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Assessment of the Buckling Behavior of Square Composite Plates with Circular Cutout Subjected to In-Plane Shear

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ABSTRACT

This paper aims at evaluating the effect of various parameters on the buckling load of square cross-ply laminated plates with circular cutouts. The parameters considered in this study are: (1) cutout size (2) cutout location (3) fiber orientation angle and (4) type of loading. Three types of in-plane loading were considered; namely, uniaxial compression, biaxial compression and shear loading. The reduction in the buckling load due to the increase of cutout size was significant in the case for shear loading as compared to uniaxial and biaxial compression. For relatively small size cutouts, a better performance was achieved if the cutout is kept close to the edge of the plate, however, for relatively large size cutouts, a higher buckling load is achieved if the cutout is kept in the middle of the plate. Several other imperative findings based upon the various parameters are also presented in this study.

KEYWORDS: Buckling, Laminates, Cutouts, Finite element analysis.

INTRODUCTION

Composite materials have such an influence on our lives that many researches invested a great deal of time and effort for a better understanding of their behavior. Composite materials have been used for a while in many industries such as: aerospace, automotive, marine and civil engineering applications – one can say that composite materials usage is limited by the individual imagination. One type of composite materials is cross-ply laminated plates with cutouts, where cutouts are introduced for accessibility reasons or to just lighten the structure. Such plates are vulnerable to buckle when subjected to various types of in-plane loadings; therefore it is of great importance to fully understand the effects of various parameters on its buckling load.

Many researchers investigated this problem over the years from different perspectives; the following paragraphs summarize their work and key findings. Vandenbrink and Kamat (1987) studied the buckling load and post-buckling behavior of isotropic and composite plate with central holes. In their study, they compared the Finite Element Method (FEM) results with experimental results and found a good agreement. In their work they concluded that there is a bit more loss in composite plates post-buckling strength as compared to isotropic plates. They also found that the buckling load of composite plates with large holes begin to increase above that of the corresponding solid plate for fiber orientation angle of 60°.

Larsson (1987, 1989) investigated the buckling load and post buckling behavior of orthotropic laminated square and circular plates with central circular cutout. Larsson utilized the FEM to solve the problem for

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uniaxial and biaxial compression for various boundary conditions. Larsson reported a reduction in the buckling load of square plates with central cutouts which was more pronounced in the case of uniaxial compression. Lee et al. (1989) used the FEM to study the buckling of orthotropic square plates with central circular hole. They observed different buckling modes in the stronger and weaker material directions and, therefore, recommended the use of full-scale configuration model in the analysis of the buckling problem. Hyer and Lee (1991) explored the improvement of the buckling load of square composite plates with central holes through the use of curvilinear fiber format. The study concluded that fiber orientation far from the hole has less effect on the buckling load as compared to the effect of fiber orientation near the hole.

Hu and Lin (1995) presented an optimization scheme with respect to fiber orientation for the buckling load of symmetrically laminated plates with central holes. They reported that the optimal buckling loads of thin square composite plates with a central circular cutout, except simply supported plates, increase with the increase of the sizes of cutouts; therefore, it is theoretically possible to come up with a cutout size and fiber angle to increase the buckling loads beyond those of corresponding plates without cutouts. Kong et al. (2001) investigated the buckling and post-buckling behavior of laminated plates with central holes through comparison of progressive failure FEM and experimental results. They concluded that the bending stiffness in the axial direction has a great effect on the buckling load and post-buckling behavior of such plates.

Akbulut and Sayman (2001) studied the buckling of laminated plates with central square holes under uniaxial and biaxial loading. In their formulation, first order shear theory was included. Angle-ply laminates with $[45/-45]_n$ orientation of plies showed the best performance under uniaxial compression. Saha et al. (2004) tested composite plates with central holes under compression loading. They found that as the hole diameter increases, the plate slenderness ratio (height/width) becomes less effective on the buckling load.

Jain and Kumar (2004) and Ghannadpour et al. (2006) investigated the critical buckling load and post-buckling behavior of laminated plates with circular and elliptical cutouts under axial compression. They found that buckling load decreases as the central circular hole diameter increases. A laminate without a cutout fails near the diagonal corner, but in the presence of a circular cutout the failure location shifts towards the cutout edge. However, with an elliptical cutout the failure takes place near the vertex of the cutout. Small cutouts can be neglected from analysis which reduced the meshing effort.

Baltaci et al. (2006) analyzed laminated circular plates with circular holes using finite element analysis. They concluded that the critical buckling load decreases as the hole gets closer to the plate center. The critical buckling load for square notched plates was more sensitive for simply supported plates than clamped ones. Mallela and Upadhyay (2006) presented a parametric study on laminated composite blade-stiffened panels subjected to in-plane shear. They proposed some design charts that can be used to selecting the optimum parameters for better stiffener proportions.

Baba (2007) and Baba and Baltaci (2007) studied the effect of support conditions on the buckling load of laminated plates with circular in semicircular cutouts under axial compression. They found that the buckling load of clamped-clamped plates is 75% higher than the buckling load of simply supported plates, and 50% for the case of clamped-pinned boundary condition. Also, an increase of the length/thickness ratio of 50% increases the buckling load in the range of 75%.

Evidently, the vast majority of the research conducted on the buckling problem of laminated plates has paid less attention to the buckling due to in-plane shear loading. Thus, the current parametric study attempts at reinforcing the literature with the lacking information on the buckling of cross-ply laminated plates with circular cutouts due to in-plane shear loading. The effect of various parameters was also investigated for comparison reasons, these parameters includes: (1) cutout size (2) cutout location or eccentricity (3) fiber orientation angle and (4) type of loading. Three types of in-plane loading were considered;

namely, uniaxial compression, biaxial compression and shear loading.

FINITE ELEMENT MODEL

Modeling composite laminated plates requires extra attention in defining the properties of the plate, including number of layers, thickness and fiber orientation of each layer, as well as mesh sizing, especially near the cutouts. In the present work, Eigen-buckling analysis is performed for the laminated composite plates which take into consideration higher-order shear deformation theory, through the use of the finite element package named ABAQUS. The plate dimension considered is $2000\ mm \times 2000\ mm$. The thickness of each layer of this eight-layer laminates is 1.5 mm. The material properties of the lamina are given in Table (1). In this table, E_1, E_2, E_3 are the modulus of elasticity, G_{12}, G_{13} are the shear modulus corresponding to the planes 1-2 and 1-3, respectively and υ_{12}, υ_{13}, υ_{23} are the corresponding Poisson ratios.

Table (1): Material properties of the lamina.

Mechanical Properties	Values
E_1	130.0 GPa
E_2	10.0 GPa
E_3	10.0 GPa
$G_{12} = G_{13}$	5.0 GPa
$\nu_{12} = \nu_{13}$	0.35
$\nu_{23} = \nu_{32}$	0.35

The model is composed of mainly four noded shell elements that have six degrees of freedom per node. Three noded shell elements are only used in irregular zones around the holes. The composite laminates are divided into sufficient number of elements to allow for free development of buckling modes and displacements. Some trial runs were also carried out to study the convergence of the results.

In this study, three types of loading were considered; namely, uniaxial compression, biaxial compression and shear loading. The plate area is located in the xy plan. For uniaxial loading, the compressive loads were applied in the x direction, while for biaxial and shear, the loads were applied in the x and y directions as shown in Figure (1). Series of preselected cases are modeled to verify the accuracy of the method of analysis. The results are compared to theoretical and numerical values, as shown in Table (2) and Table (3). In Table 2, *b*, *h* are the width and thickness of the plate, respectively, N_{cr} is the buckling load and *D* is as follows:

$$D = \frac{E\,h^3}{12\left(1-\nu\right)} \tag{1}$$

and in Table (3), *d* is the diameter of the cutout.

From these results, it can be observed that the present study and the values available in the literature are in good agreement. The buckling mode shapes obtained in the present study are similar in respect with the buckling mode shapes available in the literature as shown in Figure (1).

PLATE SUBJECTED TO UNIAXIAL LOAD

The isotropic and anisotropic $2000\ mm \times 2000\ mm$ square plate mentioned above is subjected to uniaxial load to study the effect of cutout size, cutout location and fiber orientation angle on the bucking behavior.

Effect of Cutout Size

The ratio of the cutout diameter (d) to the isotropic and anisotropic plate width (b) is varied from 0.0 to 0.5. The location of the cutout was assumed to be located at the center and at one quarter of the plate as shown in Figure (2). From this figure, it can be observed that the buckling load for anisotropic plate is decreased with different percentages when the cutout size is increased and the effect of cutout size on bucking load for the isotropic plate is considered minor as compared to that of the anisotropic.

It is worth mentioning that the decrease in buckling load in the case of fiber orientation angle [0, 90]4s is more than in the other cases where this decrease reaches 68% when changing the (d/b) ratio from 0.0 to 0.5 as shown in Figure (2).

Table (2): Comparison of buckling load between theoretical method and finite element model for isotropic plate without cutouts.

Theoretical Method	E (Pa)	ν	b (m)	h (m)	Non-Dimensional Buckling Load (Theoretical) $\frac{N_{cr}b^2}{Eh^3}$	Non-Dimensional Buckling Load (FE Model) $\frac{N_{cr}b^2}{Eh^3}$
Uniaxial Case $N_{cr}=\frac{4\pi^2 D}{b^2}$	2×10^{11}	0.3	2.0	0.02	3.615	3.623
Biaxial Case $N_{cr}=\frac{2\pi^2 D}{b^2}$	2×10^{11}	0.3	2.0	0.02	1.807	1.811
Shear Case $N_{cr}=\frac{9.34\pi^2 D}{b^2}$	2×10^{11}	0.3	2.0	0.02	8.442	8.400

Table (3): Comparison of buckling load between current finite element model and results available from Ghannadpour et al. (2006) for composite plate $[0, 90]_{4s}$ with circular cutout subjected to uniaxial load.

d/b	Non-Dimensional buckling load (Ghannadpour et al [11]) $\frac{N_{cr}b^2}{E_2h^3}$	Non-Dimensional buckling load (FE Model) $\frac{N_{cr}b^2}{E_2h^3}$
0.0	13.79	13.60
0.1	12.80	13.07
0.2	10.82	10.53
0.3	8.97	8.80
0.4	7.51	7.42
0.5	6.39	6.40
0.6	5.63	5.31
0.8	4.43	4.37

Effect of Location

The location of the cutout is changed through the square plate in combination with the size of the cutout. The changes in location are represented by the ratio of the eccentricity (e_x), i.e. distance from the edge of the plate to the center of the cutout, to the plate width (b). Uniaxial load is subjected to isotropic and anisotropic plates as shown in Figure (3).

In the case of isotropic plate and for (d/b) = 0.1, 0.2 and 0.3, the buckling load is increased with small percentages when (e_x/b) = 0.3, 0.4 and 0.5, then these percentages get higher when (e_x/b) = 0.2 and 0.1. On other hand, the buckling load is decreased for (d/b) = 0.4 and 0.5 when the cutout is moved toward the edges.

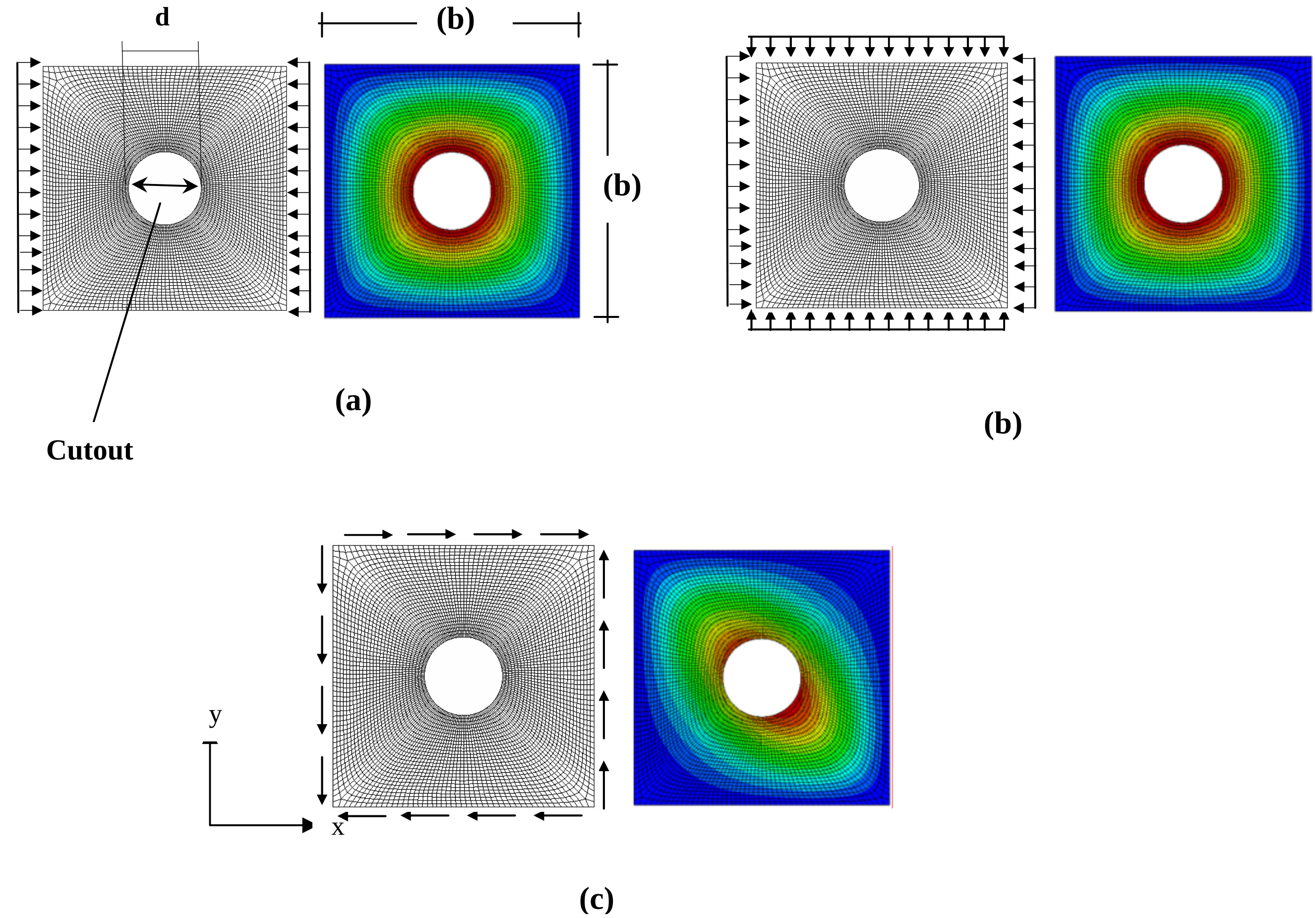


Figure (1): In-plane loading cases and first buckling mode shape for (a) uniaxial loading (b) biaxial loading (c) shear loading.

In the case of anisotropic plate with fibers orientation (45-45-90-0-0-90-45-45), the buckling load is increased for (d/b) = 0.1 and 0.2 when the cutout is moved toward the edges as shown in Figure (3). However, buckling load is decreased for (d/b) = 0.4 and 0.5 when the cutout is moved toward the edges.

PLATE SUBJECTED TO BIAXIAL LOAD

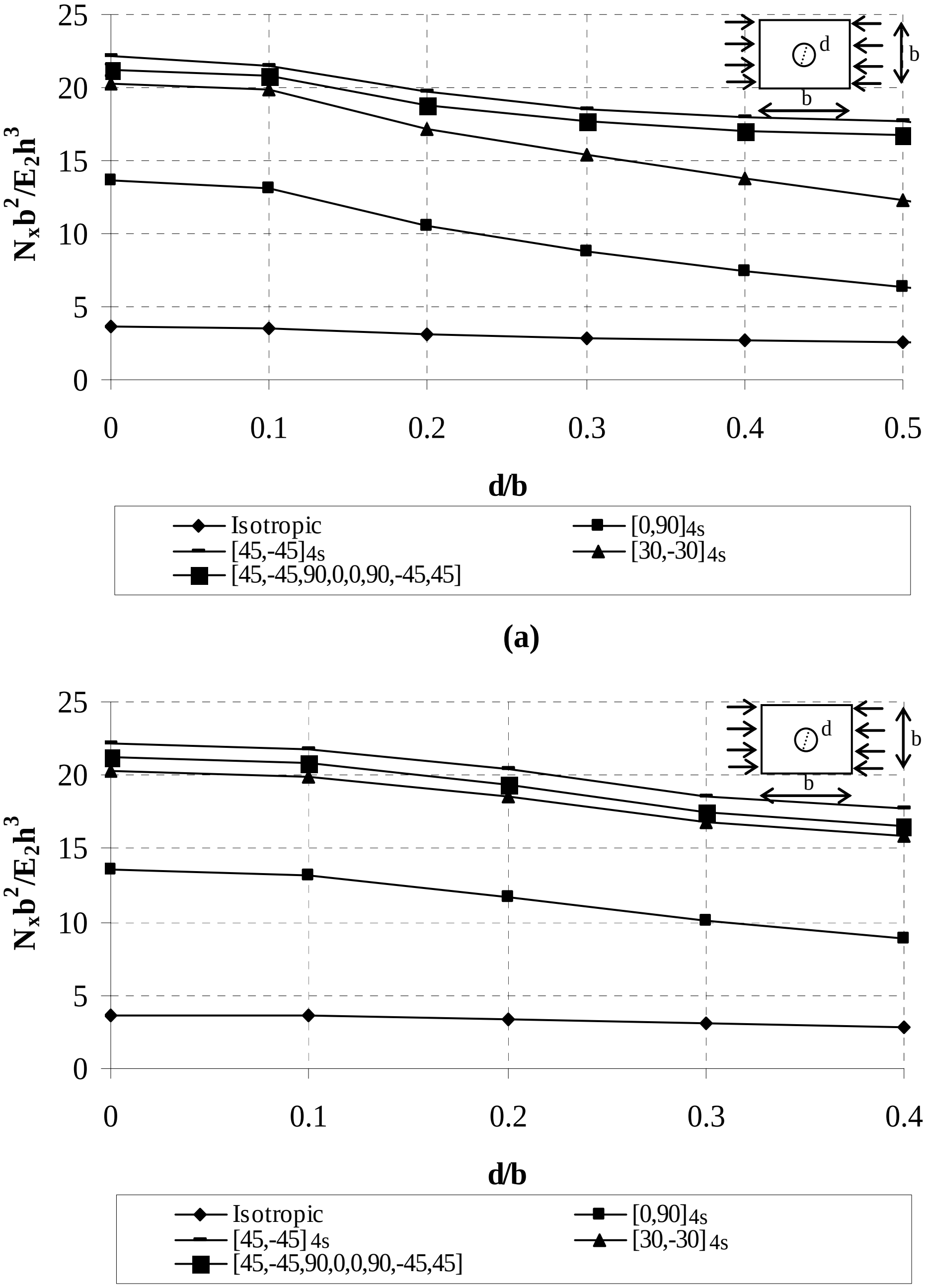
The same laminated square plate mentioned above is subjected to biaxial load to study the effect of cutout size, cutout location and fiber orientation angle on the bucking behavior.

Effect of Cutout Size

The buckling load is decreased with very small percentages for the isotropic case when the cutout size increased when it was located at the center and one quarter of the plate as shown in Figure (4).

For the case of the cutout at the center of the anisotropic plate, the buckling load is decreased for [0, 90]4s fibers orientation when the diameter of the cutout is increased, while this load is decreased with smaller percentages for the other fibers orientation until (d/b) = 0.4; then the buckling load starts increasing.

In the case of cutout at one quarter of the anisotropic plate, the buckling load is decreased when the cutout size is increased. The percentages of decrease are higher for the fibers orientation [0, 90]4s and [30,-30]4s compared with others as shown in Figure (4).



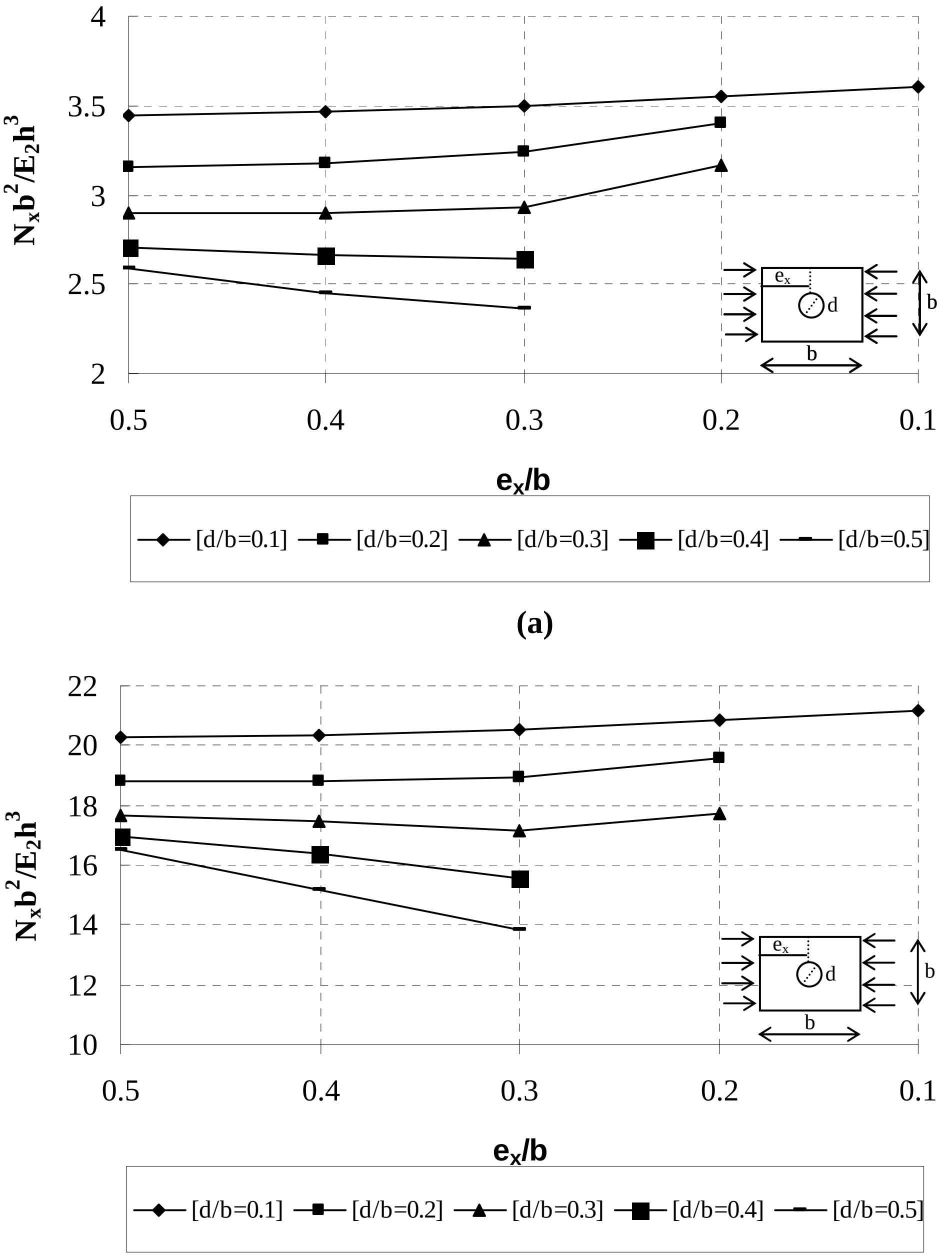
(a)

(b)

Figure (2): Variation of buckling load for laminated plate subjected to uniaxial load with different sizes of cutout located at (a) the center of the laminated plate (b) one quarter of the laminated plate.

Effect of Location

The buckling load is increased for (d/b) = 0.1, 0.2 and 0.3 when the cutout is moved toward the edges of the anisotropic plate with fibers orientation (45-45-90-0-0-90-45-45) as shown in Figure (5). On the other hand, the load is decreased for (d/b) = 0.4 and 0.5 when the cutout is moved toward the edges.



(a)

(b)

Figure (3): Variation of buckling load with different sizes of cutout for different locations of (a) isotropic plate (b) laminated [45-45-90-0-0-90-45-45] plate subjected to uniaxial load.

PLATE SUBJECTED TO SHEAR LOAD

The same square laminated plate mentioned above is subjected to shear load to study the effect of cutout size, cutout location and fiber orientation angle on the bucking behavior.

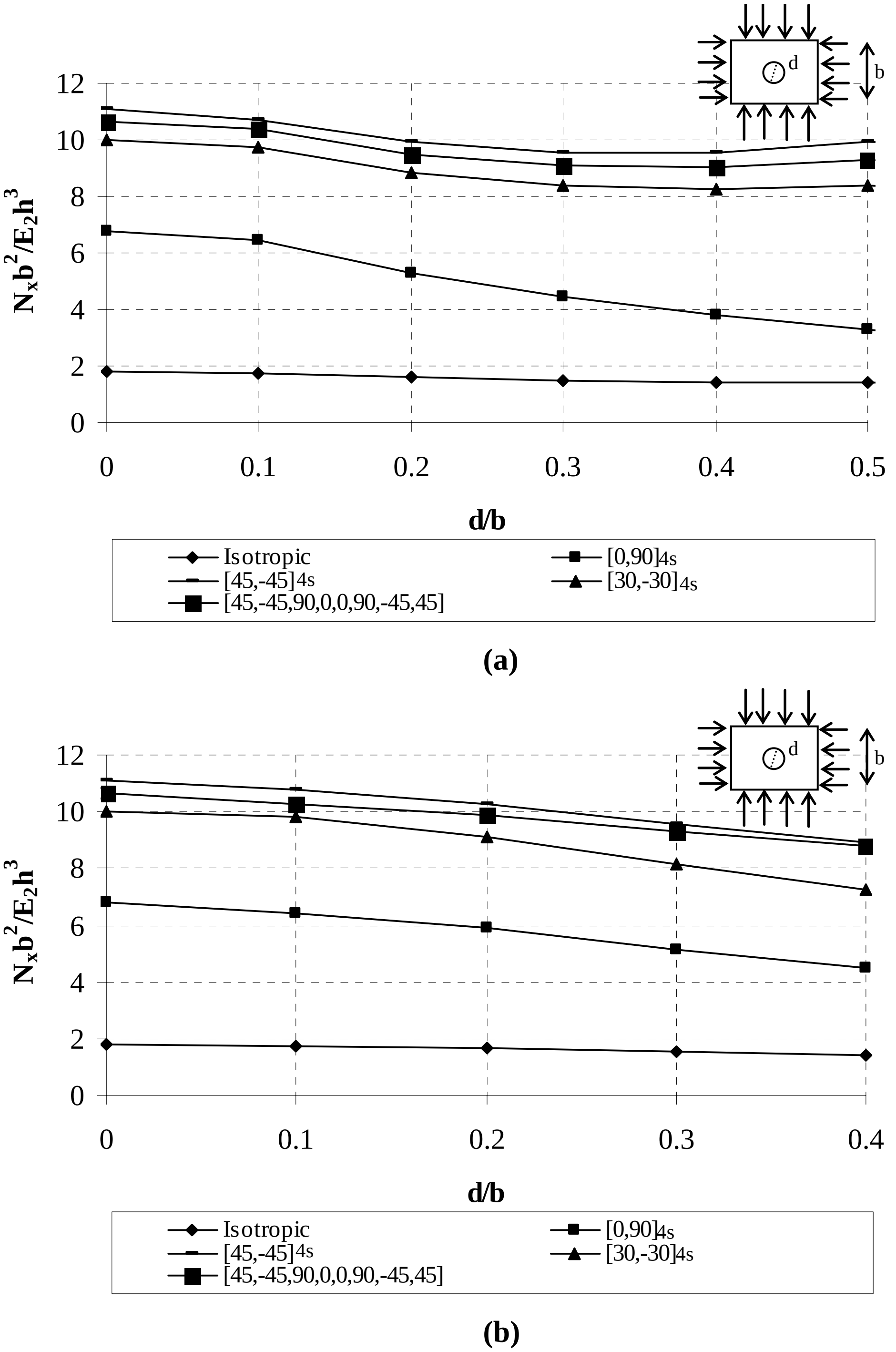


Figure (4): Variation of buckling load for laminated plate subjected to biaxial load with different sizes of cutout located at (a) the center of the laminated plate (b) one quarter of the laminated plate.

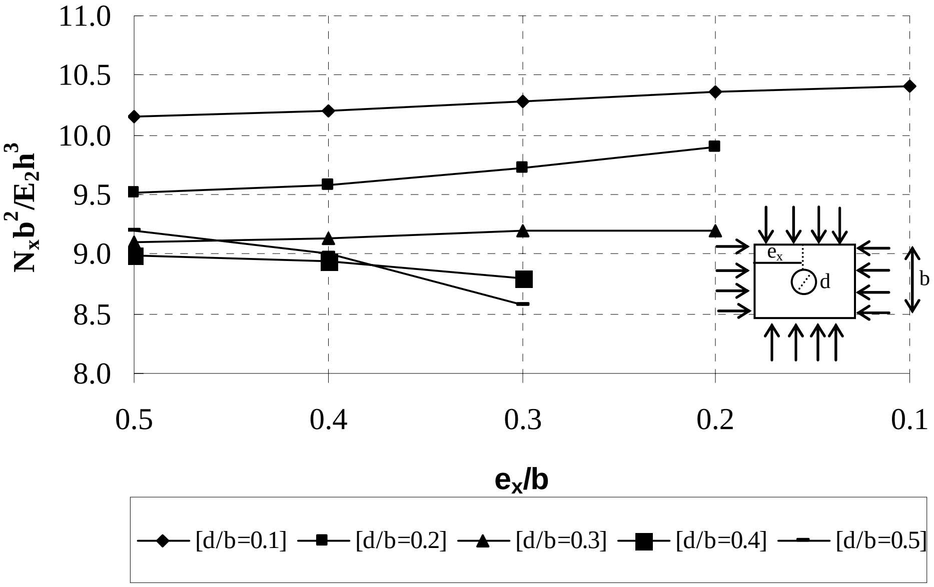
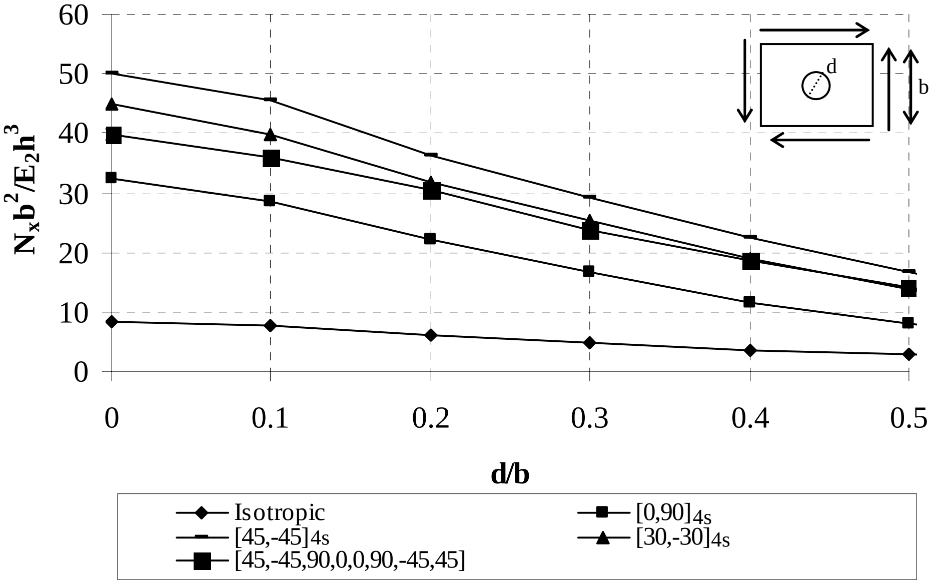
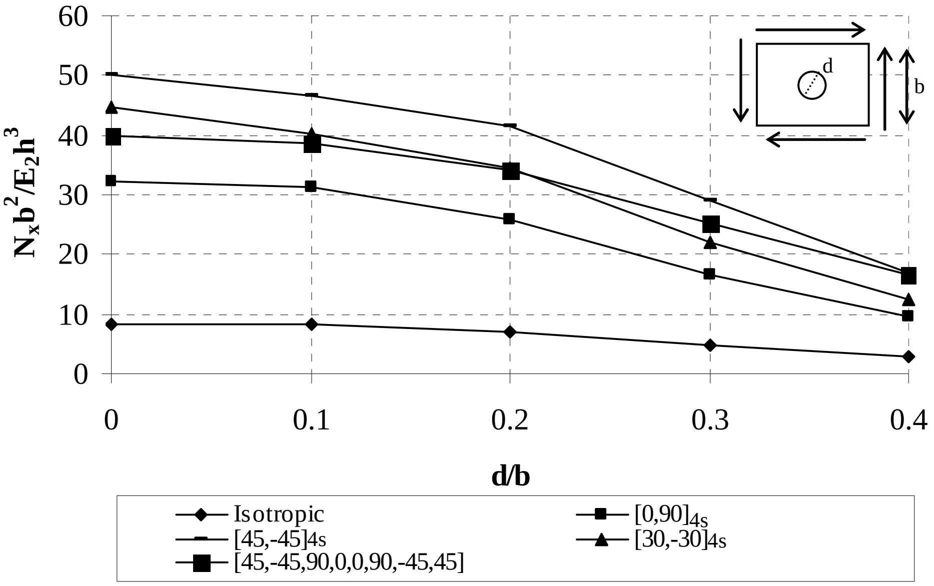


Figure (5): Variation of buckling load with different sizes of cutout for different locations of anisotropic [45-45-90-0-0-90-45-45] plate subjected to biaxial load.



(a)



(b)

Figure (6): Variation of buckling load for laminated plate subjected to shear load with different sizes of cutout located at (a) the center of the laminated plate (b) one quarter of the laminated plate.

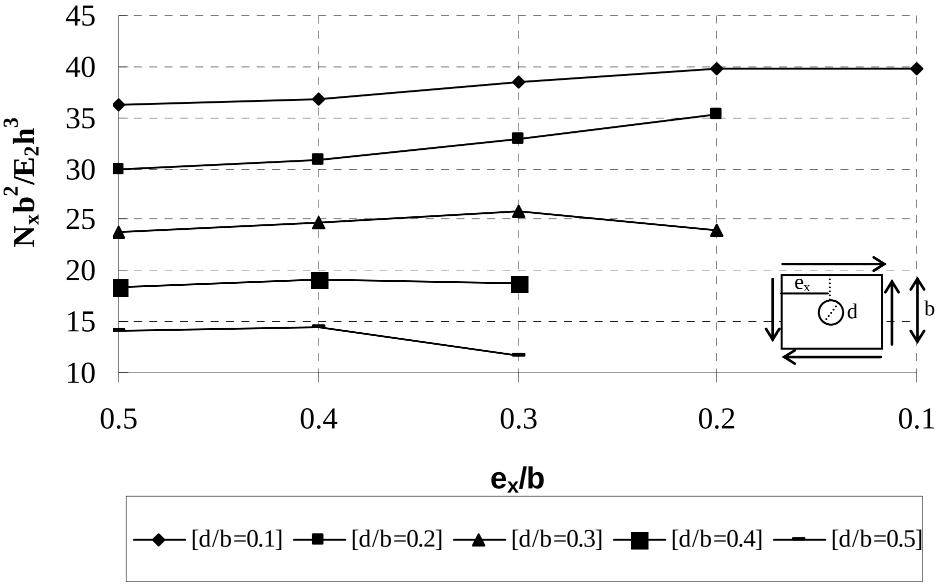


Figure (7): Variation of buckling load with different sizes of cutout for different locations of anisotropic [45-45-90-0-0-90-45-45] plate subjected to shear load.

Effect of Cutout Size

The buckling load is decreased in the isotropic plate when the cutout size is increased. Although this decrease is not significant compared with the laminated plate, it is significant when compared with the same plate but in different loading conditions such as uniaxial and biaxial.

Figure (6) shows that the buckling load is decreased with high percentages as the cutout size is increased when it is at the center and at one quarter of the laminated plate. It is worth mentioning that the decrease in buckling load in the case of fiber orientation angle [0, 90]4s is more than in the other cases where this decrease reaches 75% when changing the (d/b) ratio from 0.0 to 0.5 as shown in Figure (6).

Effect of Location

The buckling load is increased for (d/b) = 0.1 and 0.2 when the cutout is moved toward the edges as shown in Figure (7), while the load is decreased for (d/b) = 0.3, 0.4 and 0.5 when the cutout is moved toward the edges.

CONCLUSIONS

In this study, the effect of various parameters on the buckling load of square composite plates with circular cutout was investigated. These parameters included: (1) cutout size (2) cutout location (3) fiber orientation angle and (4) type of loading which also included: uniaxial and biaxial compression, as well as, in-plane shear loading. The following conclusions can be drawn:

1. Due to uniaxial and biaxial compression loading cases, cutout size has a more pronounced effect on some of the fiber orientation angles and less pronounced on others. The best performance was achieved by [45,-45]4s fiber orientation and the worst was observed in the [0, 90]4s fiber orientation.
2. The reduction in the buckling load due to increasing the cutout size was significant in the case of shear loading as compared to uniaxial and biaxial compression for the same fiber orientation angle.
3. The location of the cutout was directly related to the cutout size. For relatively small size cutout (i.e., d/b < 0.3), the buckling load increases as the cutout is shifted away from the middle of the plate, but for large size cutouts (i.e, d/b > 0.3) it was noticed that the buckling load decreases as the cutout is closer to the edge of the plate.

REFERENCES

Akbulut, H. and Sayman, O. 2001. An investigation on buckling of laminated plates with central square hole. *Journal of Reinforced Plastics and Composites*, 20 (13): 1112-1124.

Baba, B. 2007. Buckling behavior of laminated composite plates, *Journal of Reinforced Plastics and Composites*, 26 (16): 1637-1655.

Baba, B. and Baltaci, A. 2007. Buckling characteristics of symmetrically and antisymmetrically laminated composite plates with central cutout. *Applied Composite Materials*, (14): 265-276.

Baltaci, A., Sarikanat, M. and Yildiz, H. 2006. Buckling analysis of laminated composite circular plates with holes. *Journal of Reinforced Plastics and Composites*, 25 (7): 733-744.

Ghannadpour, S., Najafi, A., and Mohammadi, B. 2006. On the buckling behavior of cross-ply laminated composite plates due to circular/elliptical cutouts. *Composite Structures*, (75): 3-6.

Hyer, M. W. and Lee, H. H. 1991. The use of curvilinear fiber format to improve buckling resistance of composite plates with central circular holes. *Composite Structures*, (18): 239-261.

Hu, H. and Lin, B. 1995. Buckling optimization of symmetrical laminated plates with various geometries and end conditions. *Composites Science and Technology*, 55 (6): 277-285.

Jain, P. and Kumar, A. 2004. Postbuckling response of square laminates with a central circular/elliptical cutout. Composite Structures, (65): 179-185.

Kong, C., Hong, C. and Kim, C. 2001. Postbuckling strength of composite plate with a hole. *Journal of Reinforced Plastics and Composites*, (20): 466-481.

Larsson, P. L. 1987. On buckling of orthotropic compressed plates with circular holes. *Composite Structures*, (7): 103–121.

Larsson, P. L. 1989. On buckling of orthotropic stretched plates with circular holes. *Composite Structures,* (11): 121–134.

Lee, Y. J., Lin, H. J. and Lin, C. C. 1989. A study on the buckling behavior of an orthotropic square plate with a central circular hole. *Composite Structures*, (13): 173–188.

Mallela, U. and Upadhyay, A. 2006. Buckling of laminated composite stiffened panels subjected to in-plane shear: a parametric study. *Thin-Walled Structures*, (44): 354-361.

Saha, M., Prabhakaran, R. and Waters, W. 2004. Compressive behavior of pultruded composite plates with circular holes. *Composite Structures*, (65): 29-36.

Vandenbrink, D. J. and Kamat, M.P.1987. Post-buckling response of isotropic and laminated composite square plates with circular hole. *Finite Elements in Analysis and Design*, (3): 165-174.