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# Data-driven parameterized modeling of LTI systems with guaranteed stability

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We consider the problem of extracting a parameterized reduced-order model from a set of measurements of some underlying LTI system with (unknown) transfer function $\check{H}(s;\vartheta) \in \mathbb{C}^{P\times P}$, where $s$ is the Laplace variable and $\vartheta \in \Theta \subset \mathbb{R}^{\rho}$ is a vector of external parameters. The model is constructed using a data-driven approach starting from frequency response samples $\check{H}_{k,m} = \check{H}(j\omega_k;\vartheta_m)$ at discrete frequency $s_k = j\omega_k$ and parameter values $\vartheta_m$ for $k = 1,\ldots,\bar{k}$ and $m = 1,\ldots,\bar{m}$.
We adopt a Generalized Sanathanan-Koerner (GSK) framework [3] by representing the model as

$$H(s;\vartheta) = \frac{\mathsf{N}(s,\vartheta)}{\mathsf{D}(s,\vartheta)} = \frac{\sum_{n=0}^{\bar{n}} \sum_{\ell=1}^{\bar{\ell}} R_{n,\ell}\, \xi_\ell(\vartheta)\, \varphi_n(s)}{\sum_{n=0}^{\bar{n}} \sum_{\ell=1}^{\bar{\ell}} r_{n,\ell}\, \xi_\ell(\vartheta)\, \varphi_n(s)}, \tag{1}$$

where $R_{n,\ell} \in \mathbb{R}^{P\times P}$ and $r_{n,\ell} \in \mathbb{R}$ are the model coefficients, and where $\varphi_n(s)$, $\xi_\ell(\vartheta)$ are suitable basis functions. In particular, we use partial fractions $\varphi_n(s) = (s - q_n)^{-1}$ associated to a set of predermined stable poles $q_n$ (as in the well-known Vector Fitting scheme [2]) and tensor products of Chebychev polynomials $\xi_\ell(\vartheta)$ for frequency and parameter dependence, respectively. Model coefficients are computed through a Sanathanan-Koerner iteration [3] by setting $D^0(j\omega,\vartheta) = 1$ and solving

$$\min \sum_{k,m} |\mathsf{D}^{\mu-1}(j\omega_k,\vartheta_m)|^{-1} \left\| \mathsf{N}^{\mu}(j\omega_k,\vartheta_m) - \mathsf{D}^{\mu}(j\omega_k,\vartheta_m)\, \check{H}_{k,m} \right\|_F^2 \quad \text{for} \quad \mu = 1,2,\ldots \tag{2}$$

Our main result is a sufficient condition and an associated algorithm for enforcing uniform stability of the model $H(s;\vartheta)$ throughout the parameter domain $\vartheta \in \Theta$. This condition requires constraining the model denominator $\mathsf{D}(s,\vartheta)$ to be a Positive Real (PR) function (see [1] for the sketch of a proof). Based on the model structure (1), the PR-ness of $\mathsf{D}(s,\vartheta)$ is guaranteed when $\Re\{\mathsf{D}(j\omega,\vartheta)\} \geq 0, \forall\vartheta \in \Theta$ and $\forall\omega \in \mathbb{R}$. This is achieved by an adaptive sampling process in the parameter space $\Theta$. At GSK iteration $\mu$ and for any given $\vartheta_*$, the imaginary eigenvalues of the Hamiltonian matrix associated to a state-space realization of $\mathsf{D}^{\mu-1}$ are used to determine the frequency bands where $\Re\{\mathsf{D}^{\mu-1}(j\omega,\vartheta_*)\} < 0$, and a first-order perturbation analysis of the non-imaginary Hamiltonian eigenvalues is used to determine which directions need to be searched in the parameter space to find local minima of $\Re\{\mathsf{D}^{\mu-1}\}$. The result is an automatically determined set of discrete points $(\omega_i,\vartheta_i)$ where the constraint $\Re\{\mathsf{D}^{\mu}(j\omega_i,\vartheta_i)\} > 0$ is formulated and embedded in the GSK iteration (2). When the residual of (2) stabilizes, the model poles $p_n(\vartheta)$ (i.e., the zeros of $\mathsf{D}(s,\vartheta)$) result uniformly stable $\forall\vartheta \in \Theta$.
Several examples from Electronic Design Automation applications are provided, demonstrating the robustness and the efficiency of proposed approach. For a preview of these examples, see [1].

## References

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