Optimization of Sliding Mode Control using Particle Swarm Algorithm for an Electro-hydraulic Actuator System

C. C. Soon¹, R. Ghazali¹, H. I. Jaafar¹, S. Y. S. Hussien¹, S. M. Rozali², M. Z. A. Rashid¹

¹Faculty of Electrical Engineering, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia.
²Faculty of Engineering Technology, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia.
rozaimi.ghazali@utem.edu.my

Abstract—The dynamic parts of electro-hydraulic actuator (EHA) system are widely applied in the industrial field for the process that exposed to the motion control. In order to achieve accurate motion produced by these dynamic parts, an appropriate controller will be needed. However, the EHA system is well known to be nonlinear in nature. A great challenge is carried out in the EHA system modelling and the controller development due to its nonlinear characteristic and system complexity. An appropriate controller with proper controller parameters will be needed in order to maintain or enhance the performance of the utilized controller. This paper presents the optimization on the variables of sliding mode control (SMC) by using Particle Swarm Optimization (PSO) algorithm. The control scheme is established from the derived dynamic equation which stability is proven through Lyapunov theorem. From the obtained simulation results, it can be clearly inferred that the SMC controller variables tuning through PSO algorithm performed better compared with the conventional proportional-integral-derivative (PID) controller.

Index Terms—Electro-Hydraulic Actuator System; Particle Swarm Optimization; PID Controller; Positioning Tracking; Sliding Mode Control.

I. INTRODUCTION

In the past decades, electro-hydraulic actuator (EHA) system has been widely utilized in various heavy engineering works. The advanced design of EHA system with the integration of hydraulic and multifunctional electronic components provides a massive enhancement in an application’s performance. The integration of both electronic and hydraulic equipment that absorbed both advantages have been extensively used nowadays.

Various capability such as producing large force, torques, and high energy density making EHA system become widely used in aircraft [1], robotic [2], fatigue testing system [3], manufacturing [4, 5], and automotive application [6]. The dynamic sources such as electric motor or engine drives provide hydraulic pump dynamics that deliver fluid under pressure. The fluid delivered through pressure is used to create the necessary movements in linear or rotary.

However, the dynamic features of the EHA system is known to be highly nonlinear in nature and the existing nonlinearities and uncertainties yield to the constraint in the control of EHA system. Such characteristics appeared in the system degrade its performance significantly. These disturbances simultaneously influence the position tracking accuracy and commonly affected by the occurrences of leakage, friction and uncertainties in the system [7].

Various control approaches have been reported and proposed in the literature to overcome the problems in the control of EHA system. The raised numbers of works conducted with EHA system have been proposed over the past decades ranged from linear control, nonlinear control to intelligent control strategies such as generalized predictive control (GPC) [8], model reference adaptive control (MRAC) [9, 10], sliding mode control (SMC) [11, 12], self-tuning Fuzzy proportional-integral-derivative (PID) [13, 14], and neural network (NN) [15]. In the literature study of [11], SMC nonlinear control strategy is found to be efficient and widely applied to the nonlinear EHA system.

The SMC nonlinear control is verified to have a capability to maintain the stability in the control of various different classes of model which are exposed to the disturbances and variations in the system parameters [16]. Due to its capability, SMC has been extensively implemented into various engineering applications such as active suspension system [17], pneumatic systems [18] and active magnetic bearing systems [19]. In the literature of [20], the sliding surface has been improved by adding integral action which is simultaneously enhanced the tracking performance of the SMC control on the electromechanical plant.

In this paper, the analysis for the work done in [21] has been carried on. SMC control that is optimized through a particle swarm algorithm (PSO) has been implemented for the purpose of enhancing the trajectory tracking of an EHA system which initial value of SMC controller is taken from the trial and error tuning technique in [22]. The hydraulic actuator and servo valve integrating with the nonlinear dynamics model is derived. Afterward, the SMC control scheme is implemented to the system which the Lyapunov theorem has been used to verify its stability condition.
Subsequently, the controller tracking performance is compared with the conventional PID controller to demonstrate the significant improvement of the controller through the proposed Ziegler-Nichols (ZN) and PSO tuning technique.

The paper is composed where, Section 2 illustrates the mathematical modelling of the developed system. The SMC algorithm derivation is demonstrated in Section 3. Then, the discussion on PSO algorithm is presented in section 4. An observation results are discussed, compared, and presented in Section 5. Finally, conclusion and summary of the observation are drawn in Section 6.

II. EHA System Modelling

To produce a mechanical motion of the spool valve, the electrical current is supplied to the coil that connected to the servo valve. The torque of the motor that received the power source will drive the servo spool valve to the desired position. The configuration of typical EHA system consists of computer control unit, hydraulic actuator and servo valve as demonstrated in Figure 1.

Servo valve that is connected to the hydraulic cylinder through the pipeline formed the dynamic equation of EHA system. The oil flow in the cylinder chamber will be regulated by the servo valve and generating cylinder actuator displacement. The counter force against cylinder actuator generated from the spring and damper that attaches to the mass.

The dynamics of the servo valve are represented by a second order differential equation that related to electric current drive from the torque motor as expressed in Equation (1).

\[
\frac{d^2 x_v}{dt^2} + 2 \xi \omega_n \frac{dx_v}{dt} + \omega_n^2 = I \omega_n^2
\]  (1)

where \(\xi\) is a ratio of the damping, while the natural frequency of servo valve represented in \(\omega_n\).

In the mechanical design of each port in servo valve, the spool valve is un-exposed to the dead-zone and flow leakages problems. Critical centred on the spool valve is considered. The flow \(Q\) in each of the chambers controlled by servo valve can be modelled from the equation relates to the orifice of spool valve displacement \(x_v\) and pressure difference \(P_v\). The ideal orifice equation is written in (2).

\[ Q = K_{v} \sqrt{\Delta P_v} \]  (2)

Pressure for each chamber is obtained by defining the connection between bulk modulus, volume, and flow rate as expressed in (3) and (4).

\[
P_1 = \frac{\beta}{V_{line} + A_p (x_v + x_p)} \int (Q_1 - q_1 - q_{12} - q_{11} - \frac{dV_1}{dt}) dt
\]  (3)

\[
P_2 = \frac{\beta}{V_{line} + A_p (x_v - x_p)} \int (\frac{dV_2}{dt} - Q_2 - q_{21} - q_{22}) dt
\]  (4)

Through the overall dynamics equation of moving mass, damper, and spring, the total force produced from hydraulic actuator can be obtained in (5).

\[ F_p = A_p (P_1 - P_2) = M_p \frac{d^2 x_p}{dt^2} + B_p \frac{dx_p}{dt} + K_s x_p + F_f
\]  (5)
In a simulation study, the parameters used in a nonlinear model of EHA system have been tabulated in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Resistance of servo-valve coil</td>
<td>100 Ω</td>
</tr>
<tr>
<td>Ls</td>
<td>Inductance of Servo-valve coil</td>
<td>0.59 H</td>
</tr>
<tr>
<td>Is</td>
<td>Torque motor saturation current</td>
<td>0.02 A</td>
</tr>
<tr>
<td>ξ</td>
<td>Servo-valve damping ratio</td>
<td>0.48</td>
</tr>
<tr>
<td>ω</td>
<td>Servo-valve natural frequency</td>
<td>543 rad/s</td>
</tr>
<tr>
<td>K</td>
<td>Servo-valve natural pressure</td>
<td>2.38x10^{10} m^2/kg</td>
</tr>
<tr>
<td>β</td>
<td>Hydraulic fluid bulk modulus</td>
<td>1.4x10^{7} N/m²</td>
</tr>
<tr>
<td>P</td>
<td>Pump pressure</td>
<td>2.1x10^7 Pa</td>
</tr>
<tr>
<td>Pp</td>
<td>Return pressure</td>
<td>0 Pa</td>
</tr>
<tr>
<td>Ks</td>
<td>Spring stiffness</td>
<td>10 Nm</td>
</tr>
<tr>
<td>Xs</td>
<td>Total actuator displacement</td>
<td>0.1 m</td>
</tr>
<tr>
<td>A</td>
<td>Piston area</td>
<td>645x10^{-6} m²</td>
</tr>
<tr>
<td>M</td>
<td>Total mass</td>
<td>9 kg</td>
</tr>
<tr>
<td>B</td>
<td>Damping coefficient</td>
<td>2000 Ns/m</td>
</tr>
</tbody>
</table>

### III. THE DESIGN OF SLIDING MODE CONTROL

In the early of 60’s, SMC has been introduced which fundamental concept is extracted from variable structure control developed in Russia [16]. The most pivotal stage in the establishment of SMC control is the structure of sliding surface which is anticipated to be response to the desired control criterion. The control signal that is reached to the sliding surface is expected to be stay on the surface and slide to the origin point which is the desired position as depicted in Figure 2.

![Figure 2: The structure in the sliding mode control design](image)

In general, the sliding surface of SMC control can be formed according to the Equation (6).

\[
s(t) = \left( \lambda + \frac{d}{dt} \right)^{n-1} e(t)
\]

where \( n \) denotes the system model order to be controlled. The sliding surface for the SMC design implemented into third-order EHA system can be indicated by using the following equation:

\[
s(t) = \ddot{e}(t) + 2\dot{\lambda}\dot{e}(t) + \lambda^2 e(t)
\]

The difference between the actual actuator position and the desired trajectory yielding the tracking error as expressed in Equation (8).

\[
e(t) = x(t) - x_p(t)
\]

The third derivative of the error which is coming from the third-order model obtained through the linearized EHA system is expressed in the following equation:

\[
\dddot{e}(t) = \dddot{x}_p(t) - \dddot{x}_p(t)
\]

The general expression of SMC control structure consists of switching control and equivalent control as denoted in equation (10). Where the switching control, \( u_{sw} \), corresponding to the reaching phase when \( s(t) \neq 0 \). While the equivalent control \( u_{eq} \) corresponding to the sliding phase when \( s(t) = 0 \).

\[
u_{sw}(t) = u_{eq}(t) + u_{sw}(t)
\]

When \( \dot{s}(t) = 0 \), the equivalent control can be obtained.

Where, the derivative of the sliding surface is presented in Equation (11).

\[
\dddot{s}(t) = \dddot{e}(t) + 2\dot{\lambda}\dot{e}(t) + \lambda^{2} \dot{e}(t)
\]

Assume that the lumped uncertainty is neglected (L=0), and \( \dot{s}(t) = 0 \), the equivalent control of the SMC control can be defined as an equation below by substituting Equation (9) into (11):

\[
u_{eq}(t) = \frac{1}{C} \left( \dddot{x}_p + A_p\dddot{x}_p + B_p\dot{x}_p + 2\dot{\lambda}\dot{e}(t) + \lambda^2 \dot{e}(t) \right)
\]

By applying the sign function to the sliding surface, the switching control can be determined as:

\[
u_{sw}(t) = k_s \text{sign}(s)
\]

where \( k_s \) is a constant with positive value and \( \text{sign}(s) \) represents the signum function which have a piecewise function as below:

\[
\text{sign}(s(t)) = \begin{cases} 
1 & ; s(t) > 0 \\
0 & ; s(t) = 0 \\
-1 & ; s(t) < 0 
\end{cases}
\]

An outstanding stability analysis which is Lyapunov theorem as utilized in [20, 24-28] has been used to verify the...
stability of the controller through the following Lyapunov function:

\[ V(t) = \frac{1}{2} s^2(t) \]  \hspace{1cm} (15)

where \( V(t) > 0 \) and \( V(0) = 0 \) for \( s(t) \neq 0 \). The reaching condition as presented in equation (16) is necessary to be followed to ensure the trajectory moving from reaching phase to the sliding phase in the stable condition.

\[ \dot{V}(t) = s(t)\dot{s}(t) < 0, \text{ For } s(t) \neq 0 \]  \hspace{1cm} (16)

Substitute (9), (10) and (11) into (16) yielding:

\[ s(t)\dot{s}(t) = s(\tilde{x}(t)) + A_n\tilde{x}_p(t) + B_n\dot{x}_p(t) - \cdots \]
\[ -C_n(u_{sw}(t) + u_{sv}(t)) + 2\lambda \dot{e}(t) + \lambda^2 e(t) \]  \hspace{1cm} (17)

Then,

\[ s(t)\dot{s}(t) = s(t)[-C_n(u_{sv}(t))] \]
\[ = -s(t)C_nK_s \mathrm{sgn}(s(t)) \]
\[ = -C_nK_s|s(t)| < 0, \]  \hspace{1cm} (18)

where \( C_n, K_s \in \mathbb{R}^+ \), and \( s(t) \neq 0 \).

To ensure the stability of the switching control based on Lyapunov theorem, the chattering effect for the discontinuous function in (13) has been reduced by replace the function of hyperbolic tangent with the boundary layer of \( \varphi \) as proposed in [20, 27-28].

\[ u_{sv}(t) = k_s \tanh\left(\frac{s}{\varphi}\right) \]  \hspace{1cm} (19)

IV. PARTICLE SWARM OPTIMIZATION ALGORITHM

The inspiration of the PSO algorithm is came from the swarming behaviour in a swarm of bees, a flock of birds, or a school of fish [29]. PSO is a popular optimization algorithm since it is applicable to various types of application which is a population based optimization tool. By solving the continuous nonlinear problems, this method is found to be outperformed by simulating the simplified social system [30].

In the development of PSO technique, the XY coordinates within a two dimensional search space will be cross by each particle or other word know as an agent. The new position of each particle will be realized by the current velocity and position information [31]. The XY position and the best value obtained will be saved by each particle. The saved data are then compared to the personal experience information of each particle and formed a personal best value. Among the personal best value saved in each iteration, global best value will be concluded. The information such as the distance between the current position of each agent and its personal best position, the distance between the current position of each agent and its global best position, and the current velocity of each particle will be used to moving forward to their new position [30].

Each particle changes their position according to (20) and (21) respectively [31].

\[ v_i^{k+1} = wv_i^k + c_1 \text{rand}_{i} (p_{best_i} - s_i^k) + \cdots \]
\[ + c_2 \text{rand}_{i} (g_{best} - s_i^k) \]
\[ s_i^{k+1} = s_i^k + v_i^{k+1} \]  \hspace{1cm} (20)

where, \( w \) is the inertia weighting coefficient, \( v_i^k \) denotes the current velocity for agent \( i \) at iteration \( k \), \( v_i^{k+1} \) is a new velocity for agent \( i \) at iteration \( k+1 \), \( c_1 \) is an acceleration for cognitive or the particle itself, \( c_2 \) represent an acceleration for social or the entire swarm, \( \text{rand}_{i} \) denotes a random value between 0 and 1 for current iteration, \( p_{best} \) represent personal best value for agent \( i \), \( g_{best} \) is a global best value for agent \( i \), \( s_i^k \) indicates a current position of agent \( i \) at iteration \( k \), and \( s_i^{k+1} \) represent the position of agent \( i \) at the iteration \( k+1 \).

Simply to say that, the PSO process was done in the alteration of searching point according to the concept as illustrated in Figure 3. The \( s_i^k \) and \( s_i^{k+1} \) is the current and future searching point, \( v_i^k \) and \( v_i^{k+1} \) denotes the current and future velocity, while the \( V_{best} \) and \( V_{gbest} \) is the velocity based upon personal best and global best respectively.

![Figure 3: The concept of searching point alteration for PSO technique](image)

In this paper, the implementation of PSO is using the following parameters included, the number of particles is 10, the number of maximum iterations is 20, the dimension of the problem is 3 and 2 for PID and SMC controller respectively, the speed of convergence or known as the inertia weighting coefficient is set to be linearly decreased from 1.2 to 0.9 as proposed by the researcher in [32], and the acceleration coefficient for both \( c_1 \) and \( c_2 \) is set to 2. Integral-time-square-error (ITAE) will be used as an objective function that used to calculate the minimum error produced in each iteration.
V. RESULTS AND DISCUSSION

Simulation has been done by using MATLAB/Simulink 2013 software. For the assessment of the controller response, step input reference signal has been fed into the EHA system. The PID controller variables were first tuned by using the conventional ZN tuning method. Followed by the optimization process on the variables of $K_p$, $K_i$, and $K_d$ based on particle swarm optimization technique. Procedure has been repeated and employed to the SMC controller which initial value was taken as emphasized in the introduction section. From the simulation results as demonstrated in Figure 4, it can be seen that the response produced by the initial value produced poor performances as indicated in the red and cyan dotted line. Theoretically the high overshoot (red dotted line), and slow settling time (cyan dotted line) is an unwanted controller phenomenon applied to any engineering system. These phenomena can be occurred which is caused by the factor for instance imprecise tuning technique is applied to obtained the controller parameter. Thus, particle swarm optimization has been utilized to obtain better controller variable. As indicated in the green and purple dotted line, the overshoot of the PID controller and the slow settling time of SMC controller has been significantly improved when the optimized controller variables has been employed to the controller.

To produce much clearer view in the response of the control signal and tuning technique, the transient response analysis has been tabulated in Table 2 below. It can be seen through Table 2, the SMC control that is optimized through PSO technique produced average value in rise time and percentage overshoot, while fastest settling time and smallest steady-state error. Thus, it can be concluding that SMC optimized through PSO algorithm generated the best performances.

A set of controller variable value for PID and SMC controller tuned by using different technique has been tabulated in Table 3 below.

![Positioning Tracking (Xp = 0.03m)](#)

![Figure 4: The output response for PID and SMC control](#)

Table 2

<table>
<thead>
<tr>
<th>Controller (Tuning Technique)</th>
<th>Rise Time (s)</th>
<th>Overshoot (%)</th>
<th>Settling Time (s)</th>
<th>Steady-state Error (0.5 s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID (ZN)</td>
<td>0.0174</td>
<td>20.63</td>
<td>0.1457</td>
<td>$1.4951 \times 10^{-6}$</td>
</tr>
<tr>
<td>PID (PSO)</td>
<td>0.0188</td>
<td>5.37</td>
<td>0.1039</td>
<td>$3.2806 \times 10^{-7}$</td>
</tr>
<tr>
<td>SMC (Previous)</td>
<td>0.0409</td>
<td>0.00</td>
<td>0.1216</td>
<td>$8.3743 \times 10^{-13}$</td>
</tr>
<tr>
<td>SMC (PSO)</td>
<td>0.0255</td>
<td>1.60</td>
<td>0.0936</td>
<td>$3.5215 \times 10^{-15}$</td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Tuning Technique</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>Tuning Technique</th>
<th>$\lambda$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZN</td>
<td>1020</td>
<td>0.0150</td>
<td>0.0038</td>
<td>Previous</td>
<td>100</td>
<td>1075</td>
</tr>
<tr>
<td>PSO</td>
<td>522.2226</td>
<td>0.3180</td>
<td>0.9362</td>
<td>PSO</td>
<td>164.8521</td>
<td>739.3284</td>
</tr>
</tbody>
</table>
VI. CONCLUSION

Mathematical modelling of the EHA system has been derived and implemented in the simulation study. Then, the PSO optimized SMC control scheme is implemented to the system which the Lyapunov theorem has been used to verify its stability condition. The controller tracking performance is compared with the conventional PID controller to demonstrate the significant improvement of the controller through the proposed technique. The numerical simulations clearly inferred that the proposed PSO optimization technique was able to obtain better controller variables that leads to a better system response.

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