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Interfacial stress analysis for thin plates bonded to curved substrates

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Parole chiave: cohesive zone modeling, curved substrate, interfacial stresses

SOMMARIO Il presente lavoro è incentrato sulla modellazione analitica e numerica delle tensioni di interfaccia tra un substrato rigido a semplice curvatura e una lastra di spessore sottile. Il comportamento dell'interfaccia è descritto mediante leggi coesive nelle direzioni normale e tangenziale. Il modello analitico adotta appropriate ipotesi semplificative, mentre il modello numerico utilizza un elemento di contatto del tipo node-to-segment. Il lavoro presenta i primi risultati e confronti analitico-numerici.

ABSTRACT This paper is focused on analytical and numerical modeling of the interface between a rigid substrate with simple curvature and a thin bonded plate. The interfacial behavior is modeled by independent cohesive laws in the normal and tangential directions. The analytical model makes use of appropriate simplifying assumptions. In the numerical model the interface is modeled by zero-thickness node-to-segment contact elements. In this paper the first results and comparisons between predictions of the two models are presented.

1. INTRODUCTION

The mechanics of interfacial bond between a thin plate and a flat quasi-brittle substrate under mode-II loading has been extensively studied [1]. A typical example is given by fiber-reinforced polymer (FRP) strips bonded to concrete or masonry. Surprisingly, limited attention has been devoted to members with a curved surface, despite the fact that such members are often found in practice [2]. The reduction in bond strength due to the effect of curvature has been observed in several experiments on arches strengthened at the intrados. However, neither analytical nor numerical approaches have been proposed to tackle the problem from a mechanical standpoint.

This paper focuses on modeling of the interface between a rigid substrate with simple curvature and a thin bonded plate. The interfacial behavior is modeled by independent cohesive laws in the normal and tangential directions. The analytical model determines the interfacial stresses using appropriate simplifying assumptions. In the numerical model the interface is modeled by zero-thickness node-to-segment contact elements. The interfacial stress distributions predicted by the models are analyzed as functions of the substrate curvature prior to the onset of debonding.

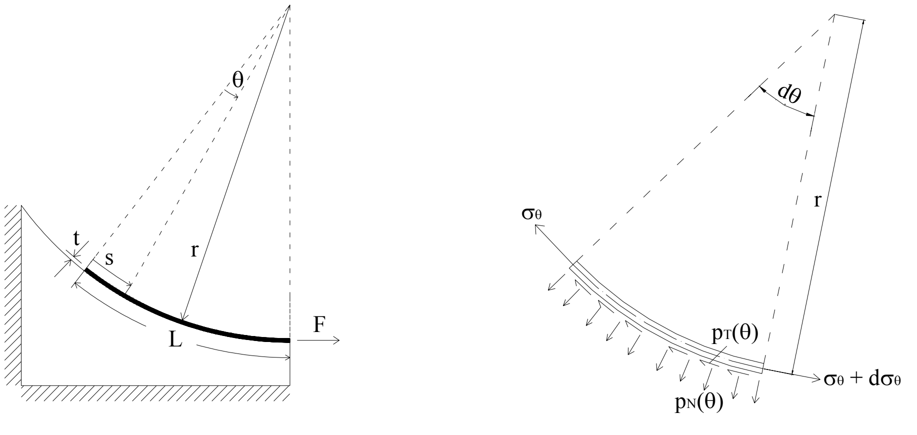
2. ANALYTICAL MODELING

2.1. Problem definition

The model considers a thin plate of thickness t, unit width and length L, made of a linearly elastic material with elastic modulus E. The plate is bonded to a rigid substrate with constant curvature radius r and loaded with a force F (Figure 1a). The force direction is tangent to the substrate surface at the loaded (right) end. To map the interface behavior, the curvilinear coordinate, s, is introduced, with origin at the plate free end. Figure 1b shows a differential element of the plate. Due to the small thickness, both the bending and shear stiffnesses of the plate are neglected. Therefore only axial forces are considered. Normal and tangential stresses, p_N and p_T, arise at the interface between the plate and the substrate. Both of them are considered uniform across the thickness of the adhesive layer.

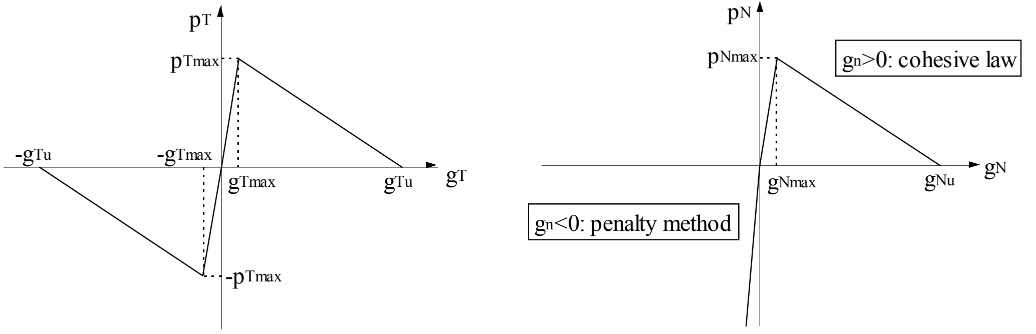
2.2. Cohesive zone modeling

Uncoupled bilinear cohesive laws are considered in the normal and tangential directions (Figure 2). Tension relates the normal relative displacement of the bonded surfaces, $g_N > 0$, and the normal stress, p_N, while shear relates the tangential relative displacement, g_T, and the tangential stress, p_T.



(a) Problem geometry (b) Differential element of the bonded plate

Figure 1. Problem definition.



(a) Tangential direction (b) Normal direction

Figure 2. Interfacial cohesive laws.

2.3. Governing equations and solutions

The governing equations are found using the equilibrium of forces of the differential element of the plate (see Figure 1b), the linearly elastic behavior of the plate material, and the compatibility equations for the plate. These are combined with the assumptions of rigid substrate, and of small thickness of the plate. For the detailed development of the model, see [3].

At small loads, the whole length of the interface is at the elastic stage in both the tangential and the normal directions, and no softening or debonding occur. Representative results for the interfacial stresses are shown in Figure 3. Note that the zero shear stress condition at the loaded end is not satisfied, which is typical of first-order solutions [2].

For practical values of the parameters, the softening stage is entered in the tangential direction first. As loading progresses, an increasingly long portion of the interface closest to the plate loaded end enters the softening stage in the tangential direction, while the rest remains at the elastic stage. In the normal direction, the interface stays at the elastic stage along the whole length of the joint. Representative curves for this case are given in Figure 4.

3. NUMERICAL MODELING

The cohesive models have been implemented into a contact element based on the node-to-segment strategy, as employed in [4], and generalized to handle cohesive forces in both the normal and tangential directions. In the normal direction under compression the non-penetration condition is enforced using the penalty method. Depending on the contact status, an automatic switching procedure is used to choose between cohesive and contact models. Each element contribution for the cohesive and contact forces is suitably added to the global virtual work equation.

Figure 5 shows two representative discretized geometries. The adherend is modeled with two-dimensional, finite deformation, linearly-elastic beam elements, whereas the substrate is discretized with 4-node isoparametric plane stress elastic elements. The non-linear problem is solved with a Newton-Raphson procedure, where the global tangent stiffness matrix is properly obtained with a consistent linearization of all the contact contributions. The model is implemented in the finite element code FEAP (courtesy of Prof. R.L. Taylor).

3. RESULTS

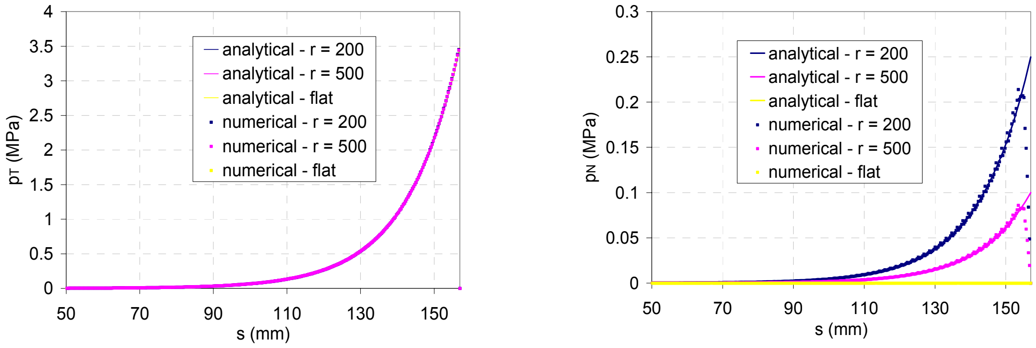
The chosen input values of the example presented are realistic for FRP sheets bonded to a concrete or masonry substrate: $E = 250$ GPa, $t = 0.165$ mm, $p_{N\max} = 2$ MPa, $p_{T\max} = 4$ MPa, $g_{N\max} = 0.01$ mm, $g_{T\max} = 0.02$ mm, $g_{Nu} = 0.1$ mm, $g_{Tu} = 0.2$ mm. The bond length is chosen as $L = 157$ mm. The curvature radii considered are equal 200 mm, 500 mm, and infinite (flat substrate). Figures 3 and 4 illustrate the results.

The curves clearly show that the substrate curvature has no appreciable effect on the magnitude and distribution of the interfacial shear stresses. This applies to both the elastic and the elastic-softening stages. Conversely, a significant influence is visible on the interfacial normal stresses. These are identically zero in the case of a flat substrate, and their magnitude increases as the substrate curvature radius decreases. As expected, tensile normal stresses are obtained as a result of the concave shape of the substrate.

An excellent agreement is found between analytical and numerical results, the respective curves being almost undistinguishable. The only difference is that the analytical model predicts non-zero shear and normal stresses at the loaded end, as a result of the first-order approximation. More correctly, in the numerical curves both interfacial stresses decrease to zero at the loaded end.

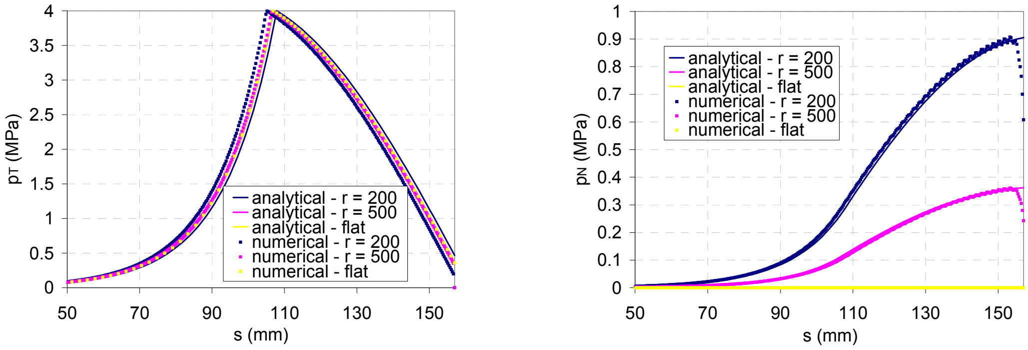
4. CONCLUSIONS

A new analytical model has been developed for the interfacial stresses between a thin plate and a rigid substrate with simple constant curvature. Also, a numerical model where the interface is modeled by zero-thickness node-to-segment contact elements has been devised. The example shown demonstrates that both models can be effectively used to analyze the response of a bonded joint in presence of substrate curvature. Further research is underway to extend the analysis to the full-range interfacial behavior, including the debonding phase, and to correlate results of the stress-based approach with those of a fracture mechanics model.



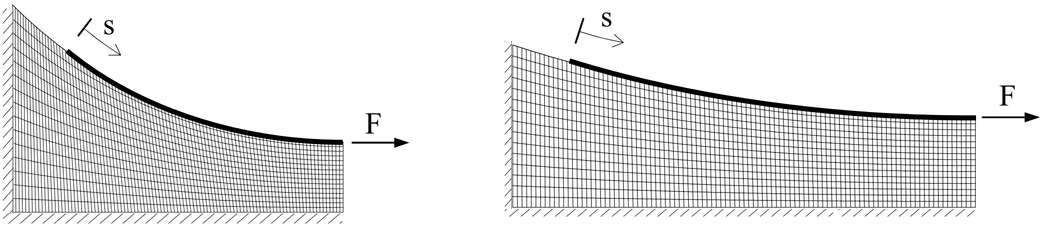
(a) Shear stresses (b) Normal stresses

Figure 3. Elastic stage in both directions (F = 50 N/mm).



(a) Shear stresses (b) Normal stresses

Figure 4. Elastic-softening stage in the tangential direction, elastic stage in the normal direction (F = 180 N/mm).



(a) $r = 200$ mm (b) $r = 500$ mm

Figure 5. Mesh used in the numerical analyses.

5. REFERENCES

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