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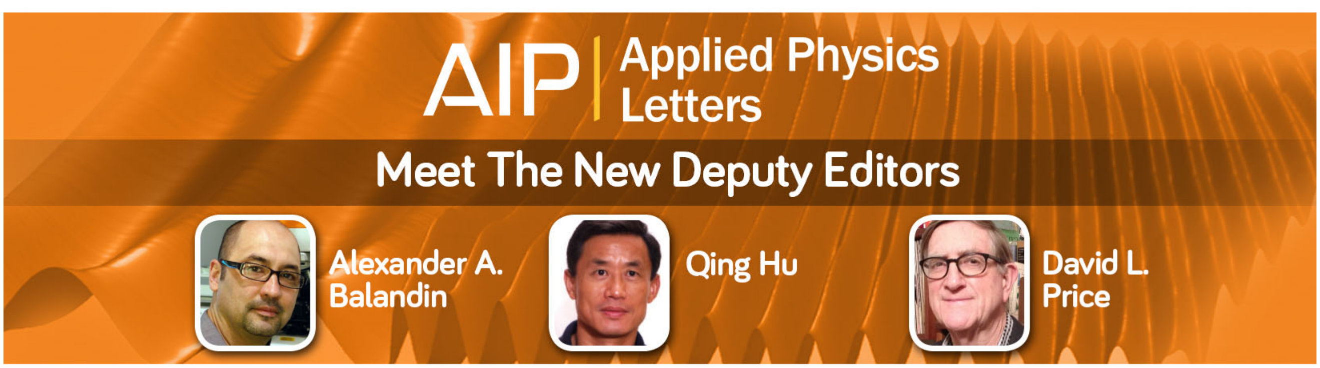
Citation: Applied Physics Letters **92**, 101912 (2008); doi: 10.1063/1.2890031
View online: http://dx.doi.org/10.1063/1.2890031
View Table of Contents: http://scitation.aip.org/content/aip/journal/apl/92/10?ver=pdfcov
Published by the AIP Publishing



A scaling method to enhance detection of a nonlinear elastic response

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(Received 17 December 2007; accepted 1 February 2008; published online 11 March 2008)

The signature of nonlinearity in the elastic response of a specimen to an impingent ultrasonic wave is usually determined through Fourier analysis, which provides low amplitude signals, often below noise level. We suggest here an alternative, based on the amplitude dependence of the response of the system. Our procedure is conceptually simple and easy to implement. In addition, it keeps simultaneously into account the nonlinear signature effects on phases, amplitudes, and frequencies of the response. The sensitivity of the approach to the presence of nonlinearity is proven experimentally. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2890031]

Often, the nonlinearity in the elastic response of a specimen to an impinging ultrasonic wave is an indication of the presence of a microstructure,[1] which is not intrinsic in the material properties and must be associated to the presence of damage.[2] To characterize the damage level of the system, it is necessary to extract from the recorded signal only the portion due to the interaction with the nonlinear scatterers. In principle, this goal can be accomplished if a "reference" signal, which corresponds to the response of the undamaged (linear) sample, is available. In practice, the definition of a "reference signal" may be arbitrary to some extent. However, in the case of a nonlinear response, a suitable criterion is readily available. If the material mechanical properties are nonlinear, a reference signal can be characterized in terms of its spectral content: the latter must not contain components such as higher order harmonics[3] or sidebands.[4] It follows that some of the features due to the nonlinear response can be extracted by bandpass filtering the recorded signal to cancel contributions at the excitation frequency. Alternatives have been developed using classical fast Fourier transform (FFT) analysis,[5] phase-coded subtraction methods,[6] or analysis of the phases of the signals.[7] These approaches do not always work satisfactorily in practice and several technical and theoretical problems have to be considered. Among them, the nonlinearity that is often intrinsic in the generation/acquisition system and the difficulty of obtaining sufficient illumination of a nonlinearity located far from the ultrasonic source.[8]

A common feature of the nonlinear acoustic response, as detected by all the methods listed above, especially when the material is affected by damage at its early stages, is the low amplitude of the individual higher-order components and/or sideband signals. Quite often, these signals are submerged within the noise and are difficult to detect unless receivers are located close to the nonlinear scatterers.[9]

To increase the signal-to-noise ratio of the detected nonlinear signature, we propose the scaling subtraction method which exploits the nonlinear contribution to the attenuation of the fundamental harmonic component of the detected signal.[10,11] This contribution breaks the proportionality between the excitation and the response at the fundamental frequency. The main mechanisms which are responsible for causing the violation of linear scaling of the response with respect to the excitation are as follows:

- The presence of nonlinear terms causes the appearance in the solution of waves with frequencies that are multiples of the frequency ω_0 of the signal emitted at the transducer. It follows that the elastic energy of the signal is redistributed among the various generated frequencies and energy losses increase with amplitude.
- The wave velocity in the nonlinear portion of the material is amplitude dependent. As a consequence, different portions of the signal travel at different velocities, with a resulting distortion of the wave, which arrives at the receiver with a phase that depends on the amplitude and extension of the nonlinear zone encountered.
- The amplitude of the response depends on the resonance curve of the specimen, which is slightly amplitude dependent. A decrease/increase of amplitude may be observed, depending on the width and shift of the resonance curve.

Formally, given a pure tone excitation with amplitude A and at frequency ω_0, the received signal $v(t)$ can be expressed in a very general form as

$$v_A(t) = \sum_{n=1} B_n(A)\cos[n\omega_0 t + \varphi_n(A)]. \quad (1)$$

In the limit of small A, the nonlinear contributions are negligible and we obtain a linear signal, considered here as the reference signal

$$v_{\text{lin}}(t) = B_1(A \to 0)\cos(\omega_0 t). \quad (2)$$

In our approach, we excite the specimen with a low amplitude (A_{lin}) and detect the reference signal v_{lin}. Then, we excite the sample with a larger amplitude $A = kA_{\text{lin}}$, detecting a signal v_A in the form of Eq. (1). We define the nonlinear response as $w_A(t) = v_A(t) - kv_{\text{lin}}(t)$. Subtraction of Eqs. (1) and (2) gives that $w_A(t)$ contains the higher order harmonics of $v_A(t)$ plus a contribution at ω_0, with amplitude that is dependent on the phase and on the nonlinear attenuation. Of course, $w_A(t)$ vanishes (except for noise effects) if the material is perfectly linear.

Evidently, $w_A(t)$ has a much larger amplitude than that of a bandpass filtered signal, which identifies nonlinear features

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through a signal $w_F=\Sigma_{n=2}B_n(A)\cos[n\omega_0 t+\varphi_n(A)]$. Besides the noticeable increase in amplitude due to the component at ω_0, we also notice that $w_A(t)$ has the additional advantage of being derived by a simple subtraction method. On the contrary, the filtering process is always based on a complex mathematical approach, where the results are often influenced by the choices of the length and sampling of the signal and by the choice of the windowing procedures. Finally, we remark that our solution also has advantages with respect to the so called phase-coded subtraction methods,[6] which allow the selection of contributions to the signal due to a single harmonic without the need of any FFT.

To validate the scaling subtraction method proposed here, we have considered a simple experiment using five samples (steel, mortar, damaged teflon, intact, and damaged concrete) in the form of 16 cm bars with diameter of 6 cm. Samples have been equipped at their tips with two identical narrow band piezoelectric transducers (PZTs), working at a resonance frequency of 55 KHz, acting as injector and receiver. The emitting transducer is connected to a function generator through an amplifier, which excites the PZT with a continuous sinusoidal wave at the same frequency as the resonance frequency of the transducer. The receiver is connected to an oscilloscope through a second amplifier and the signals in the time domain are recorded after stationary conditions have been reached. A window of 3.28 ms has been used with sampling rate of 10 Mpoints/s. Linearity of the generation/acquisition system in the voltage range considered has been verified [see Fig. 2].

Experiments have been conducted varying the amplitude of the excitation between 10 and 70 V (after amplification), with a 5 V step. The signals recorded are labeled as v_i $(i=1,\ldots,13)$. The signal v_1 (lower voltage level) has been assumed to correspond to the linear signal v_{lin}. Signals have been synchronized by recording on a second channel of the oscilloscope the input signal and using it as a time reference frame.

Each sample has been characterized by a set of signals $w_i(t)=v_i(t)-k_i v_1(t)$, where k_i are the ratios of the corresponding voltages at the generator. In Fig. 1(a), the signal $w_{13}(t)$ for the intact concrete bar is analyzed. The phase delay and amplitude differences of v_{13} with respect to the rescaled linear signal are evident. We remark again that the amplitude difference is due to the increase of attenuation at larger amplitudes. The amplitude of the scaling subtracted signal (dashed line) is of the same order of that of the recorded signal (dashed-dotted line) and much larger than that of the filtered signal (dotted line), obtained with a bandpass filter between $1.5\omega_0$ and $5.5\omega_0$. Even though very small, we believe the harmonics detected at the larger excitation amplitude [see Fig. 1(b)] to be due to the material nonlinearity. In fact, there is no trace of harmonics in the signals when the transducer and receiver are directly coupled. As expected, Fig. 1(b) shows that the difference signal $w_{13}(t)$ contains the same spectrum as the signal at the larger amplitude $v_{13}(t)$, except that the amplitude of the fundamental harmonic is smaller.

For a detailed analysis, two quantitative indicators for $w_i(t)$ have been chosen: $\alpha_i=\max[w_i(t)]$ and $\beta_i=1/(nT)\int_0^{nT}w_i^2(t)dt$, where T is the period and n an integer. The two indicators are related to the amplitude and energy of the signal, respectively. In Fig. 2, the indicator β_i is plotted versus the energy of the output signal $v_i(t)$ for the five specimens. β_i increases rapidly with increasing input voltage for all the samples considered and the plot allows to discern the small classical nonlinearity present in the steel sample from the bigger one of the other samples. The optimal linearity of the equipment is verified by the fact that the indicator remains zero when the two transducers are directly coupled to each other (left pointing triangles). A classical FFT analysis (filter around the third harmonic) allows to distinguish the behavior of the damaged samples, while the others give am-

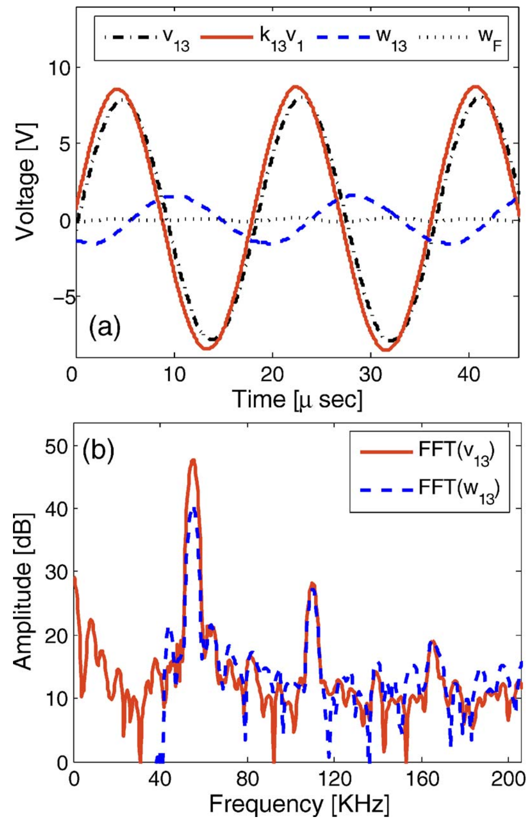

FIG. 1. (Color online) (a) Recorded signals in the time domain in the intact concrete sample at the lower (solid red line) and higher (dashed-dotted black line) input amplitudes. The signature of the nonlinearity is given by the difference of the two (dashed blue line) or by the filtered signal (dotted black line), as described in the text. (b) FFT analysis of the recorded (at higher amplitude) and subtracted signals. The y axes of (b) are in log scale (decibel).

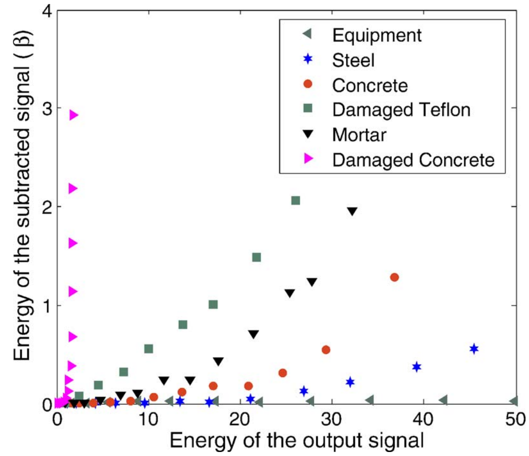

FIG. 2. (Color online) Energy indicator β of the subtracted signal vs energy of the recorded output signal.

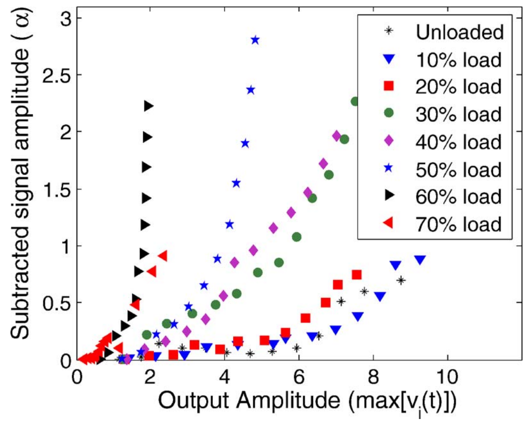

FIG. 3. (Color online) Amplitude indicator α of the subtracted signal for a concrete sample at different damage states, as indicated in the caption.

plitudes comparable with the very small one generated by the equipment. The small amplitudes of the harmonics detected may be due to the use of narrow band transducers, with a low sensitivity to the frequencies corresponding to the third harmonic.

An intact concrete bar has been subjected to loading cycles up to a given ratio with respect to the critical rupture stress σ_c (10%–70% in steps of 10%). Between two different loadings, ultrasonic measurements have been performed, using the schema described above. As a result, eight sets of signals $w_i(t)$ were available, one for each state of the bar. Given the loading amplitudes used, we expect the specimen to be subjected to distributed microdamage, while larger cracks appear only at loadings larger than 80%, when the attenuation in the material becomes so large that our measurements loose meaning. In Fig. 3, α_i is plotted versus the amplitude of the recorded signal for each case. The analysis of the data indicates no significant difference in the behavior of the bar up to a loading of $0.2\sigma_c$, in agreement with the prediction of no damage occurring at such low loading levels (here, only compaction is expected to take place). For higher stress levels, we have a significant change for a second set of curves, between $0.3\sigma_c$ and $0.5\sigma_c$, while a much stronger nonlinear behavior is found after a loading up to 60% and 70% of σ_c. We, thus, observe bands in the increase of nonlinearity of the system, in good agreement with the expectations from quasistatic damage processes in concrete. The FFT analysis does not indicate significant differences in the behavior up to a loading of 60%.

To conclude, we have shown that the effects of a nonlinear elastic feature on the amplitude of the fundamental frequency component of the signal are much stronger than those on the second and third order harmonic components. Several mechanisms concur to render the fundamental frequency more sensitive to variations in the nonlinearity of the specimen: nonlinear attenuation, nonlinear shift of the resonance curve, and amplitude dependence of the wave speed. The separation of the three effects, accounting for the energy balance, is of importance from the basic point of view. This issue can probably be addressed using the scaling subtraction method proposed here but it is beyond the scope of the present paper and the use of broadband transducers is, in our opinion, necessary to provide evidence of the link between energy transferred to higher order harmonics and loss of energy due to attenuation.

We wish to thank Professor Z. Prevorovsky (Institute of Thermo-mechanics ASCR, Prague) for having shared with us his expertise in nonlinear ultrasonic measurements, Dr. F. Bosia (University of Torino) for his precious comments and suggestions, and Mr. V. Di Vasto (Politecnico di Torino) for his support in the use of the MTS machine.

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