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Singular vector expansion functions for Finite Methods

GUIDO LOMBARDI, ROBERTO D. GRAGLIA(*)

ABSTRACT - This paper describes the fundamental properties of new singular vector bases that incorporate the edge conditions in curved triangular elements. The bases are fully compatible with the interpolatory or hierarchical high-order regular vector bases used in adjacent elements. Several numerical results confirm the faster convergence of these bases on wedge problems and the capability to model regular fields when the singularity is not excited.

1. Introduction

Several electromagnetic structures for microwave applications contain wedges or vertices of either penetrable or impenetrable materials. In the vicinity of these geometrical discontinuities the electromagnetic fields and currents could have singular behavior (1-2). These physical quantities reach great values, at worst infinite, although the energy is finite in the region of the singularity. In the scientific literature, several papers deal with singular bases for finite methods, but most of these papers are unsatisfactory. This paper presents some of the results obtained during a pluriennal research activity that concerned the analytical and numerical study of the electromagnetic interactions with structures including wedges. We have defined new singular vector bases for numerical codes based on the finite element method or on the moment method. The new singular bases are of either polynomial or hierarchical kind and incorporate the edge singularity on curvilinear elements (3-4). These functions are compatible with high-order regular vector functions of either polynomial or hierarchical kind (5-6) and they correctly model the physical behavior as described by Van Bladel and Meixner (1-2).
Several numerical results confirm the faster convergence of these new singular vector bases (3).

(*) Dipartimento di Elettronica, Politecnico di Torino- Corso Duca degli Abruzzi 24, 10129 Torino, Italy

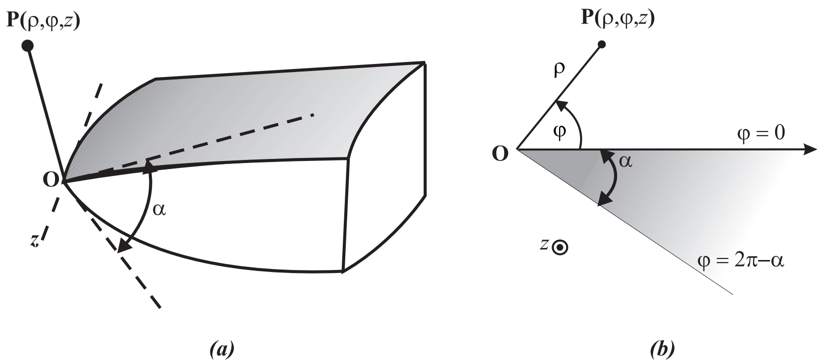

Fig. 1
(a) Cross-sectional view of the region around a sharp but curved wedge of aperture angle α and local longitudinal axis z; (b) Local straight infinite wedge model with aperture angle α and local longitudinal axis z.

$$\boldsymbol{J}_s = \frac{\nu A}{\rho^{1-\nu}}\hat{\boldsymbol{z}} + j\omega\varepsilon_0 B\rho^{\nu}\,\hat{\boldsymbol{\rho}} \pm \text{constant}\,\hat{\boldsymbol{\rho}} \qquad (1)$$

$$\left\{\begin{aligned} E_z &= j\omega\mu_0 A\rho^{\nu}\sin\nu\phi \\ \boldsymbol{H}_t &= \frac{\nu A}{\rho^{1-\nu}}\Big(\sin\nu\phi\,\hat{\boldsymbol{\phi}} - \cos\nu\phi\,\hat{\boldsymbol{\rho}}\Big) \end{aligned}\right. \qquad (2)$$

$$\left\{\begin{aligned} H_z &= j\omega\varepsilon_0 B\rho^{\nu}\cos\nu\phi + \text{constant} \\ \boldsymbol{E}_t &= -\frac{\nu B}{\rho^{1-\nu}}\Big(\cos\nu\phi\,\hat{\boldsymbol{\phi}} + \sin\nu\phi\,\hat{\boldsymbol{\rho}}\Big) \end{aligned}\right. \qquad (3)$$

2. Fundamental properties of singular bases

We investigated several ways to derive singular and complete lowest-order vector bases. We define singular bases to be lowest-order complete when the following properties are fulfilled:

1. the basis set is complete just to the regular zeroth order, and for curl (or divergence) conforming bases the curl (divergence) of the bases is also complete to regular zeroth order;
2. the singular element is fully compatible to adjacent zeroth-order regular elements attached to its nonsingular edges, and to adjacent singular elements of the same order attached to the other edges;
3. the basis functions can model the static, $\rho^{\nu-1}$ singular behavior of the transverse electromagnetic fields (curl-conforming case), or of the surface current and charge density (divergence-conforming case), in the

neighborhood of the wedge (first term of Meixner's series (2)), ρ being the radial distance from the wedge sharp-edge profile;

4. the curl-conforming bases are able to model a nonsingular transverse field with curl that vanishes at the edge of the wedge as ρ^{ν}, whereas the divergence conforming bases can model the radial component of the current density that vanishes as ρ^{ν} at the wedge sharp edge ($\nu \neq 1$ and not integer).

The singular bases reported in (3) contain as a subset the regular p-th order bases given in (5). Only for the elements attached to the edge of a wedge we introduce irrational algebraic vector subset (Meixner subset) in addiction to the regular basis subset. By using complete interpolatory polynomials of order s, in (5) we make the Meixner subset complete to arbitrary high-order s. Since these new bases are formed by the union of p-th order regular plus an s-th order singular part, there is no need to limit the size of the mesh in the neighborhood of the edge of the wedge. Above all, these new bases permit one to deal with all cases where the singularity of the fields is not excited.

3. Numerical results and modeling capabilities

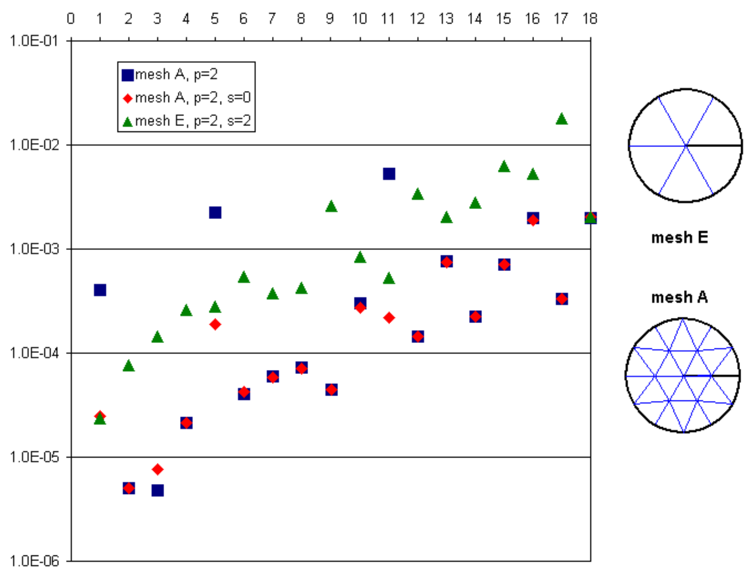

Fig. 2

Fig.2 reports the relative errors of the computed square value of the longitudinal wavenumber (kz^2) for each of the first eighteen modes of the circular vaned waveguide at ko*a = 11, where *a* is the WG radius. Errors are reported in logarithmic scale for two different kind of meshes (Mesh A is 24 triangles and mesh E is 6 triangles) and for different kind of bases (regular bases with p = 2, singular bases with p = 2 and s = 0 and finally singular bases with p = 2 and s = 2). Table 1 reports the mean values of the relative errors for the three simulations reported in Fig. 2.

Table 1.

	mesh A p=2	mesh A p=2, s=0	mesh E p=2, s=2
Mean value 1	7.738E-04	8.700E-05	5.542E-04
Mean value 2	1.991E-03	1.088E-04	2.087E-04

"Mean value 1" is the relative error among the first 11 eigenmodes and "mean value 2" is among the first 3 singular eigenmodes.

4. Acknowledgements

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