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In-network reconstruction of jointly sparse signals with ADMM

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In-network reconstruction of jointly sparse signals with ADMM

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EUCNC 2015, June 29- July 2, Paris (France)

Motivation

- Reconstruction of jointly sparse signals with innovations
- Sensing applications where information is acquired by a geographically distributed set of nodes
- \blacktriangleright No need for a fusion center \rightarrow In-network processing
- Focus on efficient ADMM techniques with low signalling overhead

Outline

Signal Model

Centralized ADMM

Distributed ADMM

Distributed ADMM with 1 bit

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Numerical results

Conclusions

Signal Model

- Consider N sensor nodes
- Observed signal at the *i*th sensor node

$$y_i = A_i x_i + \eta_i \quad ; \quad i \in \mathcal{N}$$

•
$$x_i \in \mathbb{R}^n$$
: Signal of interest
• $A_i \in \mathbb{R}^{M \times L}$ with $M \ll L$: Measurement matrix
• $\eta_i \in \mathbb{R}^L$: Acquisition noise

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• $\eta_i \in \mathbb{R}^L$: Acquisition noise

Assumption: JSM-1 model [Duarte06]

$$x_i = \Psi z_c + \Omega_i z_i \quad ; \quad i \in \mathcal{N}$$

► $z_c \in \mathbb{R}^n$: Common signal with k_c non-zero components ► $z_i \in \mathbb{R}^n$: Innovation with k_i non-zero components ► $\Psi, \Omega_i \in \mathbb{R}^{L \times L}$: Sparsity basis (w.l.g. $\Psi = \Omega_i = I_L$)

Optimization problem

Lasso formulation...

$$\min_{\{x_i, z_i\}, z_c} \frac{1}{2} \sum_{i=1}^{N} \left(\|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|z_c\|_1 \right)$$
s.t. $x_i = z_c + z_i; \quad i = 1, \dots, N$

Promotes sparsity in the innovation component

Promotes sparsity in the common component

Review

Augmented cost function

$$\min_{\{x_i, z_i\}, z_c} \frac{1}{2} \sum_{i=1}^{N} \left(\|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|z_c\|_1 + \frac{\rho}{2} \|x_i - z_i - z_c\|_2^2 \right)$$
s.t. $x_i = z_i + z_c; \quad i = 1, \dots, N$

Review

Augmented cost function

$$\min_{\{x_i, z_i\}, z_c} \frac{1}{2} \sum_{i=1}^{N} \left(\|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|z_c\|_1 + \frac{\rho}{2} \|x_i - z_i - z_c\|_2^2 \right)$$

s.t. $x_i = z_i + z_c; \quad i = 1, \dots, N$

Augmented Lagrangian

$$\mathcal{L} := \frac{1}{2} \sum_{i=1}^{N} \|y_i - A_i x_i\|_2^2 + \sum_{i=1}^{N} \tau_1 \|z_i\|_1 + N\tau_2 \|z_c\|_1 + \sum_{i=1}^{N} \frac{\rho}{2} \|x_i - z_i - z_c\|_2^2 + \sum_{i=1}^{N} \lambda_i^T (x_i - z_i - z_c)$$

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Algorithm

ADMM iterates

$$x_i(t+1) = (\rho I + A_i^T A_i)^{-1} (A_i^T y_i + \rho(z_i(t) + z_c(t)) - \lambda_i(t))$$

$$z_c(t+1), \{z_i(t+1)\} = \arg\min_{z_c,\{z_i\}} \mathcal{L}(t+1)$$

$$\lambda_i(t+1) = \lambda_i(t) + \rho (x_i(t+1) - z_i(t+1) - z_c(t+1))$$

Algorithm

ADMM iterates

$$x_{i}(t+1) = (\rho I + A_{i}^{T} A_{i})^{-1} (A_{i}^{T} y_{i} + \rho(z_{i}(t) + z_{c}(t)) - \lambda_{i}(t))$$

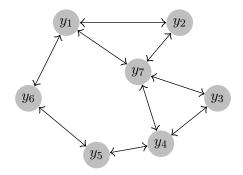
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- Difficult to compute (Algorithm 1 in the paper) \checkmark
- Centralized solution!

Distributed scenario



- Nodes only communicate with their neighbors (no fusion center)
- **Goal:** In-network reconstruction of $\{x_i\}$.

Distributed formulation

New formulation...

$$\min_{\{x_i, z_i, \zeta_i, c_i\}} \frac{1}{2} \sum_{i=1}^N \|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|\zeta_i\|_1$$

s.t. $x_i = z_i + \zeta_i; \quad i \in N$
 $\zeta_i = c_j \quad ; \quad j \in \mathcal{N}_i \cup \{i\}$

Forces consensus on the local guesses

Distributed formulation

Augmented cost function

$$\min_{\{x_i, z_i, \zeta_i, c_i\}} \frac{1}{2} \sum_{i=1}^N \|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|\zeta_i\|_1 + \frac{\rho}{2} \|x_i - z_i - \zeta_i\|_2^2 + \frac{\theta}{2} \sum_{j \in \mathcal{N}_i} \|\zeta_i - c_j\|_2^2$$

s.t. $x_i = z_i + \zeta_i; \quad i \in N$
 $\zeta_i = c_j; \quad j \in \mathcal{N}_i \cup \{i\}$

 Difficult to solve by means of classical ADMM (i.e. with 2 primal iteration blocks)

Distributed formulation

Augmented cost function

$$\min_{\{x_i, z_i, \zeta_i, c_i\}} \frac{1}{2} \sum_{i=1}^N \|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|\zeta_i\|_1 + \frac{\rho}{2} \|x_i - z_i - \zeta_i\|_2^2 + \frac{\theta}{2} \sum_{j \in \mathcal{N}_i} \|\zeta_i - c_j\|_2^2$$

s.t. $x_i = z_i + \zeta_i; \quad i \in N$
 $\zeta_i = c_j; \quad j \in \mathcal{N}_i \cup \{i\}$

- Difficult to solve by means of classical ADMM (i.e. with 2 primal iteration blocks)
- Instead, we propose to minimize the primal variables in a sequential fashion

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Algorithm

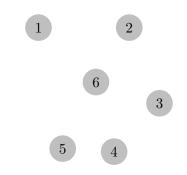
Primal iterates

$$\begin{aligned} x_i(t+1) &= (\rho I + A_i^T A_i)^{-1} (A_i^T y_i + \rho(z_i(t) + \zeta_i(t)) - \lambda_i^T) \\ z_i(t+1) &= \mathcal{S}_{\frac{\tau_1}{\rho}} \left[(x_i(t+1) - \zeta_i(t)) + \frac{\lambda_i(t)}{\rho} \right] \\ \zeta_i(t+1) &= \mathcal{S}_{\frac{\tau_2}{\rho + \theta |\mathcal{N}_i|}} \left[\frac{1}{\rho + \theta |\mathcal{N}_i|} \left(\rho \left(x_i(t+1) - z_i(t+1) \right) \right) \right. \\ &+ \theta \sum_{j \in \mathcal{N}_i} \left(c_j(t) - \frac{\mu_{i,j}(t)}{\theta} \right) + \lambda_i(t) \right) \right] \\ c_i(t+1) &= \frac{1}{|\mathcal{N}_i|} \sum_{j:i \in \mathcal{N}_j} \left(\zeta_j(t+1) + \frac{\mu_{j,i}(t)}{\theta} \right); \end{aligned}$$

Dual iterates

$$\lambda_i(t+1) = \lambda_i(t) + \rho \left(x_i(t+1) - z_i(t+1) - \zeta_i(t+1) \right) \mu_{i,j}(t+1) = \mu_{i,j}(t) + \theta \left(\zeta_i(t+1) - c_j(t+1) \right); \quad j \in \mathcal{N}_i$$

Algorithm

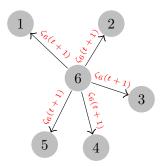


• Compute $\{x_i(t+1), z_i(t+1), \zeta_i(t+1)\}$ at each node

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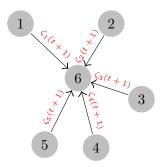
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Algorithm



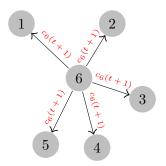
- Compute $\{x_i(t+1), z_i(t+1), \zeta_i(t+1)\}$ at each node
- Broadcast $\{\zeta_i(t+1)\}$ to your neighbors

Algorithm



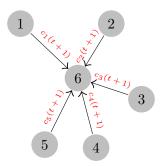
- ► Compute $\{x_i(t+1), z_i(t+1), \zeta_i(t+1)\}$ at each node
- Broadcast $\{\zeta_i(t+1)\}$ to your neighbors
- Compute $\{c_i(t+1)\}$ at each node

Algorithm



- ► Compute $\{x_i(t+1), z_i(t+1), \zeta_i(t+1)\}$ at each node
- Broadcast $\{\zeta_i(t+1)\}$ to your neighbors
- Compute $\{c_i(t+1)\}$ at each node
- Broadcast $\{c_i(t+1)\}$ to your neighbors

Algorithm



- Compute $\{x_i(t+1), z_i(t+1), \zeta_i(t+1)\}$
- Broadcast $\{\zeta_i(t+1)\}$ to your neighbors
- Compute $\{c_i(t+1)\}$ at each node
- Broadcast $\{c_i(t+1)\}$ to your neighbors
- Compute $\{\lambda_i(t+1), \mu_{j,i}(t+1)\}$ at each node

DADMM requires the exchange of analog values

- DADMM requires the exchange of analog values
- We propose to modify the updates of $\{\zeta_i(t+1)\}$ and $\{c_i(t+1)\}$ as follows:

$$\begin{aligned} \zeta_i(t+1) &= \zeta_i(t) - \epsilon \quad \text{sign}\left(g_{\zeta_i^t}\right) \\ c_i(t+1) &= c_i(t) - \epsilon \quad \text{sign}\left(g_{c_i^t}\right) \end{aligned}$$

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$$c_i(t+1) = c_i(t) - \epsilon \quad \operatorname{sign}\left(g_{c_i^t}\right)$$

• $\{g_{\zeta_i^t},g_{c_i^t}\}$ stand for the subgradients with respect to $\{c_i\}$ and $\{\zeta_i\}$ at time t

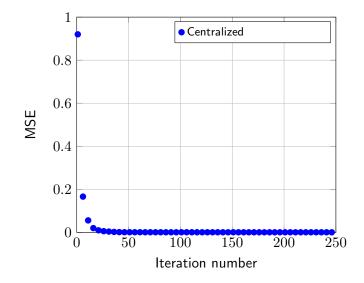
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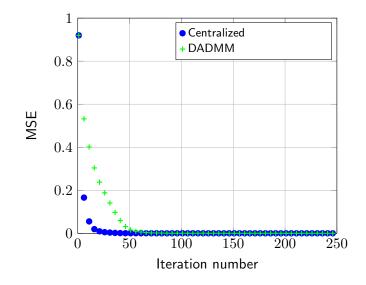
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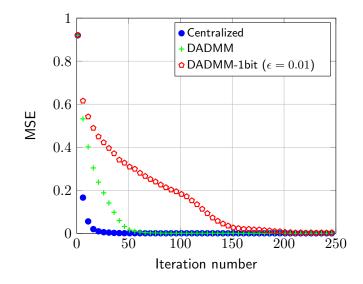
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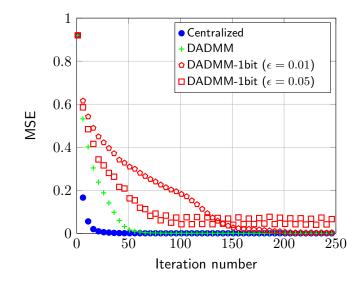
$$\zeta_{i}(t+1) = \zeta_{i}(t) - \epsilon \quad \operatorname{sign}\left(g_{\zeta_{i}^{t}}\right) \leftarrow c_{i}(t+1) = c_{i}(t) - \epsilon \quad \operatorname{sign}\left(g_{c_{i}^{t}}\right) \leftarrow c_{i}(t+1) - \epsilon \quad \operatorname{s$$

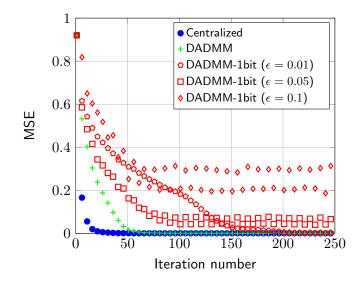
- $\{g_{\zeta_i^t}, g_{c_i^t}\}$ stand for the subgradients with respect to $\{c_i\}$ and $\{\zeta_i\}$ at time t
- Only requires the exchange of 1-bit per variable!!!! _



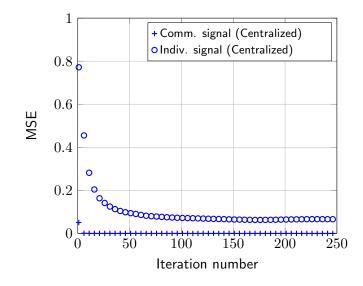




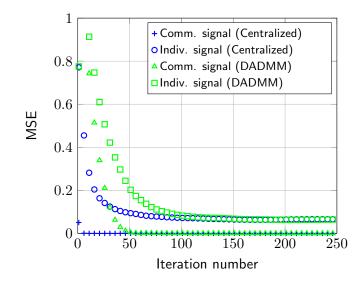




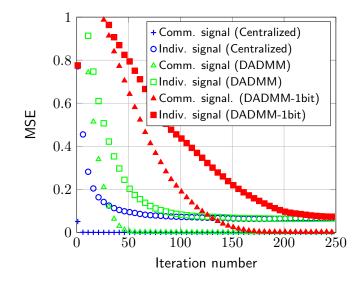
Numerical results (cont'd)



Numerical results (cont'd)



Numerical results (cont'd)



Conclusions

- We have addressed the problem of reconstruction of jointly sparse signals with innovations
- I Centralized ADMM solution and 2 distributed ADMM solution for in-network reconstruction have been proposed
- Distributed versions are shown to converge to the centralized ADMM

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The 1 bit version is shown to reduce the number of transmitted bits significantly

Thank you! Questions?

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