IMPLEMENTATION OF A COMPREHENSIVE MATHEMATICAL MODEL FOR TILT-ROTOR REAL-TIME FLIGHT SIMULATION

Federico Barra, Simone Godio, Giorgio Guglieri
federico.barra@polito.it, simone.godio@studenti.polito.it, giorgio.guglieri@polito.it
Department of Mechanical and Aerospace Engineering
Politecnico di Torino (Italy)

Pierluigi Capone, Raphael Monstein
pierluigi.capone@zhaw.ch, raphael.monstein@zhaw.ch
ZAV Centre for Aviation*
ZHAW† Zurich University of Applied Sciences (Switzerland)

Abstract

This paper aims at describing the effort performed by the joint research group of Politecnico di Torino and ZHAW (Zurich University of Applied Sciences) in achieving a novel implementation of a mathematical model for real-time flight simulation of tilt-rotors and tilt-wings aircraft. The focus is on the description of the current stage of the project, the achievements of the first version of the model, on-going improvements and future developments.
The first part of the work describes the initial development of the overall simulation model: relying on several NASA reports on the Generic Tilt Rotor Simulator (GTRS), the mathematical model is revised and the rotor dynamic model is improved in order to enhance computational performance. In particular, the model uses the conventional mathematical formulation for non-dynamic inflow modelling based on Blade Element Momentum Theory. A novel but simple numerical method is used to ensure the convergence of the non-linear equation in every tested condition. The resulting simulation model and its development and implementation in the MATLAB/Simulink® environment is described.
The second part of the work deals with the integration of the model in the ZHAW Research and Didactics Simulator (ReDSim), the replacement of the pilot controls by the introduction of a center stick and the corresponding adjustment of the force-feel system to suitable values for the tilt-rotor model. Subsequently, several pilot tests are carried out and preliminary feedbacks about the overall behaviour of the system are collected. Limits and weaknesses of the first release of the model are investigated and future necessary improvements are assessed, such as the development of a novel generic prop-rotor mathematical model.
The third part introduces the novel multi-purpose rotor mathematical model which was developed to improve the overall tilt-rotor simulation model. The multi-purpose rotor model implements non-approximated flapping dynamics and inflow dynamic based on Pitt-Peters formulation. The validation of the nover rotor model is carried out with available data of both the XV-15 Research Aircraft and the UH-60 Helicopter.

1. INTRODUCTION

Tilt-rotor is a concept which draws particular interest in the world of Vertical/Short Take Off and Landing vehicles (V/STOL): a tilt-rotor can convert its nacelles within a range of 95 degrees, from the helicopter mode (which allows the vertical take-off) to the airplane mode and therefore providing enhanced vehicle's performance in terms of speed, endurance and fuel consumption. Despite being far from mature, the tilt-rotor does present a high potential among commercial aircraft, military platforms as well as UAV/drone application. Currently, several attempts exist of developing innovative, rotary wing concepts for the future air taxi market and among them, tilt-rotor as well as tilt-wing platforms are rather recurrent [30, 26, 28]. However, such innovative, unconventional platforms indeed require suitable flight simulation models able to faithfully describe their peculiarities, such as the considerable wing download caused by the flow stream of the rotors in helicopter mode, the tilting mechanism which allows the conversion of the rotors during flight, the behaviour of the wing in backwards flight as well as the coexistence of both conventional aircraft flight controls and rotorcraft ones. Although many publications are available concerning tilt-rotor aircraft

*ZAV is the official German acronym for Zentrum für Aviatik
†ZHAW is the official German acronym for Zürcher Hochschule für Angewandte Wissenschaften

(several books have also been published recently on the topic, such as [8]) and mainly related to the NASA-Bell XV-15 project or to the Bell-Boeing V-22 Osprey [9, 7], full descriptions of a comprehensive flight simulation model are limited [15, 11]. What reported by S. W. Ferguson in [11] is the latest reference available on public domain which extensively describes the mathematical model developed for the tilt-rotor flight simulation by NASA and Bell Helicopters but does not provide a full detailed derivation of several non-standard equations, such as those describing the rotor dynamics. Nevertheless, [11] provides an extensive aerodynamic database which is crucial to develop the flight simulation model. Additional references are available of later studies on tilt-rotors flight simulation models conducted by different universities, mainly focused on handling qualities and controls ([19] and [20] seem to have benefited from XV-15 published data, as well). Based on what described in these documents, thought, none of the later studies seems to have aimed for or achieved yet the development of a model suitable for piloted simulations.

Although piloted flight simulation is, to some extent, a trade-off between performance and accuracy [14], state-of-the-art computational power and model-based design tools provide room for improvements in the development of more detailed physical models to be implemented in flight simulation platforms. For instance, the direct use of MATLAB/Simulink® comes with much potential in terms of flexibility and modularity of models and allows easy modifications and new developments. As a matter of fact, intepreted and non compiled MATLAB/Simulink® models are already extensively exploited at the ZAV for airplanes, helicotpers and convertiplanes.

2. TILT-ROTOR MODEL

As already mentioned, the initial mathematical model of the tilt-rotor was mainly derived from [11], which is referred as the first release of the GTRS, a simulation code which is believed to be still in use in its updated and extensively modified version within several US programs. The implementation of the simulation code, instead, is novel and Matlab/Simulilnk® based. Additional information concerning the novel tilt-rotor simulation model can be found in [1, 2].

2.1. First version of the prop-rotor model

What reported in this section is the initial version of the rotors mathematical model as derived from [11], which was meant to improve what presented in [15] by introducing a customised formulation of the rotor hub forces as well as a first order flapping dynamics model as shown in Eq. (1). In [15] the flapping dynamic was neglected and the steady-state result for the tilt of the rotors' tip path planes were computed. The first order flapping dynamic model is derived from the simplified formulation introduced by Chen in [6] and further developed in [4], but customized terms appear for the particular case of tilt-rotors and the authors cannot trace the derivation of those terms back to any additional reference. However, the flap motion model can be expressed in compact form, for each rotor, as

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{\bar{a}}_1 \\ \dot{\bar{b}}_1 \end{Bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} \bar{a}_1 \\ \bar{b}_1 \end{Bmatrix} = \begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix} \quad (1)$$

where $\bar{a}_1$, $\bar{b}_1$ are the lateral and longitudinal cyclic angles of the ideal tip-path plane of each rotor. According to Eq. (1), the first derivative of each angle can be expressed in explicit form and integrated numerically using simple discrete Euler integrators. Since the flap dynamics was reduced and expressed in terms of disk angles by exploiting the Multi Blade Coordinate Transformation, discrete Euler integrators can be used and are numerically stable already at relatively low sampling frequencies so that the real-time simulation is allowed. The equations describing the flapping dynamics were implemented as reported in [11].

As far as the rotor forces are concerned, the formulation reported in [11] is essentially an integration of the Blade Element Theory along the blade span as proposed in [4], but once again non-standard terms appear to account for highly twisted blades such as those of prop-rotors. The inflow dynamics, instead, is reduced to the uniform, constant formulation (a rather useful insight in the different inflow models generally adopted for flight dynamics and control applications can be found in [5]) and computed resolving numerically the inflow-thrust implicit equation

$$\lambda_{i+1} = \lambda_0 + \frac{C_{T_i}\left[1-(1-G)(K_w)\right](1+X_{SS}+X_{SF})}{\sqrt{\mu^2+0.866\lambda_i^2}+\frac{0.6\sqrt{C_{T_i}^3}+|C_{T_i}|-\frac{8}{3}\lambda_i|\lambda_i|}{(|C_{T_i}|+8\mu^2)(|C_{T_i}|+8\lambda_i^2)}} \quad (2)$$

where C_T is the thrust coefficient of the rotor, G is the in-ground-effect coefficient, K_w is the washout coefficient, X_{SS} and X_{SF} account for the side-by-side and tandem effects, while λ and λ_0 are the inflow terms. Evidently, Eq. (2) recalls the conventional expression of the inflow ratio as also reported by Johnson in [18], but several semi-empirical corrections are introduced to adapt the formulation to experimental data. Eq. (2) was implemented as reported in [11] but the iterative method was modified: instead of using two nested loops to ensure the convergence of first the inflow (2) and then the thrust equation, a single loop was implemented so that the iteration is performed for both the equations at the same time until the old values of λ, C_T and the new converge with a tolerance of 10^{-6}. The algorithm representing the modified loop is shown in Fig. 1, while the original one can be found in [11].

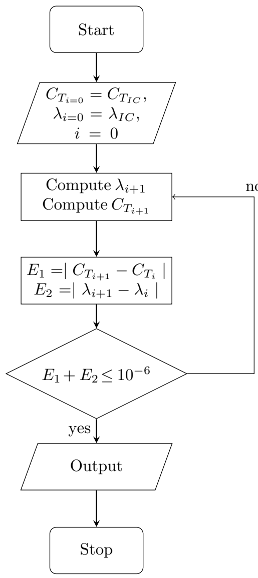

Fig. 1 Flow-chart of the Inflow-Thrust solution algorithm

2.2. Aerodynamic model

The aerodynamic model accounts for all the forces and moments generated by the fuselage, the wing-pylons and tail surfaces. The model is mainly based on the aerodynamic database of the XV-15 reported in [11]. The novel Simulink® model mainly exploits what reported in [11] with no major modification, but the aerodynamic database is modified according to [12] and then interpolated and organized in a series of multidimensional look-up tables (Fig. 2 shows the data used for the aerodynamic lift coefficient of the wing, further data can be found in [2]). The model accounts for the interaction between the wakes of both rotors and the main aerodynamic surfaces in terms of induced downwash/upwash angles as well as the download exerted on the wing. The interaction between the fuselage and the rotor, instead, is neglected based on the assumption that unlike conventional helicopters the rotors of the XV-15 are shifted from the centre-line of the fuselage and mainly affect the behaviour of the wing from both a structural and an aerodynamic point of view.

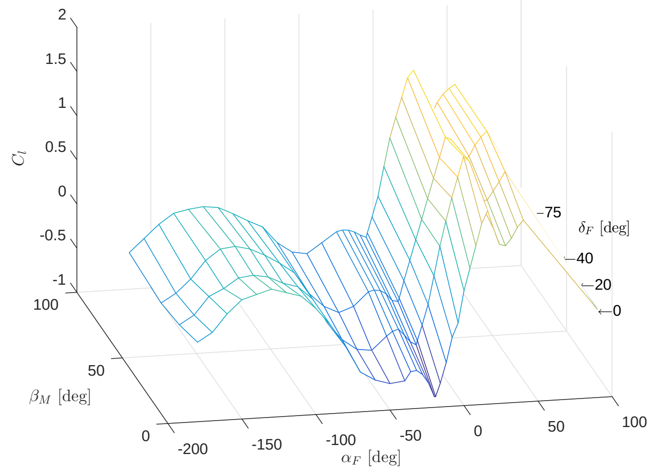

Fig. 2 XV-15 experimental data of the aerodynamic lift coefficient (C_{L_W}) of the wing as a function of the mast tilt-angle (β_M), flap deflection (δ_F) and angle of attack of the aircraft (α_F)

2.3. Rigid Body Equations of Motion

What described in [11] is the typical formulation of the equations of motion of a rigid body with six-degrees-of-freedom, expressed using Euler angles. Moments equations, though, are expressed in the implicit form, which is identical to the one provided by Etkin in [10] and not as convenient as the explicit one for the software implementation (algebraic loops may occur and additional modification shall be made, see [24]). Therefore, within the new simulation model, the explicit formulation was introduced in a form which essentially resembles the one provided by Stevens in [29].

2.4. Software architecture

The model was implemented in Matlab/Simulink® environment: a Simulink® library of the tilt-rotor model was created and can be connected to either the trim model (which is exploited by a proprietary trim algorithm to define the initial condition of the aircraft before the simulation) or the overall simulation model which interacts with the simulation platform (Displays, Visual System, and Control Loading System). What shown in Fig. 3 is the top-level view of the tilt-rotor model, which is structured in subsystems according to what described before in this section (additional information can be found in [1]).

2.5. Limits of the rotor model derived from literature

As anticipated, when testing the first release of the mathematical model, weaknesses were highlighted in some specific flight conditions in helicopter mode. Precisely,

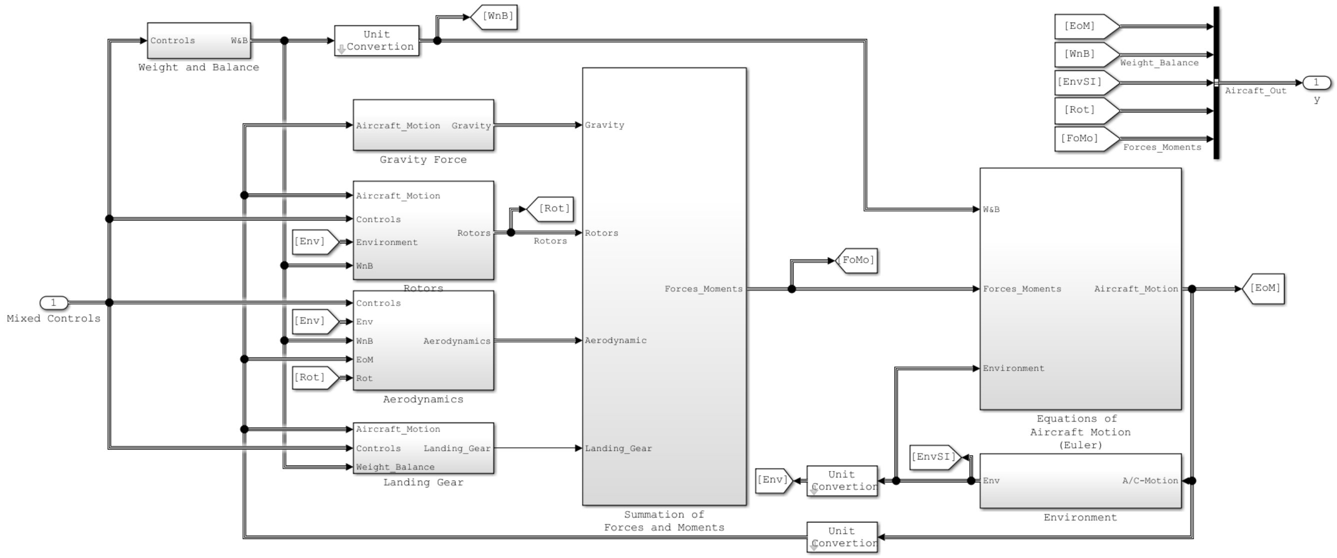

Fig. 3 Top-level view of the Simulink® Tilt-Rotor Model

a first set of simulations performed by licensed helicopter pilots showed that the model was affected by severe instabilities when trying to fly at low speed in helicopter mode with no Stability and Control Augmentation System engaged and the hover condition could not be reached. As shown in Fig. 4 the model behaves as expected when flying at higher speeds, but when the pilot tries to reduce the forward speed and reach hover, at a speed of approximately 28 kts the aircraft suddenly incur some strong instabilities which yield to loss of authority on the lateral control and eventually loss of the aircraft. What experimented during the simulation could also be highlighted with some off-line stability analysis of the model: once trimmed, the model can be linearised around the trim condition so that an equivalent state-space system and corresponding eigenvalues can be obtained according to [3]. Tab. 1 reports the most unstable pole of the system and highlights how, if reducing forward speed below 35 knots, a strongly positive, real pole compromises the stability of the system the closer to the hover condition the analysis is carried at. Conventional helicopters are generally affected by moderate unstable dynamics at low speed and hover (mainly related to the rotor and the variation of its average angle of attack, [22]), but what experienced in the simulation of the first version of the novel tilt-rotor model is considered unrealistic.

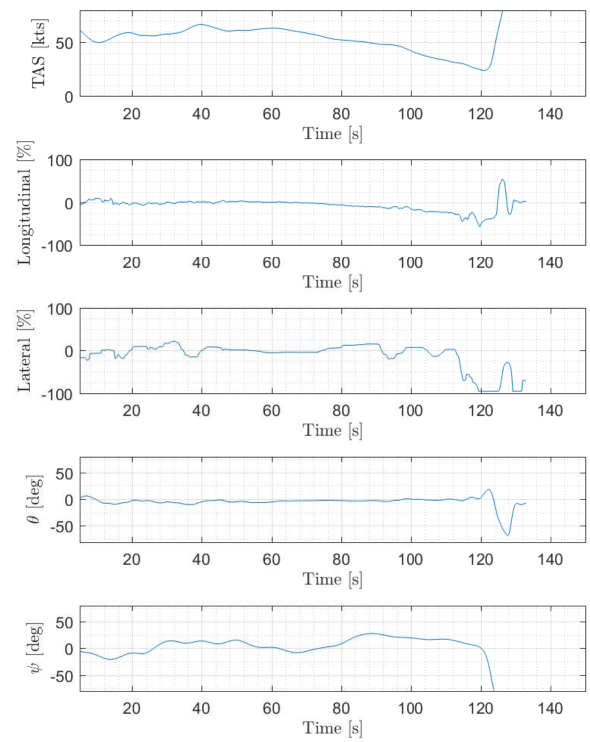

Fig. 4 Time history of a piloted simulation. Simulation starts in forward flight at 80 kts and the pilot tries to slow down to hover

V [kts]	T_2 [s]	$\Re(z)$	$\Im(z)$
5	0.89	0.78	0
7	0.96	0.72	0
10	1.07	0.65	0
12.5	1.19	0.58	0
15	1.31	0.53	0
17.5	1.43	0.48	0
20	1.56	0.44	0
22.5	1.75	0.40	0
25	1.96	0.35	0
27.5	2.14	0.32	0
30	2.51	0.28	0
32.5	3.26	0.21	0
35	12.71	0.05	0

Table 1 Most unstable pole of the system analysed at several low forward speeds. For each flight speed the table reports time-to-double, real part and imaginary part of the relative pole

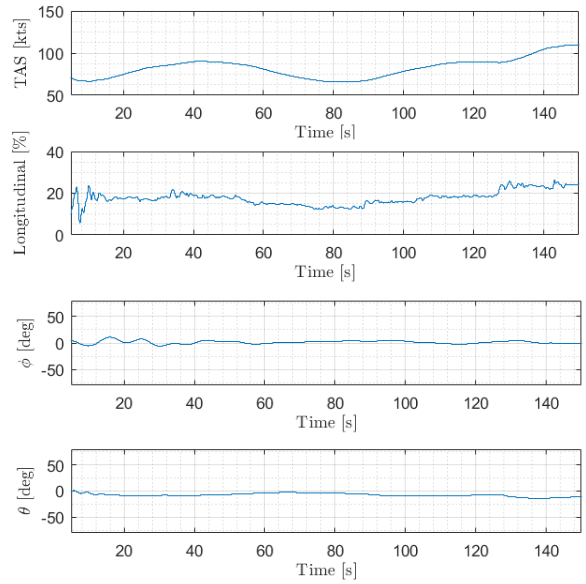

Fig. 5 Time history of a piloted simulation. Simulation starts in forward flight at 80 kts and the pilot performs a series of speed capture tasks

On the contrary, when simulating flight at higher speeds the model behaves as intended, so that a simple tasks as a series of speed captures like the one shown in fig. 5 can be performed as expected. Detailed results of the off-line stability analyses for the whole flight envelope of the aircraft are not reported here, but additional data can be found in [1] and referred to [12] for validation). Eventually, the model was also tested in aircraft mode with positive results.

Several attempts have been made to identify the source of the behaviour at low-speed/hover (which is believed to be unrealistic since [12] does not refer to similar conditions during validation). Since the source of instability seems to be located in the rotor models, the authors tend to believe the non-standard rotor model described in [11] has been either wrongly interpreted or contains some documentation mistakes. Unfortunately, the issue with this model is that the whole mathematical derivation of the equations of motion is not fully available.

Consequently, the authors decided to develop a more reliable mathematical model of the prop-rotor aiming at improving the overall simulation model of the generic tiltrotor aircraft. The choice of developing a new one seem to be in line with previous works such as [19], [20] and [21], in which the aerodynamic database and the overall aircraft model was preserved, while the rotor model was replaced with different standard versions from literature or new developments, too.

3. NEW GENERIC PROP-ROTOR MATHEMATICAL MODEL

3.1. Assumptions

What described below is the novel generic prop-rotor mathematical model developed for real-time flight simulation. For this particular reason, a trade-off is needed between the level of detail the model provides and its performance. Thus, according to [14] and previous works such as [4] and [19], the rotor dynamics was limited to the sole flapping degree of freedom, while lead-lag dynamics and torsion dynamics are neglected. The model implements the full, second-order flapping dynamics of each blade with no approximation for the reduced order tip-path-plane dynamics and a full calculation of the air-loads along the blade span as function of the azimuthal position. The resulting model is based on the following assumptions:

- air-loads are expressed by exploiting the Blade Element Theory and integrating along each blade span, so no numerical integration must be performed during the simulation, but mere algebraic calculations must be computed at each time step;
- the blade section's aerodynamics is expressed with no approximation and therefore extended to large inflow angles (which are generally typical for prop-rotors and propellers, [16]).
- the blade section's aerodynamic data are derived from conventional 2-D analysis based on equipoten-

tial methods for the linear part and then expanded for large angles of attack exploiting the semi-empirical formulation proposed by Hoerner in [17]. This method allows to properly tune the airfoil characteristics to better suit the real blade and reproduce stall and post-stall behaviours.

- each blade is considered rigid in bending and torsion, an equivalent spring is placed at the flap hinge and can be properly tuned to reproduce the behaviour of different rotor designs (rigid, fully articulated, hingeless, gimballed rotors);
- the blade twist distribution is expressed using a third-order polynomial which can be tuned to well suit prop-rotor as well as conventional rotor designs;
- the chord distribution is also introduced using a third-order polynomial so that a wider range of blade designs can be implemented;
- the approximation of small flapping angles β is used only in the mathematical formulation of the flapping dynamics. This assumption is needed to express the flapping dynamic analytically and it is believed to be acceptable based on previous literature [6] and safety limitations of tilt-rotor designs (XV-15 flapping is limited within ± 12 degrees, as reported in [11]). However, the approximation is not made when expressing the overall air-loads generated on the blades and then transferred to the rotor hub.
- as shown later in this paper, the effect of $\dot{\beta}$ on the sectional angle of attack is assumed to be small and therefore linearised to preserve the analytical formulation (theoretically it should be instead expressed by $\arctan\left(\frac{V_P}{V_T}\right)$ but then it would not be possible to isolate the terms in $\dot{\beta}$ within the flapping equation, see later Eqs. 4 and 5) and therefore simplified as $\frac{\dot{\beta}r}{\Omega(e+r)}$ by neglecting, for this term only, the effect of the hub components of velocities which might now lead to singularities in the model for blade stations close to the hub at which, in some translational flight regimes, the denominator might become zero. Nevertheless, such a contribution is preserved for all the other terms in the $\arctan\left(\frac{V_P}{V_T}\right)$;
- terms in $\ddot{\beta}$ and the Coriolis contribution due to $\dot{\beta}$ are also considered small and neglected only when calculating the accelerations acting on the blade, as in [6];
- a dynamic inflow model is implemented based on Pitt-Peters formulation [23];
- an analytical formulation of the tip loss factor is implemented according to what suggested in [25]

3.2. Mathematical model

1. Elementary Angle of Attack
According to the Blade Element Theory and referring to Fig. 6, the sectional aerodynamic angle of attack (α) is defined as

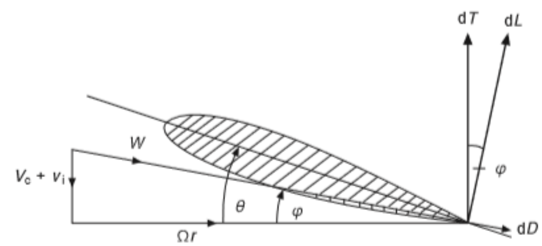

Fig. 6 Blade Element

$$\alpha = K_1\beta + \theta_0 - a1\cos(\psi) - b1\sin(\psi) + t_3r^3 + t_2r^2 + t_1r + t_0 + \arctan(\frac{V_P}{V_t}) - \alpha_0 - \frac{\dot{\beta}r}{\Omega(e+r)} \tag{3}$$

where β is the flapping angle of a single blade, a_1 and b_1 are the lateral and longitudinal cyclic inputs, K_1 is the pitch-flap coupling term, t_3, t_2, t_1, t_0 are the coefficients of the twist distribution law, α_0 is the zero-lift angle and θ_0 is the collective pitch input referred the blade root, while perpendicular and tangential velocities (V_P, V_T) are expressed as

$$\begin{aligned} V_P = & \frac{\left(q_h(e+r)R - V_{tip}r\lambda_{1c}\right)\cos(\psi)}{R} \\ & + \frac{\left(p_h(e+r)R - V_{tip}r\lambda_{1s}\right)\sin(\psi)}{R} \\ & + \frac{R(-V_{tip}\lambda_0 + w_h)}{R} \\ & + x_{uo}(a1\Omega\sin(\psi) - b1\Omega\cos(\psi)) \end{aligned} \tag{4}$$

$$V_T = \sin(\psi)u_h + \cos(\psi)v_h + \Omega(e+r) \tag{5}$$

The last term of Eq. (4) is a first-order contribution which introduces an approximated unsteady component which is connected to $\dot{\theta}$ through the offset x_{uo}, according to a formulation derived from Theodorsen's theory and reported by Johnson in [18]. Furthermore, the effect of the hub's rotational (p_h,q_h, r_h) and translational rates (u_h,v_h, w_h) are also taken into account.

2. Inflow Dynamics
In Eqs. (4) and (5) the periodic and constant terms of inflow (λ_{1_c}, λ_{1_s}, λ_0) are also introduced according to Pitt-Peters classical inflow model as shown in Eq. (6),

$$\begin{Bmatrix} \dot{v}_0 \\ \dot{v}_{1s} \\ \dot{v}_{1c} \end{Bmatrix} = -[M]^{-1}[L]^{-1} \begin{Bmatrix} v_0 \\ v_{1s} \\ v_{1c} \end{Bmatrix} + [M]^{-1} \begin{Bmatrix} C_t \\ C_{la} \\ C_{ma} \end{Bmatrix}. \quad (6)$$

Matrices $[M]$ and $[L]$ are respectively the apparent mass matrix and the static gain matrix, as introduced in [23] and reported below in eqs. (7) and (8), in which χ is the wake skew angle, while v_m and v_t are the mass flow parameters, functions of the advance ratio and static inflow coefficient. C_t, C_l and C_m are, respectively, the coefficients of thrust, rolling moment and pitching moment of the equivalent rotor disk generated by the rotation of the blades.

$$[M] = \begin{bmatrix} \frac{128}{75\pi} & 0 & 0 \\ 0 & -\frac{16}{45\pi} & 0 \\ 0 & 0 & -\frac{16}{45\pi} \end{bmatrix} \quad (7)$$

$$[L] = \begin{bmatrix} \frac{1}{2v_t} & 0 & \frac{15}{64v_m}\tan\frac{\chi}{2} \\ 0 & -\frac{4}{v_m(1+\cos\chi)} & 0 \\ \frac{15}{64v_m}\tan\frac{\chi}{2} & 0 & -\frac{4\cos\chi}{v_m(1+\cos\chi)} \end{bmatrix} \quad (8)$$

3. Elementary Airloads
According to Eq. (3), the elementary aerodynamic moment acting on the blade with respect to the blade root can be expressed as

$$dm = \frac{1}{2}\rho V^2 c(r)(C_d \sin(\phi) + C_{l\alpha}\alpha\cos(\phi))r\,dr \quad (9)$$

in which $C(r)$ is the chord distribution, C_d is the airfoil drag coefficient, r is the blade elementary station, ρ is the air density and V the overall speed acting on the blade element. What referred as C_{l_α} is not to be mistaken for the conventional lift-curve slope, but it is actually a curve expressing the derivative of the lift curve with respect to the zero-lift angle of the airfoil. This coefficient is not constant and allows to account for stall and post-stall conditions. By using such a formulation for the elementary aerodynamic angle of attack as showed in Eq. (3), numerical integration method like trapezoid (which is in some sort linear, too) can be performed symbolically without losing the linearity of the terms in β and $\dot{\beta}$, terms which can be grouped firstly in dm_1 , dm_2 , dm_3, yielding to M_1 , M_2 , M_3 after the integration:

$$dm = dm_1\dot{\beta} + dm_2\beta + dm_3 \quad (10)$$

$$M = M_1\dot{\beta} + M_2\beta + M_3 \quad (11)$$

As already mentioned, the blade elementary forces and moments are integrated along each blade using a simple trapezoidal numerical method and considering 20 blade elements which can be evenly or progressively distributed between root and tip. The same structure of Eq. (11) applies to all the aerodynamic forces and moments.

4. Flapping Dynamics
The flapping motion, is derived according to Eq. (11), yielding to

$$m\ddot{\beta} + c\dot{\beta} + k\beta = F \quad (12)$$

where

$$\begin{aligned} m &= I_\beta \\ c &= -M_1 \\ k &= K_\beta - M_2 a_{z_1} m_b r_{cg} \\ F &= M_3 - m_b a_{z_2} r_{cg} - m_b g_{hz} r_{cg} \end{aligned} \quad (13)$$

Referring to Eq. (13), I_β is the blade's flapping inertia, K_β the equivalent flapping spring, r_{cg} the distance of the flapping hinge from the centre of gravity of the aircraft, m_b is the blade mass and g_{z_h} the gravity contribution. The acceleration $a_{z_{blade}}$ along z_{blade}, instead, is collected in the form:

$$a_{z_{blade}} = a_{z_1}\beta + a_{z_2} \quad (14)$$

For the sake of brevity further details about the mathematical model are omitted in this paper but will be the subject of a forthcoming paper which is currently being prepared.

3.3. Software architecture

The new mathematical model is implemented in a Simulink® library which is then linked to the overall tiltrotor model and replaces the previous version of the rotor model as described above in this paper (see the block called Rotors, in Fig. 3). Within the so called Rotors-block, both rotors are implemented separately, then the overall forces and induced velocities on the wing and tail are computed in two separate sub-blocks as shown in the simplified flow chart in Fig. 7.

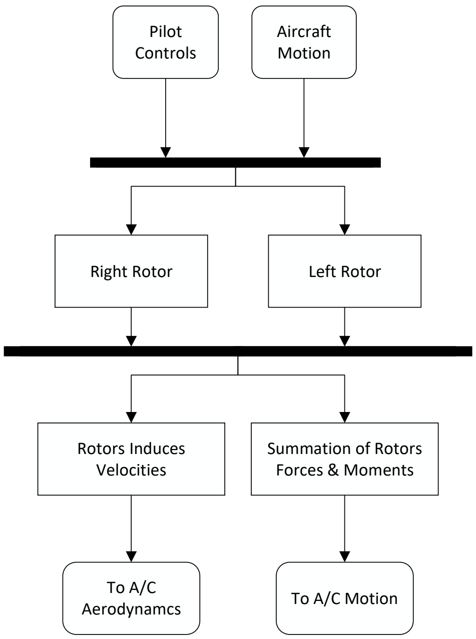

Fig. 7 Simplified flowchart of the new Rotors-Block of the Simulink® Model

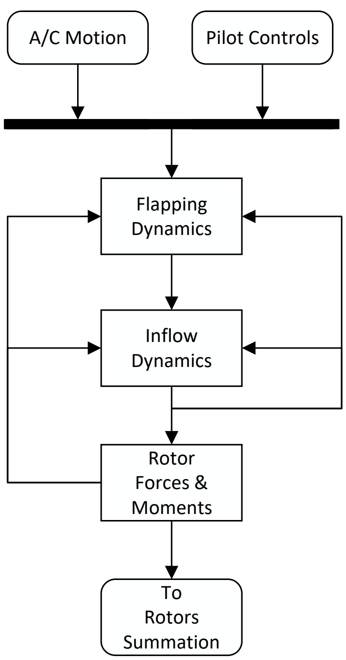

Fig. 8 Simplified flowchart of the new Rotors-Block of the Simulink® Model

The single Rotor-Block is structured as shown in Fig. 8: at each time step pilot controls and the aircraft motion are taken as inputs so the flapping dynamics can be evaluated. Thus, the inflow dynamics is computed and inflow parameters can be used to calculate all the air-loads generated by each blade and the overall forces and moments at each rotor hub. Actually, each of the blocks shown in Fig. 8 requires the values of the states computed in the other blocks, so each signal is fed back to previous block paying attention not to generate un-wanted algebraic loops.

3.4. Discrete Time integration

The model is implemented in Simulink® in discrete time domain. Unlike the previous model, which used a simplified formulation of the rotor dynamics and required a much lower computational effort, the implementation of the second order flapping dynamics in discrete domain poses some issues related to the representation of fast dynamics with relatively low damping ratio. Precisely, if Euler-discrete integrators were used as in the previous model, a sampling frequency of 10 KHz would be needed to properly reproduce the expected dynamics. Clearly, a sampling frequency of 10 KHz compromises the code computing performance preventing real-time applications.

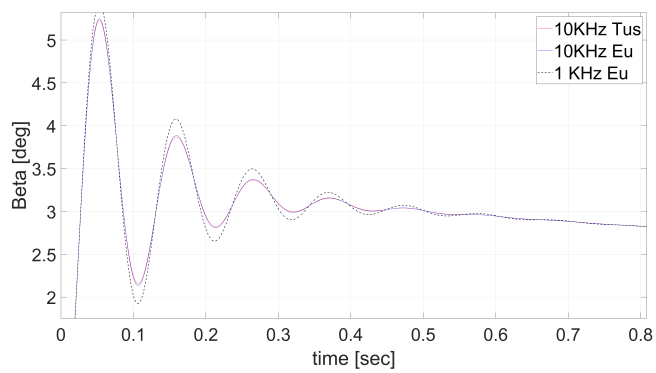

Fig. 9 Comparison between different sampling frequencies for Euler Integrators and the reference Tustin integrator (1 of 3)

To represent this dynamics correctly, Eq. (12) was modeled in Simulink® by using an ad hoc developed second order system based on Tustin integration method. This integration scheme allows quite accurate results with relatively low sampling rate. Thus, the discrete Tustin integrators were modelled according to [13] and introduced within the model with positive results: as shown in Fig. 9 and 10, what obtained by using discrete Euler integrators with a sampling frequency of 10 KHz can be reproduced with Tustin integrators running at 500 Hz and even a sampling frequency of 200 Hz can be considered acceptable.

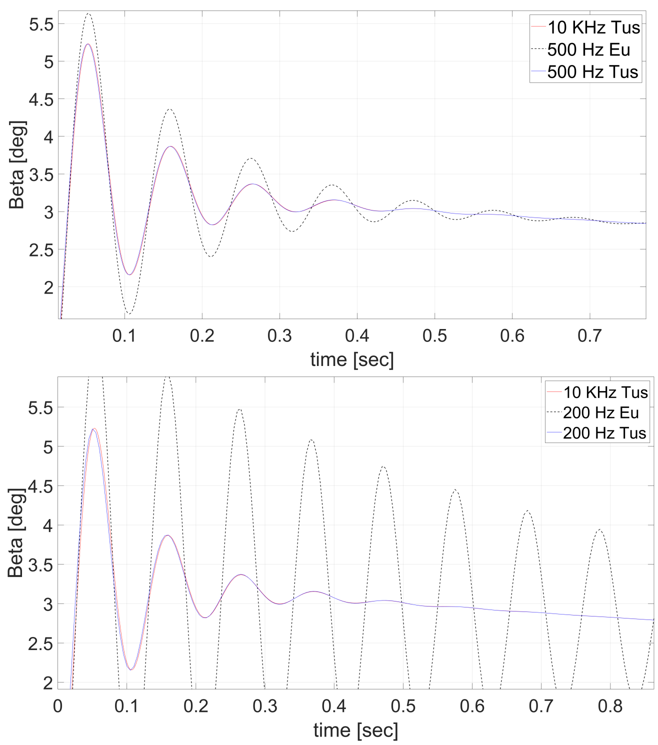

Fig. 10 Comparison between different sampling frequencies for Euler Integrators and the reference Tustin integrator (2 of 2)

4. VALIDATION OF THE GENERIC PROP-ROTOR MATHEMATICAL MODEL

A first validation of the generic prop-rotor mathematical model was carried out and results are considered satisfactory. Given the fact that the novel mathematical model is meant to be generic and therefore suitable for a wide range of rotor designs, two substantially different rotor layouts were considered. First, to account for tilt-rotor designs, the XV-15 prop-rotor was considered; second, the Sikorsky UH-60 was used as a reference for a conventional heavy utility class of helicopter design. A comparison between the results of the novel rotor mathematical model. The results of the preliminary validation seem to demonstrate the correctness of the novel model for the two deeply different configurations.

4.1. XV-15 Case Validation

In this case, the reference data set is derived from a series of experimental tests conducted at the NASA Langley Research Center on several prop-rotor designs, as reported by Harris in [16]. Precisely, the original layout of the XV-15 is considered, also in accordance with the reference data found in [11] and used for the development of the overall tilt-rotor simulation model. From the data provided in [16]), all reference conditions were set at equal Mach Number at the Blades' tip (M_{tip}), in environmental conditions referred to sea level on a standard day. As shown below, both coefficients of thrust and power predicted by the model match the reference data for the hover condition with quite a good degree of fidelity.

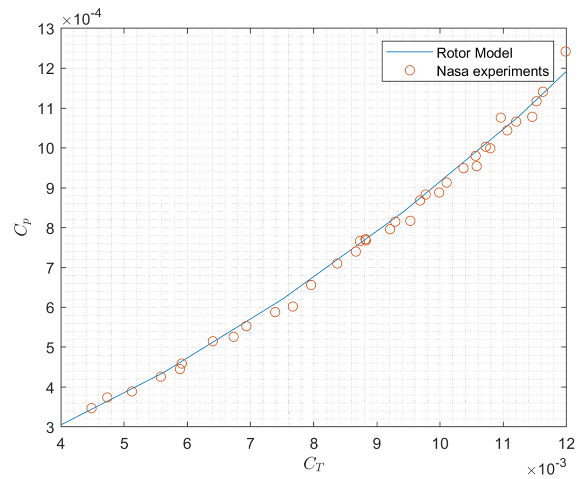

Fig. 11 XV-15 C_p - C_t trend in Hover

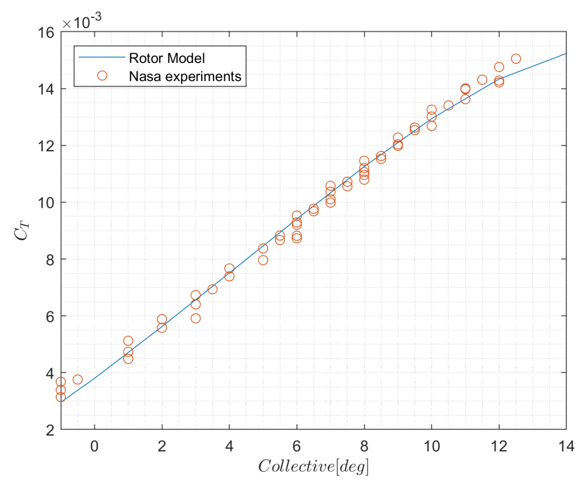

Fig. 12 XV-15 C_t - θ trend in Hover

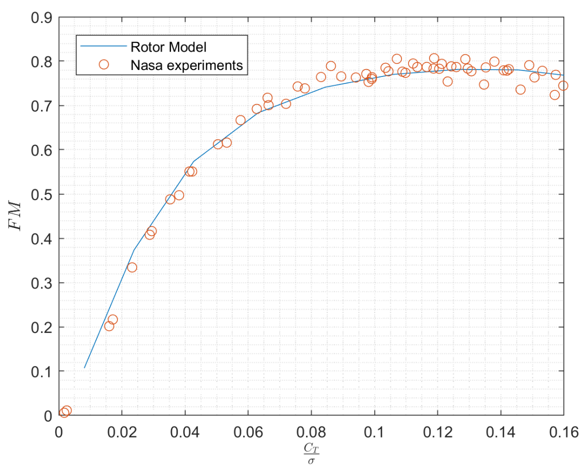

Fig. 13 XV-15 FM - C_t trend in Hover

4.2. UH-60 Case Validation

For the UH-60 case, the reference data set was derived once again from experimental tests conducted at NASA Langley Research Centre and reported in [31], all data was selected with equal M_{tip}. What reported in [31] refers to tests conducted on a scaled rotor model (due to the reduced width of the wind tunnel) so all geometric and environmental factors were adapted to ensure the Reynolds numbers similarity and the validity of the comparison performed. As shown in Figs. 14, 15 and 16 the novel mathematical model seems to well predict the rotor's general performance. Instead (Fig. 17), at higher advance ratios the model seems to overestimate the power coefficient. This will be the subject of further investigation in the near future.

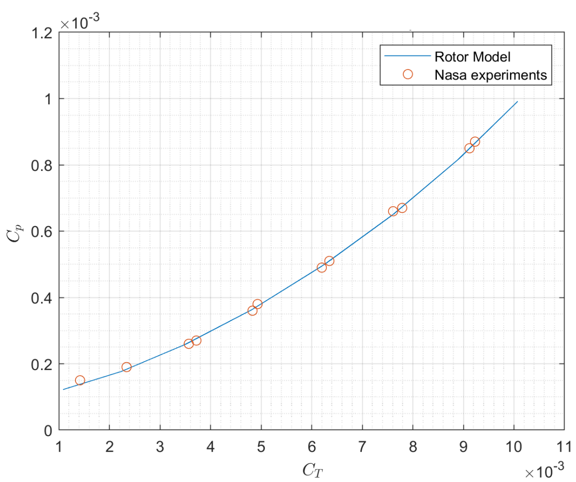

Fig. 14 UH-60 C_p - C_t trend in Hover

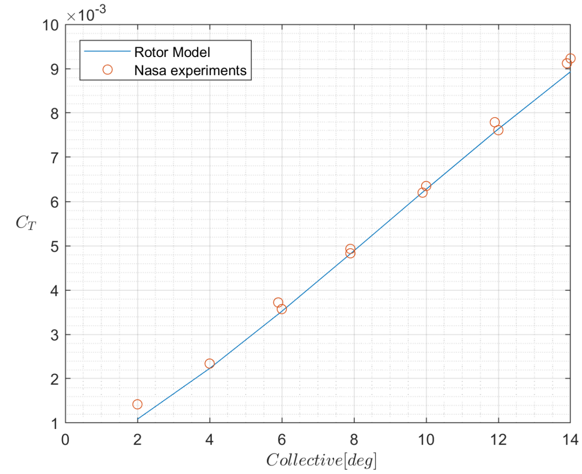

Fig. 15 UH-60 C_T - θ trend in Hover

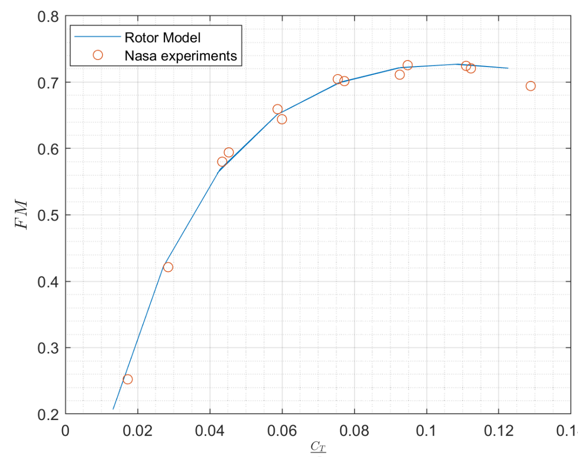

Fig. 16 UH-60 FM - C_T trend in Hover

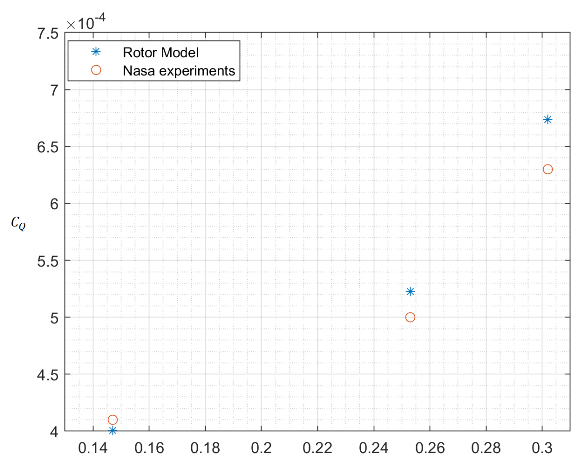

Fig. 17 UH-60 C_Q - μ trend in forward flight

5. THE FLIGHT SIMULATION PLATFORM

The ZHAW started the development of ReDSim (Research and Didactics Simulator) in 2010 and its functionality has since then been continuously expanded (Fig. 18 shows the current state of the ReDSim). ReDSim was originally designed as a flexible and universal platform for fixed-wing aircraft. It has a 170-degree out-of-the-window-view projection system and a three-channel control loading system (CLS) to simulate the forces on the pilot controls (yoke/centre stick and pedals). The ReDsim is also equipped with a professional throttle quadrant. The tilt-rotor model presented in this paper was the first non-fixed-wing model to be simulated in ReDSim. To this end, an additional collective lever was installed in the cockpit to control the thrust.

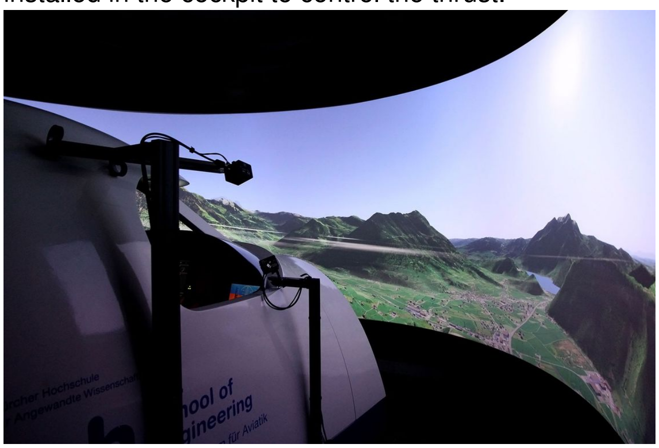

Fig. 18 ReDSim. Courtesy of ZHAW.

The mathematical models of the aircraft to be simulated in ReDSim are implemented in MATLAB/Simulink® 2019a on a Windows 10 computer. To provide the best flexibility, the Simulink® models are not compiled and can be run directly from MATLAB. This allows the user to quickly modify and test the model in the simulator. To allow pilot-in-the-loop simulations, the Simulink® model must be able to maintain real-time. This is done with a proprietary C^{++} S-Function that, for each iteration step, compares the time of the simulation with the time of the computer. If the time of the simulation has caught up with the computer's time, the simulation is blocked and waits for the computer time to pass before the next iteration step is started. This simple approach does not guarantee real-time in the strict sense that, as reported in [27], "computer applications or processes [. . .] can respond with low bounded latency to user requests" , but assures that the simulation time does not run faster that the computers time. Warnings are generated in case the real-time cannot be respected. The difference of the simulation time and the computer time is shown in Fig. 19.

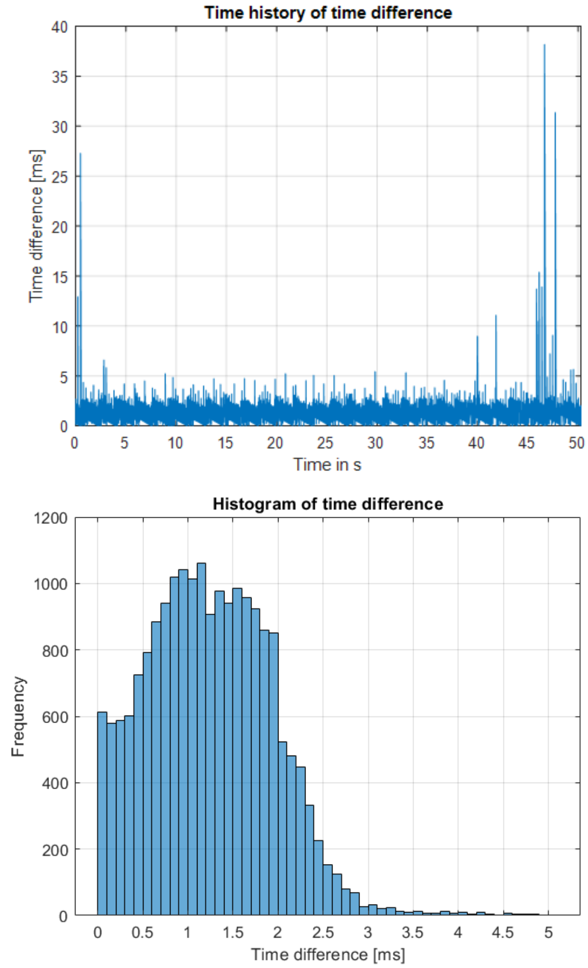

Fig. 19 Time History and histogram of the difference between simulation and computer time.

For this particular test, the function was called at a 400Hz (every 2.5 ms). The histogram of time difference in Fig. 19 shows that about 97.5% of the samples have less than 2.5 ms of time difference. The mean value of the distribution will also depend on the resolution of the function used to suspend the execution of the current thread until the time-out elapses. This is deemed to be acceptable for pilot-in-the-loop simulation, where the main requirement is represented by the time delay between the pilot input and aircraft response (as per "Allowable airplane response delay" in MIL-F-8785C). The time history in Fig. 19 shows a periodic increase in the time difference with a peak value of approximately 37 ms. These spikes are assumed to be caused by other services that are running on the Windows computer and will be investigated in the future. Several mitigations are possible such as: removal of windows services, stop antivirus at run time, increase task priority, etc. Anyhow, it is obvious from Fig. 19 that the execution of one iteration of the model can be achieved in less than 2.5 ms, since the simulation is always able to recover from a delay caused by the operating system.

6. CLOSING REMARKS

The development of a realistic and efficient simulation model of a tilt-rotor / tilt-wing aircraft to be used for design, development and real-time tests is a challenging task. This led to the development of a novel multi-purpose rotor simulation model able to guarantee the required level of fidelity together with acceptable computational loads. The novel model preliminary validation seems to confirm its suitability for the envisaged tasks. An extensive validation of the model will follow in the near future and is expected to be the subject of further publications. Forthcoming activities involve further validation tests, improvements of the reconfigurable flight simulator, pilot-in-the-loop simulations with the aim to use the generic model for flight control system design and development as well as for handling qualities assessments.

ACKNOWLEDGMENTS

This activity is part of a doctoral research program (Ph.D. in Aerospace Engineering at Politecnico di Torino) supported by Zurich University of Applied Sciences, Centre for Aviation.

Copyright Statement

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