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# MACROMODELING OF ELECTRICAL INTERCONNECTS AND PACKAGES VIA PEEC APPROACH

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**Abstract** - This paper deals with model order reduction of multi-input-multi-output structures described by Partial Element Equivalent Circuits (PEEC). A Krylov-based algorithm is adopted, that guarantees the stability and the passivity of the obtained reduced circuit with good accuracy.

Our procedure is validated by means of a simple example made of two parallel stacked traces with air filling, representing an ideal interconnect. Comparisons showing the frequency behaviour of the interconnect scattering parameters with analytical expressions based on Telegrapher's equations are presented. We then apply the proposed modeling process to analyze, both in the time and in the frequency domain, the crosstalk between two microstrip traces either in the case of air dielectric and in the case of relative dielectric constant of 4.7. The influence of the model order on the results is also discussed.

## I. INTRODUCTION

The continuous progress in computer-aided design techniques and in microelectronics technology are leading to ever denser and geometrically more complex layouts and to the use of ever-increasing operating frequencies, at both the chip and package level. Signal rise times of hundreds of picoseconds are becoming quite common and the previously negligible effects of interconnects represent the major limiting factor of the system performances.

Increased speeds and reduced geometric distances between the traces and the planes lead, in fact, to high signal couplings and distortions, which can cause the electronic device not to meet the required Signal Integrity (SI) and EMC specifications. Engineers must then consider the phenomena of crosstalk, ringing, signal delay and reflections during early design stages, by means of efficient and low computational costs models. Unfortunately, at high frequencies, conventional signal integrity tools, being based on a circuit approach that only considers the length of the traces and not their shape, become inaccurate and 3D techniques are necessary to take into account all the geometric features of the structure under examination.

The Partial Element Equivalent Circuit (PEEC) method applied in this paper represents a well-suited modeling technique for analyzing interconnects performances,

because of its ability to handle 3D complex irregular structures. This formulation transforms the geometry under examination into a passive RLC equivalent circuit, where the signal propagation phenomena are modeled by means of resistances, inductances and capacitances. The mutual coupling between the traces and the planes are taken into account by mutual inductances and by current controlled current generators. Besides, we can easily connect electrical model of drivers and receivers to the ports and perform simulations within a SPICE-like environment, both in time and in frequency domain.

Because of the large equivalent circuit complexity of the PEEC approach, a model order reduction (MOR) technique must be applied to perform circuit simulations in a reasonable amount of time. Recently, several moment-matching algorithms based on Padé direct approximations (AWE [1], CFH [2]) or on Krylov subspaces (PVL [3], Arnoldi [4]) were proposed. However, they suffer of ill-conditioning problems or do not ensure the passivity of the reduced model. Consequently, we choose a different Krylov-space technique: the Passive Reduced-order Interconnect Macromodeling Algorithm (PRIMA) [5], because it guarantees the stability and the passivity of the obtained reduced circuit with good result accuracy. Besides, as this approach handles model order reduction on multi-input-multi-output structures, its application to macromodeling of general electrical interconnects and packages will be easy and immediate.

## II. PEEC MODEL

The PEEC method allows the electromagnetic analysis of all structures composed by finite size dielectric and conductor regions. It transforms the structure under examination into a passive RLC equivalent circuit. In this section, we briefly give an overview of the method and of the MNA formulation of the equivalent circuit with the ports connected.

In the absence of an applied external field, the Electric Field Integral Equation (EFIE) holds in a volume  $V'$ , with  $S'$  representing the external surface and the interfaces between different material regions. By subdividing the volume  $V'$  and the surface  $S'$  into elementary cells, as shown in Figure 1, and by substituting the unknowns (the current and the charge

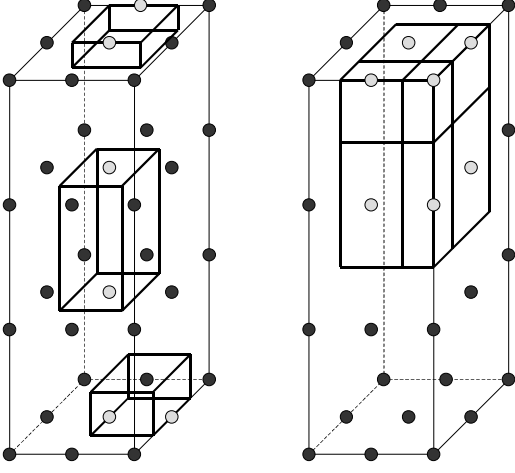


Figure 1: Elementary volume and surface cells

density) with their average values on the respective cells, we come up with equation (1) below describing the equivalent circuit of a volume cell in the generic  $\gamma$  direction.  $N_{v\gamma}$  denotes the number of volume cells in the  $\gamma$  direction,  $N_s$  is the total number of surface cells,  $a_i$  is the cross-section of the  $i$ -th cell and  $t_{ij}$  is the time-of-flight between the center points of the  $i$ -th and the  $j$ -th cell. The first term of equation (1) describes the electromagnetic behaviour of the cell (conductor or dielectric); the second one takes into account the coupling with the other volume cells and the last one

expresses the potential difference between the cell nodes.

By replacing the  $\gamma$ -component of the total electric field with its well-known expression for conductor or dielectric cells, we can interpret the equation as an equivalent circuit in terms of resistances, inductances, capacitances and controlled generators [6], getting the partial models shown in Figure 2.

$R_{\gamma i}$ ,  $L_{\gamma ii}$ ,  $C_{\gamma i}^+$  and  $C_{\gamma i}$  are the resistance, the inductance and the excess capacitance of a volume cell and the pseudocapacitance of a surface cell, respectively;  $M_{\gamma m}$  and  $p_{\gamma m}$  denote the mutual inductance between volume cells and the coefficient of potential between surface cells.

The development of the overall PEEC circuit is performed by properly connecting the partial models of the elementary cells, i.e. by enforcing the Kirchoff's current law at each electric node of the resulting PEEC circuit. This means that we comply with the discrete version of the conservation of charge equation for the examined structure.

As the partial elements method produces non-sparse matrices of large dimensions, the solution of the problem, necessarily based on matrix inversion, is very expensive in terms of computational efforts. Therefore, an efficient and accurate MOR technique becomes necessary to perform real-world structure analysis in a reasonable run time.

In order to prepare the formulation for the MOR algorithm, we need to connect voltage sources to the examined structure ports and assemble the mathematical model by using the MNA approach for the equivalent

$$\frac{1}{a_{\gamma i}} \int_{V_{\gamma i}} E_{\gamma}(\vec{r}, t) dV + \sum_{n=1}^{N_{v\gamma}} \left\{ \frac{\mu_0 \mu_r}{4\pi a_{\gamma i} a_{\gamma n}} \frac{\partial I_{\gamma n}(t_{\gamma in})}{\partial t} \int_{V_{\gamma i}} \int_{V_{\gamma n}} \frac{1}{|\vec{r} - \vec{r}'|} dV' dV \right\} + \sum_{m=1}^{N_s} \left\{ \frac{1}{4\pi \epsilon_0 a_{\gamma i} a_{\gamma m}} \left[ Q_m^T(t_{\gamma^+ m}) \int_{S^+_{\gamma i}} \int_{S^+_{\gamma m}} \frac{1}{|\vec{r} - \vec{r}'|} dS' dS - Q_m^T(t_{\gamma^- m}) \int_{S^-_{\gamma i}} \int_{S^-_{\gamma m}} \frac{1}{|\vec{r} - \vec{r}'|} dS' dS \right] \right\} = 0 \quad (1)$$

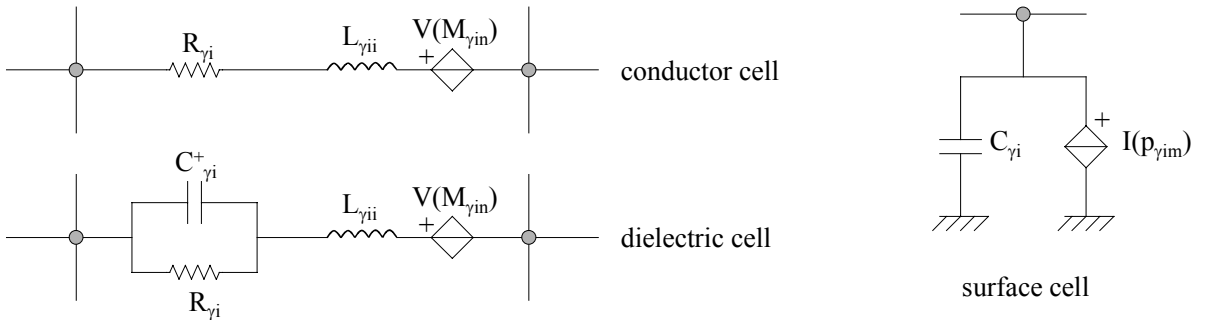


Figure 2: PEEC models for volume and surface cells

circuit [7], i.e.

$$\begin{aligned} \mathbf{C} \dot{\mathbf{z}} + \mathbf{G} \mathbf{z} &= \mathbf{B} \mathbf{v}_p \\ \mathbf{i}_p &= \mathbf{B}^T \mathbf{z} \end{aligned} \quad (2)$$

where  $\mathbf{z}$  is the vector of the MNA variables, i.e. the unknown node voltages and the branch currents for voltage sources and inductors. The vectors  $\mathbf{v}_p$  and  $\mathbf{i}_p$  denote respectively the voltages and currents at the  $N$  ports of the structure and the matrices  $\mathbf{C}$ ,  $\mathbf{G}$  and  $\mathbf{B}$  of dimension  $n \times n$  describe the circuit topology. The independence of these matrices from the time variable indicates that retardation, i.e. time-of-flight between cells, has been neglected and the assumed model is the lumped circuit known in literature as  $(L_p, P, R)$  PEEC.

### III. MODEL ORDER REDUCTION ALGORITHM

From the variety of approaches for reducing the PEEC network order, we select the PRIMA method [5], because of its robustness and its ability to preserve the stability and the passivity of the circuit. Such technique is a Krylov-space order reduction method that performs MOR on multi-input-multi-output structures and that overcomes the ill-conditioning associated with the direct Padé approximation.

The algorithm is based on the construction of a matrix  $\mathbf{X}$  whose columns constitute an orthonormal basis for the corresponding Krylov subspace [5], i.e.

$$\text{col sp}(\mathbf{X}) = \text{Ker} \left( -\mathbf{G}^{-1} \mathbf{C}, \mathbf{G}^{-1} \mathbf{B}, \left[ \frac{q}{N} \right] \right) \quad (3)$$

The matrix  $\mathbf{X}$  has  $q$  columns, where  $q$  is the order of reduction, while the number of rows matches the size of the original MNA system;  $N$  is the number of ports. Through the matrix  $\mathbf{X}$ , which defines a change of variable ( $\mathbf{z} \rightarrow \tilde{\mathbf{z}}$ ) in the MNA system, the corresponding congruence transformation of the model is computed

$$\begin{aligned} \tilde{\mathbf{C}} &= \mathbf{X}^T \mathbf{C} \mathbf{X} \\ \tilde{\mathbf{G}} &= \mathbf{X}^T \mathbf{G} \mathbf{X} \\ \tilde{\mathbf{B}} &= \mathbf{X}^T \mathbf{B} \end{aligned} \quad (4)$$

To the reduced order circuit, defined through its MNA formulation,

$$\begin{aligned} \tilde{\mathbf{C}} \dot{\tilde{\mathbf{z}}} + \tilde{\mathbf{G}} \tilde{\mathbf{z}} &= \tilde{\mathbf{B}} \mathbf{v}_p \\ \mathbf{i}_p &= \tilde{\mathbf{B}}^T \tilde{\mathbf{z}} \end{aligned} \quad (5)$$

we can connect electrical, linear or non-linear, model of drivers and receivers and perform its simulation in a reasonable run time, since the system dimension is reduced to  $q \ll n$ .

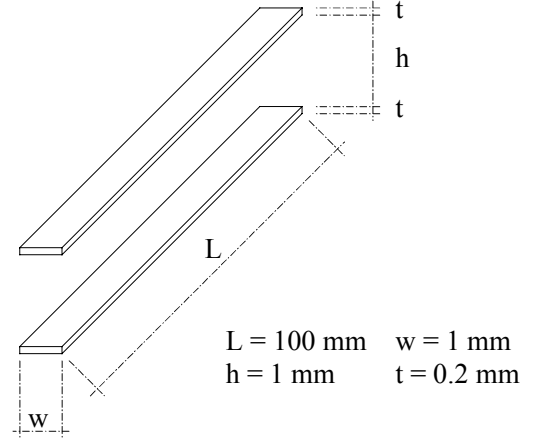


Figure 3: Two parallel stacked traces

On the other hand, we can extract a macromodel of the structure by representing the N-port by its reduced order admittance matrix  $\tilde{\mathbf{Y}}(s)$

$$\tilde{\mathbf{Y}}(s) = \tilde{\mathbf{B}}^T (\tilde{\mathbf{G}} + s \tilde{\mathbf{C}})^{-1} \tilde{\mathbf{B}} \quad (6)$$

or its reduced scattering parameters  $\tilde{\mathbf{S}}(s)$

$$\tilde{\mathbf{S}}(s) = \mathbf{Y}_0^{-1/2} (\mathbf{Y}_0 - \tilde{\mathbf{Y}}(s)) (\mathbf{Y}_0 + \tilde{\mathbf{Y}}(s))^{-1} \mathbf{Y}_0^{1/2} \quad (7)$$

provided that the proper matrix of reference admittances  $\mathbf{Y}_0$  is defined.

## IV. COMPUTATIONAL RESULTS

The proposed method, based on PEEC methodology and PRIMA algorithm, is tested on two simple structures for which an analytical solution exists. The first example considers the electromagnetic behaviour of two parallel stacked traces placed in air, by analyzing the scattering parameters of the structure. The second one studies the crosstalk between two microstrip traces: in the time domain for no dielectric substrate, and in the frequency domain for a relative dielectric constant of 4.7.

### IV.1 Two Parallel Stacked Traces

The geometrical characteristics of the examined interconnection are shown in Figure 3: the traces are 100-mm long, 1-mm wide and 0.2-mm thick, while their distance is 1 mm. The  $\mathbf{S}$  matrix is computed with respect to reference resistances of  $50 \Omega$ .

To validate our results, we first calculate the analytical expression of the scattering parameters using the Telegrapher's equations and the characteristic impedance value computed according to the conventional equations [8]. In parallel, we perform the simulation of the original PEEC circuit using Spice. We next apply the PRIMA algorithm to the MNA formulation to generate the simplified equivalent circuit and we evaluate the reduced scattering matrix through the equation (7) above.

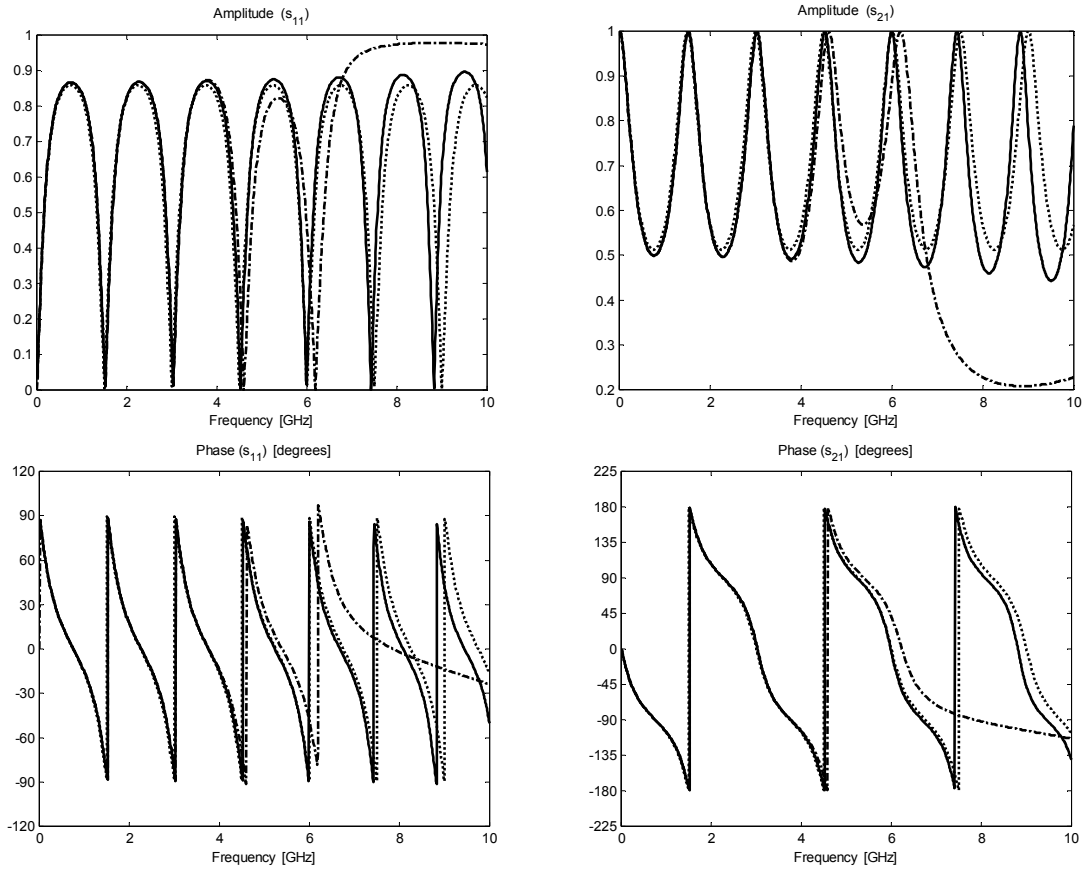


Figure 4: Scattering parameters of two parallel stacked traces, evaluated by Telegrapher's equations (dotted line), by original PEEC circuit (solid line) and by reduced order PEEC model (dash-dotted line)

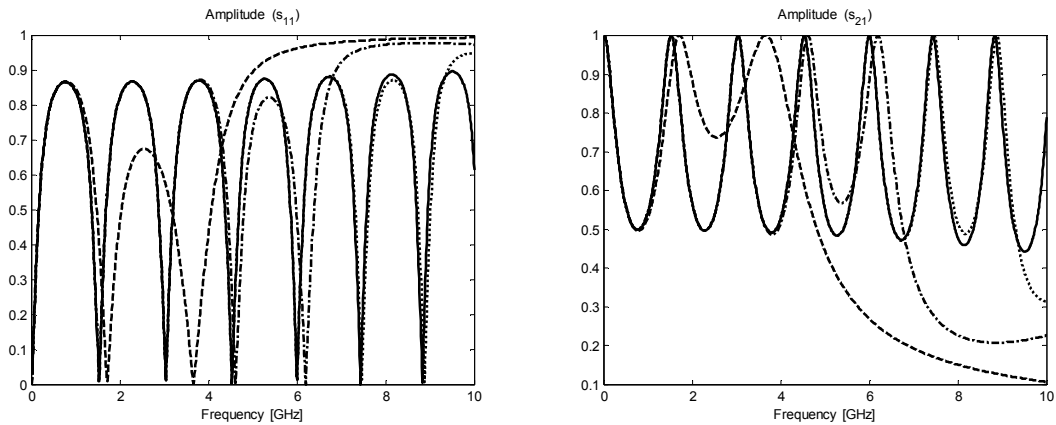


Figure 5: Effect of the reduced order  $q$  on the amplitude of the scattering parameters: original PEEC circuit (solid line),  $q = 8$  (dashed line),  $q = 16$  (dash-dotted line) and  $q = 24$  (dotted line)

As we discretize each trace with 21 nodes, generating 20 volume cells and 21 surface cells, and the structure has 2 ports, the original MNA system has size 84. The order of the performed reduced system is instead 16.

Figure 4 shows the obtained results and validates our PEEC original model: neglecting the time-of-flight between the cells causes, beyond 5 GHz, the mismatches of amplitude and of resonance frequencies in comparison to the Telegrapher's equations solution.

These differences obviously increase as frequency increases. Besides, the scattering parameters evaluated via the original non-reduced PEEC circuit and via the reduced order model agree very well in the low frequency range, because the implemented PRIMA algorithm guarantees the transfer function moment matching in the origin of the Laplace domain. In particular, having the structure two ports, an order reduction with  $q = 16$  guarantees the matching of the

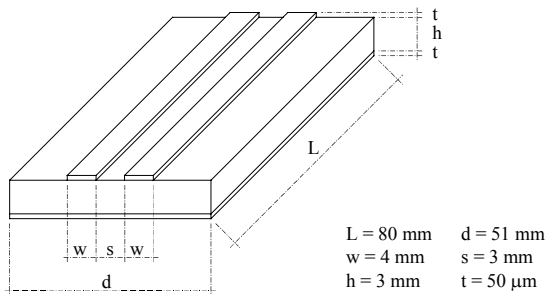


Figure 6: Microstrip structure

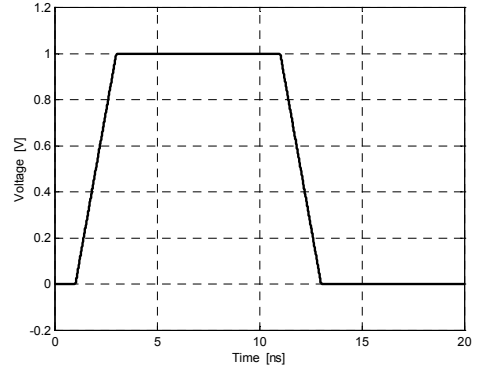


Figure 7: Active line excitation

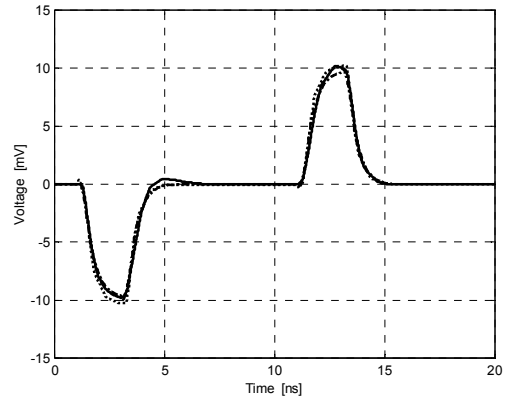
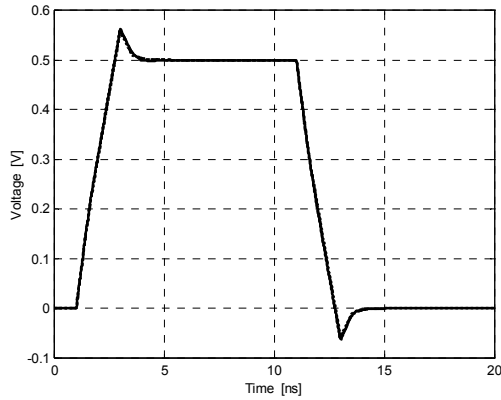


Figure 8: Voltage waveform on active line input termination and on victim line far end evaluated by transmission line model (dotted line), by original PEEC circuit (solid line) and by reduced order PEEC model (dash-dotted line)

first eight moments.

Increasing the order  $q$  of reduction allows extending the frequency range of validity for the simplified model, as shown in Figure 5: with  $q = 24$  PRIMA captures almost the entire spectrum. Establishing a priori a relationship between  $q$  and the maximum frequency is not easy, since it depends on the location of original PEEC poles, whose computation we want to avoid, being prohibitive in terms of memory requirements. In fact, after having captured the dominant poles in the low frequency range, extending the frequency spectrum of validity requires a remarkable increment of the order reduction: the PEEC discretization generates a large quantity of poles in a limited frequency range and PRIMA tends to capture redundant poles [9]. In order to overcome this issue, a block order reduction algorithm should be applied.

## IV.2 Microstrip Structure

The second example firstly studies the crosstalk voltage waveforms between two microstrip traces with no dielectric substrate, whose geometric characteristics are reported in Figure 6. Obviously, only one line is excited by an 8 ns quasi-square pulse with 2 ns rise and fall time

(see Figure 7), whereas the other one is the coupling victim. All terminations are loaded by a  $50 \Omega$  impedance.

We construct the PEEC model, which is of dimension 263, as expressed by equation (2) and, by applying PRIMA algorithm, we generate the simplified equivalent circuit of order  $q = 16$ , according to equation (5). We then compute the port voltages by solving the respective ODE system. Results validation is obtained through a transmission line model based on per-unit-length parameter evaluation via the software LINPAR and on circuit solution via Spice.

Figure 8 shows obtained voltage waveform on active line input termination and on victim line far end: results agree very well. And it is important to note that the solutions of original PEEC circuit and of reduced order model are performed, by means of a computer with 512 MB RAM and 1.8 GHz clock, in 577 seconds and in 3.125 seconds of CPU time, respectively, with a speed-up of 185.

We then add to the examined structure a substrate of relative dielectric constant 4.7, between the plane and the traces, and we analyze the crosstalk coefficients in the frequency domain. We follow the same approach as above, unless that the excitation is provided by an ideal pulse and we compute the port voltages by matrix

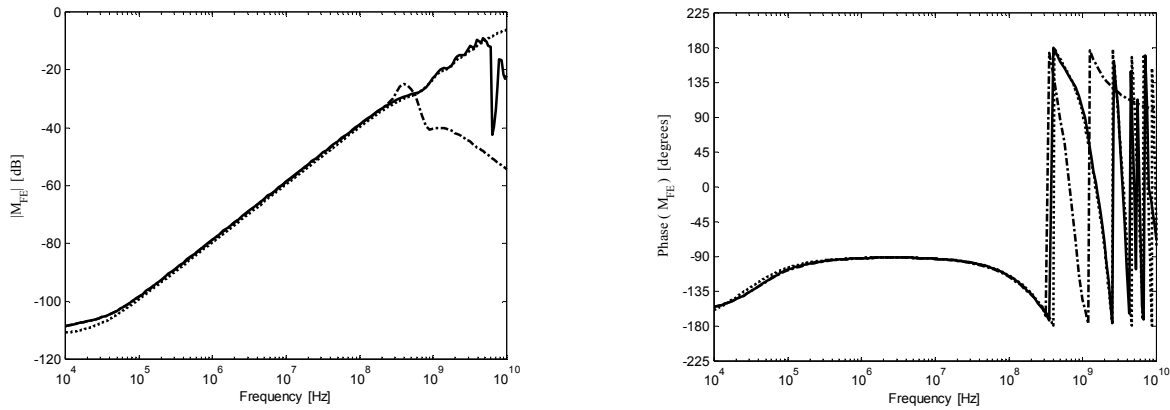


Figure 9: Amplitude and phase of the far end crosstalk coefficient evaluated by transmission line model (dotted line), by original PEEC circuit (solid line) and by reduced order PEEC model (dash-dotted line).

inversion, as suggested by equation (6), instead of solving the ODE systems.

The original PEEC circuit and the reduced order model are respectively of dimension 2126 and 150 and their solutions at each frequency value are performed in 209 seconds and in 72 ms of CPU time, respectively, with a speed-up of 2900. As shown in Figure 9, a transmission line model validates the obtained results.

## V. CONCLUSION

PEEC method, in conjunction with the PRIMA algorithm, has been used to study the electromagnetic behaviour of two parallel stacked traces placed in air. The amplitude of the scattering parameters obtained before and after the model reduction has been compared to that calculated via the analytical expression extracted by the Telegrapher's equations.

Analyzing both in the time domain and in the frequency domain the crosstalk voltages between two microstrip traces has further tested the proposed method. It has also shown the possibility to perform simulations of the equivalent circuit in a much reasonable time, without losing accuracy in a limited frequency range close to the expansion point.

Future works will analyze more complex and real-world structures and will investigate the possibility of reducing the PEEC model at expansion points different from the origin of the Laplace domain. We could then combine portions of each reduced circuit to obtain a lower-order macromodel approximating the original system over a larger frequency range.

## VI. REFERENCES

- [1] L. T. Pillage, R. A. Rohrer, "Asymptotic waveform evaluation for timing analysis", *IEEE Transactions on Computer-Aided Design*, vol. 9, No. 4, pp. 352-366, April 1990.
- [2] E. Chiprout, M. Nakhla, "Analysis of interconnect

networks using complex frequency hopping (CFH)", *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 14, No. 2, pp. 186-200, February 1995.

- [3] P. Feldmann, R. W. Freund, "Efficient linear cut analysis by Padé via Lanczos process", *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 14, No. 5, pp. 639-649, May 1995.

- [4] M. Silveria, M. Kamon, I. Elfadel, J. White, "A coordinate-transformed Arnoldi algorithm for generating guaranteed stable reduced-order models of arbitrary RLC circuits", *Proceedings of IEEE ICCAD'96*, pp. 288-294, November 1996.

- [5] A. Odabasioglu, M. Celik, L. T. Pileggi, "PRIMA: Passive Reduced-order Interconnect Macromodeling Algorithm", *Proceedings of IEEE ICCAD'97*, pp. 58-65, November 1997.

- [6] A. E. Ruehli, H. Heeb, "Circuit Models for Three-Dimensional Geometries Including Dielectrics", *IEEE Transactions on Microwave Theory and Techniques*, vol. 40, no. 7, pp. 1507-1516, July 1992.

- [7] A. E. Ruehli, U. Miekala, H. Heeb, "Stability of Discretized Partial Element Equivalent EFIE Circuit Models", *IEEE Transactions on Antenna and Propagation*, vol. 43, no. 6, pp. 553-559, June 1995.

- [8] C. R. Paul, "Introduction to Electromagnetic Compatibility", John Wiley & Sons, Inc., New York, 1992.

- [9] R. Achar, P. K. Gunupudi, M. Nakhla, E. Chiprout, "Passive Interconnect Reduction Algorithm for Distributed/Measured Networks", *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 47, no. 4, pp. 287-301, April 2000.