Route Stability in MANETs under the Random Direction Mobility Model

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Abstract—A fundamental issue arising in mobile ad hoc networks (MANETs) is the selection of the optimal path between any two nodes. A method that has been advocated to improve routing efficiency is to select the most stable path so as to reduce the latency and the overhead due to route reconstruction. In this work, we study both the availability and the duration probability of a routing path that is subject to link failures caused by node mobility. In particular, we focus on the case where the network nodes move according to the Random Direction model, and we derive both exact and approximate (but simple) expressions of these probabilities. Through our results, we study the problem of selecting an optimal route in terms of path availability. Finally, we propose an approach to improve the efficiency of reactive routing protocols.

Index Terms—Mobile ad hoc networks, routing, modeling and analysis.

1 Introduction

Mobile wireless networks are receiving an increasing interest due to the possibility of ubiquitous communications they offer. In particular, mobile ad hoc networks (MANETs) enable users to maintain connectivity to the fixed network or exchange information when no infrastructure, such as a base station or an access point, is available. This is achieved through multihop communications, which allow a node to reach far away destinations by using intermediate nodes as relays.

The selection and maintenance of a multihop path, however, is a fundamental problem in MANETs. Node mobility, signal interference, and power outages make the network topology frequently change; as a consequence, the links along a path may fail and an alternate path must be found. To avoid the degradation of the system performance, several solutions have been proposed in the literature, taking into account various metrics of interest. A method that has been advocated to improve routing efficiency is to select the most stable path [1], [2], [3], [30] so as to avoid packet losses and limit the latency and overhead due to path reconstruction.

In this work, we focus on the stability of a routing path, which is subject to link failures caused by node mobility. We define the path duration as the time interval from when the route is established until one of the links along the route becomes unavailable, while we say that a path is available at a given time instant t when all links along the path are active at time t. Then, our objective is to derive the probability of *path duration* till time t and the probability of *path availability* at time t.

Clearly, the probabilities of path duration and path availability strongly depend on the mobility pattern of the network nodes. Indeed, the path duration (availability) is determined by the duration (availability) of its links, which on its turn depends on the movement of a node with respect to the other. To characterize the nodes position with respect to each other, we need the spatial distribution of a single node over time. One would like to be able to evaluate these quantities in presence of various mobility models, however the analysis is extremely difficult even under simple mobility patterns [4]. (Please see Section 2.2 for a detailed discussion of related work and previous results.)

Here we focus on bidimensional random mobility [5], and we consider nodes moving according to the Random Direction (RD) mobility model, which was first introduced in [6], [7]. According to such model, each node alternates periods of movement (*move* phase) to periods during which it pauses (*pause* phase); at the beginning of each *move* phase, a node independently selects its new direction and speed of movement [6]. Speed and direction are kept constant for the whole duration of the node *move* phase.

The main contributions of our work are as follows:

- We derive for the *first* time an expression for the transform of the distribution of a node moving according to the RD model. This expression can be numerically inverted to obtain the temporal evolution of the probability density function of the node position, given an assigned initial condition. Closed-form expressions for the temporal evolution of the distribution moments can also be derived directly from the transform (Section 4).
- We propose a simple, approximate expression for the probability of *link availability* under the RD

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model, which leverages the derivation of the second moment of the node spatial distribution (Section 4.2). Our findings suggest that as time proceeds, the probability of link availability under a generic mobility model can be obtained through a similar approximation. The same approach can be applied to the computation of the probability of *path duration* (Section 4.3).

- Based on our results on the probabilities of link availability and link duration, we study the same metrics for multihop paths, again in the case of RD mobility. We discuss the validity of the link independence assumption, which is widely used, and compare it against a refined assumption that accounts for link correlation (Section 5). We observe that the link independence assumption provides sufficiently accurate results.
- We show how our analysis can be exploited to improve the efficiency of traffic routing in MANETs. In particular, we show how to select the optimal route in terms of path availability and how to determine the optimal number of hops between source and destination, taking into account the initial distance between the nodes (Section 7). We then propose an approach to find and select routes, which accounts for the expected data transfer time over the path and allows to reduce the overhead of reactive routing protocols.

2 Motivation and Related Work

In this section, we highlight the reasons why a theoretical analysis of route stability in MANETs is useful, and how our contribution differs from previous work.

2.1 Why Is Route Stability Analysis Needed?

To meet the quality-of-service requirements of mobile users, several metrics can be considered for selecting a source destination routing path. Some examples can be found in [8], [9], [10] where the critical issue of energy consumption is considered, or in [11], [12] where the selection of high throughput routes is addressed. Here, we focus on route stability, which is an aspect of fundamental importance as one can judge from the following considerations.

Stable routes. To maximize throughput and reduce traffic latency, it is essential to ensure reliable source-destination connections over time [30]. A route should therefore be selected based on some knowledge of the nodes motion and on a probability model of the path future availability.

Efficient route repair. If an estimate of the path duration is available, service disruption due to route failure can be avoided by creating an alternative path before the current one breaks [3]. Note that having some information on the path duration avoids waste of radio resources due to preallocation of backup paths.

Network connectivity. Connectivity and topology characteristics of a MANET are determined by the link dynamics. These are fundamental issues to network design, since they determine the system capability to support user communications and their reliability level.

Performance evaluation. The performances achieved by high-layer protocols, such as transport and application protocols, heavily depend on the quality-of-service metrics obtained at the network layer. As an example, the duration and frequency of route disruptions have a significant impact on TCP behavior, as well as on video streaming and VoIP services. Thus, characterizing route stability is the basis to evaluate the quality of service perceived by the users.

As a last remark, we would like to stress the importance of random mobility models and the usefulness of an analytical approach for studying the transient behavior of the network routes. Although more realistic mobility models and simulation studies are needed for a detailed assessment of network performances, a theoretical analysis is able to provide insights on general problems and to identify possible solutions under general network conditions. Also, a theoretical approach enables us to effectively analyze the link and path transient behavior, which is essential to the study of the communication performance between mobile nodes. The use of random mobility models is justified not only by the need for analytically tractable results, but also by their capability to capture in a simple manner the aggregate behavior of independent users.

2.2 Our Contribution with Respect to Previous Work

The problem of link and route stability has been widely addressed in the literature. Routing protocols accounting for route stability while selecting the source-destination path can be found in [1], [2], [3], [30], just to name a few. In particular, the work in [3] considers nodes moving along nonrandom patterns and exploits some knowledge of the nodes motion to predict the path duration.

Studies on link and path availability and duration are presented in [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24]. In [13], a partially deterministic and a Brownian motion, where nodes start moving from the same location, are considered. Note that our analysis for the RD mobility is instead carried out under general initial conditions. This fact makes the analysis more complex but at the same time permits to relate probability of link duration and/or availability at time t to the initial distance between nodes.

In [14], [15], [16], [17], nodes move with random direction and at random velocity, but both direction and velocity are kept constant over time so that the link [14], [15] and the path [16], [17] duration can be analyzed using geometric observations. A similar approach cannot be exploited under our more general assumptions. Link and path availability under random mobility models that consist of a sequence of mobility epochs, each of them corresponding to a new value of node speed and direction, are studied in [18]. There, the central limit theorem is applied with respect to the summation of the Cartesian components of the node movement over all epochs during a given time interval. However, in [18], the mobility model does not include movement pauses and only the availability metric is evaluated.

Most studies analyzing path duration make the assumption of independent behavior of the links along the path. This assumption is removed in [19], where the *joint* probability distribution of the path duration is derived, but only for the case of a discrete random walk model. In our work, we discuss the validity of the assumption on link independence under general conditions, and we also derive some results on optimal path selection that hold for various system scenarios.

Of particular relevance to our work are the studies in [20], [21], where the impact of mobility on message latency is analyzed in the case of multihop paths. The authors consider a simple linear topology where nodes move along adjacent segments with reflecting boundaries according to unidimensional Brownian motion. Transfer of messages between adjacent nodes can take place only when nodes are in contact, i.e., they are within the radio range of each other. By assuming that the message transmission is instantaneous, they evaluate the expected latency of a message transferred from the first to the last node of a sequence. In [21], a similar problem is addressed. There, the statistics of the encounter time between nodes are derived under Random Waypoint and Random Direction mobility; these statistics are then used to evaluate the delay performance of mobility-assisted routing schemes. Note, however, that the focus in [20], [21] is on traffic relaying rather than on link and path stability.

In [22], it is shown via simulation that the probability distribution of the path duration is accurately approximated by an exponential distribution when the number of hops along the path is sufficiently large, no matter which mobility model is considered. The parameter of the exponential distribution is empirically obtained from the map layout, node density, and other detailed parameters of the mobility scenario. This observation on the path duration distribution is confirmed by the analytical studies in [23], [24]. The work in [23] exploits Palm's calculus by assuming that links along a path are independent and that steady state is reached; in [24], the assumption on link independence is relaxed and the parameter of the exponential distribution is determined using the expected duration of the links in the path. We highlight that in [23], [24] the dependency on the initial distance between the nodes forming the path is not taken into account. Also, even if the results in [22], [23], [24] have a significant theoretical value, they hold only for paths including a large number of hops.

Finally, the work in [25], [26] shows that under the RD model, the time evolution of the node position can be described through a system of partial differential equations, and that under mild conditions, a (weak) solution of these equations over a rectangular area can be found. Here, differently from [25], [26], we study the dynamics of nodes moving over an infinite bidimensional domain, and we obtain a closed expression for the general (weak) solution of the RD equations in [25], [26] in the frequency domain (i.e., the moment-generating function). Then, we analyze the availability and duration of links and paths (with paths of any length and given the initial distance between the nodes). We also show how to use these results to optimally select a routing path.

3 Assumptions and Definitions

While studying path duration and availability in MANETs, we make the following assumptions:

1. The network comprises homogeneous nodes moving over a bidimensional area; in particular, all nodes have a common radio range, R, and have the same mobility pattern.
2. Nodes move independently of each other.
3. A free space propagation model is considered, i.e., the received signal only depends on its distance from the transmitter.

Let us consider two generic nodes, A and B, and let $\mathbf{X}_A(t)$ and $\mathbf{X}_B(t)$ be their positions, respectively, at time t. We define the distance between the two nodes at time t as $d_{A,B}(t) = \|\mathbf{X}_A(t) - \mathbf{X}_B(t)\|$. According to assumption (3), a communication link between A and B exists if the two nodes are within the radio range of each other. Then, considering assumption (1), we say that a link between A and B exists at time t if $d_{A,B}(t) < R$, and this link is bidirectional.

Let us define the probability of link availability $A_{link}(d_{A,B}(0), t)$ as the probability that the link between nodes A and B is active at time t, given that the initial distance between the two nodes is equal to $d_{A,B}(0), 0 \leq d_{A,B}(0) < R$, i.e.,

$$A_{link}(d_{A,B}(0), t) = \mathbb{P}(d_{A,B}(t) < R | d_{A,B}(0)). \tag{1}$$

We define the link duration probability, $D_{link}(d_{A,B}(0), t)$, as the probability that the link between A and B has been uninterruptedly active till time t, given that their initial distance is $d_{A,B}(0), 0 \leq d_{A,B}(0) < R$,

$$D_{link}(d_{A,B}(0), t) = \mathbb{P}(\inf\{\tau \text{ s.t. } d_{A,B}(\tau) > R\} > t | d_{A,B}(0)). \tag{2}$$

Now, consider $n+1$ mobile nodes, and let $\mathbf{X}_i(t)$ be the position of node i with $1 \leq i \leq n+1$ at time t. Assume that $d_{i,i+1}(0) < R$ for $1 \leq i \leq n$ and let us denote by $\mathbf{d}_0 = [d_{1,2}(0), \ldots d_{n,n+1}(0)]$ the vector of initial nodes distances $d_{i,i+1}(0), 1 \leq i \leq n$. Then, consider a path of n hops obtained by visiting the $n+1$ nodes in sequence: $1 \rightarrow 2 \rightarrow 3 \rightarrow \cdots \rightarrow n+1$. The probability of path availability at time t is defined as

$$A_{path}(\mathbf{d}_0, t) = \mathbb{P}(d_{i,i+1}(t) < R, \forall i \leq n | \mathbf{d}_0), \tag{3}$$

while the path duration probability $D_{path}(\mathbf{d}_0, t)$ is the probability that the path has been uninterruptedly active till time t

$$\begin{aligned} &D_{path}(\mathbf{d}_0, t) \\ &= \mathbb{P}(\inf\{\tau \text{ s.t. for some } i, 1 \leq i \leq n \ d_{i,i+1}(\tau) > R\} > t | \mathbf{d}_0). \end{aligned} \tag{4}$$

4 Link Availability and Link Duration under the RD Model

We consider the Random Direction model [6], i.e., each node alternates periods of movement (*move* phase) to periods during which it pauses (*pause* phase); at the beginning of each *move* phase, a node independently selects its new direction and speed of movement. Speed and direction are kept constant for the whole duration of the node *move* phase; the durations of *move* and *pause* phases are, in general, distributed according to independent random variables.

Under the RD model, the temporal evolution of the node position, either in the *move* or in the *pause* phase, can be described through a system of partial differential equations (PDE's) [25]. In [25], (weak) solution of these equations has been obtained over a finite rectangular area. Here, instead, we consider the dynamics of nodes moving over an infinite bidimensional domain, and we obtain a closed expression for the general (weak) solution of the RD equations in the frequency domain (i.e., the moment-generating function), under the assumption that *move* and *pause* times are exponentially distributed. Even if a direct analytical inverse transform of the obtained moment-generating function appears to be prohibitive, closed expressions for the moments of the spatial probability density function (pdf) can easily be derived. By using the node spatial distribution, we write an exact expression for the probability of link availability, and then propose a simple approximation to evaluate this metric based on the second moment of the spatial distribution, which provides satisfactory results.

4.1 Node Spatial Distribution

The dynamics of the node movement can be described in terms of a Markov Process over a general state space [27], in which the instantaneous node state is characterized by 1) the phase $P(t) \in \mathcal{P} = \{move, pause\}$, 2) the instantaneous position $\mathbf{X}(t)$, and 3) the current speed $\mathbf{V}(t)$ (to be specified only when $P(t) = move$).

Let $m(\mathbf{x}, \mathbf{v}, t)$ be the pdf at time t of the node in the *move* phase, over the state space originated by pairs $(\mathbf{x} \in \mathbb{R}^2, \mathbf{v} \in \mathbb{R}^2)$:

$$m(\mathbf{x}, \mathbf{v}, t) \triangleq \frac{\partial^2 \mathbb{P}(P(t) = move, \mathbf{X}(t) \leq \mathbf{x}, \mathbf{V}(t) \leq \mathbf{v})}{\partial x \partial v}.$$

Let $p(x, t)$ be the pdf at time t over the state space $\mathbf{x}$ for the node in the *pause* phase:

$$p(\mathbf{x}, t) \triangleq \frac{\partial \mathbb{P}(P(t) = pause, \mathbf{X}(t) \leq \mathbf{x})}{\partial x}.$$

We assume that *move* and *pause* phases are exponentially distributed with parameters μ and λ, respectively, and that at the beginning of each *move* phase a node selects a speed from the generic distribution $f_V(\mathbf{v})$, with the absolute speed value being upper bounded by a constant $V_{\max}$ (a reasonable assumption).

The evolution of Markov Processes over a general (uncountable) state space can be described, similarly to what happens for Markov Chains, by a set of dynamical equations, relating the probability distribution functions at different time instants [27]. These equations, known in the literature as Chapman-Kolmogorov equations of the associated Markov Process, can be expressed either in terms of integral equations or in terms of differential equations (in the latter case, they are also called Master equations).

For the RD model, the Chapman-Kolmogorov equations in differential forms have been first obtained in [25]:

$$\begin{aligned} \frac{\partial m(\mathbf{x}, \mathbf{v}, t)}{\partial t} &= -\mathbf{v} \cdot \nabla_{\mathbf{x}} m(\mathbf{x}, \mathbf{v}, t) \\ &\quad + \lambda f_V(\mathbf{v}) p(\mathbf{x}) - \mu\, m(\mathbf{x}, \mathbf{v}, t), \end{aligned} \tag{5}$$

$$\frac{\partial p(\mathbf{x}, t)}{\partial t} = -\lambda p(\mathbf{x}, t) + \mu \int m(\mathbf{x}, \mathbf{v}, t) \mathrm{d}\mathbf{v}, \tag{6}$$

being $\mathbf{v} \cdot \nabla_{\mathbf{x}} m(\mathbf{x}, \mathbf{v}, t)$ the inner product between $\mathbf{v}$ and $\nabla_{\mathbf{x}} m(\mathbf{x}, \mathbf{v}, t)$, which is the gradient of $m(\mathbf{x}, \mathbf{v}, t)$ with respect to $\mathbf{x}$. Since the solution of the above equations lies in L^1 (space of absolutely summable functions) at any time, we apply a double Fourier transform over the space coordinates and a unilateral Laplace transform over time. We obtain

$$\begin{aligned} sm(\mathbf{k}, \mathbf{v}, s) - m_0(\mathbf{k}, \mathbf{v}) &= -\mathbf{v} \cdot j\mathbf{k} m(\mathbf{k}, \mathbf{v}, s) \\ &\quad + \lambda f_V(\mathbf{v}) p(\mathbf{k}) - \mu m(\mathbf{k}, \mathbf{v}, s), \end{aligned} \tag{7}$$

$$sp(\mathbf{k}, s) - p_0(\mathbf{k}, \mathbf{v}) = -\lambda p(\mathbf{k}, t) + \mu \int m(\mathbf{k}, \mathbf{v}, s) \mathrm{d}\mathbf{v}, \tag{8}$$

where

$$\begin{aligned} m(\mathbf{k}, \mathbf{v}, s) &= \int_{\mathbf{x}} \int_{t \geq 0} m(\mathbf{x}, \mathbf{v}, t) e^{-st - j\mathbf{k} \cdot \mathbf{x}} \mathrm{d}t \, \mathrm{d}\mathbf{x}, \\ p(\mathbf{k}, s) &= \int_{\mathbf{x}} \int_{t \geq 0} p(\mathbf{x}, t) e^{-st - j\mathbf{k} \cdot \mathbf{x}} \mathrm{d}t \, \mathrm{d}\mathbf{x}, \\ m_0(\mathbf{k}, \mathbf{v}) &= \int m(\mathbf{x}, \mathbf{v}, 0) e^{-j\mathbf{k} \cdot \mathbf{x}} \, \mathrm{d}\mathbf{x}, \\ p_0(\mathbf{k}) &= \int p(\mathbf{x}, 0) e^{-j\mathbf{k} \cdot \mathbf{x}} \, \mathrm{d}\mathbf{x}. \end{aligned}$$

After some calculations, defining

$$M(\mathbf{k}, s) = \int m(\mathbf{k}, \mathbf{v}, s) d\mathbf{v},$$

we have

$$M(\mathbf{k}, s) = \frac{\int \frac{\lambda f_V(\mathbf{v}) p_0(\mathbf{k})}{(s+\lambda)(s+\mu+j\mathbf{k}\cdot\mathbf{v})} \mathrm{d}\mathbf{v} + \int \frac{m_0(\mathbf{k}, \mathbf{v})}{s+\mu+j\mathbf{k}\cdot\mathbf{v}} \mathrm{d}\mathbf{v}}{1 - \int \frac{\lambda\mu f_V(\mathbf{v})}{(s+\lambda)(s+\mu+j\mathbf{k}\cdot\mathbf{v})} \mathrm{d}\mathbf{v}} \tag{9}$$

and

$$p(\mathbf{k}, s) = \frac{\mu M(\mathbf{k}, s) + p_0(\mathbf{k})}{s + \lambda}. \tag{10}$$

Unfortunately, we were unable to analytical invert the transform in (9); moreover, some care is required even to numerically invert the transforms in (9) and (10) due to the presence of a singularity in the origin. However, by manipulating (9) and (10), it is possible to get a different expression for the transforms in terms of move-pause cycles, that can easily be numerically inverted (please see the Appendix for further details). Furthermore, closed-form expressions for the temporal evolution of the moments of the node spatial distribution can be obtained. Let $Y^l(t)$ be the lth central moment of the node position distribution at time t (irrespective of the phase), and let $Y^l(s)$ be its Laplace transform. We have

$$\begin{aligned} Y^l(t) &= \sum_{i=1}^{2} \left[\int_{\mathbf{x}} \int_{\mathbf{v}} (x_i)^l m(\mathbf{x}, \mathbf{v}, t) \mathrm{d}\mathbf{v} \mathrm{d}\mathbf{x} + \int (x_i)^l p(\mathbf{x}, t) \, \mathrm{d}\mathbf{x} \right], \\ Y^l(s) &= j^l \sum_{i=1}^{2} \frac{\partial^l}{\partial^l k_i} [M(\mathbf{k}, s) + p(\mathbf{k}, s)]|_{\mathbf{k}=0}. \end{aligned}$$

Since $Y^l(s)$ is a rational expression in s, for every l a direct inverse transform can be obtained.

In particular, we report here the formula for the variance, which will be of essential importance to derive the approximation for the probability of link availability presented in the following section. The formula is derived assuming that the node starts at $t = 0$ in steady-state conditions:[1]

1. In steady-state conditions, $\mathbb{P}(P(0) = move) = \frac{\lambda}{\lambda+\mu}$ and the node speed distribution is equal to $f_V(\mathbf{v})$. Expressions similar to (11) can be obtained for different initial conditions.

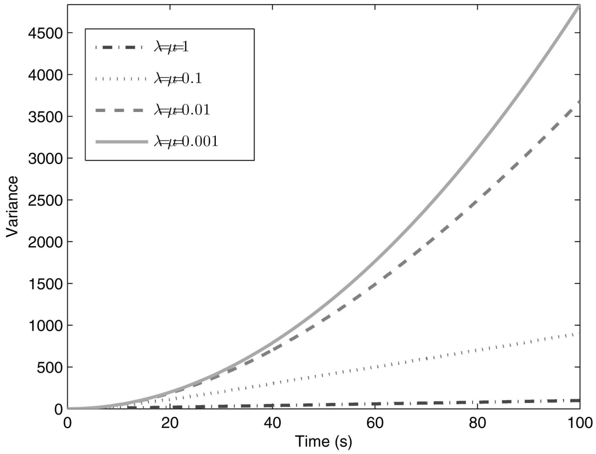

Fig. 1. Variance of the node spatial distribution when the node starts at $t = 0$ in $\mathbf{x} = 0$. The values of λ and μ are expressed in s^{-1}, while the variance values are expressed in m^2.

$$\sigma^2(t) = Y^2(t) = 2\frac{\lambda\sigma_V^2}{\mu^2(\mu+\lambda)}(\mu t + e^{-\mu t} - 1), \qquad (11)$$

where σ_V^2 is the variance of the node speed distribution $f_V(\mathbf{v})$.

The variance as a function of time and for different values of $\lambda = \mu$ (i.e., equal average duration of the *move* and *pause* phases) is shown in Fig. 1, for $\sigma_V^2 = 1\ \mathrm{m}^2/\mathrm{s}$. Note that as time increases, all curves in the plot tend to exhibit a linear behavior with t, since in (11) μt becomes the dominant term. For long durations of the *move* and *pause* phases (i.e., small values of $\lambda = \mu$), the variance increases and the linear behavior appears for larger values of t.

4.2 Link Availability

We can write an exact expression for the probability of link availability at time t, by using the spatial pdf of nodes moving according to the RD model.

Consider two nodes A and B. Let $M_A(\mathbf{x},t) = \int m_A(\mathbf{x},\mathbf{v},t)\mathrm{d}\mathbf{v}$ and $p_A(\mathbf{x},t)$ be the spatial distributions of A at time t in the *move* and *pause* phases, respectively, and let $M_B(\mathbf{x},t) = \int m_B(\mathbf{x},\mathbf{v},t)\mathrm{d}\mathbf{v}$ and $p_B(\mathbf{x},t)$ be the spatial distributions of B at time t in the *move* and *pause* phases, respectively. The probability of link availability between the two nodes can be expressed as

$$A_{link}(d_{A,B}(0),t) = \int_{\mathbf{x}_A}\int_{\mathbf{x}_B}[M_A(\mathbf{x}_A,t) + p_A(\mathbf{x}_A,t)]\cdot[M_B(\mathbf{x}_B,t) + p_B(\mathbf{x}_B,t)]\mathbb{1}_{\|\mathbf{x}_A-\mathbf{x}_B\|<R}\,\mathrm{d}\mathbf{x}_A\,\mathrm{d}\mathbf{x}_B, \qquad (12)$$

which stems directly from its definition (1). The above expression can be numerically evaluated, however the solution is computationally intensive. Indeed, it requires first to numerically evaluate the spatial distribution of nodes A and B at time t through a tridimensional inverse transform (two spatial dimensions and one temporal dimension), then to numerically compute the integral in (12) over a four-dimensional domain. For these reasons, we propose an approximate methodology for the evaluation of $A_{link}(d_{A,B}(0),t)$, which relies on approximating the spatial distribution of each node at time t with a normal distribution having the same mean and variance. This approximation is justified by the fact that at time t, the total movement of a node with respect to its initial position is essentially determined by the vectorial sum of the elementary movements associated to its accomplished *move* phases. Elementary movements being independent and identically distributed, the central limit theorem can be invoked to claim that the marginal spatial distribution of the nodes tends to be a normal distribution for sufficiently large t.

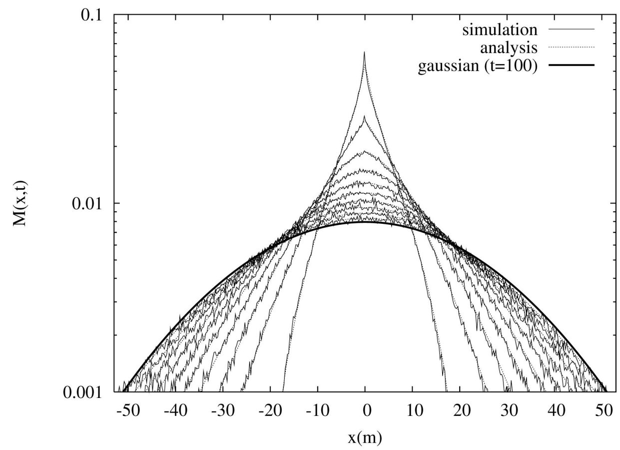

Fig. 2. Node spatial distribution in the *move* phase sampled at $t = 10, 20, 30, \ldots, 100$ s.

As an example, in Fig. 2 we show the numerical inverse transform of $M(\mathbf{k},s)$ in (9), in case of a unidimensional RD model in which *move* and *pause* times have an average of 5 s while the speed is uniformly distributed in $[-2,2]$ m/s. The plot presents on a log scale the spatial distribution $M(\mathbf{x},t)$ in the *move* phase of a node, which starts at time $t = 0$ in $\mathbf{x} = 0$ in the *pause* phase. The node spatial distribution is sampled every 10 s, i.e., the average duration of a cycle including one *move* phase and one *pause* phase. We compare simulation results with the numerical inverse transform obtained following the methodology described in the Appendix. We observe that after a few cycles, the node spatial distribution takes a bell shape. On the same plot we also reported, for $t = 100$ s only, a normal distribution having the same variance of the node spatial distribution. We notice that after 10 cycles, the normal approximation is indeed very good.

Under the proposed approximation, denoting by $\mathcal{N}(\mu,\sigma^2,\mathbf{x})$ a normal distribution with average μ and variance σ^2, we have that (12) becomes

$$\begin{aligned} A_{link}&(d_{A,B}(0),t) \\ &= \int_{\mathbf{x}_A}\int_{\mathbf{x}_B}\mathcal{N}(\mathbf{x}_0^A,\sigma^2(t),\mathbf{x}_A)\mathcal{N}(\mathbf{x}_0^B,\sigma^2(t),\mathbf{x}_B) \\ &\quad\times \mathbb{1}_{\|\mathbf{x}_A-\mathbf{x}_B\|<R}\,\mathrm{d}\mathbf{x}_A\,\mathrm{d}\mathbf{x}_B \\ &= \int_{\|\mathbf{x}\|<R}\mathcal{N}(\mathbf{x}_0^A-\mathbf{x}_0^B,2\sigma^2(t),\mathbf{x})\mathrm{d}\mathbf{x} \\ &= \frac{1}{2\sigma^2(t)}\int_0^R e^{-\frac{\rho^2+d_{A,B}^2(0)}{4\sigma^2(t)}} I_0\left(\frac{\rho d_{A,B}(0)}{2\sigma^2(t)}\right)\rho\,\mathrm{d}\rho, \end{aligned} \qquad (13)$$

where we exploited the fact that the relative position of nodes A and $B, \mathbf{X}_A(t) - \mathbf{X}_B(t)$, is still normally distributed,

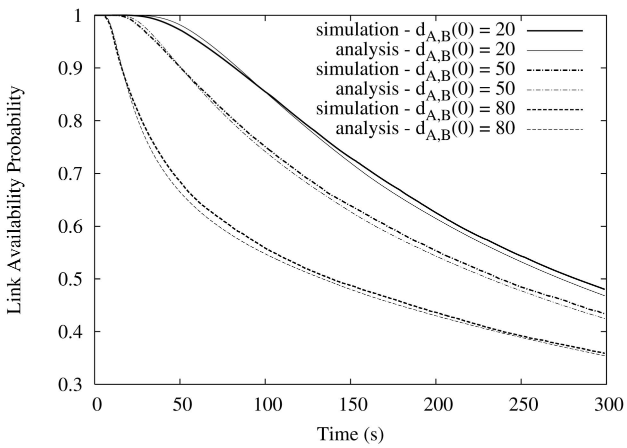

Fig. 3. Probability of link availability versus time in case of the RD model, for different values of the initial distance between the nodes forming the link. Analytical results have been obtained using (13).

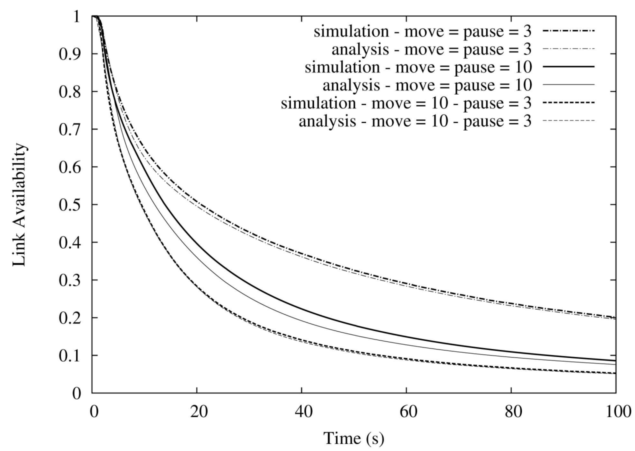

Fig. 4. Probability of link availability versus time in case of the RD model. Different curves correspond to different values of the average duration of the move and pause phases. Analytical results have been obtained using (13).

being the individual position of nodes A and B described by independent normal random variables.

Fig. 3 presents the probability of link availability computed through the approximation in (13), for a bidimensional RD model. The *move* and *pause* times have an average of 10 s, the speed is uniformly distributed in $[-2, 2]$ m/s, and the radio range[2] is $R = 100$ m. We consider three different values of $d_{A,B}(0)$, namely, 20, 50, and 80 m. We observe that the link availability is quite sensitive to the initial distance between the nodes forming the link. As expected, the smaller the initial distance, the higher the link availability. As time goes to infinity, the impact of the initial condition tends to vanish. Indeed, when t becomes large, the expression in (13) tends to $\frac{R^2}{4\sigma^2(t)}$, independently of $d_{A,B}(0)$. Combined with the result in (11), we conclude that under the RD model, the link availability behaves, asymptotically, as $1/t$. Moreover, comparing with simulation, we observe that our approximation provides excellent results; in particular, even when the normal approximation is not accurate (i.e., for small values of t), simulation and analytical results are very close. This behavior is confirmed by the results in Fig. 4, which reports the probability of link availability as a function of time, when the speed is uniformly distributed in $[-5, 5]$ m/s, $R = 50$, and $d_{A,B}(0) = 40$. The different curves correspond to different average durations of the move and pause phases. Again, the comparison between analytical and simulation results shows the accuracy of our approximation, which becomes slightly looser only when the average duration of pause phases increases. Hence, in the following, we use (13) to compute the probability of link availability under the RD model.

4.3 Link Duration

An exact expression for the link duration probability under the RD model appears prohibitive for the following reasons:

- The relative motion between two nodes moving according to independent RD motions is no longer an RD motion. In principle, the relative motion between nodes A and B can be described in terms of a Markov Process over a general state space [27] and obtained by solving the corresponding Chapman-Kolmogorov equations. However, the dimensionality of the state space increases since the instantaneous system state is characterized by 1) the phases of the nodes $P_A(t) \in \mathcal{P} = \{move, pause\}$ and $P_B(t) \in \mathcal{P} = \{move, pause\}$; 2) the instantaneous relative position $\mathbf{X}_A(t) - \mathbf{X}_B(t)$; and 3) the current speed of the two nodes $\mathbf{V}_A(t)$ and $\mathbf{V}_B(t)$.
- The obtained Chapman-Kolmogorov equations should be solved over a circular spatial domain of radius R, with absorbing boundary conditions. However, the structure of the equations describing the relative motion becomes much more complex when they are expressed in polar coordinates, and any attempt to apply the methodology of separation of variables fails.

For these reasons, some approximations are needed to evaluate the link duration for the RD model. A rough approximation could be to plug the instantaneous variance of the node spatial distribution (which can be computed exactly by (11)) directly into the expression of the link duration which holds under the Brownian motion.[3] Note that although being a rough approximation, this is a typical second order approximation, i.e., an approximation that matches the first two moments of the original stochastic process. Second order approximations have successfully been applied in several contexts, such as in queuing theory, where a queue workload is approximated by a reflected Brownian motion with same mean and variance [28]. We

2. This choice of parameters could well represent a scenario of human mobility in a crowded environment, such as a market place, where individuals carry a wireless communication device in their pocket.

3. Consider two generic nodes A and B moving according to a Brownian motion with variance $\sigma^2 t$, whose relative distance at time $t = 0$ is $d_{A,B}(0)$. The link duration probability at time t is given by $D_{link}(d_{A,B}(0), t) = 2\pi \sum_{i=0}^{\infty} \beta_i e^{-(\frac{z_i}{R})^2 \sigma^2 t} \int_0^R J_0(z_i \frac{\rho}{R}) \rho \mathrm{d}\rho$, where J_0 is the 0th order Bessel function of first kind, z_i is the ith zero of the considered Bessel function, and $\beta_i = J_0(z_i \frac{d_{A,B}(0)}{R}) / (2\pi \int_0^R J_0^2(z_i \frac{\rho}{R}) \rho \mathrm{d}\rho)$.

observe that in contrast to the link availability, the probability of link duration decays exponentially as time goes to infinity. This is because the dominant term in the link duration obtained under Brownian motion becomes $e^{-(\frac{z^*}{R})^2\sigma^2 t}$, where $z^* \simeq 2.405$ is the first zero of the Bessel function J_0.

5 Multihop Paths

Consider $n+1$ nodes moving according to an RD motion, and assume that $d_{i,i+1}(0) < R, 1 \le i \le n$. Then, consider the path of n (bidirectional) links obtained traversing the $n+1$ nodes in sequence.[4] Since nodes' movements are assumed to be independent, processes $\mathbf{X}_i(t), 1 \le i \le n+1$, are independent as well. It follows that $d_{i,i+1}(t)$ and $d_{k,k+1}(t)$ are independent if $|k-i| > 1$, but the lengths of adjacent hops, such as $d_{i,i+1}(t)$ and $d_{i+1,i+2}(t)$, are not independent.

The existing correlation between adjacent hop lengths makes an exact analysis of the path dynamics very difficult. We propose two different approximations to evaluate $A_{path}(\mathbf{d}_0, t)$ (or $D_{path}(\mathbf{d}_0, t)$). For simplicity, in the following we refer to the probability of path availability, but we emphasize that a similar discussion can be done for the probability of path duration.

The first approximation is to assume that links' dynamics are independent. In this case, $A_{path}(\mathbf{d}_0, t)$ can be easily expressed in terms of $A_{link}(d_{i,i+1}(0), t)$ as follows:

$$A_{path}(\mathbf{d}_0, t) = \prod_{i=1}^{n} A_{link}(d_{i,i+1}(0), t), \tag{14}$$

being $A_{link}(d_{i,i+1}(0), t)$ the probability that the link between nodes i and $i+1$ is available at t. The above expression follows immediately from the definition in (3).

A more accurate approximation can be obtained by accounting for the correlation between adjacent links. First, consider the case of a 3-link path:

$$\begin{aligned} A_{path}(\mathbf{d}_0, t) &= \mathbb{P}(d_{1,2}(t) < R, d_{2,3}(t) < R, d_{3,4}(t) < R) \\ &= \mathbb{P}(d_{1,2}(t) < R, d_{3,4}(t) < R | d_{2,3}(t) < R) \\ &\quad \times \mathbb{P}(d_{2,3}(t) < R), \end{aligned} \tag{15}$$

where the latter expression can be approximated by considering the events $d_{1,2}(t) < R$ and $d_{3,4}(t) < R$ to be conditionally independent given the event[5] $d_{2,3}(t) < R$. It results that

$$\begin{aligned} A_{path}(\mathbf{d}_0, t) \simeq\ & \mathbb{P}(d_{1,2}(t) < R | d_{2,3}(t) < R) \\ & \cdot \mathbb{P}(d_{3,4}(t) < R | d_{2,3}(t) < R) \mathbb{P}(d_{2,3}(t) < R). \end{aligned} \tag{16}$$

Note that (16) takes into account the correlation between the links $(1,2)$ and $(2,3)$, as well as between the links $(2,3)$ and $(3,4)$. Iterating (16), we can obtain an approximate expression of $A_{path}(\mathbf{d}_0, t)$ for paths of any length. For example, in the case of a 4-link path

4. Note that we consider a sequence of nodes, but we do not assume a line topology. Indeed, when the route connecting a source with a destination is fixed, the sequence of nodes just represents the sequence of hops between the source and the destination.

5. Note that even if the events $d_{1,2}(t) < R$ and $d_{3,4}(t) < R$ are independent, they are not conditionally independent given the event $d_{2,3}(t) < R$.

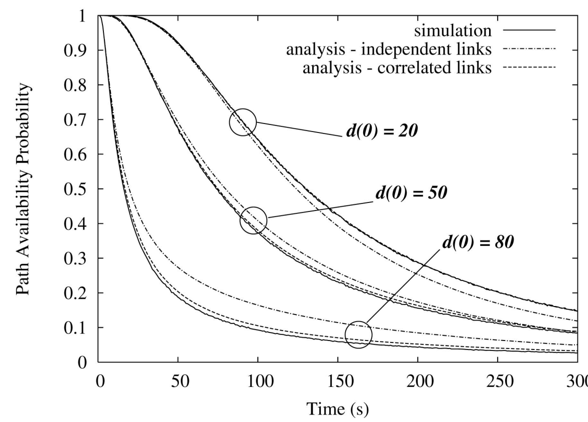

Fig. 5. Probability of path availability under the Brownian motion, in case of three links and different values of initial hop lengths.

$$\begin{aligned} & A_{path}(\mathbf{d}_0, t) \\ &= \mathbb{P}(d_{1,2}(t) < R, d_{2,3}(t) < R, d_{3,4}(t) < R, d_{4,5}(t) < R) \\ &\approx \mathbb{P}(d_{1,2}(t) < R, d_{2,3}(t) < R, d_{3,4}(t) < R) \\ &\quad \cdot \mathbb{P}(d_{4,5}(t) < R | d_{3,4}(t) < R), \end{aligned} \tag{17}$$

and in (17), we recognize the probability of path availability for a 3-link path, written in (16). Then, it should be clear that we can generalize, by induction, the expression of $A_{path}(\mathbf{d}_0, t)$ to any number n of hops.

In short, we can do better than the independence assumption among the links by accounting at least for the correlation between adjacent links. The improved approximation requires evaluating the joint probability $\mathbb{P}(d_{i,i+1}(t) < R, d_{i+1,i+2}(t) < R)$ that can be expressed in terms of the spatial pdf's $f_i(\mathbf{x}, t)$, $f_{i+1}(\mathbf{x}, t)$, and $f_{i+2}(\mathbf{x}, t)$ of nodes i, $i+1$, and $i+2$, respectively, at time t

$$\begin{aligned} & \mathbb{P}(d_{i,i+1}(t) < R, d_{i+1,i+2}(t) < R) \\ &= \int_{\mathbf{x}_A} \int_{\mathbf{x}_B} \int_{\mathbf{x}_C} f_i(\mathbf{x}_A, t) f_{i+1}(\mathbf{x}_B, t) f_{i+2}(\mathbf{x}_C, t) \\ &\quad \times \mathbb{1}_{\|\mathbf{x}_A - \mathbf{x}_B\| < R} \mathbb{1}_{\|\mathbf{x}_B - \mathbf{x}_C\| < R}\, d\mathbf{x}_A d\mathbf{x}_B d\mathbf{x}_C. \end{aligned}$$

We now evaluate the accuracy of our approximations for the path availability, comparing analytical and simulation results. Fig. 5 presents the probability that a path consisting of three hops is available, with all of the hops having the same initial length, namely, 20, 50, and 80 m. We first consider a Brownian motion with $\sigma^2 = 12\ \mathrm{m}^2/\mathrm{s}$ (a typical value for a scenario of human mobility) and $R = 100$ m so as to assess the impact of the approximations introduced for multihop paths without any additional source of error. Simulation results are compared with the approximation based on the link independence assumption, and with our improved approximation that partially accounts for the correlation among links. We observe that the improved approximation gets very close to the simulation results in all considered cases. The independence assumption slightly underestimates the probability of path availability for an initial hop length of 20 m, whereas overestimates it in the other two cases. From extensive experiments, we have arrived at the conclusion that, in general, the results obtained under the independence assumption are less accurate when the initial length of the hops is close to the radio range (see the case $d(0) = 80$ m). However, the

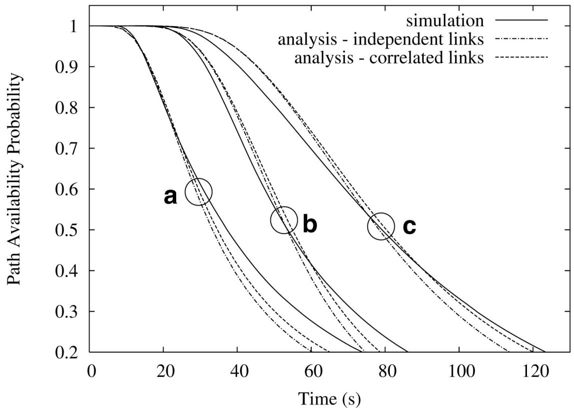

Fig. 6. Probability of path availability in case of the RD model for three different values of initial hop lengths.

independence assumption performs fairly well whenever the probability of path availability is above, say, 0.5, i.e., in the cases of more practical interest for the applications.

Next, we present the results of a few experiments with the RD model. Note that in this case, we have the combination of two approximations: one due to the normal approximation to compute the probability that a single link is available, the other due to our approximations (considering either independent or pairwise correlated links) to compute the probability that a multihop path is available. This time, we keep the initial length of each hop fixed to 20 m and vary the parameters for the RD model considering three scenarios:

- 2 links, E[*move*] $=$ E[*pause*] $= 10$ s, speed uniformly distributed in $[-5, 5]$ m/s.
- 5 links, E[*move*] $=$ E[*pause*] $= 30$ s, speed uniformly distributed in $[-2, 2]$ m/s.
- 10 links, E[*move*] $=$ E[*pause*] $= 10$ s, speed uniformly distributed in $[-2, 2]$ m/s.

Fig. 6 presents the results obtained in the above three scenarios, comparing simulation results with those obtained by our two approximations for the probability that a multihop path is available. We observe that both approximations are satisfactory when the probability of path availability is not too low. Indeed, the use of the refined approximation that partially accounts for link correlation does not provide significant improvements with respect to the one based on the independence assumption.

6 Summary of Results

The main results derived in previous sections can be summarized as follows:

- We have obtained, for the first time, a closed-form expression (see (9) and (10)) for the transform of the node spatial distribution over an infinite bidimensional domain in the case in which nodes move according to the RD model. Using the above distribution, we have written an exact expression (12) for the link availability. Moreover, we have obtained an exact expression for the variance of the node spatial distribution (11).
- Knowledge of the variance of the spatial distribution has enabled us to derive an approximate expression of the probability of link availability. The approximation relies on the observation that the total movement of a node with respect to its initial position is determined by the sum of its elementary movements. The central limit theorem can therefore be invoked to claim that after some time the node spatial distribution tends to become a normal distribution. Hence, we have plugged into (12) a normal distribution with variance given by (11). A similar approximation can be applied to obtain the link duration probability.
- Having analyzed the probabilities of link availability and duration, we have moved to the study of the same metrics in the case of multihop paths, again under the RD mobility model. We have discussed the validity of the link independence assumption, which is widely used, and compared it with a refined approximation that accounts for the correlation between adjacent links (Section 5).

Based on the above results, in the rest of the paper we address the problem of finding optimal paths in a MANET in terms of path availability.

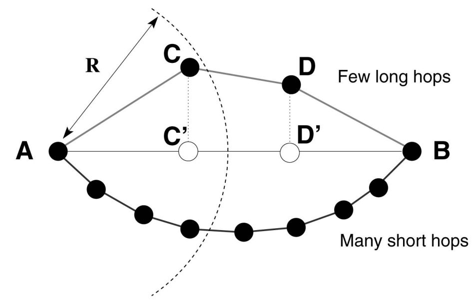

Fig. 7. Path selection between nodes A and B.

7 Optimal Path Selection

In the following discussion, we assume that the parameters of the underlying mobility model of the nodes are given (e.g., obtained through measurements) and that all nodes have a fixed, common radio range R. To introduce the problem of path selection, suppose node A wants to communicate with node B (see Fig. 7), possibly using intermediate nodes as relays. To maximize the stability of the route in response to node mobility, one can think of two different strategies: 1) a few long hops and 2) many short hops. On the one hand, considering that the entire path fails if just a single link fails and that nodes move independently of each other, it seems better to minimize the number of hops. On the other hand, short links are much more stable than long links (see Fig. 7).

These simple considerations suggest that there could be an optimal choice in between the extreme solutions of using few long hops (of length close to the nodes radio range), or many short hops (in the limit, an infinite number of infinitesimal hops).

Let us assume, for simplicity, that the node density is large enough that we can virtually select any point in the area to act as relay between nodes A and B. Under this assumption, it is possible to show that the above intuition is correct: For a given source-destination pair, there exists an optimal number of hops to use, which depends, besides the distance between the two end points, on the desired duration of the path.

To make the problem analytically tractable, we choose as route selection criterion the probability of path availability, as defined in Section 3, because it allows simple closed-form expressions. Moreover, we rely on the independence assumption among links to compute the availability of the entire path (14).

The rest of the section is organized as follows: We first establish in Section 7.1 some properties that must be satisfied by an optimal path having a given number of hops. Next, we turn to the problem of finding the optimal number of hops (Section 7.2). In Section 7.3, we check our analytical results against simulation in a practical example, and we outline a routing scheme to apply our results to the problem of path selection in a real network.

7.1 Properties of an Optimal Path with Given Number of Hops

In a nutshell, we are going to prove the following basic result: the links forming an optimal path (with a given number of hops) have the same length and lie on the segment connecting the source to the destination nodes.

First, consider the probability of link availability $A_{link}(d_{A,B}(0),t)$. Let $a(z,t)$ be the pdf of the absolute distance z reached by a node at time t, which starts moving at $t=0$ from an arbitrary initial position. To make the validity of our result more general, we only require $a(z,t)$ to be a decreasing function of z (for any t). This is true for the RD and the Brownian model, and it holds for a more general class of mobility models than the one considered in this paper.

Lemma 1. *The probability of link availability $A_{link}(d_{A,B}(0),t)$, at any time t, is a decreasing function of $d_{A,B}(0)$, provided that $a(z,t)$ is a decreasing function of z for any t.*

Proof. The proof can be found in the Appendix. □

Proposition 1. *The optimal path between two nodes A and B lies on the segment connecting the two nodes, provided that $a(z,t)$ is a decreasing function of z for any t.*

Proof. By contradiction, suppose the optimal path is outside the segment $\overline{AB}$. Then, we could build another feasible path through the projections of the relay nodes on segment $\overline{AB}$ (in Fig. 7, path $ACDB$ can be replaced by path $AC'D'B$), obtaining a path in which all hops are shorter than or equal to the hops of the original path. Based on (14) and the previous lemma, the constructed path has higher availability than the original path. □

Proposition 2. *For a given number of hops n, the optimal path has all hops of equal length, provided that $A_{link}(d_{A,B}(0),t)$ is a decreasing and log-concave function of $d_{A,B}(0)$.*

Proof. The proof can be found in the Appendix. □

7.2 Finding the Optimal Number of Hops

We now focus on the problem of finding the optimal number of hops n. This is a discrete optimization problem; to proceed analytically, we relax n to a continuous quantity x and look for the maximum of $A_{link}(L/x,t)^x$ with respect to x. At first, we consider uni- or bidimensional Brownian motion and obtain useful approximations for the optimal value of x, which we denote by x^*, to be later used for the case of the RD mobility model.

Let us consider a unidimensional Brownian motion with infinitesimal variance σ^2, for which the expression of the link availability is given by [29]

$$A_{link}(d_{A,B}(0),t) = 1 - Q\left(\frac{R + d_{A,B}(0)}{\sqrt{2\sigma^2 t}}\right) - Q\left(\frac{R - d_{A,B}(0)}{\sqrt{2\sigma^2 t}}\right), \tag{18}$$

where $Q(\cdot)$ is the tail of a normal distribution with zero mean and unit variance.

We notice that reasonable values of link availability (say larger than 0.5) require $2\sigma^2 t$ to be small with respect to R^2. When this is the case, $A_{link}(d_{A,B}(0),t)$ can be made close to 1 by setting the hop length L/x much smaller than R. Thus, we can assume $2\sigma^2 t \ll R^2, L \ll Rx$. It follows that in (18), we can approximate the Q function by $Q(z) \approx \frac{1}{\sqrt{2\pi}z} e^{-z^2/2}$, valid for $z > 1$ (true in our case). By considering that $(1-y)^n \approx 1 - ny$ when y is small (as in our case), after some calculations we can reduce to find the minimum of

$$\frac{e^{-\frac{R^2x^2+L^2}{2\sigma^2 t x^2}}}{\frac{R^2x^2-L^2}{2\sigma^2 t x^2}} \left[\frac{Rx}{\sqrt{2\sigma^2 t}} \cosh\left(\frac{RL}{2\sigma^2 t x}\right) + \frac{L}{\sqrt{2\sigma^2 t}} \sinh\left(\frac{RL}{2\sigma^2 t x}\right)\right].$$

Neglecting L^2 with respect to R^2x^2, we are left to minimizing the term in square brackets in the above expression. Approximating $\cosh(z) \approx 1 + \frac{1}{2}z^2$ and $\sinh(z) \approx z$, we obtain the minimum at $x^* = \frac{L}{\sqrt{2\sigma^2 t}}\sqrt{1 + \frac{R^2}{4\sigma^2 t}}$. Finally, considering that the number of hops must be an integer number and that it cannot be smaller than the minimum possible number of hops $\lceil L/R \rceil$, we approximate the optimal number of hops in the unidimensional Brownian motion as

$$n^*_{1-\dim} = \max\left(\lceil L/R \rceil, \left\lfloor \frac{L}{\sqrt{2\sigma^2 t}} \sqrt{1 + \frac{R^2}{4\sigma^2 t}} \right\rfloor\right). \tag{19}$$

A similar approximation can be carried out for the bidimensional Brownian motion. After some tedious computations, we have found the following approximation:

$$n^*_{2-\dim} = \max\left(\lceil L/R \rceil, \left\lfloor \frac{L}{\sqrt{2\sigma^2 t}} \sqrt{1 + \frac{R^2}{4\sigma^2 t}} - \frac{L}{2\sqrt{2\sigma^2 t}} \right\rfloor\right), \tag{20}$$

which turns out to be very good.

In Fig. 8, we show the accuracy of our approximations for the optimum number of hops in the case of uni and bidimensional Brownian motion with $\sigma^2 = 12\ \mathrm{m}^2/\mathrm{s}$, $R = 100$ m, and for two different values of L, namely, $2R$ and $6R$. We compare the results provided by (19) and (20) with the actual optimum obtained after exploring numerically all values of n. As predicted by our approximations, the optimum number of hops is proportional to L, the initial distance between A and B; for the same value of L, it is smaller in the bidimensional case than in the unidimensional case. Moreover, the optimum number of hops

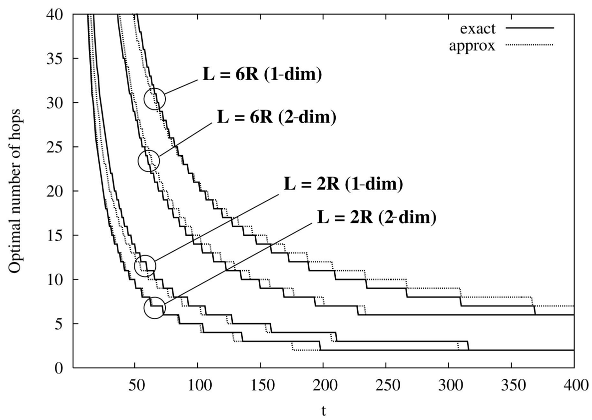

Fig. 8. Approximations and actual values of the optimum number of hops for the unidimensional and the bidimensional Brownian motion as a function of time.

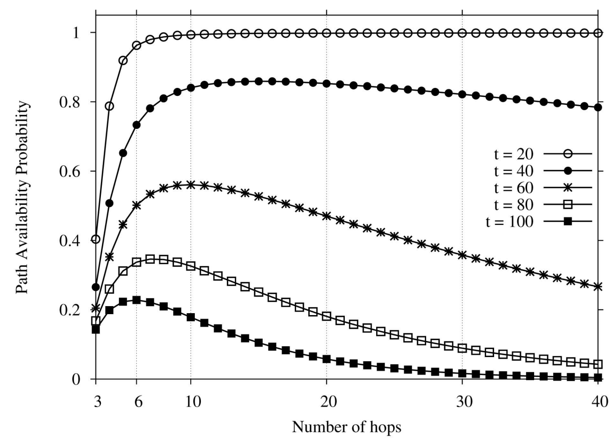

Fig. 10. Path availability at different time instants, as a function of n; bidimensional Brownian motion with $L = 2.5R$.

decreases with the passing of time. In particular, for large t (i.e., large $\sigma^2 t$) it approaches the minimum number of hops $\lceil L/R \rceil$.

Note that when $t \to 0$ ($2\sigma^2 t \to 0$), surprisingly the optimal number of hops tends to infinity. This is clearly more a theoretical curiousness than something of practical interest. Indeed, when $2\sigma^2 t$ is very small, the path is available with probability close to 1 for any feasible number of hops. This is shown in Fig. 9, where we report the path availability as a function of n and t, in the case of a bidimensional Brownian motion with $R = 100$ m and $L = 2.5R$. On this surface, we plotted three curves obtained using (15): one corresponding to the minimum number of hops ($n = 3$), one with twice the minimum ($n = 6$), and the one corresponding to the actual optimum number of hops. We observe that the value of the path availability obtained when the optimum number of hops gets large (above 30) is practically granted also for much smaller values of n, such as $n = 5, 6$ (see the plateau of the surface in Fig. 9).

We also notice that a significant improvement can be achieved by just doubling the number of hops with respect to the minimum (i.e., passing from 3 to 6). To better show this fact, in Fig. 10 we plotted a few cuts of the surface in Fig. 9, at different time instants. We observe that much of the gain can be achieved without increasing n too far beyond the minimum number of hops. Values of n around 2-3 times the minimum can dramatically improve the path availability for any value of t, up to the time at which the path availability becomes very small (below 0.2).

Similar conclusions can be drawn under the RD mobility model. As an example, Fig. 11 reports the path availability obtained using approximation (13) in the same scenario of Fig. 10. This time, we use an RD model in which *move* and *pause* times have an average of 10 s while the speed is uniformly distributed in $[-5, 5]$ m/s. Again, we observe that the optimal number of hops decreases with time, and that significant improvements of the path availability can be obtained using a number of hops a few times larger than the minimum.

7.3 Applications

First, we check the validity of our analytical results against simulation. We consider the same RD model used to derive the results in Fig. 11. Fig. 12 reports the path availability as a function of time obtained by simulation for three different values of n, namely, 3, 5, 12. As expected, the path achieving the best performance varies over time. Up to about $t = 20$ s, the 12-hops path provides the best availability, significantly outperforming the minimum hop path of $n = 3$. Between 20 and 50 s, the optimal path is the one with $n = 5$. Only after

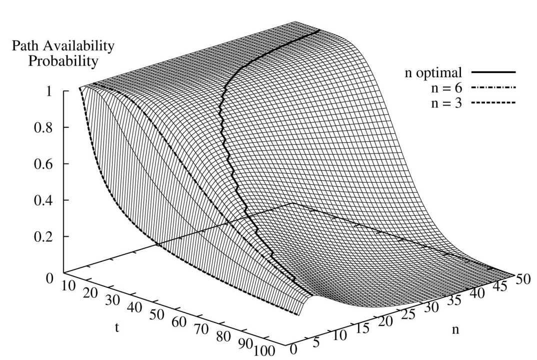

Fig. 9. Path availability as a function of t and n, in case of a bidimensional Brownian motion with $L = 2.5R$.

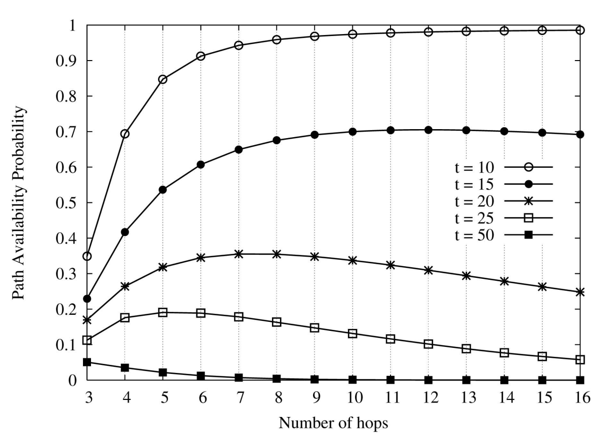

Fig. 11. Path availability at different time instants, as a function of n; bidimensional RD model with $L = 2.5R$.

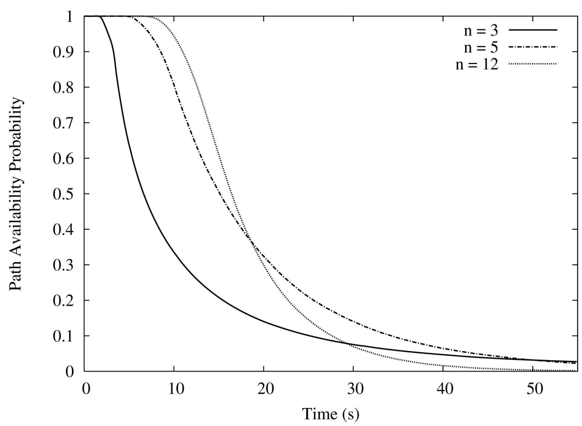

Fig. 12. Path availability as a function of time according to simulation, for different number of hops; bidimensional RD model with $L = 2.5R$.

$t = 50$ s the minimum hop path becomes the best one, but at this point the path availability has already dropped to very low values. These results closely agree with the analytical predictions reported in Fig. 11.

We conclude that *the desired duration of a path plays a crucial role in the selection of the path itself*. In a proactive routing protocol, one would like the routes maintained in the routing table of a node to be available with high probability till a refresh timer expires and routes are recomputed. In a reactive routing protocol, one would like a route to remain stable with high probability for the expected duration of the data communication along the path. For example, in the scenario of Fig. 12, if the amount of data to send requires the connection to last for 10 s, then the optimal path would be the one with 12 hops.

Our proposed approach to find and select routes in a MANET thus takes the desired path duration into account. In particular, let us consider the case of a reactive routing protocol. Our proposal, during the path discovery phase, is to include a $desired\_duration$ field into the route request message sent by the source node. When a reply message is sent back to the source, each node along the reverse path makes an estimate of the probability that the link just traversed by the message will still be available after $desired\_duration$. Based on the independence assumption (14), the node updates an $available\_probability$ field (initialized to one by the node that first sends the route reply message), multiplying the current value by the locally computed probability. When the source node receives the route reply message, it obtains an estimate of the overall path availability, which can be used to select the best available route.

Finally, in case the node density is very large (for example, an area crowded of people carrying communication devices) and links of any length can be assumed to be always found, the source node can optimize the above procedure, under the additional assumption that it can estimate the distance from the destination. Indeed, the source node can precompute an optimal number of hops (and thus an optimal link length) using (20) (evaluated at $t = desired\_duration$). This information can help significantly in reducing the number of messages to be propagated in the network during the path discovery phase: Messages are sent only to neighbors at a distance close to the optimal precomputed one.

8 Conclusions

We studied the duration and availability probabilities of routing paths in MANETs—a fundamental issue to provide reliable routes and short route disruption times. We focused on the Random Direction mobility model and derived both exact and approximate (but simple) expressions for the probability of path duration and availability. We used these results to determine the optimal path in terms of route stability; in particular, we showed some properties of the optimal path and we provided an approximate yet accurate expression for the optimal number of hops. Finally, based on our findings, we proposed an approach to find and select routes, which accounts for the expected data transfer time over the path and allows to reduce the overhead of reactive routing protocols.

Appendix

Here, we show how (9) and (10) can be manipulated to get interesting insights on the mobile dynamics, and obtain a different expression for $M(\mathbf{k}, s)$ and $p(\mathbf{k}, s)$ that can easily be numerically inverted.

Let us define $I(\mathbf{k}, s)$ as

$$I(\mathbf{k}, s) = \int \frac{\lambda\mu f_V(\mathbf{v})}{(s+\lambda)(s+\mu+j\mathbf{k}\cdot\mathbf{v})} d\mathbf{v}.$$

By construction, for every $\mathbf{k}$ or s we have $|I(\mathbf{k}, s)| \leq 1$. Moreover, $|I(\mathbf{k}, s)| = 1$ only when $\mathbf{k} = 0, s = 0$; thus, for $\mathbf{k} \neq 0$ or $s \neq 0$, we have $\frac{1}{1-I(\mathbf{k},s)} = \sum_{i=0}^{\infty} I^i(\mathbf{k}, s)$. It follows that $M(\mathbf{k}, s)$ can be rewritten as

$$M(\mathbf{k}, s) = \left(p_0(\mathbf{k}) \frac{I(\mathbf{k}, \mathbf{v})}{\mu} + \int \frac{m_0(\mathbf{k}, \mathbf{v})}{s+\mu+j\mathbf{k}\cdot\mathbf{v}} d\mathbf{v} \right) \sum_{i=0}^{\infty} I^i(\mathbf{k}, s), \tag{21}$$

which allows us to provide a physical interpretation for $I(\mathbf{k}, s)$ as the Green's function related to the first cycle evolution of the system dynamics for a node that at time $t = 0$ is in the *pause* phase at $\mathbf{x} = 0$. $I(\mathbf{k}, s)$ can be inverted, obtaining

$$I(\mathbf{x}, t) = \lambda\mu \left[e^{-\lambda t} \left(\int_{0_+}^{t} \frac{f_V(\mathbf{x}/\tau)}{\tau} e^{-(\mu-\lambda)\tau} d\tau \right) \mathbb{1}_{\|x\|>0} + \delta(\mathbf{x}) e^{-\lambda t} \right],$$

which provides the node spatial distribution at time t for a node that is ending its first cycle. For finite t, $M(\mathbf{x}, t)$ can be efficiently evaluated by directly inverting the transform in (22) after an appropriate truncation of the series there in.

Proof of Lemma 1. Consider the relative motion of one node with respect to the other. Let $f(z, t)$ be the pdf of the distance reached by this node from its initial location. It results that $f(z, t)$ is also a decreasing function of z. Let $F(z, t)$ be the corresponding cumulative function. In the unidimensional case, we can write

$$A_{link}(d_{A,B}(0), t) = F(R - d_{A,B}(0), t) + F(R + d_{A,B}(0), t) - 1.$$

If we compute the derivative of the above expression with respect to $d_{A,B}(0)$, we obtain

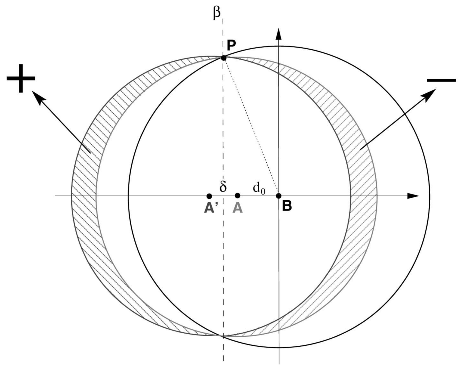

Fig. 13. Illustration of why the path availability is a decreasing function of $d_{A,B}(0)$ in the bidimensional case.

$$\frac{\partial A_{link}(d_{A,B}(0),t)}{\partial d_{A,B}(0)} = \tag{22}$$

$$f(R + d_{A,B}(0),t) - f(R - d_{A,B}(0),t), \tag{23}$$

which is negative because $f(R + d_{A,B}(0),t) \leq f(R - d_{A,B}(0),t)$. Hence, $A_{link}(d_{A,B}(0),t)$ is a decreasing function of $d_{A,B}(0)$.

In the bidimensional case, the same property can be verified with the help of the diagram in Fig. 13. Consider an arbitrary value $d_{A,B}(0)$ of the initial distance between nodes A and B, $0 \leq d_{A,B}(0) < R$. In the coordinates system with origin at point B, the path availability $A_{link}(d_{A,B}(0),t)$ is obtained integrating $f(z,t)$ in the disc of radius R centered at point A. Now let us increase $d_{A,B}(0)$ by an infinitesimal amount δ. The value of $A_{link}(d_{A,B}(0)+\delta,t)$ is obtained integrating $f(z,t)$ in the disc of radius R centered at point A' (see Fig. 13), and we need to show that $A_{link}(d_{A,B}(0)+\delta,t) < A_{link}(d_{A,B}(0),t)$. When we move from $d_{A,B}(0)$ to $d_{A,B}(0)+\delta$, the domain of integration is reduced by the area marked with the "minus" sign, while it is increased by the area marked with the "plus" sign, which is the symmetric of the "minus" area with respect to line β. Now, consider the circle centered at point B with radius $\overline{BP}$, where P is the point marked on Fig. 13. From simple geometric considerations, it results that the "minus" area is entirely contained within this circle, whereas the "plus" area is entirely outside the same circle. Since $f(z,t)$ is a decreasing function of z, the integral of $f(z,t)$ over the "plus" area is less than the integral of $f(z,t)$ over the "minus" area, and we can conclude that $A_{link}(d_{A,B}(0)+\delta,t) < A_{link}(d_{A,B}(0),t)$. □

Proof of Proposition 2. Let L be the initial distance between A and B, and recall that $\mathbf{d_{A,B}(0)} = \{d_{i,i+1}(0)\}$ is the vector of initial hop lengths $d_{i,i+1}(0), 1 \leq i \leq n$. Having fixed n, the optimal path is the solution of the optimization problem

$$\max \prod_{i=1}^{n} A_{link}(d_{i,i+1}(0),t), \tag{24}$$

$$\text{s.t.} \sum_{i=1}^{n} d_{i,i+1}(0) = L \quad \text{and} \quad d_{i,i+1}(0) \geq 0, 1 \leq i \leq n,$$

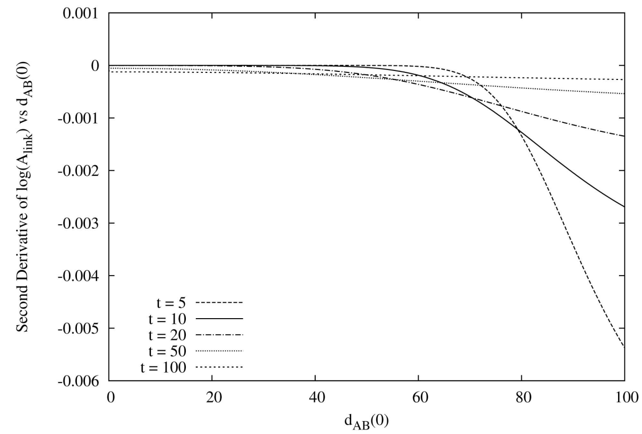

Fig. 14. Second derivative of $\log A_{link}(d_{A,B}(0),t)$ as a function of $d_{A,B}(0)$, for $R = 100, \sigma^2 = 12$, and different values of t.

where we have used the property that the optimal path must lie on the segment $\overline{AB}$ of length L. Taking the logarithm of the objective function, we can alternatively find the maximum of $\sum_{i=1}^{n} \log\left[A_{link}(d_{i,i+1}(0),t)\right]$. Therefore, if the link availability is a log-concave function of $d_{A,B}(0)$, we obtain a convex optimization problem over a convex set in $\mathbb{R}^n$, which admits a unique global optimum. Setting to zero the gradient of the Lagrangian function:

$$\mathcal{L}(\mathbf{d},\lambda) = \sum_{i=1}^{n} \log\left[A_{link}(d_{i,i+1}(0),t)\right] - \lambda\left(\sum_{i=1}^{n} d_{i,i+1}(0) - L\right),$$

we obtain the solution in which $d_{i,i+1}(0) = L/n, \forall i$. □

Remark. Finding the conditions under which $A_{link}(d_{A,B}(0),t)$ is a log-concave function of $d_{A,B}(0)$ is a difficult task. In the special case of a unidimensional mobility model, the only requirement is that $a(z,t)$ is a decreasing function of z. Indeed, we can derive (24) with respect to $d_{A,B}(0)$, yielding

$$\frac{\partial^2 A_{link}(d_{A,B}(0),t)}{\partial d_{A,B}(0)^2} = f'(R - d_{A,B}(0),t) + f'(R + d_{A,B}(0),t), \tag{25}$$

which is negative if $A_{link}(d_{A,B}(0),t)$ is a decreasing function of $d_{A,B}(0)$. In this case, $A_{link}(d_{A,B}(0),t)$ would be a concave function of $d_{A,B}(0)$, thus also log-concave. In the bidimensional case, the analysis is more difficult; however, we have verified numerically (using a discrete approximation for the second derivative of $\log(d_{A,B}(0))$, which has been evaluated at a very large number of points) that $A_{link}(d_{A,B}(0),t)$ is log-concave for any choice of parameters and for any t. As an example, Fig. 14 shows the second derivative of $\log A_{link}(d_{A,B}(0),t)$ with respect to $d_{A,B}(0)$, for $R = 100, \sigma^2 = 12$, and different values of t.

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