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# Flexible aggregation operators to support hierarchization of Engineering Characteristics in QFD 

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#### Abstract

Quality Function Deployment (QFD) is a management tool for organizing and conducting design activities of new products and/or services together with their relevant production and/or supply processes, starting from the requirements directly expressed by the end-users. It is organized in a series of operative steps which drive from the collection of the customer needs to the definition of the technical characteristics of the production/supply processes. The first step entails the construction of the House of Quality ( HoQ ), a planning matrix translating the Customer Requirements (CRs) into measurable product/service technical characteristics (Engineering Characteristics - ECs). One of the main goals of this step is to transform CR importances into an EC prioritization. A robust evaluation method should consider the relationships between CRs and ECs while determining the importance levels of ECs in the HoQ. In traditional approaches, such as for example Independent Scoring Method, ordinal information is arbitrarily converted in cardinal information introducing a series of controversial assumptions. Actually, the current scientific literature presents a number of possible solutions to this problem, but the question of attributing scalar properties to information collected on ordinal scales is far from being settled. This paper proposes a method based on ME-MCDM techniques (Multi Expert / Multiple Criteria Decision Making), which is able to compute EC prioritization without operating an artificial numerical codification of the information contained in the HoQ. After a general description of the theoretical principles of the method, a series of application examples are presented and discussed.


Keywords: Quality Function Deployment, House of Quality, Engineering Characteristics, Customer Requirements, Independent Scoring Method, ordinal scale, MCDM operators.

## 1 Introduction

Quality Function Deployment (QFD) is a practical and effective tool for structuring design activities of a new product or service and the related production/supply process according to the real exigencies of the end-user [Akao, 1988; Franceschini, 2001].
Since its introduction, QFD has been recognized as a strategic approach to pursue customer satisfaction. Its large diffusion and effectiveness is widely witnessed by the
large amount of scientific literature produced over the years [Carnevalli and Cauchick Miguel, 2008].
Many empirical studies demonstrated that the correct implementation of QFD may bring significant improvements in the process of product/service development, including earlier and fewer design modifications, fewer start-up issues, improved cross-functional communications, improved product/service quality, reduced product/service development time and cost, etc. [Biren, 1998; Chan and Wu, 2002.a, 2002.b; Lager, 2005; Carnevalli and Cauchick Miguel, 2008].

From the procedural point of view, QFD is based on four phases which deploy Customer Requirements (CRs) throughout the planning process [Akao, 1988]. Each phase is supported by a specific Quality Table, representing and relating the variables concurring to the design. A schematic structure of these four phases and the relevant Quality Tables is reported in Fig. 1 [Akao, 1988; Franceschini, 2001].


Figure 1. Scheme of the four phases of QFD. Adapted from [Lager, 2005].
Special attention is given to Phase I, whose goal is to transform CR importances into EC (Engineering Characteristic) prioritization. In this process, customer ordinal information is usually converted in cardinal information introducing two controversial assumptions:

- The importance of each CR, normally expressed on ordinal scales, is artificially encoded on a cardinal scale (interval or ratio scales), i.e. in the form of a number [Wasserman, 1993; Franceschini and Rupil, 1999; Franceschini, 2001].
- The prioritization of ECs is traditionally carried out by methods, such as the Independent Scoring Method [Akao, 1988; Franceschini, 2001], that generally require the numerical conversion of the symbols contained in the House of Quality (HoQ) again into numbers.
In order to deal with this problem, during the years, many alternative techniques have been proposed in the scientific literature. Examples are: Multi Criteria Decision Aid (MCDA) techniques, Borda's method and equivalent techniques based on pairwise
comparison, techniques based on fuzzy logic, hybrid methods [Franceschini and Rossetto, 1995; Dym and Wood, 2002; Han et al., 2004; Yan et al., 2013; Franceschini et al. 2014; Chen and Chen, 2014; Iqbal et al., 2015; Jianga et al., 2015]. This work proposes an alternative approach to hierarchize ECs with the aim of overcoming the aforementioned controversial assumptions. The method is able to deal with information expressed on ordinal scale with no need to resort to an artificial numerical conversion of the scale. Inspired by the work of Yager, for the solution of multi-criteria decision-making problems, it can be classified within the class of MEMCDM techniques (Multi Expert / Multiple Criteria Decision Making) [Yager and Filev, 1994].
From the conceptual point of view, the method (i) extends the logic of Boolean operators "Min" and "Max" to multilevel ordinal scales and (ii) uses the importances of CRs as linguistic quantifiers for weighting the impacts of the relationship coefficients [Yager and Filev, 1994]. The final result is the hierarchization of ECs according to the importances of the CRs to which each EC is related.
Since the method considers only the ordinal properties of the gathered information, it does not require any arbitrary and artificial scaling of data. It can routinely be applied according to typical techniques for CR investigation in QFD and it can be easily computerized.
After a brief review of QFD basic concepts in Section 2, this paper presents a conceptual and formal description of the method in Section 3, as well as its advantages and limitations. Furthermore, in order to demonstrate its potentiality and effectiveness, some practical examples of application are reported and discussed in Section 4. Sections 5 and 6 are respectively dedicated to a discussion of the obtained results and to the final conclusions.


## 2 Basics of QFD

The QFD approach consists of four phases which deploy the customer requirements throughout the planning process (see Fig. 1). In the first phase, through the HoQ (usually also indicated as Product Planning Matrix), CRs are related to ECs of the product/service. In the second phase, ECs are associated to critical part characteristics through the Part Deployment Matrix. Then, the Process Planning Matrix relates the characteristics of the single subsystem with its respective production process. Finally, the Process and Quality Control Matrix defines inspection and quality control parameters and methods to be used in the production process. The conduction of each phase is assigned to a cross-functional design team.
The first phase is fundamental and strategic for the success of QFD implementation [Franceschini 2001; Tontini 2007; Li, Tang et al. 2009; Li, Tang et al. 2010]. Errors made at this stage can propagate throughout all the subsequent phases.
With reference to Fig. 2, the construction of the HoQ can be broadly structured into ten steps, which are deeply described in Franceschini et al. (2014).


Figure 2. Main steps of House of Quality [Franceschini et al., 2014].
The focus of the present paper is concentrated on Step 8, which has the goal of prioritizing ECs. To this purpose, several approaches are possible. The traditional and most used method is the Independent Scoring Method [Akao 1988]. Basing on the ratings of CRs and the relationship matrix, it provides a rating of ECs. It requires two operative steps. The first and more controversial step consists in converting the relationship matrix into a cardinal matrix according to an arbitrary convention: the most typical approach is that of evaluating the relationships between CRs and ECs on four levels - i.e. strong, medium, weak and absent relationships - and then encode them into four numeric values, respectively, $9,3,1$ and 0 . Then the so called relative importance (or the relative weight) of each EC is evaluated as a linear function of the relative importance of CRs and the transformed relationship matrix coefficients [Akao 1988]. The typical model used is:

$$
\begin{equation*}
w_{j}=\sum_{i=1}^{n} d_{i} \cdot r_{i j} \tag{1}
\end{equation*}
$$

where:
$w_{j}$ is the importance of the $j$-th EC,
$d_{i}$ is the importance of the $i$-th CR ,
$r_{i j}$ is the numerical value corresponding to the relationship coefficient between the $j$-th EC and the $i$-th CR.

Alternatively, other approaches have been presented in the scientific literature, ranging from Multi Criteria Decision Aid (MCDA) to fuzzy logics [Carnevalli and Cauchick Miguel, 2008; Franceschini et al., 2014]. In general, all these techniques differ from each other for many reasons: (i) typology of data, (ii) hypothesis on the data properties, (iii) aggregation models used for the project information, (iv) models used for linking CRs with relationship coefficient of the HoQ [Carnevalli and Cauchick Miguel, 2008].

## 3 The proposed approach

EC prioritization is aimed at drawing the designer's attention towards the technical elements which most impact the major CRs as expressed by the customers [Akao, 1988; Franceschini, 2001].
The conversion of CR importances into EC prioritization must not alter the meaning of the collected information [Franceschini et al., 2014]. The proposed method is able to deal with information expressed on an ordered qualitative scale with no need to resort to an artificial numerical conversion of the scale. As anticipated, it can be classified within the class of the so called ME-MCDM techniques (Multi Expert - Multiple Criteria Decision Making) [Yager, 1993].
The use of qualitative scales raises a few issues for data processing. For example, while on cardinal scales the distance between two scale elements is defined (hence, sum and product operators may be applied), this cannot be defined for qualitative scales, which have ordinal properties only.
The method is inspired by the work of Bellman and Zadeh, lately "enriched" by Yager for the solution of multi criteria decision-making problems [Yager and Filev, 1994]. In fact, EC prioritization can be considered as a decision-making problem. The decision consists of defining the EC order of importance for the considered product or service. Referring to the decision-making theory, in the QFD context, the decisional problem is implemented according to the following assumptions: the CRs stand for the "decision criteria", and the ECs represent the "alternatives" ("courses of action") [Yager and Filev, 1994]. In the Relationship Matrix the symbols that qualify $r_{i j}$ coefficients are interpreted as the "assessment" that each $i$-th CR assigns to the each $j$-th EC. The proposed method allows to carry out an overall synthesis of the "assessments" of the CRs over the set of ECs, considering as weighing elements the CR importances. The approach can be organized on a four-step procedure:
i) Definition of the scale levels for the importances associated to each $\mathrm{CR}_{i},(i=1 \ldots . . n)$ and for the relationship coefficient $\left(r_{i j}\right)$ between each $\mathrm{CR}_{i}$ and each $\mathrm{EC}_{j}(j=1 \ldots m)$.

For simplicity, it is assumed that the importance associated to each CR is defined on a $s$-point ordinal scale similar to that used for relationship coefficients and with the same number of levels. If scales with different number of levels are needed, the method may still be adopted, but mappings can become a bit more complex.
Table 1 shows an example of a correspondence map between CR importances and relationship coefficients expressed on a 3-levels ordinal scale $(s=3)$.

| Scale <br> level | CR importance <br> $\left(d_{\boldsymbol{i}}\right)$ | Importance <br> value | Relationship coefficient <br> $\left(r_{i j}\right)$ | Symbol |
| :---: | :---: | :---: | :---: | :---: |
| $L_{1}$ | not (or weakly) <br> important | 1 | no (or weak) relationship | (empty cell) |
| $L_{2}$ | important | 2 | medium relationship | 0 |
| $L_{3}$ | very important | 3 | strong relationship | 0 |

Table 1. Example of correspondence map between CR importances and relationship coefficients expressed on a 3-levels ordinal scale ( $s=3$ ).
ii) Data collection and filling-in of House of Quality matrix.
iii) Implementation of the $\mathrm{EC}_{j}$ prioritization model:

$$
\begin{equation*}
w_{j}=\operatorname{Min}_{i=1 \ldots n}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i j}\right]\right\} \tag{2}
\end{equation*}
$$

where
$w_{j}$ is the computed importance of the $j$-th EC $(j=1 \ldots m)$,
$d_{i}$ is the importance of the $i$-th CR $(i=1 \ldots n)$,
$r_{i j}$ is the relationship coefficient between $\mathrm{CR}_{i}$ and $\mathrm{EC}_{j}$,
Min is the Minimum operator,
Max is the Maximum operator,
$\operatorname{Neg}\left(d_{i}\right)$ is the negation operator, defined as follows [Yager, 1993]:

$$
\begin{equation*}
\operatorname{Neg}\left(L_{k}\right)=L_{s-k+1} \tag{3}
\end{equation*}
$$

where $L_{k}$ is the $k$-th level of the evaluation scale ( $k=1 \ldots s$ ).
It is worth noting that $w_{j}$ values are defined on the same ordinal scale as those utilized for rating $d_{i}$ importances and $r_{i j}$ coefficients.

Furthermore, the expression of $w_{j}$, obtained by Eq. (2), synthetizes the concept that "if a given criterion CR is important, then the evaluation of the alternative EC must be high in order to obtain an high importance $w_{j}$ ".
iv) Determination of EC total ordering (ranking):

If two or more ECs have the same importance level, a more detailed selection may be obtained by considering the following further indicator:

$$
\begin{equation*}
T\left(\mathrm{EC}_{j}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{j}\right)\right] \tag{4}
\end{equation*}
$$

where the operator $\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{j}\right)\right]$ gives the number of elements contained in the set $\mathrm{A}\left(\mathrm{EC}_{j}\right)$, with $\mathrm{A}\left(\mathrm{EC}_{j}\right)=\left\{\mathrm{CR}_{i} \mid r_{i j}>w_{j}\right\}$.
This term represents a second-step investigation for establishing an indication of the dispersion of the obtained value of EC importance. It counts how many CRs with high relationship coefficient, in comparison with the obtained value of EC importance, are present in the evaluation of each EC.
The meaning of $T\left(\mathrm{EC}_{j}\right)$ will be further described by some practical examples in the next section.
For the same level of $w_{j}$, ECs with higher values of $T\left(\mathrm{EC}_{j}\right)$ are considered more important, hence a further sub-ordering can be obtained according to $T\left(\mathrm{EC}_{j}\right)$ values

From Eq.(2) it is possible to observe that low-importance CRs have little effect on the overall "score" (i.e. the computed importance of the $j$-th EC). Consider, for example, a CR that has little importance, it will get a low importance rating $L_{k}$ on the scale. After computing the negation of this score, the consequent value is high. Then, with the application of the Max operator, the higher value between the negation of the importance and the relationship coefficient is selected. For a given EC, all the values related to the whole set of CRs are computed. Then, the Min operator extracts the smallest of these values. In this way, all the contributions to the overall importance of a given EC, related to CRs with little importance, are automatically cut off.
The result of the application of Eq. (2) is a balanced tradeoff between high-value relationship coefficients related to CRs with little importance, and low-value relationship coefficients related to CRs with high importance.
It can be shown that the formulation suggested in Eq.(2) satisfies the properties of Pareto optimality, independence to irrelevant alternatives, positive association of individual scores with overall score and symmetry [Yager, 1993].
It must be highlighted that an essential feature of this approach is that there is no need for numeric values and it does not force undue precision on the experts of the QFD design team.
The rationale of the procedure is to consider as the most important ECs those with the highest relationship coefficients on the most important CRs. When two or more ECs have the same ranking a more detailed selection may be performed using the $T\left(\mathrm{EC}_{j}\right)$ indicator.

It must be stressed that the logic proposed in Eq.(2) is just the interpretation of one of the possible ways that a decision maker can undertake for aggregating relationship coefficients and importances without distorting the information collected in the HoQ. Different decision makers may decide to implement different aggregation logics with the only constraints of being consistent with the gathered information.
In this specific case, ECs associated with even one CR with a high importance but with a low correlation coefficient are penalized in comparison to ECs associated with CRs with the same importance, but with higher values of correlation coefficients. Furthermore, if all the CR importances are on the lower level, the consequent calculated values of all the EC importances will result on the same high level, independently from the values of the related relationship coefficients. On the contrary, if all the CR importances are on the higher level, the ECs that will obtain the higher calculated values of importance are those with all the relationship coefficients on high levels, and the penalized ECs are those which present even one correlation coefficient on the lower levels.
An opposite logic may be selected, for example, by exchanging the positions of Min and Max operators in Eq.(2). In this case, ECs with at least one relationship coefficient on high level are preferred. If all the CR importances are on the higher level, the ECs will obtain all the same importance in the lower level. If all the CR importances are on the lower level, the preferred ECs are those which have at least one relationship coefficient with high level.
Also the logic in Eq.(4) may be changed according to the objective of the decision maker. In the present case, ECs with the higher number of relationship coefficients over the obtained value of the relevant importance are preferred, without taking into account the importance level of related CRs, which at this decision stage are not considered significant, since their contribution has just been considered in the first step using Eq.(2). As an alternative logic, it may be decided to assign high positions in the final ranking only to those ECs which are correlated to CRs with maximum importance.

## 4 Application examples

Let us consider the example of a design of a new model of climbing safety harness. This example is often considered in the scientific literature and may represent an helpful benchmark for the application of the proposed method [Hunt, 2013; Franceschini et al., 2014].
The CRs and ECs identified during customer interviews and the technical analysis are reported respectively in Tabs. 2 and 3.

| Customer Requirements (CRs) |  |  |
| :---: | :---: | :---: |
|  | Easy to put on | $\mathrm{CR}_{1}$ |
|  | Confortable when hanging | $\mathrm{CR}_{2}$ |
|  | Fits over different clothes | $\mathrm{CR}_{3}$ |
|  | Accessible gear loops | $\mathrm{CR}_{4}$ |
|  | Does not restrict movement | $\mathrm{CR}_{5}$ |
|  | Lightweight | $\mathrm{CR}_{6}$ |
|  | Safe | $\mathrm{CR}_{7}$ |
|  | Attractive | $\mathrm{CR}_{8}$ |

Table 2. CRs for the design of a new model of climbing safety harness [Hunt, 2013].

| Engineering Characteristics (ECs) |  |
| :---: | :---: |
| Meets safety standards | $\mathrm{EC}_{1}$ |
| Harness weight | $\mathrm{EC}_{2}$ |
| Webbing strength | $\mathrm{EC}_{3}$ |
| No. of clours | $\mathrm{EC}_{4}$ |
| No. of sizes | $\mathrm{EC}_{5}$ |
| Padding thickness | $\mathrm{EC}_{6}$ |
| No. of gear loops | $\mathrm{EC}_{7}$ |

Table 3. ECs for the design of a new model of climbing safety harness [Hunt, 2013].
Different choices of $s$ (i.e. the number of levels of the ordinal scale) have an obvious impact on the results of the HoQ analysis. For that reason, four distinct situations are analyzed and discussed in the following sub-sections.
For all the analyzed cases, the relevant HoQ have been constructed by the same QFD design team, hence guaranteeing complete coherence between the CR importances and the relationship coefficients considered in the different situations and evaluated using scales with different number of levels.

### 4.1 CR importances and relationship coefficients expressed on a 3-level scale

Considering the correspondence map between CR importances and relationship coefficients expressed on a 3-level ordinal scale ( $s=3$ ) reported in Tab.1, the HoQs reported in Figs. 3 and 4 are obtained.


Figure 3. Traditional HoQ for the design of a new model of climbing safety harness. See Tabs.1, 2 and 3 for the meaning of the symbols.

|  |  | $d_{i}$ | Engineering Characteristics |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{EC}_{1}$ | $\mathrm{EC}_{2}$ | $\mathrm{EC}_{3}$ | $\mathrm{EC}_{4}$ | EC5 | $\mathrm{EC}_{6}$ | $\mathrm{EC}_{7}$ |
|  | $\mathrm{CR}_{1}$ |  | $L_{3}$ | $L_{1}$ | $L_{2}$ | $L_{1}$ | $L_{1}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ |
|  | $\mathrm{CR}_{2}$ | $L_{3}$ | $L_{1}$ | $L_{2}$ | $L_{1}$ | $L_{1}$ | $L_{3}$ | $L_{3}$ | $L_{1}$ |
|  | $\mathrm{CR}_{3}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{2}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ |
|  | $\mathrm{CR}_{4}$ | $L_{2}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{3}$ |
|  | $\mathrm{CR}_{5}$ | $L_{3}$ | $L_{2}$ | $L_{2}$ | $L_{3}$ | $L_{1}$ | $L_{3}$ | $L_{2}$ | $L_{1}$ |
|  | $\mathrm{CR}_{6}$ | $L_{2}$ | $L_{1}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{2}$ | $L_{2}$ |
|  | $\mathrm{CR}_{7}$ | $L_{3}$ | $L_{3}$ | $L_{1}$ | $L_{2}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ |
|  | $\mathrm{CR}_{8}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ | $L_{2}$ |

Figure 4. HoQ for the design of a new model of climbing safety harness, derived from Fig. 3 and using a 3-level ordinal scale both for CR importances and relationship coefficients. See Tabs.1, 2 and 3 for the meaning of the symbols.

According to Eq. (3), the negations of a 3-point ordinal scale are:

$$
\operatorname{Neg}\left(L_{1}\right)=L_{3}, \quad \operatorname{Neg}\left(L_{2}\right)=L_{2}, \quad \operatorname{Neg}\left(L_{3}\right)=L_{1} .
$$

Hence, the importance of $\mathrm{EC}_{1}$ may be computed according to Eq. (2) in the following way:

$$
\begin{aligned}
w_{1} & =\operatorname{Min}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 1}\right]\right\}= \\
& =\operatorname{Min}\left\{\begin{array}{l}
\operatorname{Max}\left[\operatorname{Neg}\left(L_{3}\right), L_{1}\right], \operatorname{Max}\left[\operatorname{Neg}\left(L_{3}\right), L_{1}\right], \operatorname{Max}\left[\operatorname{Neg}\left(L_{1}\right), L_{1}\right], \operatorname{Max}\left[\operatorname{Neg}\left(L_{2}\right), L_{1}\right], \\
\operatorname{Max}\left[\operatorname{Neg}\left(L_{3}\right), L_{2}\right], \operatorname{Max}\left[\operatorname{Neg}\left(L_{2}\right), L_{1}\right], \operatorname{Max}\left[\operatorname{Neg}\left(L_{3}\right), L_{3}\right], \operatorname{Max}\left[\operatorname{Neg}\left(L_{1}\right), L_{1}\right]
\end{array}\right\}= \\
& =\operatorname{Min}\left\{\begin{array}{l}
\operatorname{Max}\left[L_{1}, L_{1}\right], \operatorname{Max}\left[L_{1}, L_{1}\right], \operatorname{Max}\left[L_{3}, L_{1}\right], \operatorname{Max}\left[L_{2}, L_{1}\right], \\
\operatorname{Max}\left[L_{1}, L_{2}\right], \operatorname{Max}\left[L_{2}, L_{1}\right], \operatorname{Max}\left[L_{1}, L_{3}\right], \operatorname{Max}\left[L_{3}, L_{1}\right]
\end{array}\right\}= \\
& =\operatorname{Min}\left\{L_{1}, L_{1}, L_{3}, L_{2}, L_{2}, L_{2}, L_{3}, L_{3}\right\}=L_{1}
\end{aligned}
$$

The importances for the other ECs may be computed in the same way, obtaining the following result:

$$
\begin{aligned}
& w_{2}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 2}\right]\right\}=L_{1} \\
& w_{3}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 3}\right]\right\}=L_{1} \\
& w_{4}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 4}\right]\right\}=L_{1} \\
& w_{5}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 5}\right]\right\}=L_{1} \\
& w_{6}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 6}\right]\right\}=L_{1} \\
& w_{7}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 7}\right]\right\}=L_{1}
\end{aligned}
$$

In this specific case all the ECs obtain the same importance, hence the ranking obtained after this step is:

$$
\mathrm{EC}_{1} \approx \mathrm{EC}_{2} \approx \mathrm{EC}_{3} \approx \mathrm{EC}_{4} \approx \mathrm{EC}_{5} \approx \mathrm{EC}_{6} \approx \mathrm{EC}_{7}
$$

This "flattening effect" is mainly due to the low discriminating power of the method when applying Eq. (2) to scales with an exiguous number of levels.
With the aim of discriminating the EC relative ranking, the corresponding values of the $T\left(\mathrm{EC}_{j}\right)$ indicator may be computed:

```
\(T\left(\mathrm{EC}_{1}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{1}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 1}>w_{1}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{5}, \mathrm{CR}_{7}\right\}\right]=2\)
\(T\left(\mathrm{EC}_{2}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{2}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 2}>w_{2}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{1}, \mathrm{CR}_{2}, \mathrm{CR}_{5}, \mathrm{CR}_{6}\right\}\right]=4\)
\(T\left(\mathrm{EC}_{3}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{3}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 3}>w_{3}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{5}, \mathrm{CR}_{7}\right\}\right]=2\)
\(T\left(\mathrm{EC}_{4}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{4}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 4}>w_{4}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{3}, \mathrm{CR}_{8}\right\}\right]=2\)
\(T\left(\mathrm{EC}_{5}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{5}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 5}>w_{5}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{1}, \mathrm{CR}_{2}, \mathrm{CR}_{3}, \mathrm{CR}_{5}\right\}\right]=4\)
\(T\left(\mathrm{EC}_{6}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{6}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 6}>w_{6}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{2}, \mathrm{CR}_{5}, \mathrm{CR}_{6}\right\}\right]=3\)
\(T\left(\mathrm{EC}_{7}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{7}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 7}>w_{7}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{4}, \mathrm{CR}_{6}, \mathrm{CR}_{8}\right\}\right]=3\)
```

The final ranking of ECs is:
$\mathrm{EC}_{2} \approx \mathrm{EC}_{5} \succ \mathrm{EC}_{6} \approx \mathrm{EC}_{7} \succ \mathrm{EC}_{1} \approx \mathrm{EC}_{3} \approx \mathrm{EC}_{4}$
In this way the QFD design team obtains the level of attention to devote to each EC for the technical design of a new climbing safety harness.
4.2 CR importances and relationship coefficients expressed on a 10-level scale

Consider now the correspondence map between CR importances and relationship coefficients expressed on a 10-level ordinal scale ( $s=10$ ) reported in Tab.4. The related HoQs are sketched in Figs. 5 and 6.

| Scale <br> level | CR importance <br> $\left(d_{\boldsymbol{i}}\right)$ | Importance <br> value | Relationship coefficient <br> $\left(r_{i j}\right)$ | Symbol |
| :---: | :---: | :---: | :---: | :---: |
| $L_{1}$ | not important | 1 | no relationship | (empty <br> cell) |
| $L_{2}$ | $\ldots$ | 2 | $\ldots$ | $\diamond$ |
| $L_{3}$ | $\ldots$ | 3 | $\ldots$ | $\ominus$ |
| $L_{4}$ | moderately important | 4 | medium relationship | $\diamond$ |
| $L_{5}$ | $\ldots$ | 5 | $\ldots$ | $\square$ |
| $L_{6}$ | $\ldots$ | 6 | $\ldots$ | $\boxtimes$ |
| $L_{7}$ | important | 7 | strong relationship | $\square$ |
| $L_{8}$ | $\ldots$ | 8 | $\ldots$ | $\bigcirc$ |
| $L_{9}$ | $\ldots$ | 9 | $\ldots$ | $\otimes$ |
| $L_{10}$ | very important | 10 | very strong relationship | $\bullet$ |

Table 4. Correspondence map between CR importances and relationship coefficients expressed on a 10 -level ordinal scale ( $s=10$ ).


Figure 5. Traditional HoQ for the design of a new model of climbing safety harness See Tabs. 2, 3 and 4 for the meaning of the symbols.

|  |  | $d_{i}$ | Engineering Characteristics |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{EC}_{1}$ | $\mathrm{EC}_{2}$ | $\mathrm{EC}_{3}$ | $\mathrm{EC}_{4}$ | $\mathrm{EC}_{5}$ | $\mathrm{EC}_{6}$ | $\mathrm{EC}_{7}$ |
|  | $\mathrm{CR}_{1}$ |  | $L_{9}$ | $L_{3}$ | $L_{3}$ | $L_{2}$ | $L_{1}$ | $L_{9}$ | $L_{3}$ | $L_{2}$ |
|  | $\mathrm{CR}_{2}$ | $L_{8}$ | $L_{2}$ | $L_{5}$ | $L_{2}$ | $L_{1}$ | $L_{9}$ | $L_{9}$ | $L_{1}$ |
|  | $\mathrm{CR}_{3}$ | $L_{2}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{4}$ | $L_{8}$ | $L_{2}$ | $L_{1}$ |
|  | $\mathrm{CR}_{4}$ | $L_{5}$ | $L_{2}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{10}$ |
|  | $\mathrm{CR}_{5}$ | $L_{9}$ | $L_{7}$ | $L_{4}$ | $L_{10}$ | $L_{1}$ | $L_{10}$ | $L_{7}$ | $L_{1}$ |
|  | $\mathrm{CR}_{6}$ | $L_{7}$ | $L_{3}$ | $L_{10}$ | $L_{1}$ | $L_{1}$ | $L_{2}$ | $L_{7}$ | $L_{7}$ |
|  | $\mathrm{CR}_{7}$ | $L_{10}$ | $L_{10}$ | $L_{3}$ | $L_{6}$ | $L_{1}$ | $L_{2}$ | $L_{1}$ | $L_{2}$ |
|  | $\mathrm{CR}_{8}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{10}$ | $L_{1}$ | $L_{2}$ | $L_{7}$ |

Figure 6. HoQ for the design of a new model of climbing safety harness, derived from Fig. 5 and using a 10-level ordinal scale both for CR importances and relationship coefficients. See Tabs. 2, 3 and 4 for the meaning of the symbols.

According to Eq. (3), the negations of a 10-point ordinal scale are:

$$
\begin{array}{lllll}
\operatorname{Neg}\left(L_{1}\right)=L_{10}, & \operatorname{Neg}\left(L_{2}\right)=L_{9}, & \operatorname{Neg}\left(L_{3}\right)=L_{8}, & \operatorname{Neg}\left(L_{4}\right)=L_{7}, & \operatorname{Neg}\left(L_{5}\right)=L_{6}, \\
\operatorname{Neg}\left(L_{6}\right)=L_{5}, & \operatorname{Neg}\left(L_{7}\right)=L_{4}, & \operatorname{Neg}\left(L_{8}\right)=L_{3}, & \operatorname{Neg}\left(L_{9}\right)=L_{2}, & \operatorname{Neg}\left(L_{10}\right)=L_{1} .
\end{array}
$$

According to Eq. (2) the importances related to each of the 7 ECs are:

$$
\begin{aligned}
& w_{1}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 1}\right]\right\}=L_{3} \\
& w_{2}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 2}\right]\right\}=L_{3} \\
& w_{3}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 3}\right]\right\}=L_{2} \\
& w_{4}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 4}\right]\right\}=L_{1} \\
& w_{5}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 5}\right]\right\}=L_{2} \\
& w_{6}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 6}\right]\right\}=L_{1} \\
& w_{7}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 7}\right]\right\}=L_{2}
\end{aligned}
$$

The obtained ranking is:
$\mathrm{EC}_{1} \approx \mathrm{EC}_{2} \succ \mathrm{EC}_{3} \approx \mathrm{EC}_{5} \approx \mathrm{EC}_{7} \succ \mathrm{EC}_{4} \approx \mathrm{EC}_{6}$
Comparing these results with those of the previous example, it is worth noting that increasing the number of scale levels ( $s$ ) the "flattening effect" of the method tends to disappear with contemporary increase of its discriminating power. However, it must be observed that scales with too many levels may cause ambiguity of interpretation between contiguous levels. For this reason, in the scientific literature it is often suggested to not exceed 5 levels [Franceschini and Rupil, 1999; Franceschini, 2001]. Again, in order to further discriminate among the ECs with the same importance, the $T\left(E C_{j}\right)$ indicator may be computed according to Eq. (4):

```
\(T\left(\mathrm{EC}_{1}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{1}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 1}>w_{1}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{9}, \mathrm{CR}_{10}\right\}\right]=2\)
\(T\left(\mathrm{EC}_{2}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{2}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 2}>w_{2}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{2}, \mathrm{CR}_{7}, \mathrm{CR}_{9}\right\}\right]=3\)
\(T\left(\mathrm{EC}_{3}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{3}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 3}>w_{3}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{5}, \mathrm{CR}_{7}\right\}\right]=2\)
\(T\left(\mathrm{EC}_{4}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{4}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 4}>w_{4}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{3}, \mathrm{CR}_{8}\right\}\right]=2\)
\(T\left(\mathrm{EC}_{5}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{5}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 5}>w_{5}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{1}, \mathrm{CR}_{2}, \mathrm{CR}_{3}, \mathrm{CR}_{5}\right\}\right]=4\)
\(T\left(\mathrm{EC}_{6}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{6}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 6}>w_{6}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{1}, \mathrm{CR}_{2}, \mathrm{CR}_{3}, \mathrm{CR}_{5}, \mathrm{CR}_{6}, \mathrm{CR}_{8}\right\}\right]=6\)
\(T\left(\mathrm{EC}_{7}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{7}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 7}>w_{7}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{4}, \mathrm{CR}_{7}, \mathrm{CR}_{8}\right\}\right]=3\)
```

The new final ranking is:
$\left(\mathrm{EC}_{2} \succ \mathrm{EC}_{1}\right) \succ\left(\mathrm{EC}_{5} \succ \mathrm{EC}_{7} \succ \mathrm{EC}_{3}\right) \succ\left(\mathrm{EC}_{6} \succ \mathrm{EC}_{4}\right)$
A first interesting result with this second codification is that, even if the HoQ in Fig. 5 is coherent with that in Fig. 3 (that is, relationship coefficients and CR importances of Fig. 5 are obtaining by splitting in a further detail the corresponding coefficients and

CR importances in Fig. 3), some significant rank reversal of ECs are observed. See, for example, $\mathrm{EC}_{1}$ and $\mathrm{EC}_{6}$.

### 4.3 CR importances and relationship coefficients expressed on a 5-level scale

This case considers the situation in which both CR importances and relationship coefficients are expressed on a 5-level ordinal scale ( $s=5$ ) (see Tab.5).
The choice of this number of scale levels is the result of a tradeoff aimed at avoiding the discussed flattening effect while concurrently preventing the problem of disambiguation between contiguous levels.
The related HoQs are reported in Figs. 7 and 8.

| Scale <br> level | CR importance <br> $\left(d_{i}\right)$ | Importance <br> value | Relationship coefficient <br> $\left(r_{i j}\right)$ | Symbol |
| :---: | :---: | :---: | :---: | :---: |
| $L_{1}$ | not important | 1 | no relationship | (empty cell) |
| $L_{2}$ | weakly important | 2 | weak relationship | $\square$ |
| $L_{3}$ | moderately important | 3 | medium relationship | $\square$ |
| $L_{4}$ | important | 4 | strong relationship | 0 |
| $L_{5}$ | very important | 5 | very strong relationship | $\square$ |

Table 5. Correspondence map between CR importances and relationship coefficients expressed on a 5-level ordinal scale ( $s=5$ ).

|  |  | $d_{i}$ |
| :---: | :---: | :---: |
|  | $\mathrm{CR}_{1}$ | 5 |
|  | $\mathrm{CR}_{2}$ | 4 |
|  | $\mathrm{CR}_{3}$ | 1 |
|  | $\mathrm{CR}_{4}$ | 3 |
|  | $\mathrm{CR}_{5}$ | 5 |
|  | $\mathrm{CR}_{6}$ | 4 |
|  | $\mathrm{CR}_{7}$ | 5 |
|  | $\mathrm{CR}_{8}$ | 2 |


| Engineering Characteristics |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{EC}_{1}$ | $\mathrm{EC}_{2}$ | $\mathrm{EC}_{3}$ | $\mathrm{EC}_{4}$ | $\mathrm{EC}_{5}$ | $\mathrm{EC}_{6}$ | $\mathrm{EC}_{7}$ |


| $\square$ | $\square$ |  |  | 0 | $\square$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\square$ |  |  | 0 | 0 |  |
|  |  |  | $\square$ | 0 |  |  |
|  |  |  |  |  |  | 0 |
| 0 | $\square$ | 0 |  | 0 | 0 |  |
| $\square$ | 0 |  |  |  | 0 | 0 |
| 0 | $\square$ | $\square$ |  |  |  |  |
|  |  |  | 0 |  |  | 0 |

Figure 7. Traditional HoQ for the design of a new model of climbing safety harness. See Tabs. 2, 3 and 5 for the meaning of the symbols.

|  |  | $d_{i}$ | Engineering Characteristics |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | EC ${ }_{1}$ | $\mathrm{EC}_{2}$ | $\mathrm{EC}_{3}$ | $\mathrm{EC}_{4}$ | EC5 | $\mathrm{EC}_{6}$ | $\mathrm{EC}_{7}$ |
|  | $\mathrm{CR}_{1}$ |  | $L_{5}$ | $L_{2}$ | $L_{2}$ | $L_{1}$ | $L_{1}$ | $L_{5}$ | $L_{2}$ | $L_{1}$ |
|  | $\mathrm{CR}_{2}$ | $L_{4}$ | $L_{1}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ | $L_{5}$ | $L_{5}$ | $L_{1}$ |
|  | $\mathrm{CR}_{3}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{2}$ | $L_{4}$ | $L_{1}$ | $L_{1}$ |
|  | $\mathrm{CR}_{4}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{5}$ |
|  | $\mathrm{CR}_{5}$ | $L_{5}$ | $L_{4}$ | $L_{2}$ | $L_{5}$ | $L_{1}$ | $L_{5}$ | $L_{4}$ | $L_{1}$ |
|  | $\mathrm{CR}_{6}$ | $L_{4}$ | $L_{2}$ | $L_{5}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{4}$ | $L_{4}$ |
|  | $\mathrm{CR}_{7}$ | $L_{5}$ | $L_{5}$ | $L_{2}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ |
|  | $\mathrm{CR}_{8}$ | $L_{2}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{5}$ | $L_{1}$ | $L_{1}$ | $L_{4}$ |

Figure 8. HoQ for the design of a new model of climbing safety harness, derived from Fig. 7 and using a 5 -level ordinal scale both for CR importances and relationship coefficients. See Tabs. 2, 3 and 5 for the meaning of the symbols.

According to Eq. (3), the negations of a 5-point ordinal scale are:

$$
\operatorname{Neg}\left(L_{1}\right)=L_{5}, \quad \operatorname{Neg}\left(L_{2}\right)=L_{4}, \quad \operatorname{Neg}\left(L_{3}\right)=L_{3}, \quad \operatorname{Neg}\left(L_{4}\right)=L_{2}, \quad \operatorname{Neg}\left(L_{5}\right)=L_{1} .
$$

Hence, according to Eq. (2), the obtained EC importances are:
$w_{1}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 1}\right]\right\}=L_{2}$
$w_{2}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 2}\right]\right\}=L_{2}$
$w_{3}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 3}\right]\right\}=L_{1}$
$w_{4}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 4}\right]\right\}=L_{1}$
$w_{5}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 5}\right]\right\}=L_{1}$
$w_{6}=\operatorname{Min}_{i=1 . . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 6}\right]\right\}=L_{1}$
$w_{7}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 7}\right]\right\}=L_{1}$
and the obtained ranking is:
$\mathrm{EC}_{1} \approx \mathrm{EC}_{2} \succ \mathrm{EC}_{3} \approx \mathrm{EC}_{4} \approx \mathrm{EC}_{5} \approx \mathrm{EC}_{6} \approx \mathrm{EC}_{7}$
The related values of $T\left(\mathrm{EC}_{j}\right)$ indicator are (see Eq. (4)):

$$
\begin{aligned}
& T\left(\mathrm{EC}_{1}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{1}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 1}>w_{1}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{5}, \mathrm{CR}_{7}\right\}\right]=2 \\
& T\left(\mathrm{EC}_{2}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{2}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 2}>w_{2}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{2}, \mathrm{CR}_{6}\right\}\right]=2 \\
& T\left(\mathrm{EC}_{3}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{3}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 3}>w_{3}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{5}, \mathrm{CR}_{7}\right\}\right]=2 \\
& T\left(\mathrm{EC}_{4}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{4}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 4}>w_{4}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{3}, \mathrm{CR}_{8}\right\}\right]=2 \\
& T\left(\mathrm{EC}_{5}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{5}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 5}>w_{5}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{1}, \mathrm{CR}_{2}, \mathrm{CR}_{3}, \mathrm{CR}_{5}\right\}\right]=4 \\
& T\left(\mathrm{EC}_{6}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{6}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 6}>w_{6}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{1}, \mathrm{CR}_{2}, \mathrm{CR}_{5}, \mathrm{CR}_{6}\right\}\right]=4 \\
& T\left(\mathrm{EC}_{7}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{7}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 7}>w_{7}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{4}, \mathrm{CR}_{6}, \mathrm{CR}_{8}\right\}\right]=3
\end{aligned}
$$

The new final ranking is:
$\left(\mathrm{EC}_{1} \approx \mathrm{EC}_{2}\right) \succ\left(\mathrm{EC}_{5} \approx \mathrm{EC}_{6} \succ \mathrm{EC}_{7} \succ \mathrm{EC}_{3} \approx \mathrm{EC}_{4}\right)$
Also in this case a first interesting result with this codification is that, even if the HoQ in Fig. 7 is coherent with that in Figs. 3, and 5 (that is, relationship coefficients and CR importances of Fig. 7 are obtaining by splitting in a further detail the corresponding coefficients and CR importances in Fig. 3), a significant rank reversal is observed.

### 4.4 CR importances and relationship coefficients evaluated on scales with different number of levels

In typical QFD applications, CR importances and relationship coefficients may be coded on different ordinal scales. Specifically, CR importances are usually evaluated on a 5-level scale (see Tab. 6), while for relationship coefficients a symbolic 4-level ordered scale is often used (see Tab. 7) [Akao, 1988; Franceschini, 2001].

| Scale <br> level | CR importance <br> $\left(d_{\boldsymbol{i}}\right)$ | Importance <br> value |
| :---: | :---: | :---: |
| $L_{1}$ | not important | 1 |
| $L_{2}$ | weakly important | 2 |
| $L_{3}$ | moderately important | 3 |
| $L_{4}$ | important | 4 |
| $L_{5}$ | very important | 5 |

Table 6. Correspondence map of CR importances evaluated on a 5-level ordinal scale ( $s=5$ ) [Akao, 1988; Franceschini, 2001].

| Relationship coefficient <br> $\left(r_{i j}\right)$ | Symbol |
| :---: | :---: |
| no relationship | (empty cell) |
| weak relationship | $\Delta$ |
| medium relationship | 0 |
| strong relationship | 0 |

Table 7. Example of relationship coefficients evaluated on a symbolic 4-level ordinal scale $(s=4)$.

This case may not be faced simply using the aggregation method proposed in Eq. (2). For this kind of problem more complicated algorithms must be introduced [Yager and Filev, 1994]. A practical approximated solution to this problem may be obtained by coding with the same scale levels two or more contiguous importance levels or by using an adapted mapping of the relationship coefficient. This second solution is implemented in the following example (see Tab. 8). Of course, this approach leaves a discretionary power to the QFD design team in the definition of the criteria for the adapted mapping of the CR importances or of the relationship coefficients. These criteria must be previously defined according to the meaning of the information collected in the HoQ and the interpretation that must be attributed to data aggregation. It must be said that this level of discretionary power, if correctly managed, highlights the versatility of the proposed method, which is one of its main advantages.

| Scale level | Relationship coefficient <br> $\left(r_{i j}\right)$ | Symbol |
| :---: | :---: | :---: |
| $L_{1}$ | no relationship | (empty cell) |
| $L_{2}$ | no relationship | (empty cell) |
| $L_{3}$ | weak relationship | $\Delta$ |
| $L_{4}$ | medium relationship | $\bigcirc$ |
| $L_{5}$ | strong relationship |  |

Table 8. Example of a possible correspondence map of the relationship coefficients evaluated on a symbolic 4-level ordinal scale ( $s=4$ ).

According to the mappings of Tabs. 6 and 8, the HoQs for the design of a new model of climbing safety harness reported in Figs. 9 and 10 are prepared.

|  |  | $d_{i}$ | Engineering Characteristics |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{EC}_{1}$ | $\mathrm{EC}_{2}$ | $\mathrm{EC}_{3}$ | $\mathrm{EC}_{4}$ | $\mathrm{EC}_{5}$ | $\mathrm{EC}_{6}$ | $\mathrm{EC}_{7}$ |
|  | $\mathrm{CR}_{1}$ |  | 4 |  |  |  |  | 0 |  |  |
|  | $\mathrm{CR}_{2}$ | 4 |  | $\Delta$ |  |  | 0 | 0 |  |
|  | $\mathrm{CR}_{3}$ | 2 |  |  |  | $\Delta$ | 0 |  |  |
|  | $\mathrm{CR}_{4}$ | 3 |  |  |  |  |  |  | $\bigcirc$ |
|  | $\mathrm{CR}_{5}$ | 4 | 0 | $\Delta$ | $\bigcirc$ |  | $\bigcirc$ | 0 |  |
|  | $\mathrm{CR}_{6}$ | 3 |  | - |  |  |  | 0 | 0 |
|  | $\mathrm{CR}_{7}$ | 5 | - |  | $\Delta$ |  |  |  |  |
|  | $\mathrm{CR}_{8}$ | 2 |  |  |  | $\bigcirc$ |  |  | 0 |

Figure 9. Traditional HoQ for the design of a new model of climbing safety harness. See Tabs.2, 3, 6 and 8 for the meaning of the symbols.


| $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{4}$ | $L_{1}$ | $L_{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{1}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ | $L_{4}$ | $L_{4}$ | $L_{1}$ |
| $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{3}$ | $L_{4}$ | $L_{1}$ | $L_{1}$ |
| $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{5}$ |
| $L_{4}$ | $L_{3}$ | $L_{5}$ | $L_{1}$ | $L_{5}$ | $L_{4}$ | $L_{1}$ |
| $L_{1}$ | $L_{5}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{4}$ | $L_{4}$ |
| $L_{5}$ | $L_{1}$ | $L_{3}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{1}$ |
| $L_{1}$ | $L_{1}$ | $L_{1}$ | $L_{5}$ | $L_{1}$ | $L_{1}$ | $L_{4}$ |

Figure 10. HoQ for the design of a new model of climbing safety harness, derived from Fig. 9 and using a 5-level ordinal scale for CR importances and an adapted 4-level ordinal scale for relationship coefficients. See Tabs.2, 3, 6 and 8 for the meaning of the symbols.

The negations of a 5-point ordinal scale have just been reported in the example of Section 4.3.
By Eq. (2) the EC importances become the following:
$w_{1}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 1}\right]\right\}=L_{2}$
$w_{2}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 2}\right]\right\}=L_{1}$
$w_{3}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 3}\right]\right\}=L_{2}$
$w_{4}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 4}\right]\right\}=L_{1}$
$w_{5}=\operatorname{Min}_{i=1 \ldots 8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 5}\right]\right\}=L_{1}$
$w_{6}=\operatorname{Min}_{i=1 . . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 6}\right]\right\}=L_{1}$
$w_{7}=\operatorname{Min}_{i=1 . .8}\left\{\operatorname{Max}\left[\operatorname{Neg}\left(d_{i}\right), r_{i 7}\right]\right\}=L_{1}$
The obtained ranking is:
$\mathrm{EC}_{1} \approx \mathrm{EC}_{3} \succ \mathrm{EC}_{2} \approx \mathrm{EC}_{4} \approx \mathrm{EC}_{5} \approx \mathrm{EC}_{6} \approx \mathrm{EC}_{7}$
And, using Eq. (4), the $T\left(\mathrm{EC}_{j}\right)$ indicator may be computed:

$$
\begin{aligned}
& T\left(\mathrm{EC}_{1}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{1}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 1}>w_{1}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{5}, \mathrm{CR}_{7}\right\}\right]=2 \\
& T\left(\mathrm{EC}_{2}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{2}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 2}>w_{2}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{2}, \mathrm{CR}_{5}, \mathrm{CR}_{6}\right\}\right]=3 \\
& T\left(\mathrm{EC}_{3}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{3}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 3}>w_{3}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{5}, \mathrm{CR}_{7}\right\}\right]=2 \\
& T\left(\mathrm{EC}_{4}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{4}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 4}>w_{4}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{3}, \mathrm{CR}_{5}\right\}\right]=2 \\
& T\left(\mathrm{EC}_{5}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{5}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 5}>w_{5}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{1}, \mathrm{CR}_{2}, \mathrm{CR}_{3}, \mathrm{CR}_{5}\right\}\right]=4 \\
& T\left(\mathrm{EC}_{6}\right)=\operatorname{Dim}\left[\mathrm{A}\left(\mathrm{EC}_{6}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 6}>w_{6}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{2}, \mathrm{CR}_{5}, \mathrm{CR}_{6}\right\}\right]=3 \\
& T\left(\mathrm{EC}_{7}\right)=\operatorname{Dim}\left[\mathrm{A}_{2}\left(\mathrm{EC}_{7}\right)\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{i} \mid r_{i 7}>w_{7}\right\}\right]=\operatorname{Dim}\left[\left\{\mathrm{CR}_{4}, \mathrm{CR}_{6}, \mathrm{CR}_{8}\right\}\right]=3
\end{aligned}
$$

The new final ranking is:

$$
\left(\mathrm{EC}_{1} \approx \mathrm{EC}_{3}\right) \succ\left(\mathrm{EC}_{5} \succ \mathrm{EC}_{2} \approx \mathrm{EC}_{6} \approx \mathrm{EC}_{7} \succ \mathrm{EC}_{4}\right)
$$

Also in this case a first interesting result with this codification is that, even if the HoQ in Fig. 9 is coherent with that in Figs. 3, 5 and 7 (that is, relationship coefficients and CR importances of Fig. 9 are obtaining by splitting in a further detail the corresponding coefficients and CR importances in Fig. 3), a significant rank reversal is observed comparing the previous codifications.

## 5 Discussion

Analyzing the results of Section 4 and considering the mathematical properties of the proposed method, the following observations may be highlighted:

- Lowering the importance assigned to a specific CR produces a decrease of its influence on the relevant ECs.
- If two or more ECs have the same importance, it is possible to perform a more detailed selection with the help of the $T\left(\mathrm{EC}_{j}\right)$ indicator. In such a way "tie" situations, in which the computed importance gives the same result, may be discriminated.
- The mapping of ECs on the $w_{j}$ scale gives their relative importance only. The absolute value of EC importances is not significant. The goal is to obtain a final ranking of ECs. So, for example, an EC with level $L_{4}$ means that it has a lower importance than an EC with level $L_{5}$ and higher importance of an EC with level $L_{3}$.
- It must be emphasized that, in comparison to traditional approaches (such as for example Independent Scoring Method), the proposed method allows a more flexible procedure for combining CR importances and relationship coefficients, and defining different technical logics of analysis.
- The method may generate a "flattening effect" when applying Eq. (2) to scales with an exiguous number of levels. This suggests the use of scales with higher number of levels, such as, for example, 10 or more. However scales with too many levels may cause difficulty in discrimination between contiguous levels. The scientific literature and the performed tests (see Section 4) suggest 5 -level scale as an acceptable compromise [Franceschini and Rupil, 1999; Franceschini, 2001]. The "flattening effect" after the application of Eq. (2) can also occur when working with large numbers of CRs.
- The importance associated to each EC is defined on a $s$-point ordinal scale similar to that used for the CR importance and the relationship coefficients, and with the same number of levels. As a consequence, the final ordering of the ECs may be obtained on no more than $s$ ordered categories.
- Considering the two steps respectively related to Eqs. (2) and (3), the method tends to flatten upwards all the importance values $w_{j}$ for all the ECs when both CR importances and relationship coefficients have high values. This is coherent with the aim of the method, because it indicates to the QFD design team that all the ECs are important and no one of them must be neglected. In the same way, the method tends to flatten downwards all the computed EC importances when both CR importances and relationship coefficients have low values.
- In the end, it must be stressed that the schemes proposed in Eq. (2) and, subsequently, in Eq.(4) may be replaced by other possible aggregation logics according to specific interpretations of data contained in the Relationship Matrix (for example, it may be decided to assign high positions in the final ranking only to those ECs which are correlated to CRs with maximum importance, and so on). Especially this aspect emphasizes the great flexibility and versatility of the method.


## 6 Conclusions

This paper introduces and discusses the application of a new method to compute the EC prioritization in QFD. Data processing is performed working exclusively on the ordinal features of qualitative scales used to collect information related to CR importances and relationship coefficients between CRs and ECs. The method processing simplicity is comparable with traditional approaches, such as for example Independent Scoring Method.
The main novel elements of the method are:

- it is able to aggregate design information evaluated on ordinal qualitative scales, overcoming the controversial assumptions of data cardinality, usually introduced by other traditional approaches;
- it does not require any arbitrary and artificial "scalarization" of collected information;
- it is able to deal with situations in which both CR importances and relationship coefficients are rated on different ordinal scales;
- it is easily automatable.

Future developments of the methodology will be addressed to the development of specific strategies of aggregation in order to verify its degree of flexibility and versatility.

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