In-network reconstruction of jointly sparse signals with ADMM

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Motivation

- Reconstruction of jointly sparse signals with innovations
- Sensing applications where information is acquired by a geographically distributed set of nodes
- No need for a fusion center $\rightarrow$ In-network processing
- Focus on efficient ADMM techniques with low signalling overhead

Outline

Signal Model

- Consider N sensor nodes
- Observed signal at the ith sensor node

$$y_i = A_i x_i + \eta_i \quad ; \quad i \in \mathcal{N}$$

  - $x_i \in \mathbb{R}^n$: Signal of interest
  - $A_i \in \mathbb{R}^{M \times L}$ with $M \ll L$: Measurement matrix
  - $\eta_i \in \mathbb{R}^L$: Acquisition noise

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- **Assumption**: JSM-1 model [Duarte06]

$$x_i = \Psi z_c + \Omega_i z_i \quad ; \quad i \in \mathcal{N}$$

  - $z_c \in \mathbb{R}^n$: Common signal with k_c non-zero components
  - $z_i \in \mathbb{R}^n$: Innovation with k_i non-zero components
  - $\Psi, \Omega_i \in \mathbb{R}^{L \times L}$: Sparsity basis (w.l.g. $\Psi = \Omega_i = I_L$)

Centralized ADMM

Optimization problem

- Lasso formulation...

$$\min_{\{x_i, z_i\}, z_c} \frac{1}{2} \sum_{i=1}^{N} \left(\|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|z_c\|_1 \right)$$

$$\text{s.t.} \quad x_i = z_c + z_i; \quad i = 1, \ldots, N$$

  - Promotes sparsity in the innovation component
  - Promotes sparsity in the common component

Centralized ADMM

Review

- Augmented cost function

$$\min_{\{x_i,z_i\},z_c} \frac{1}{2}\sum_{i=1}^{N}\Big(\|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|z_c\|_1 + \frac{\rho}{2}\|x_i - z_i - z_c\|_2^2 \Big)$$

$$\text{s.t.} \quad x_i = z_i + z_c; \quad i = 1, \ldots, N$$

Centralized ADMM

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$$\text{s.t.} \quad x_i = z_i + z_c; \quad i = 1, \ldots, N$$

- Augmented Lagrangian

$$\mathcal{L} := \frac{1}{2}\sum_{i=1}^{N}\|y_i - A_i x_i\|_2^2 + \sum_{i=1}^{N}\tau_1\|z_i\|_1 + N\tau_2\|z_c\|_1 + \sum_{i=1}^{N}\frac{\rho}{2}\|x_i - z_i - z_c\|_2^2 + \sum_{i=1}^{N}\lambda_i^T(x_i - z_i - z_c)$$

Centralized ADMM

Algorithm

- ADMM iterates

$$x_i(t+1) = (\rho I + A_i^T A_i)^{-1}(A_i^T y_i + \rho(z_i(t) + z_c(t)) - \lambda_i(t))$$

$$z_c(t+1), \{z_i(t+1)\} = \arg\min_{z_c, \{z_i\}} \mathcal{L}(t+1)$$

$$\lambda_i(t+1) = \lambda_i(t) + \rho\left(x_i(t+1) - z_i(t+1) - z_c(t+1)\right)$$

Centralized ADMM

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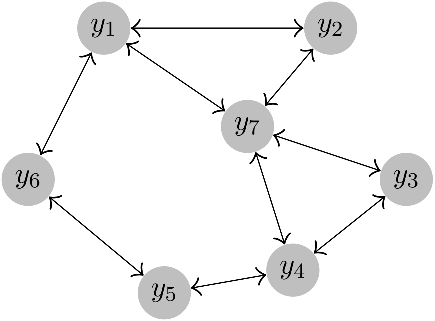
  - Difficult to compute (Algorithm 1 in the paper)
  - Centralized solution!

Distributed ADMM

Distributed scenario



- Nodes only communicate with their neighbors (no fusion center)
- **Goal:** In-network reconstruction of $\{x_i\}$.

Distributed ADMM

Distributed formulation

- New formulation...

$$\min_{\{x_i, z_i, \zeta_i, c_i\}} \frac{1}{2} \sum_{i=1}^{N} \|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|\zeta_i\|_1$$

$$\text{s.t.} \quad x_i = z_i + \zeta_i; \quad i \in N$$

$$\zeta_i = c_j; \quad j \in \mathcal{N}_i \cup \{i\}$$

  - Forces consensus on the local guesses

Distributed ADMM

Distributed formulation

- Augmented cost function

$$\min_{\{x_i, z_i, \zeta_i, c_i\}} \frac{1}{2} \sum_{i=1}^{N} \|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|\zeta_i\|_1 + \frac{\rho}{2} \|x_i - z_i - \zeta_i\|_2^2 + \frac{\theta}{2} \sum_{j \in \mathcal{N}_i} \|\zeta_i - c_j\|_2^2$$

$$\text{s.t.} \quad x_i = z_i + \zeta_i; \quad i \in N$$

$$\zeta_i = c_j; \quad j \in \mathcal{N}_i \cup \{i\}$$

- Difficult to solve by means of classical ADMM (i.e. with 2 primal iteration blocks)

Distributed ADMM

Distributed formulation

- Augmented cost function

$$
\begin{aligned}
\min_{\{x_i,z_i,\zeta_i,c_i\}} \frac{1}{2}\sum_{i=1}^{N} \|y_i - A_i x_i\|_2^2 + \tau_1 \|z_i\|_1 + \tau_2 \|\zeta_i\|_1 \\
+ \frac{\rho}{2}\|x_i - z_i - \zeta_i\|_2^2 + \frac{\theta}{2}\sum_{j\in\mathcal{N}_i} \|\zeta_i - c_j\|_2^2 \\
\text{s.t.} \quad x_i = z_i + \zeta_i; \quad i \in N \\
\zeta_i = c_j; \quad j \in \mathcal{N}_i \cup \{i\}
\end{aligned}
$$

- Difficult to solve by means of classical ADMM (i.e. with 2 primal iteration blocks)
- Instead, we propose to minimize the primal variables in a sequential fashion

Distributed ADMM

Algorithm

- Primal iterates

$$x_i(t+1) = (\rho I + A_i^T A_i)^{-1}(A_i^T y_i + \rho(z_i(t) + \zeta_i(t)) - \lambda_i^T)$$

$$z_i(t+1) = \mathcal{S}_{\frac{\tau_1}{\rho}}\left[(x_i(t+1) - \zeta_i(t)) + \frac{\lambda_i(t)}{\rho}\right]$$

$$\zeta_i(t+1) = \mathcal{S}_{\frac{\tau_2}{\rho+\theta|\mathcal{N}_i|}}\left[\frac{1}{\rho+\theta|\mathcal{N}_i|}\left(\rho\left(x_i(t+1) - z_i(t+1)\right)\right.\right.$$
$$\left.\left. + \theta \sum_{j\in\mathcal{N}_i}\left(c_j(t) - \frac{\mu_{i,j}(t)}{\theta}\right) + \lambda_i(t)\right)\right]$$

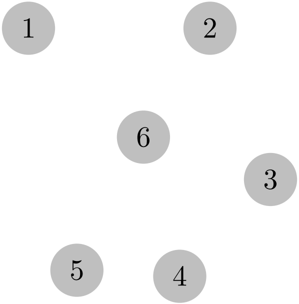
$$c_i(t+1) = \frac{1}{|\mathcal{N}_i|}\sum_{j:i\in\mathcal{N}_j}\left(\zeta_j(t+1) + \frac{\mu_{j,i}(t)}{\theta}\right);$$

- Dual iterates

$$\lambda_i(t+1) = \lambda_i(t) + \rho\left(x_i(t+1) - z_i(t+1) - \zeta_i(t+1)\right)$$
$$\mu_{i,j}(t+1) = \mu_{i,j}(t) + \theta\left(\zeta_i(t+1) - c_j(t+1)\right); \quad j \in \mathcal{N}_i$$

Distributed ADMM (cont'd)

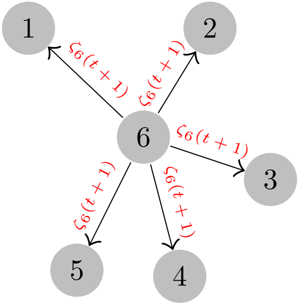
Algorithm



- Compute $\{x_i(t+1), z_i(t+1), \zeta_i(t+1)\}$ at each node

Distributed ADMM (cont'd)

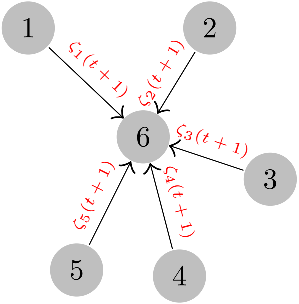
Algorithm



- Compute $\{x_i(t+1), z_i(t+1), \zeta_i(t+1)\}$ at each node
- Broadcast $\{\zeta_i(t+1)\}$ to your neighbors

Distributed ADMM (cont'd)

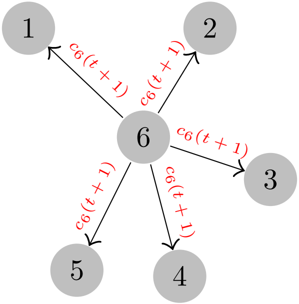
Algorithm



- Compute $\{x_i(t+1), z_i(t+1), \zeta_i(t+1)\}$ at each node
- Broadcast $\{\zeta_i(t+1)\}$ to your neighbors
- Compute $\{c_i(t+1)\}$ at each node

Distributed ADMM (cont'd)

Algorithm



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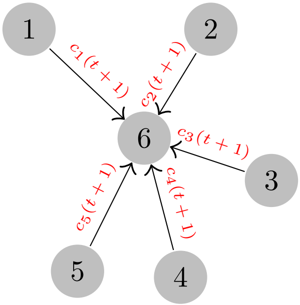
Algorithm



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- Broadcast $\{\zeta_i(t+1)\}$ to your neighbors
- Compute $\{c_i(t+1)\}$ at each node
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- Compute $\{\lambda_i(t+1), \mu_{j,i}(t+1)\}$ at each node

Distributed ADMM with 1 bit

- DADMM requires the exchange of analog values

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- We propose to modify the updates of $\{\zeta_i(t+1)\}$ and $\{c_i(t+1)\}$ as follows:

$$\zeta_i(t+1) = \zeta_i(t) - \epsilon \, \text{sign}\left(g_{\zeta_i^t}\right)$$

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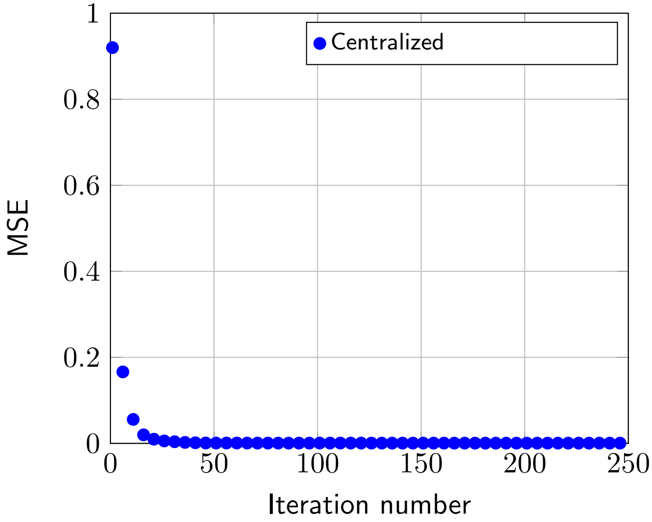
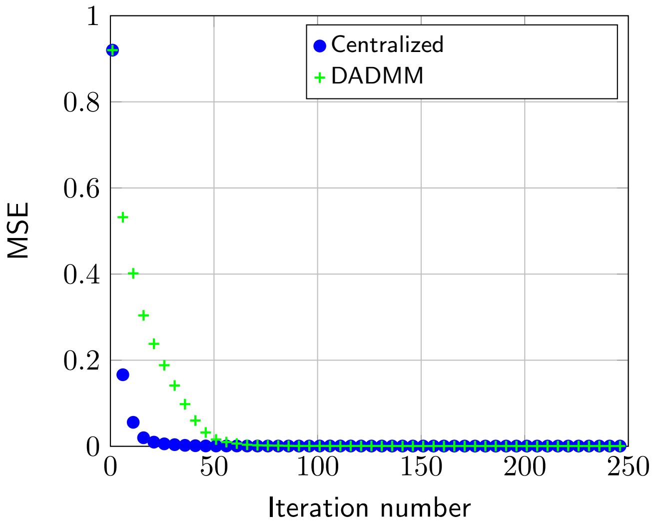
- $\{g_{\zeta_i^t}, g_{c_i^t}\}$ stand for the subgradients with respect to $\{c_i\}$ and $\{\zeta_i\}$ at time t
- Only requires the exchange of 1-bit per variable!!!!

Numerical results



Numerical results



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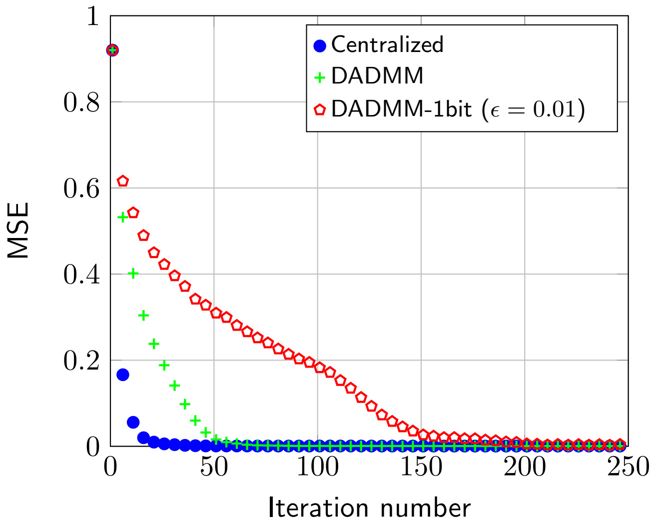

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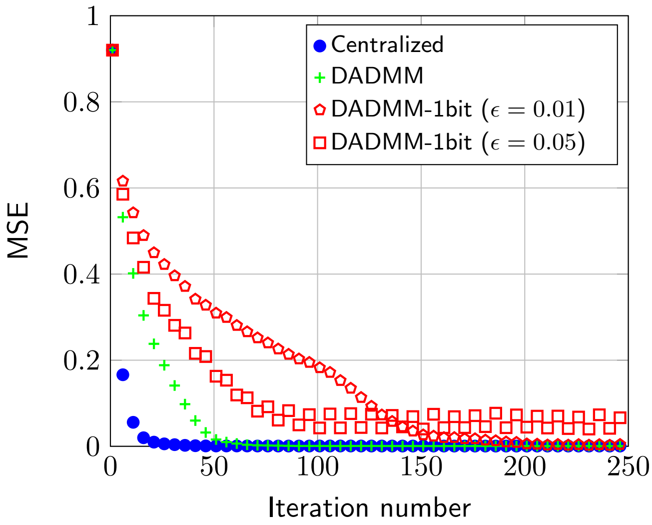

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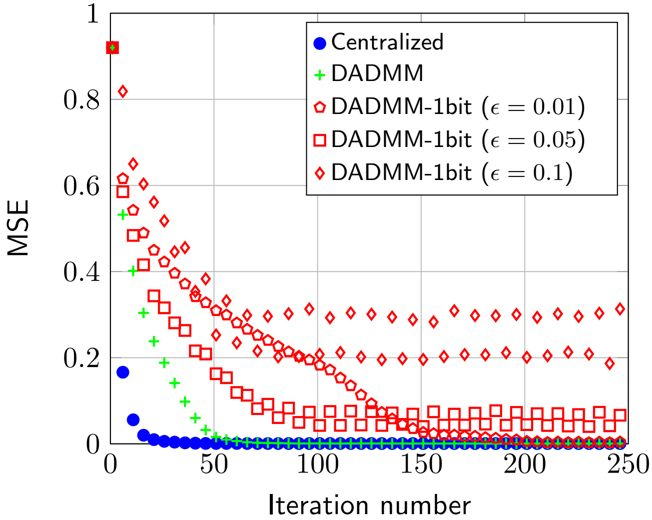
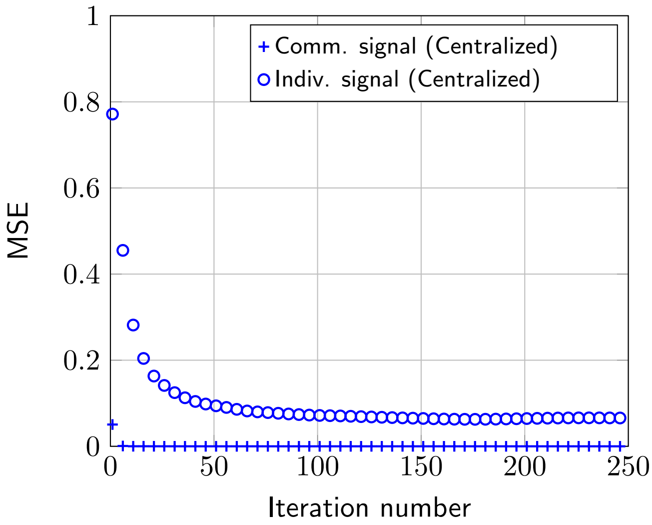
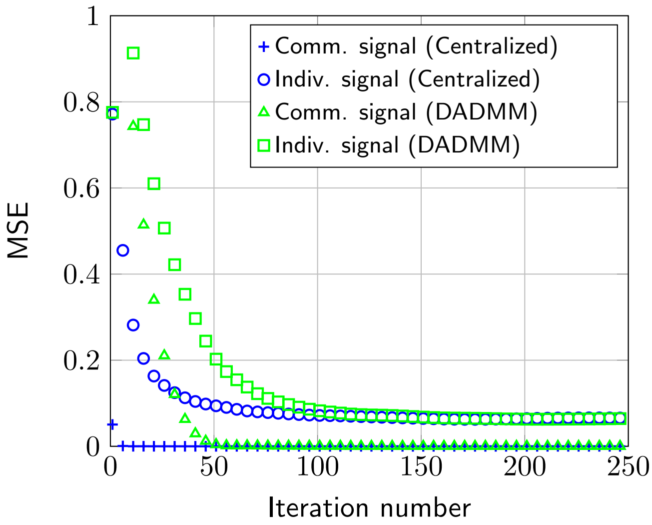


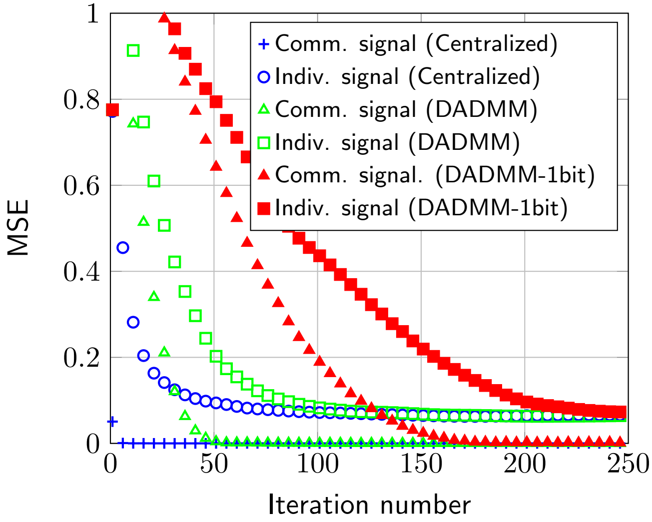
Numerical results (cont'd)



Numerical results (cont'd)



Numerical results (cont'd)



Conclusions

- We have addressed the problem of reconstruction of jointly sparse signals with innovations
- 1 Centralized ADMM solution and 2 distributed ADMM solution for in-network reconstruction have been proposed
- Distributed versions are shown to converge to the centralized ADMM
- The 1 bit version is shown to reduce the number of transmitted bits significantly

Thank you!
Questions?