.

Cusping (**Figure 23**) was another type of Gothic decoration that was used for decorating arches or circles by triangular projections on their inner edge in a fractal-like repetition all over the facades.

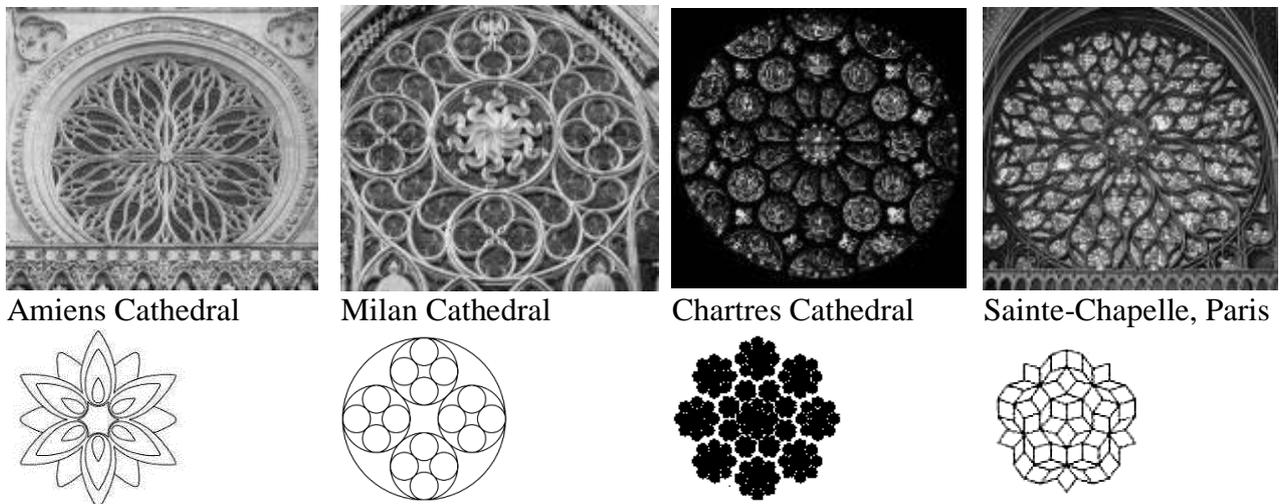


Figure 24: Fractal patterns, language of Nature and geometric progressions in Gothic rose windows. Photos: author.

Fractal components of Gothic cathedrals also speak from the rose windows (**Figure 24**). They all depict an overall circular form, in which fractal-like shapes are inscribed. Around this circle, further circles of varying sizes (geometrical progression) are usually placed, and some of these contain more fractal-like patterns.

3.2.3. Fractals in Gothic Interiors:

The properties of fractal hierarchical self similarity is also evident in the interiors of Gothic buildings, where all the verticals (arches, windows, moldings, etc.) are repeated on different scales,

within the same bay and then in the adjacent bays all around (**Figure 25 –a, b**). Even the structural system of Gothic cathedral shows fractal-like skeleton of arches, vaults and buttresses (**Figure 25-c**).

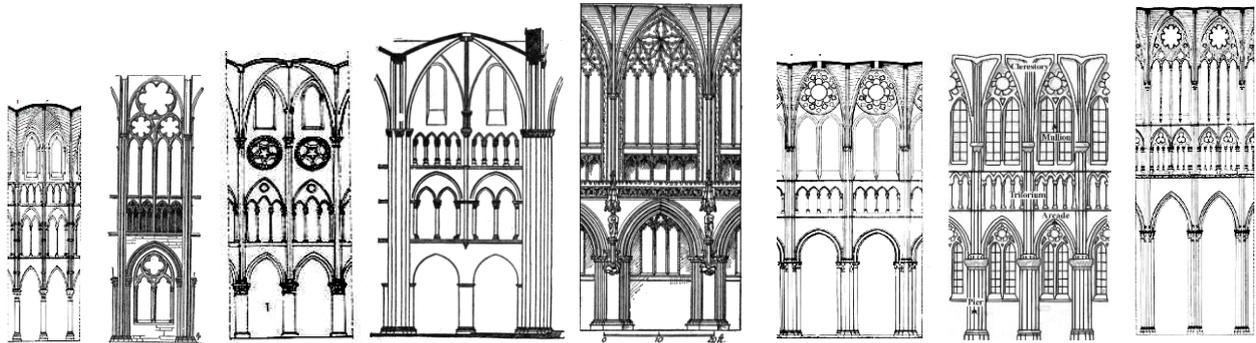


Figure 25-a: One bay at Laon, St. Denis, Notre Dame, St. George in Limburg and Lichfield cathedrals

Figure 25-b: Two bays at Charters, Reims and Amiens, cathedrals

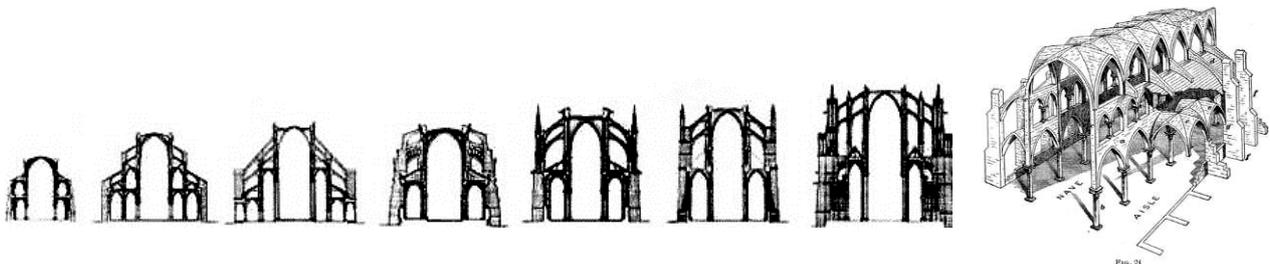


Figure 25-c: Fractal-like structure in Gothic cathedrals: (left) cross sections in Laon, Notre Dame, Bourges, Charters, Reims, Amiens, and Beauvais cathedrals (right) typical Gothic structure Figures after Fletcher (1905), p. 366, 395; Smith, (1908), p. 81, 94; Bloxam (1882), p. 44, and Charles Jencks, “Architecture Becomes Music,” (2013) (<http://www.architectural-review.com/essays/architecture-becomes-music/8647050.article>)

In this structure, shafts and supports sweep unbroken from floor to ceiling to meet the ribs of the vaults, like a tall tree spreading into branches, in one of the most common fractal patterns (**Figure 26**). By springing a large number of ribs from each point of support, the vaulting treatment took different shapes and techniques. At first, intermediate ribs, known as tiercerons (**Figure 26-a**), were introduced between the transverse and diagonal ribs. An increase and elaboration of the tiercerons led to a new set of ribs known as Stellar (from star) or Lierne (from the French *lien*), which means to bind or hold (**Figure 26-b**). These vaults led, by a succession of trials and phases, to a shape of vaulting known as fan, palm or conoidal vaulting (**Figure 26-c**) that was especially popular in England.⁶¹

⁶¹ Fletcher, 1905, 335-41.

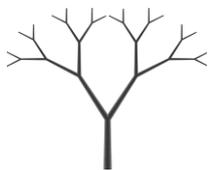
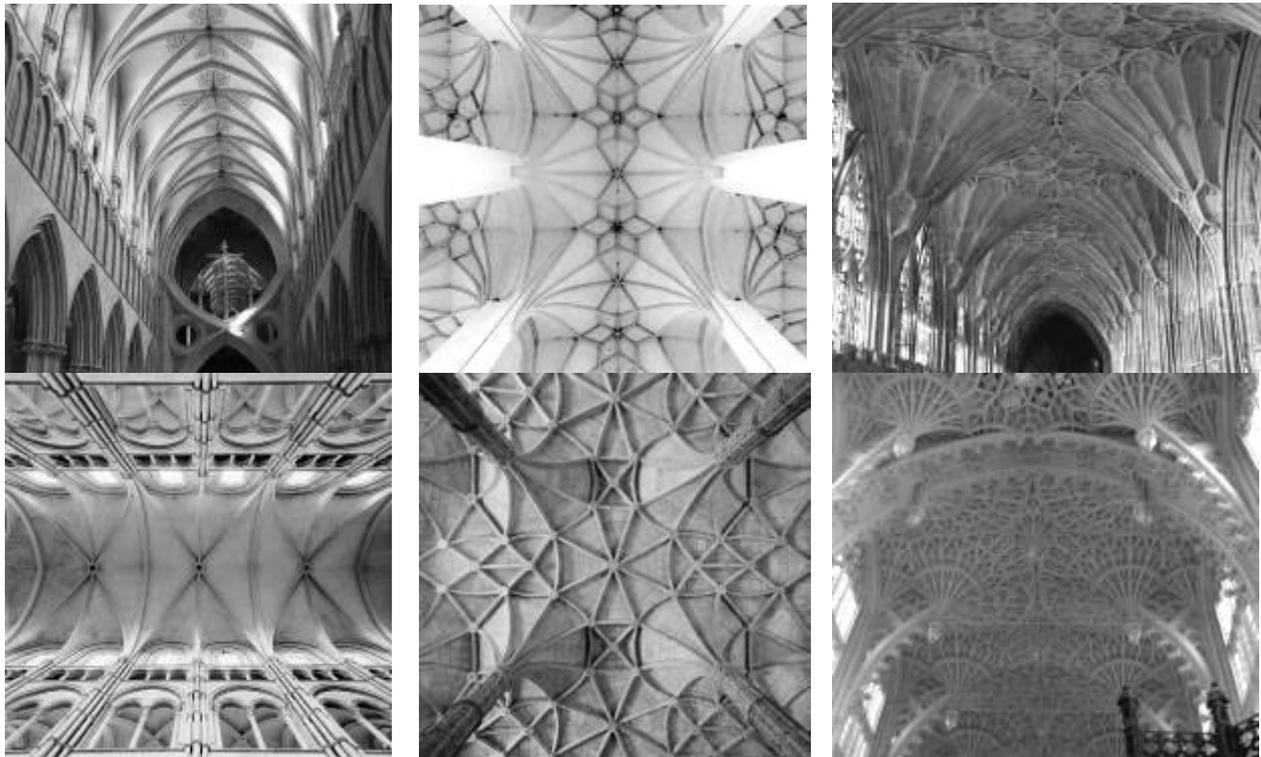


Figure 26-a: Shapes of Gothic vaults as variations of fractal trees (language of Nature): Ribbed vaulting: (above) Wales Cathedral, England and (below) Laon Cathedral, France.

Photos: author.

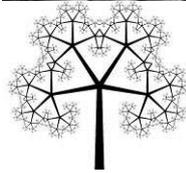


Figure 26-b: Shapes of Gothic vaults as variations of fractal trees (language of Nature): Lierne ribs: (above) Church of Hieronymite Monastery, Portugal and (below) Frauenkirche, Munich, Germany.



Figure 26-c: Shapes of Gothic vaults as variations of fractal trees (language of Nature): fan-vaulting: (above) Gloucester Cathedral, England and (below) Chapel of Henry VII, Westminster, England.

3.2.4. Results:

From the discussions above, as illustrated by **figures 17 to 26**, it is to realize that **no imaginary or working lines were needed** to grasp the fractal patterns on the plans, façades and structural elements of these buildings (as it was the case with the *ad quadratum* and *ad triangulum* Euclidian geometry in part 3.1), **nor are there any more symbolic indications**, but rather direct visual expression. In these examples, fractal orders and patterns were used to vividly visualize the language of Nature (its laws, shapes, and patterns) through a wide spectrum of visual expressions including: Cosmatesque or Cosmati tiles on plans; Arkhitektoiniki scaling, trceries, cusping, rose windows, etc. on façades, as well as hierarchical growth and variations of fractal branching

structures in the interiors of the vaulting systems, which allowed a visual dialogue to be established with these elements at each inch of the building. **Modular rhythm, qualities of Nature, and extensive use of geometrical progressions** were also common features demonstrated in these elements and self-evident in fractal structures in general.

4. Discussion

From the discussions, the figures, and the results of part 3.1, it had been shown that Euclidian geometry in Gothic architecture was not merely a drawing tool, but rather a high art explored by people of the highest intelligence and greatest abilities working in a period of religious intensity. The analyses and the examples in this part show that Gothic architecture developed under strict geometric rules that was based on the architect's understanding of natural forms, geometric principles and mathematical ratios that were thought to be the dominant ratios of the Universe. As such, **the viewer was not meant to be a mere observer; but rather a recipient of the force and metaphysics that was intended to be embedded in the cathedral through this geometry.** Analytical illustrations in the previous section (particularly **Figures 8, 9, 14, 24 and 26**) show that in both cases: Euclidian applications and fractal applications, geometry aimed at reproducing forms and patterns that are present in Nature, which was considered to be the **underpinning language of the Universe**. Medieval theologians believed that God spoke through these forms and it is through such forms that they should appeal to him, thus Nature became the principal book that made the *Absolute Truth* visible. So, even when they applied the abstract Euclidian geometry, the Golden Mean and the proportional roots, which they found in the proportions of living forms, governed their works.

In doing this, Gothic designers distinguished between the abstract proportional lines of plans and elevations (form language), and the ornamental patterns on walls, ceiling, openings, and pavements (pattern language). As shown in 3.1.6 and 3.2.4., only the latter was meant to be directly perceived by human beings, while the former was largely hidden from view and incomprehensible by the beholder. Not only due to their nature, as imaginary working lines, but also because in a

time, when a good many people were illiterate, it is hard to think that the proportional systems of the surds or the Golden Mean were expected to be realized by the eye of the worshipers, neither were their connotations, such as the Light-Matrix or the tree of life known to them, even though their pleasing influence must have been unconsciously sensed. Therefore, it is likely that these lines were not aimed at beauty, though they often arrived at it, but were rather related to a certain internal logic of the building or to a symbolic or metaphorical expression that aimed to address the eye that may comprehend this symbolic system with its invisible lines, complex ratios, and theological connotations; this is the eye of Heaven.

To answer the question of how these theological concepts had been conveyed to the masons, it should be emphasized here that during the Middle Ages, the monasteries and convents were the main centers of instruction, and the clergy were the guardians of treasuries of civilization. Beside teaching religion, they were the masters of science, literature, diplomacy, and all kinds of art including the art of war.⁶²

In his book, *The Gothic Enterprise: A Guide to Understanding the Medieval Cathedral*, Robert Scott explores the various factors that shaped cathedral construction process. Scott delves into historical, social, political, and theological causes and concludes that cathedrals were fundamentally a product of the religious mindset of the various social classes of that time period. Scott gives a perfect example of this in recounting Abbot Sugar's theological vision for the Abbey Church of St. Denis in France, which was the initial model of Gothic style.

At this time, the bishops and priests did not take for granted that God would inhabit any church building; they strove to make the cathedral mirror their concept of heaven as much as possible, so that it becomes entitled to God's presence. Therefore, filling the cathedral with bountiful light and constructing it according to the heavenly geometrical design were major priorities. Scott says, "In essence, new structures and forms were invented to solve problems created by theological purposes," he adds then that "Because the designers were trying to reconstruct an

⁶² Ben Crenshaw, *The Gothic Cathedral as a Product of the Religious Medieval Mind*, 2009: 12-13.

imitation of heaven according to the clerical understanding of heaven, the cathedral was a model of symmetry, perfect geometric proportions, and repeated patterns and volumes." Scott explains in some depth that since the designers and masons, who oversaw the building projects, had been educated in religious schools, they would have been well-versed in these concepts and eager to implement them in their work. ⁶³

On the other hand, the applications of Fractal geometry (part 3.2) were much more visible, comprehensible and related to human scale, erasing the difference in scale between the huge structures and the human scale and establishing a visual dialogue that externalizes the laws of Nature to the eye of the beholders. Spires and pinnacles emphasize the sense of upward thrust, while leafy crockets and other foliate patterns literalize the organic growth metaphor and reproduced the patterns of Nature in another language that was easily realized by the human eye. These notions are summarized in **Table 1**, which illustrates the different logic and contexts in which each of these two types of geometry was employed. In his study of the geometric proportioning strategies in Gothic Architecture, Robert Bork supports this idea when he demonstrates how geometric manipulation was responsible for the placement of decoration, such as string courses and crockets, as a cross-over point between the large scale and the more visible. ⁶⁴

From the results in 3.1.6 and 3.2.4, one can conclude that three basic elements were the bridges that linked these two types of geometry to each other: (a) modular order, (b) the language of Nature, and (c) geometrical progressions.

Context	Euclidian geometry	Fractal geometry
Language	Form language	Pattern Language
Terminologies	Proportional ratios of Nature	Floral, foliage and ornamental details
Dialogue	Symbolic expression	Visual expression

⁶³ Robert Scott, *The Gothic Enterprise*, 2003: 166-168.

⁶⁴ Robert Bork, "Dynamic Unfolding and the Conventions of Procedure," 2014. <http://journal.eahn.org/articles/10.5334/ah.bq>

Setting	The overall structure of the building: plans, elevations, structural elements on a huge scale	Ornaments and details: spires, stone carving, vaulting ribs, windows, tailings on human scale
Lines	Working imaginary lines that do not refer to walls or actual elements	Actual lines representing walls and visible architectural elements

Table 1: A comparison between the contexts in which each type of geometry was used

5. Conclusion:

The new science of complexity and Chaos theories are leading architects now into a new era of scientific design, where science will serve to create the technology, while art will provide the spirit. This paper shows that the pleasant atmosphere of Gothic cathedrals was not an outcome of a mere design process that aimed at creating beautiful buildings, but rather at reproducing the geometry and proportions of Nature, which theologians and master masons thought to be the divine language of the Universe. In these buildings Euclidian geometry was used to symbolize the cosmic order of the Universe for the eye of Heaven, whereas Fractal geometry was used to visualize this order for the eye of the worshipers.

As seen in the analysis and results in part 3, the overall geometry of the building was based on Euclidian geometry, where a mesh of proportional and working lines were not meant to be built into walls or actual element, but acted like reinforcing steel-mesh within the concrete slabs of a modern building. These lines were neither visible, as they were not actually built into walls or structural elements. They were not comprehensible to the normal viewer, as they reflected complex geometrical ratios. Nonetheless, they determined where visible elements, such as outer walls, piers, ornaments, were placed. The symbolic connotations of these ratios, which were always related in a way or another to cosmological issues, were also unknown out of the circles of theologians and clergymen. Because of this, it is likely that the purpose of this geometrical structure was essentially to sanctify the building by integrating it geometrically with divine geometry; thereby making the building receptive to divine grace and preserving it from the destructive forces of evil. That this geometry produces proportions that were pleasing to the human eye is due to the inherent integrity

of the Euclidean geometry and that the proportions of Nature are fundamental to the sensation of beauty. This mesh of reinforcement was then enclosed by a beautifully ornamented envelop of Fractal geometry that visualized this language of Nature in a glorious variety of visual expressions.

From the results in 3.1.6 and 3.2.4, it is also to conclude that there are three basic design strategies that worked as the bridges, by which these two kinds of geometry were thought to become integrated with each other (and with that of the Heaven); these are: (a) modular order, (b) the language of Nature, and (c) geometrical progressions:

(a) *Geometrical Progression*, which sets up a geometry that extends outward to the limits of the Heavens and inward to the smallest particle, but never vanishes. This characteristic is self evidently built-in in fractal patterns. Through its system of scaling down, it brought these huge buildings down to the size of people. On the other hand, the discussions in part 3.1 and the illustrations in **Figure 3** and **Figures 8–14** show the extensive use of such progressions in the Euclidian *ad quadratum* and the *ad triangulum* geometries of these buildings.

(b) It is important here to also refer to the module as main organizational element that was used by Gothic designers, who worked "almost exclusively in modular fashion."⁶⁵ The geometry governing the architecture created with the aid of modules remains Euclidean, but they were so organized to allow a fractal organization/hierarchy in these buildings.

(c) The tendency to follow the laws of Nature's language and copy its geometry was another linking bridge; the **unintentional adoption** of fractal repetition is evidence of this fact. It was believed that God spoke through these forms and it was through these forms that people should appeal to him. Architects, who worked within this tradition, may well have been aware of this awe-inspiring concept, though none of them would ever have read one word of Euclid, neither did they know

⁶⁵ Hecht, 1979.

anything about the recently discovered principles of Fractal geometry. Thus, it is likely to refer here to an "unintentional" use of it that was adopted from what was seen as the **geometry of Nature**.

From the results in 3.1.6 and 3.2.4 and the discussion in part 4, as summarized in table 1, the paper concludes that the language of Nature was interpreted in Gothic Cathedrals into two types of architectural expression, symbolic language of form and visual language of patterns. Euclidian geometry was used to compose a form language, using invisible proportional lines that were meant to address the eye of the Heaven through its symbolic expression, while fractal patterns were used to reproduce the principles of Nature in a more visual expression that was fully integrated into the overall Euclidian structure via geometrical progressions, modular order, and the tendency to express the language of Nature.

This visual dialogue, that directly addresses the human's fundamental sensation of beauty, certainly played a role in endowing Gothic cathedrals with this pleasant effect and helped make their huge dimensions more acceptable to human scale. It is also still able to inspire contemporary architects an innovative approach to insert their own legacy and metaphors into their buildings through science and geometry. 🐼

Abstract: As new trends in architecture tend to depend upon scientific rules rather than stylistic dictates, they tend to pursue to duplicate the positive, pleasant feelings of the great historical buildings, without copying, neither their form nor their style. Geometry, which was traditionally considered the sacred language of Nature ever since the classical times, played an important role in these styles. This paper aims at analyzing the geometric principles of Gothic architecture from the perspective of the originators, where geometry, with its related metaphysics, acted as the vocabulary underpinning both form and pattern languages. The paper tries to find the context and the settings, in which two different kinds of geometry were employed in this style; Euclidian geometry, as form language, and Fractal geometry, as pattern language, as well as find the bridge(s) that linked them. In this, it comes to a conclusion that the Gothic cathedral, with its unlimited scale, yet very detailed, structure, was an externalization of a dual language that was meant to address human cognition through its details, while at the same time send appeals of protection to Heaven through the overall structure, using what was thought to be the language of the Universe; geometry.

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