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# Prioritization of QFD Customer Requirements based on the Law of Comparative Judgments 

Fiorenzo Franceschini ${ }^{1}$ and Domenico Maisano<br>${ }^{1}$ fiorenzo.franceschini@polito.it Politecnico di Torino, DIGEP (Department of Management and Production Engineering), Corso Duca degli Abruzzi 24, 10129, Torino (Italy)


#### Abstract

Quality Function Deployment (QFD) is a useful tool to improve the design/development process of products and services. The initial phases of the QFD process - i.e., those concerning the collection and analysis of the so-called Voice of the Customer - are probably the most critical, because any distortion can propagate to the whole process results, making it ineffective or even misleading. The focus of this paper is on the phase of prioritization of customer requirements (CRs). There are numerous techniques for this task; however (i) the simplest often introduce questionable or unrealistic assumptions, while (ii) the most sophisticated often require too much elaborate and repetitious information from customers, which may lead to inconsistencies.

This paper introduces a new prioritization technique based on the Thurstone's Law of Comparative Judgment. This technique makes it possible to aggregate the evaluations by multiple respondents and transform them into an interval scale, which depicts the relative importance of CRs. The greatest strength of this technique is combining a refined theoretical model with a simple and userfriendly data collection process. The description is supported by a realistic application example concerning the prioritization of QFD's CRs in the design of an aircraft seat.


Keywords: QFD, Customer requirements, Prioritization, Relative importance ratings, Law of Comparative Judgment, Thurstone scaling, Interval scale.

## 1. Introduction and literature review

Quality Function Deployment (QFD) is a powerful technique to increase the customer satisfaction of product and services. The implementation of QFD may generate significant improvements in the design/development process, such as fewer and earlier design changes, improved cross-functional communications, improved product/service quality, and reduced development time and cost (Hauser and Clausing, 1988; Griffin and Hauser, 1993; Tran and Sherif, 1995; Franceschini, 2002). These improvements are critical success factors to companies in a global marketplace characterized by intense international competition.

The great diffusion of QFD is demonstrated by the literally thousands of scientific publications illustrating a variety of industrial applications, methodological improvements, new variants, and possible integration with other tools.

Typically, QFD utilizes four sets of matrices - the so called Houses of Quality (HoQs). The four HoQs respectively translate (i) customer requirements (CRs) into engineering characteristics and, in turn, into (ii) parts characteristics, (iii) process plans, and (iv) production requirements (Franceschini, 2002). For detailed information, we refer the reader to the vast literature and extensive reviews, e.g., (Chan and Wua, 2002; Sharma et al., 2008).
The customer input, also defined as Voice of the Customer (VoC), is the key starting point for QFD process; if it does not accurately reflect what the customer expects from the product/service of interest, the process may lead to incorrect conclusions (Sireli et al, 2007). Therefore, the first HoQ, also defined as Product Planning HoQ, is of fundamental and strategic importance (Gonzalez et al. 2003). The Product Planning HoQ construction process can be summarized into ten phases, as shown in Fig. 1.


Fig. 1. Main phases of the Product Planning HoQ construction process.
Among these phases - described in detail in the literature, e.g., see (Chan and Wua, 2002; Franceschini, 2002; Franceschini and Rossetto, 2002) - particularly significant are those related to the VoC collection and analysis. The initial phase (i.e., Phase 1, "Customer Requirements", in the scheme in Fig. 1) concerns the VoC collection -through interviews and questionnaires - and analysis, in order to determine an exhaustive list of CRs. For this task, it is necessary to select a representative sample of (potential) customers, with reasonable knowledge of the product/service to be designed. It was found empirically that samples consisting of 20 to 30 respondents are sufficient to cover most CRs; also, for data collected to be reasonable and applicable, respondents have to gain a full understanding of the task required (Urban and Hauser, 1993).
In the second part of this phase, a cross-functional team of experts - composed of members from
marketing, design, quality, finance and production - have to review, reorganize and insert the CRs into the Product Planning HoQ.
The next stage, which is the focus of this paper, is that of the prioritization of CRs (i.e., Phase 2, "Relative Importance Ratings", in the scheme in Fig. 1), presuming that the main CRs related to the product/service to be designed have already been identified in Phase 1. The expression "Relative Importance Ratings" indicates that this prioritization is aimed at discriminating a CR based on its importance over the others. On the other hand, Phase 4, "Final Importance Ratings" (in Fig. 1), denotes a prioritization that also takes into account the comparison of quality performance of the products/services of the company and those of its competitors.
In Phase 2, a sample of customers - generally the same involved in Phase 1 - have to prioritize the QFD's CRs using several possible approaches. Some of them are point direct scoring method (Hauser and Clausing, 1988; Griffin and Hauser, 1993), analytic hierarchy process (AHP) (Li et al., 2009; Chuang, P.T., 2001), analytic network process (ANP) (Karsak et al., 2002; Lee et al., 2008), outranking methods (Franceschini and Rossetto, 1995; Figueira et al., 2005), fuzzy variants (Chan et al., 1999; Kwong and Bai, 2002; Buyukozkan et al., 2004), and techniques derived from the Kano model (Matzler and Hinterhuber, 1998; Sireli et al., 2007; Chaudha et al., 2011). Without going into these techniques in detail, we remark that they may use different kinds of response data and elaborations from respondents. Even though all these techniques are supposed to reflect the VoC, sometimes they may lead to misleading results, especially when the data collection approach is too complex and elaborate. Here are some examples:

- Techniques based on the AHP and ANP method require CR judgments in the form of paired comparison data, defined on a ratio scale; e.g. " $\mathrm{CR}_{1}$ is twice as important as $\mathrm{CR}_{2}$ " (Chuang, 2001; Franceschini, 2002; Kwong and Bai, 2002; Lee et al., 2008 , Li te al., 2009). These evaluations are inevitably arbitrary and subjective as respondents may find it difficult to express their judgments on this scale. Techniques that integrate the Kano model in the QFD environment require relatively complex questionnaires (Nahm et al., 2013) and the definition of arbitrary weights for the (qualitative) Kano categories (i.e., basic or must-be (B), one dimensional (O), attractive $(\mathrm{A})$, indifferent $(\mathrm{I})$, reverse $(\mathrm{R})$ and questionable $(\mathrm{Q})$ (Tan and Shen, 2000)).
- Other sophisticated techniques for the CR prioritization, such as that proposed by Nahm et al. (2013), model the uncertainty in customer requirements, taking into account the uncertainty of customer's judgment. Unfortunately, they generally include complex and structured questionnaires and, sometimes, introduce questionable assumptions in the response data processing.
- In the classical questionnaires for prioritizing QFD's CRs, respondent judgements are defined on a 5 -level rating response scale ( $1=$ Not at all important, 2=Low importance, 3=Medium
importance, $4=$ High importance and $5=$ Very high importance). This response scale has two inherent limitations:

1. Since it is an ordinal scale, it only allows comparisons like " $\mathrm{CR}_{1}$ is more important than $\mathrm{CR}_{2}$ ". Unfortunately, a typical abuse is "promoting" this scale to an interval or even ratio scale, so as to make incorrect comparisons like "the distance, in terms of importance, between $\mathrm{CR}_{1}$ and $\mathrm{CR}_{2}$ is greater than that between $\mathrm{CR}_{3}$ and $\mathrm{CR}_{4}$ " or " $\mathrm{CR}_{1}$ is three times more important than CR2" (Stevens, 1946; Berko, Kloeber, Deckro, 2002; Franceschini, 2007).
2. These scales are used subjectively, as there is no absolute reference shared by all respondents. In general, "indulgent" respondents will tend to assign higher levels of importance, while "severe" respondents will tend to assign lower ones. For example, let us consider the ratings about three CRs (i.e., $\mathrm{CR}_{1}, \mathrm{CR}_{2}$ and $\mathrm{CR}_{3}$ ) by two fictitious respondents ( A and B ). These ratings on a 5 -level scale are respectively A: 3, 2, 1, and B: 5, 4, 2. Despite the relative rankings are identical (i.e., $\mathrm{CR}_{1}>\mathrm{CR}_{2}>\mathrm{CR}_{3}$ ), judgments by A (severe respondent) are concentrated in the lowest levels of the scale, while those of B (indulgent respondent) in the highest. For this reason, it is questionable to aggregate judgments by different respondents through indicators of central tendency, such as the median or the mean value.

The objective of this paper is to introduce a simple technique for the CR prioritization, based on the so-called Thurstone's "Law of Comparative Judgement", for aggregating the judgments by multiple respondents and transforming them into a numerical interval scale (Thurstone, 1927).

An important benefit of this technique is combining a simple and user-friendly data collection process - based on the definition of respondent judgements on a 5-level ordinal scale - with a refined theoretical model.
The remainder of this paper is structured into three sections. Sect. 2 provides some background information, which is helpful to grasp the logic of the novel prioritization technique: (i) basic concepts concerning the Thurstone model and (ii) description of a process for deriving response data suitable to this model, keeping data collection as simple and user-friendly as possible. Sect. 3 shows a realistic application example concerning the prioritization of the QFD's CRs in the design of an aircraft seat for passengers. The concluding section summarizes the original contributions of the paper, focusing on the benefits and limitations of the proposed technique, and possible future research.

## 2. Background Information

### 2.1 Basics of Thurstone's Law of Comparative Judgement

In 1927, Thurstone presented his Law of Comparative Judgement (LCJ), i.e., a mathematical model
to estimate scale values based on binary choices between stimuli (Thurstone, 1927). The explanation of this model will refer to the problem of the relative importance prioritization of QFD's CRs, on the basis of the VoC.

Thurstone postulated that each stimulus (CR in this case) will possess some attribute (importance level in this case) in varying but unknown degrees. For each of the CRs and among all subjects, it is assumed that a preference will exist. These two conditions imply the assumption of unidimensionality of the scale representing the importance of CRs (McIver and Carmines, 1981). It is also assumed that, for each $i$-th CR, the preference will be distributed normally, i.e., $\mathrm{CR}_{i} \sim N\left(\mu_{i}, \sigma_{i}^{2}\right)$, being $\mu_{i}$ and $\sigma_{i}^{2}$ the unknown mean value and variance of that CR. A person's preference for each CR versus every other CR is thereby obtained. The more persons who select one CR of a pair over the other CR, the greater the importance for that CR , and thus the greater its scale weight (Edwards, 1957).

Thurstone's LCJ is an indirect form of measurement based on a transformation of individual preferences (input data) into scale values on a psychological continuum. Such indirect approaches are referred to as scaling processes. There are many scaling models; the most well known are the Rasch model (Rasch, 1966; Jansen, 1984) and conjoint analysis (Luce and Tukey, 1964). In addition, the LCJ model is based on deriving group scale values from dispersed individual choice data. Therefore, it can be also considered as a statistic choice model.
In Thurstone's terminology, choices are mediated by a discriminal process. He defines this as the process by which an individual identifies, distinguishes, or reacts to stimuli. Let us consider the theoretical distributions of the discriminal process for any two $\mathrm{CRs}, \mathrm{CR}_{i}$ and $\mathrm{CR}_{j}$ (see Fig. 2(a)). In the LCJ model, the distribution associated with a given CR is characterized by a dispersion (or variance) of that CR , which reflects the subject-to-subject variability. Dispersions may be different for different CRs. Let $\mu_{i}$ and $\mu_{j}$ correspond to the (unknown) scale values of the two CRs and $\sigma_{i}^{2}$ and $\sigma_{j}^{2}$ the (unknown) variances.

The difference $\left(\mathrm{CR}_{i j}=\mathrm{CR}_{i}-\mathrm{CR}_{j}\right)$ will follow a normal distribution with parameters:

$$
\begin{equation*}
\mu_{i j}=\mu_{i}-\mu_{j} \text { and } \sigma_{i j}=\sqrt{\sigma_{i}^{2}+\sigma_{j}^{2}-2 \cdot \rho_{i j} \cdot \sigma_{i} \cdot \sigma_{j}} . \tag{1}
\end{equation*}
$$

where:
$\mu_{i}$ and $\mu_{j}$ denote the (unknown) mean values of $\mathrm{CR}_{i}$ and $\mathrm{CR}_{j}$;
$\sigma_{i}^{2}$ and $\sigma_{j}^{2}$ denote the (unknown) variances of $\mathrm{CR}_{i}$ and $\mathrm{CR}_{j}$;
$\rho_{i j}$ denotes the (unknown) correlation between the pairs of discriminal processes $\mathrm{CR}_{i}$ and $\mathrm{CR}_{j}$.
Considering the area subtended by the distribution of $\mathrm{CR}_{i j}$, let us draw a vertical line passing through the point with $\mathrm{CR}_{i j}=\mathrm{CR}_{i}-\mathrm{CR}_{j}=0$ (see Fig. 2(b)). The area to the right of the line depicts the observed proportion of times $\left(p_{i j}\right)$ that $\mathrm{CR}_{i j} \geq 0$. Of course, the area to the left depicts the
complementary proportion ( $1-p_{i j}$ ).


Fig. 2. (a) Theoretical distributions of the discriminal process for two $\mathbf{C R s}$ (i.e., $\mathbf{C R}_{i}$ and $\mathbf{C R}$ ). (b) Link between $\mathbf{C R}_{i j}=\mathbf{C R}_{i}-\mathbf{C R}_{j}$, and $z_{i j}$, i.e., the unit normal deviate corresponding to the probability $1-p_{i j}$, being $p_{i j}=\operatorname{Pr}\left(\mathrm{CR}_{i j} \geq 0\right)$.

In the standard method of Thurstone scaling, the paired comparison approach is used to collect response data. Under the protocol, respondents are forced to express a preference for one CR over another (i.e., by asking them to rank order CRs two at a time rather than all at once). All possible $\binom{n}{2}=\frac{n \cdot(n-2)}{2}$ pairs are assessed, $n$ being the number of CRs of interest.

Paired comparison data of each respondent are reported into a "binary" matrix (B). For the purpose of example, Fig. 3(a) shows three matrices $\left(B_{1}, B_{2}\right.$ and $\left.B_{3}\right)$ related to three fictitious respondents. The element of the single respondent's matrix is 1 when the CR in the $i$-th row is preferred to that in the $j$-th column. If two CRs have identical level of importance (e.g., $\mathrm{CR}_{3}$ and $\mathrm{CR}_{4}$ in matrix $\mathrm{B}_{1}$ ), their mutual paired comparisons are conventionally 0.5 .

After the total pairs of CRs have been determined for a large number of respondents $(N)$, respondents' matrices can be summed into a single frequency matrix $\left(\mathrm{F}\right.$ ), whose general element $f_{i j}$ represents the number of times that $\mathrm{CR}_{i}$ was preferred to $\mathrm{CR}_{j}$. Fig. 3(b) reports the matrix F aggregating the judgment matrices $\mathrm{B}_{1}, \mathrm{~B}_{2}$ and $\mathrm{B}_{3}$. The general element $f_{i j}$, which appears in the $i$-th row and $j$-th column, denotes the observed number of times that $\mathrm{CR}_{i}$ was judged better or worse than $\mathrm{CR}_{j}$.

Matrix P (Fig. 3(c)) is constructed from matrix F $\left(p_{i j}=\frac{f_{i j}}{N}\right)$. The element $p_{i j}$ is the observed proportion of times that $\mathrm{CR}_{i}$ was chosen over $\mathrm{CR}_{j}$. Symmetric cells now sum to unity.

Interpreting $p_{i j}$ in probabilistic terms, it can be stated that $p_{i j}=\operatorname{Pr}\left(\mathrm{CR}_{i j} \geq 0\right)$. Since $\mathrm{CR}_{i j}$ follows a normal distribution, a standardized variable can be defined:

$$
\begin{equation*}
z_{i j}=\frac{\mathrm{CR}_{i j}-\mu_{i j}}{\sigma_{i j}}, \tag{2}
\end{equation*}
$$

where the element $z_{i j}$ is the unit normal deviate. For $\mathrm{CR}_{i j}=0$ the unit normal deviate is determined by the theoretical proportion $\left(1-p_{i j}\right)$, i.e., $z_{i j}=\Phi^{-1}\left(1-p_{i j}\right), \Phi$ being the cumulative distribution function of the standard normal distribution (see Fig. 2(b)). The element $z_{i j}$ will be positive for all values of $\left(1-p_{i j}\right)$ over 0.50 and negative for all values of $\left(1-p_{i j}\right)$ under 0.50 .
(a) Respondent paired comparison matrices


Fig. 3. Main steps of Thurstone scaling.

In detail, for $\mathrm{CR}_{i j}=0$, Eq. 2 becomes:

$$
\begin{equation*}
z_{i j}=\frac{0-\mu_{i j}}{\sigma_{i j}}=-\frac{\mu_{i j}}{\sigma_{i j}} \rightarrow \mu_{i j}=-z_{i j} \cdot \sigma_{i j} \text {, with } z_{i j}=\Phi^{-1}\left(1-p_{i j}\right) . \tag{3}
\end{equation*}
$$

Combining the second formula in Eq. 3 with the expression of $\sigma_{i j}$ in Eq.1, we obtain:

$$
\begin{equation*}
\mu_{i j}=\mu_{i}-\mu_{j}=-z_{i j} \cdot \sqrt{\sigma_{i}^{2}+\sigma_{j}^{2}-2 \cdot \rho_{i j} \cdot \sigma_{i} \cdot \sigma_{j}}, \tag{4}
\end{equation*}
$$

Matrix P is used to construct matrix Z (see Fig. 3(d)), the basic transformation matrix. Zeros are entered in the diagonal cells in matrix Z because we can ordinarily assume that here $\mu_{i}-\mu_{i}=0$. Apart from the aforementioned assumptions, the Thurstone model considered here is based on the
following further hypotheses:

- CRs are judged differently by subjects; if all subjects would express the same preference for each outcome, the model would not be viable (proportions of 1.00 and 0.00 in the matrix P cannot be used because the $z$ values corresponding to these proportions are $\pm \infty$ ). This is the case for the pair-wise comparisons $\mathrm{CR}_{1}$ and $\mathrm{CR}_{3}, \mathrm{CR}_{1}$ and $\mathrm{CR}_{4}$, and $\mathrm{CR}_{2}$ and $\mathrm{CR}_{4}$ in the matrix P in Fig. 3(c): in every comparison the second CR is unanimously preferred to the first. A simplified approach for tackling this problem is associating values of $p_{i j} \leq 0.001$ with $z_{i j}=\Phi^{-1}(1-0.001)=3.09$ and values of $p_{i j} \geq 0.999$ with $z_{i j}=\Phi^{-1}(1-0.999)=-3.09$ (see the items marked with "*" in the matrix P in Fig. 3(c)). More sophisticated solutions to deal with this issue have been proposed (Edwards, 1957; Krus and Kennedy, 1977).
- As a further practical assumption, it is assumed that the CRs standard deviations are all equal ( $\sigma_{i}$ $=\sigma_{j}=\ldots=\sigma$ ). Therefore Eq. 4 turns into:

$$
\begin{equation*}
\mu_{i j}=-z_{i j} \cdot \sqrt{2 \cdot \sigma^{2} \cdot\left(1-\rho_{i j}\right)}, \tag{5}
\end{equation*}
$$

- It is further assumed that the intercorrelations are all equal to one another ( $\left.\rho_{i j}=\rho, \forall i, j\right)$, so that Eq. 5 turns into:

$$
\begin{equation*}
\mu_{i j}=-z_{i j} \cdot \sqrt{2 \cdot \sigma^{2} \cdot(1-\rho)} \tag{6}
\end{equation*}
$$

More precisely, Thurstone (1927) states that in a paired judgment in which the evaluation of one of the stimuli has no influence on the evaluation of the other stimulus, the correlation $\rho_{i j}$ is likely to be very low and possibly even zero. Also, the assumption that the intercorrelations are all equal to zero is relatively safe when (i) the set of stimuli is rather variegated ${ }^{1}$ and (ii) the group of respondents is not too small. Since, in the case of the QFD's CR prioritization, both these conditions are generally satisfied, it does not seem unreasonable to assume that $\rho_{i j}=\rho=0, \forall i, j$.

Then, under the assumptions we have made, $\sqrt{2 \cdot \sigma^{2} \cdot(1-\rho)}$ (or $\sqrt{2 \cdot \sigma^{2}}$ in the case $\rho$ is assumed to be zero) will be a constant and is the common scale factor of the various arithmetic mean pairs of CRs. Without any loss of generality, this common scale factor is set to 1 , so that:

$$
\begin{equation*}
\mu_{i j}=\mu_{i}-\mu_{j}=-z_{i j} . \tag{7}
\end{equation*}
$$

Eq. 7, with the assumptions involved in its derivation, is commonly referred to as Case V of the LCJ (Thurstone, 1927).

Now we can show that Thurstone scale values for each CR can be obtained from the elements of the matrix Z. Actually, if we sum the entries in the $j$-th column of the matrix $Z$, we obtain:

[^0]\[

$$
\begin{equation*}
\sum_{i=1}^{n} z_{i j}=\sum_{i=1}^{n}\left(\mu_{j}-\mu_{i}\right)=n \cdot \mu_{j}-\sum_{i=1}^{n} \mu_{i}, \tag{8}
\end{equation*}
$$

\]

where $\sum_{i=1}^{n} z_{i j}$ means that the $j$-th column is held constant and the summation is over the $n$ rows of the table. The first term on the right is the sum of the scale value of the $j$-th CR and the second term is the sum of the scale values of all $n$ CRs on the psychological continuum. Dividing both sides of Eq. 8 by $n$, we have:

$$
\begin{equation*}
\bar{z}_{\cdot j}=\frac{\sum_{i=1}^{n} z_{i j}}{n}=\mu_{j}-\frac{\sum_{i=1}^{n} \mu_{i}}{n}=\mu_{j}-\mu, \tag{9}
\end{equation*}
$$

being:
$\bar{z}_{. j}$ the arithmetic mean of the entries in the $j$-th column of the matrix $Z$;
$\mu \quad$ the arithmetic mean of the $(n) \mu_{i}$ values;
$\mu_{j}$ the mean value of the $j$-th CR.
Thus we see that the mean of the $z$ values in the $j$-th column of the matrix Z expresses the mean value of $j$-th CR in terms of its deviation from the mean of all the $\mu_{i}$ values (i.e., $\mu$ ). This procedure can be applied to every column of matrix $Z$, in order to obtain the scale values of every CR. These values are shown in the second row at the bottom of matrix Z (Fig. 3(d)) and graphically represented in Fig. 3(e).
As a check upon calculations, we observe that the sum of the scale values in deviation form is equal to zero ( $\sum_{j=1}^{n} \bar{z}_{. j}=\sum_{j=1}^{n} \mu_{j}-\sum_{j=1}^{n} \mu=n \mu-n \mu=0$ ). CRs with negative scale values are thus judged to be less favourable than the average of the scale values of all CRs and those with positive scale values are judged to be more favourable than the average. Since the scale origin - taken as the mean of the scale values of the CRs on the psychological continuum - is arbitrary, we can apply a permissible scale transformation (i.e., monotonically increasing linear function (Stevens, 1946)), so as to obtain numerical values easier to handle. This will not change the relative position of the scale values on the psychological continuum.

### 2.2 Practical response mode

As shown in Sect. 2.1, in the standard method of Thurstone scaling, the paired comparison approach is used to collect response data. A drawback of this approach is that it can be tedious and complex to manage for $n$ greater than 4 or 5 , since it requires so much repetitious information from respondents. An alternative response mode, which also yields data suitable for Thurstone scaling, is based on two steps:

1. Turning each respondent's judgments, typically expressed on a 5 -level rating scale (see Fig. 4(a)), into rank order data (see Fig. 4(b)).
2. For each respondent, rank order data can be transformed into paired comparison data and reported into a matrix (see Fig. 4(c)). The element of the single respondent's matrix is 1 when the CR in the $i$-th row is preferred to that in the $j$-th column. If two CRs have identical level of importance, their mutual paired comparisons are conventionally 0.5 .

The response mode based on a 5-level rating scale - as well as being less tedious and time consuming than the paired comparison approach - forces the respondent to be transitive (e.g., if $\mathrm{CR}_{1}>\mathrm{CR}_{2}$ and $\mathrm{CR}_{2}>\mathrm{CR}_{3}$, then $\mathrm{CR}_{1}>\mathrm{CR}_{3}$ ). Also, it is generally familiar to respondents and therefore less subject to misinterpretation.
(a) Respondent judgments

(b) Rank order data
(c) Respondent's matrix with paired comparison data

|  | $\mathrm{CR}_{1}$ |  |  | $\mathrm{CR}_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{CR}_{3}$ | $\mathrm{CR}_{4}$ |  |  |  |
|  | 0.5 | 1 | 0 | 0 |
|  | 0 | 0.5 | 0 | 0 |
| $\mathrm{CR}_{2}$ | 0 | 0.5 |  |  |
| $\mathrm{CR}_{3}$ | 1 | 1 | 0.5 | 0.5 |
|  | $\mathrm{CR}_{4}$ | 1 | 1 | 0.5 |
|  |  |  |  |  |



Fig. 4. Process for deriving paired comparison data based on respondents' judgments, defined on a 5-level rating scale. The binary matrices $\left(B_{1}, B_{2}\right.$ and $\left.B_{3}\right)$ in (c) are derived from the results of the questionnaires in (a). The same three fictitious respondents introduced in Fig. 3 are considered. In (b), symbols " $\sim$ " and " $>$ " respectively mean "indifferent to" and "preferred to".

## 3. Application example

To exemplify the performance of the proposed approach, this section illustrates an example about the CR prioritization for a civilian aircraft seat, from the perspective of passengers.

Through market survey, a sample of 30 respondents - i.e., regular air passengers - are selected to identify the CRs by individual interview, focus groups and existing information. Finally, 12 major CRs (reported in Tab. 1) are identified to represent the major concerns of customers.
Then, a questionnaire for assessing the level of importance of each of the 12 CRs is submitted to each of the respondents. Results, defined on a 5-level rating scale, are reported in Tab. A. 1 (in the appendix).

| Abbr. | Description |
| :---: | :--- |
| $\mathrm{CR}_{1}$ | Comfortable (does not give you back ache) |
| $\mathrm{CR}_{2}$ | Enough leg room |
| $\mathrm{CR}_{3}$ | Comfortable when you recline |
| $\mathrm{CR}_{4}$ | Does not hit person behind when you recline |
| $\mathrm{CR}_{5}$ | Comfortable seat belt |
| $\mathrm{CR}_{6}$ | Seat belt feels safe |
| $\mathrm{CR}_{7}$ | Arm rests not too narrow |
| $\mathrm{CR}_{8}$ | Arm rest folds right away |
| $\mathrm{CR}_{9}$ | Does not make you sweat |
| $\mathrm{CR}_{10}$ | Does not soak up a spilt drink |
| $\mathrm{CR}_{11}$ | Hole in tray for coffee cup |
| $\mathrm{CR}_{12}$ | Magazines can be easily removed from rack |

Tab. 1. List of the major CRs related to an aircraft seat, from the perspective of passengers.
For each respondent, judgements are then transformed into ranked order data and, in turn, into paired comparison data, according to the procedure described in Sect. 2.2. For example, as regards the respondent 1, rank order data are $\left(\mathrm{CR}_{5} \sim \mathrm{CR}_{7}\right)>\left(\mathrm{CR}_{1} \sim \mathrm{CR}_{3} \sim \mathrm{CR}_{6}\right)>\left(\mathrm{CR}_{2} \sim \mathrm{CR}_{8} \sim \mathrm{CR}_{9}\right)>$ $\mathrm{CR}_{10}>\left(\mathrm{CR}_{4} \sim \mathrm{CR}_{11} \sim \mathrm{CR}_{12}\right.$ ), which are transformed into the matrix in Fig. A. 1 (in the appendix).

We remark that, consistently with the convention introduced in Sect. 2.1, the mutual paired comparisons of two CRs with identical importance are both 0.5 (e.g., see $\mathrm{CR}_{1}$ and $\mathrm{CR}_{8}$ or $\mathrm{CR}_{1}$ and $\mathrm{CR}_{12}$ in the matrix in Fig. A. 1 (in the appendix).
Next, the paired comparison data matrices relating to the 30 respondents are summed into a single frequency matrix ( F ), in Fig. A. 2 (in the appendix).
The matrix F is transformed into the matrix P (in Fig. A.3, in the appendix) and subsequently into the matrix Z (in Fig. A.4, in the appendix).

According to the convention illustrated in Sect. 2.1, for $p_{i j} \leq 0.001$ and $p_{i j} \geq 0.999, z_{i j}$ values have been set to 3.090 and -3.090 respectively (see the $p_{i j}$ values marked with "*", in the matrix P (Fig. A.3, in the appendix).

Thurstone scale values for each CR are finally calculated through the mean value of the column elements of the matrix $Z$ (see the second row at the bottom of matrix $Z$, in Fig. A. 4 in the appendix). Since the unit and the origin of the resulting interval scale are both arbitrary, we can transform the scale values so that they are included in the interval $[1 ; 5]$, according to the transformation:

$$
\begin{equation*}
\frac{\mu_{j}^{\prime}-1}{5-1}=\frac{\mu_{j}-\min \left(\mu_{j}\right)}{\max \left(\mu_{j}\right)-\min \left(\mu_{j}\right)} \tag{10}
\end{equation*}
$$

being:
$\mu_{j}$ the scale value related to the $j$-th CR, resulting from Thurstone scaling;
$\mu_{j}^{\prime}$ the transformed scale value related to the $j$-th CR in the interval scale $[1,5]$.
Fig. 5 provides a graphical representation of the Thurstone scale values, before and after the transformation in Eq. 10. This transformation is nothing else than a monotonically increasing linear function of the type:

$$
\begin{equation*}
\mu_{j}^{\prime}=a+b \cdot \mu_{j}, \tag{11}
\end{equation*}
$$

being:
$a=\frac{\max \left(\mu_{j}\right)-5 \cdot \min \left(\mu_{j}\right)}{\max \left(\mu_{j}\right)-\min \left(\mu_{j}\right)}$ and $b=\frac{4}{\max \left(\mu_{j}\right)-\min \left(\mu_{j}\right)}>0$.
This transformation eases Phases 4 and 8 of the Product Planning HoQ construction process (in Fig. 1), since they are traditionally based on CR importance levels defined on a 1 to 5 scale.
(a)

(b)


Fig. 5. Resulting Thurstone scale values (a) before and (b) after the scale transformation in Eq. 10.

## 4. Concluding remarks

The following three subsections respectively discuss (i) the benefits of the proposed procedure, (ii) its limitations and (iii) some ideas for future research.

### 4.1 Benefits

- The proposed procedure allows aggregation of the typical CR judgments - generally expressed on ordinal response scales - into a continuous interval scale, avoiding the typical abuses (e.g., arbitrary promotion of the scale properties) of the classical approaches (Franceschini et al., 2007).
- Unlike other methods, such as the AHP, ANP or Kano model, the proposed procedure does not require complicated elaborations by respondents. Particularly in populations in which educational attainment and numeracy are limited, a simple measurement strategy may have considerable practical advantages over more complex techniques, such as ease of comprehension and greater reliability due to reduced measurement error. However, the fact remains that the Thurstone model can be extended to more complex response modes, such as questionnaires in
which CRs are ordered or compared in pairs by each respondent.
- The Thurstone model is relatively robust to incomplete data, like the omission of a portion of judgments by respondents. When the incidence of incomplete data is high, the model presented can be replaced by more refined ones, and it is also possible to check the internal consistency of the results obtained (Thurstone, 1927; Edwards, 1957). However, if CRs were identified correctly, the amount of omitted judgments should not be too large. The opposite could mean that the CRs in use do not reflect the real needs of the customer.


### 4.2 Limitations

- Like any model, that of Thurstone is based on several assumptions, such as, (i) the phenomena to be scaled must lie on a latent unidimensional scale, (ii) the model is based on the normal distribution of stimuli, and (iii ) dispersion and correlation of the stimuli are assumed to be equal. Some of these assumptions can be relaxed when using more sophisticated but also complex variants of the proposed model (Maydeu-Olivares and Böckenholt, 2008).
- The 5-level rating scale for CR judgments is simple and intuitive for the respondent but has a relatively limited resolution. In some cases, this can make the analysis uncertain, since it may generate a significant number of "ties" (i.e., CRs with identical levels of importance), when judgments are transformed into paired comparison data. The problem can be solved by using alternative response scales with a larger number of levels, or questionnaires in which the CRs are ranked or compared in pairs by each respondent.
- As for most of the statistical models, the larger the sample of respondents, the more reasonable and robust results will be. Thurstone (1927) recommends that there be at least a few tens of respondents. This seems to be in line with the typical amount of customers involved in the initial phases of QFD process.


### 4.3 Ideas for future research

Further research should investigate the relationship between results acquired from different techniques for CR prioritization. Moreover, the use of Thurstone LCJ could be extended to other prioritization processes within QFD, such as Phases 3 and 9 in Fig. 1.

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## Appendix

See the following Figures and Tables.

| Respondent No. | $\mathrm{CR}_{1}$ | $\mathrm{CR}_{2}$ | $\mathrm{CR}_{3}$ | $\mathrm{CR}_{4}$ | $\mathrm{CR}_{5}$ | $\mathrm{CR}_{6}$ | $\mathrm{CR}_{7}$ | $\mathrm{CR}_{8}$ | $\mathrm{CR}_{9}$ | $\mathrm{CR}_{10}$ | $\mathrm{CR}_{11}$ | $\mathrm{CR}_{12}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 3 | 4 | 1 | 5 | 4 | 5 | 3 | 3 | 2 | 1 | 1 |
| 2 | 4 | 3 | 3 | 2 | 4 | 3 | 5 | 1 | 4 | 1 | 1 | 2 |
| 3 | 5 | 5 | 4 | 3 | 5 | 5 | 4 | 3 | 4 | 1 | 1 | 1 |
| 4 | 5 | 5 | 4 | 2 | 5 | 5 | 4 | 1 | 3 | 1 | 1 | 2 |
| 5 | 5 | 4 | 2 | 1 | 4 | 4 | 5 | 2 | 3 | 2 | 2 | 2 |
| 6 | 5 | 5 | 4 | 3 | 5 | 3 | 3 | 2 | 3 | 1 | 2 | 1 |
| 7 | 5 | 3 | 3 | 1 | 5 | 4 | 5 | 3 | 4 | 2 | 1 | 2 |
| 8 | 4 | 3 | 1 | 3 | 5 | 3 | 2 | 1 | 3 | 1 | 1 | 1 |
| 9 | 5 | 5 | 4 | 4 | 4 | 5 | 2 | 3 | 3 | 1 | 1 | 1 |
| 10 | 3 | 5 | 2 | 1 | 5 | 2 | 3 | 1 | 2 | 1 | 1 | 2 |
| 11 | 5 | 3 | 5 | 3 | 3 | 4 | 3 | 1 | 3 | 1 | 2 | 2 |
| 12 | 5 | 4 | 3 | 1 | 5 | 3 | 3 | 2 | 3 | 1 | 1 | 2 |
| 13 | 5 | 4 | 3 | 2 | 5 | 3 | 5 | 1 | 1 | 1 | 2 | 1 |
| 14 | 4 | 5 | 1 | 1 | 3 | 3 | 3 | 2 | 3 | 1 | 1 | 1 |
| 15 | 4 | 5 | 4 | 3 | 4 | 4 | 5 | 3 | 4 | 1 | 1 | 2 |
| 16 | 5 | 4 | 3 | 3 | 5 | 5 | 5 | 3 | 3 | 2 | 3 | 1 |
| 17 | 4 | 4 | 4 | 2 | 5 | 5 | 5 | 3 | 2 | 2 | 2 | 1 |
| 18 | 5 | 5 | 4 | 2 | 3 | 3 | 2 | 2 | 4 | 1 | 1 | 1 |
| 19 | 5 | 4 | 4 | 2 | 3 | 4 | 3 | 2 | 3 | 1 | 1 | 2 |
| 20 | 4 | 5 | 3 | 3 | 5 | 3 | 5 | 2 | 4 | 2 | 1 | 1 |
| 21 | 5 | 4 | 4 | 2 | 3 | 3 | 3 | 1 | 1 | 1 | 1 | 1 |
| 22 | 5 | 5 | 4 | 4 | 5 | 4 | 3 | 2 | 3 | 1 | 2 | 1 |
| 23 | 5 | 4 | 3 | 2 | 4 | 4 | 5 | 2 | 4 | 2 | 3 | 1 |
| 24 | 5 | 5 | 3 | 3 | 3 | 5 | 3 | 2 | 2 | 2 | 2 | 1 |
| 25 | 5 | 4 | 2 | 1 | 4 | 3 | 2 | 2 | 4 | 1 | 1 | 1 |
| 26 | 4 | 4 | 1 | 2 | 5 | 3 | 4 | 2 | 2 | 1 | 1 | 2 |
| 27 | 4 | 5 | 2 | 2 | 4 | 3 | 3 | 3 | 3 | 1 | 1 | 1 |
| 28 | 4 | 4 | 2 | 3 | 5 | 3 | 4 | 2 | 4 | 2 | 3 | 1 |
| 29 | 5 | 5 | 2 | 1 | 4 | 5 | 3 | 1 | 3 | 2 | 1 | 1 |
| 30 | 5 | 4 | 4 | 1 | 5 | 3 | 5 | 1 | 3 | 2 | 1 | 1 |

Tab. A.1. Levels of importance assigned by 30 respondents to the CRs, through a 5-level rating scale ( $1=$ Not at all important, $5=$ Very high importance).

| $\mathrm{CR}_{1}$ |  | $\mathrm{CR}_{1}$ | $\mathrm{CR}_{2}$ | $\mathrm{CR}_{3}$ | $\mathrm{CR}_{4}$ | $\mathrm{CR}_{5}$ | $\mathrm{CR}_{6}$ | $\mathrm{CR}_{7}$ | $\mathrm{CR}_{8}$ | $\mathrm{CR}_{9}$ | $\mathrm{CR}_{10}$ | $\mathrm{CR}_{11}$ | $\mathrm{CR}_{12}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.5 | 1 | 0.5 | 1 | 0 | 0.5 | 0 | 1 | 1 | 1 | 1 | 1 |
| $\mathrm{B}_{1}=$ | $\mathrm{CR}_{2}$ | 0 | 0.5 | 0 | 1 | 0 | 0 | 0 | 0.5 | 0.5 | 1 | 1 | 1 |
|  | $\mathrm{CR}_{3}$ | 0.5 | 1 | 0.5 | 1 | 0 | 0.5 | 0 | 1 | 1 | 1 | 1 | 1 |
|  | $\mathrm{CR}_{4}$ | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0.5 |
|  | $\mathrm{CR}_{5}$ | 1 | 1 | 1 | 1 | 0.5 | 1 | 0.5 | 1 | 1 | 1 | 1 | 1 |
|  | $\mathrm{CR}_{6}$ | 0.5 | 1 | 0.5 | 1 | 0 | 0.5 | 0 | 1 | 1 | 1 | 1 | 1 |
|  | $\mathrm{CR}_{7}$ | 1 | 1 | 1 | 1 | 0.5 | 1 | 0.5 | 1 | 1 | 1 | 1 | 1 |
|  | $\mathrm{CR}_{8}$ | 0 | 0.5 | 0 | 1 | 0 | 0 | 0 | 0.5 | 0.5 | 1 | 1 | 1 |
|  | $\mathrm{CR}_{9}$ | 0 | 0.5 | 0 | 1 | 0 | 0 | 0 | 0.5 | 0.5 | 1 | 1 | 1 |
|  | $\mathrm{CR}_{10}$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0.5 | 1 | 1 |
|  | $\mathrm{CR}_{11}$ | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0.5 |
|  | $\mathrm{CR}_{12}$ | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0.5 | 0.5 |

Fig. A.1. Paired comparison data relating to the judgments by respondent 1, in Tab. A.1.


Fig. A.2. Matrix F, obtained from the paired comparison data originated from the respondents' judgments (in Tab. A.1).

| $\mathrm{CR}_{1}$ | $\mathrm{CR}_{1}$ | $\mathrm{CR}_{2}$ | $\mathrm{CR}_{3}$ | $\mathrm{CR}_{4}$ | $\mathrm{CR}_{5}$ | $\mathrm{CR}_{6}$ | $\mathrm{CR}_{7}$ | $\mathrm{CR}_{8}$ | $\mathrm{CR}_{9}$ | $\mathrm{CR}_{10}$ | $\mathrm{CR}_{11}$ | $\mathrm{CR}_{12}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.500 | 0.650 | 0.933 | 1.000* | 0.567 | 0.833 | 0.683 | 1.000* | 0.933 | 1.000* | 1.000* | 1.000* |
| $\mathrm{CR}_{2}$ | 0.350 | 0.500 | 0.833 | 0.967 | 0.467 | 0.667 | 0.617 | 0.967 | 0.833 | 1.000* | 1.000* | 1.000* |
| $\mathrm{CR}_{3}$ | 0.067 | 0.167 | 0.500 | 0.783 | 0.183 | 0.317 | 0.317 | 0.800 | 0.500 | 0.900 | 0.867 | 0.900 |
| $\mathrm{CR}_{4}$ | 0.000* | 0.033 | 0.217 | 0.500 | 0.050 | 0.083 | 0.167 | 0.567 | 0.267 | 0.733 | 0.733 | 0.717 |
| $\mathrm{CR}_{5}$ | 0.433 | 0.533 | 0.817 | 0.950 | 0.500 | 0.667 | 0.667 | 1.000* | 0.850 | 1.000* | 1.000* | 1.000* |
| $\mathrm{P}=\mathrm{CR}_{6}$ | 0.167 | 0.333 | 0.683 | 0.917 | 0.333 | 0.500 | 0.483 | 0.983 | 0.667 | 1.000* | 0.983 | 0.983 |
| $\mathrm{CR}_{7}$ | 0.317 | 0.383 | 0.683 | 0.833 | 0.333 | 0.517 | 0.500 | 0.917 | 0.700 | 1.000* | 1.000* | 1.000* |
| $\mathrm{CR}_{8}$ | 0.000* | 0.033 | 0.200 | 0.433 | 0.000* | 0.017 | 0.083 | 0.500 | 0.167 | 0.733 | 0.667 | 0.717 |
| $\mathrm{CR}_{9}$ | 0.067 | 0.167 | 0.500 | 0.733 | 0.150 | 0.333 | 0.300 | 0.833 | 0.500 | 0.933 | 0.900 | 0.933 |
| $\mathrm{CR}_{10}$ | 0.000* | 0.000* | 0.100 | 0.267 | 0.000* | 0.000* | 0.000* | 0.267 | 0.067 | 0.500 | 0.467 | 0.517 |
| $\mathrm{CR}_{11}$ | 0.000* | 0.000* | 0.133 | 0.267 | 0.000* | 0.017 | 0.000* | 0.333 | 0.100 | 0.533 | 0.500 | 0.500 |
| $\mathrm{CR}_{12}$ | 0.000* | 0.000* | 0.100 | 0.283 | 0.000* | 0.017 | 0.000* | 0.283 | 0.067 | 0.483 | 0.500 | 0.500 |

Fig. A.3. Matrix P, obtained from the matrix F (in Fig. A.2).


Fig. A.4. Matrix $\mathbf{Z}$ containing the unit normal deviates $\left(z_{i j}\right)$ corresponding to the complementary probabilities $\left(1-p_{i j}\right)$ to those in matrix $P\left(p_{i j}\right)$. Values of $p_{i j} \leq 0.001$ and $\geq 0.999$ (marked with "*" in Fig. A.3) have been conventionally associated with $z_{i j}=3.090$ and $\mathbf{- 3 . 0 9 0}$ respectively.


[^0]:    ${ }^{1}$ The adjective "variegated" indicates that the stimuli of interest represent different basic concepts, not the same one, just stated in different ways.

