The Newsboy Problem In Determining Optimal Quantity In Stochastic Inventory Problem For Fixed Demand

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ABSTRACT

This paper describes optimization problem related to newsboy model using the famous stochastic inventory problem in determining optimal order–up–to quantity when demand is a continuous random variable. This study was done by the use of a stochastic model. In view of this, data were collected and collated from cocoa board, Ilaro, Egbado-South, Nigeria and used to test the validity and applicability of the model.

Keywords: Stochastic, demand uncertainty, concave, optimal order-up-to quantity

1.0: INTRODUCTION

The newsboy problem, a famous stochastic inventory replenishment problem of "perishable" goods can be described as follows: Given a known stochastic distribution $G(x)$ for the demand of a product, the challenge is what is the optimal order quantity if only one order can be placed before actual demand can be observed? This problem is classic in management science and operational research and has an analytical solution that is quite elegant and robust.

2.0: LITERATURE REVIEW

The literature mentions a large number of extensions to the classical problem. David E. Bell (2001) mentions that the vendor may change its price as customer value higher availability. Julien Mostard and Ruud Teunter (2002) in their Technical report on "The newsboy problem with resalable returns" discussed a situation when the customer may return back the product. A. Ridder, E. Vander Lean, and M. Solomon (1998) observed how larger demand variability may lead to lower costs in the newsvendor problem. Erwin Kalvelagen (2003) in his newsboy model approximates the continuous uniform distribution by a discrete distribution. Gerchak and Mossaman (1992) show that a more variable demand may lead to a higher or lower optimal order quantity.

However, this paper examines and develops optimal order-up-to quantity when demand is a continuous random variable with annual supply of cocoa in Ilaro, Egbado-South Local Government Area, Ogun State Nigeria.

3.0: PROBLEM STATEMENT

The structure and assumption of the model include one period and one selling season of the product. The known demand distribution is *D* and unknown actual demand is *d*. Supply quantity has to be determined at the beginning of the period when actual demand is unknown. Each unit has a cost price of *C* and selling price *P*. Each unit unsold at the end of the period can be salvaged at a value v which may be negative hence $v < c < p$. Each unit of unmet demand induces a penalty cost *h* called the holding cost.

In this basic formulation, a decision maker facing random demand for a product for one period must decide how many units of the product to stock in order to maximize his expected profit. The optimal solution to this problem is to strike a balance between the expected shortage cost (overage cost) and leftover cost (underage cost) when price is fixed.

3.2: DETERMINING THE PROFIT FUNCTION

The standard newsvendor profit function is $\pi = E\left[P\min\left(q,\, D\right)\right] - c\,q$,

where *D* is a random variable with probability distribution G representing demand, each unit is sold for price *P* and purchased for price *C,* and *E* is the expectation operator. We assume that we have already decided our supply *q*. Now as demand *d* is observed two cases emerge. First, if $d < q$ then we can only sell or supply *d* units with Profit $g\left(q,\, d\right) = pd + v\left(q-d\right) - c\,q$. When $d \geq q,$ then we can sell or supply *q* units fully with some demand unmet with the Profit $g\left(q,\, d\right) = p\,q + h\left(d-q\right) - c\;q$. Hence the profits function for the two possible cases become:

$$g\left(q,\, d\right) = -c\,q + \begin{cases} p\,d & if\ d < q \\ p\,q & if\ d \geq q \end{cases} \quad \text{on assumption that } v = h = 0.$$

For the single product and single demand product, the challenge is if the total demand is greater than the quantity demanded, there will be a stock out at a shortage cost of $C_c\left(D-Q\right)^{+}$ and if total demand is less than quantity demanded, then we have overstock at a cost of $C_o(Q-D)^+$. Hence, our challenge is to decide an optimal order quantity q^* that will maximize the profit of the supplier. Since *d* is not known but a realization of the random variable *D,* the profit function $g(q, d)$ is also a random variable which depends on q and d. We use expected values for the random variable and since the objective function is linear, the expected profit is given as:

$$e(q)=\mathrm{E}\left[Z(q,d)\right]=\int_a^b Z(q,d)g(d)dd,$$

where $g(d)$ is the density function of D. The expected profit function, denoted by $e(q)$ is concave in q because the integrand

$$\mathrm{Z}(q,d)=-c\,q+\begin{cases}p\,d & \text{if } d\le q\\ p\,q & \text{if } d>q\end{cases}=-cq+\min\{pd,\,pq\}$$

is concave in q for any fixed value of d and the expectation operator (integration over d) preserves concavity. One of Fermat's theorems states that optimal of unconstrained problems are found at stationary points. For maximum profit, a necessary condition is to find $e'(q)=\dfrac{d}{dq}e(q)=0$

That is, given $e(q)=\mathrm{E}_D\left[Z(q,d)\right]=\int_a^b Z(q,d)g(d)dd$

$$=-c\,q+\int_a^q Pdg(d)dd+\int_q^b Pqg(d)dd$$

$$=-c\,q+P\int_a^q dg(d)dd+Pq\int_q^b g(d)dd.$$

For unique maximum for q^*, $\dfrac{d}{dq}e(q)=e'(q)=0$

Hence $-c\,q+p\left[1-G(q)\right]=0$, $\therefore\ q^*=G^{-1}\left(\dfrac{p-c}{p}\right)$

Moreover $q^*(p)$ is strictly increasing in p, when $c>h$. Since $g(q)$ is continuous, and $e(q)$ is twice differentiable in q, for concavity of q we have:

$$\frac{d^2e(q)}{dq^2}=e''(q)=-p\,g(q)\le 0.$$

Thus the solution to the optimal order quantity of the newsboy problem, with lead time zero, $q^* = G^{-1}\left(\frac{p-c}{p}\right)$, where G^{-1} denotes the inverse cumulative distribution function of *D* maximizes the expected profits. The ratio $\frac{p-c}{p}$, referred to as the critical ratio balances the cost of under-stocked and the total costs of being either overstocked or under-stocked. This critical ratio point determines the optimum order point and affects the direction and magnitude of the order-up-to quantity. Since $|v| < c$ and $p > c > 0$, then $0 \le \frac{p-c}{p} \le 1$. The inverse function of G, denoted by G^{-1} is continuous and strictly decreasing. This indicates that $G^{-1}\left(q^*\right) = \frac{p-c}{p}$ is feasible. Due to non negativity of demand, $G^{-1}\left(q^*\right)$ is nonnegative. Thus $q^* \ge 0$

The cumulative distribution function of the continuous random variable D is $G(q) = P[D \le q]$, where $g(q) = \frac{d}{dq}G(q)$ is the density function or probability density function (PDF) of q. The expected value of the continuous random variable *D* is given by its means as: $E[D] = \int_o^\infty qg(q)dq$

3.4: DETERMINING THE OPTIMAL-ORDER-UP-TO QUANTITY

Suppose that the demand during the period is *D*. If the retailer stocks *q* tons of cocoa at the beginning of the period, the profit for that period is given by: Expected Profits, $e(q) = (p-c)\min(D,q)^+ - (c-v)(q-D)^+$, where $(p-c)\min(D,q)^+$ is the total profit made on each ton of cocoa sold. Also $(c-v)(q-D)^+$ is the total loss incurred on leftover unsold tons of cocoa. In order to determine the best order-up-to-quantity q^*, we need to set up appropriate objective. In this paper, we considered the case when demand is continuous. We assume that demand, *D* is a continuous random variable for purpose of mathematical tractability. We also assume that demands are nonnegative continuous random variables. Continuity of demand is an abstraction that is used to simplify the analysis since in practice demand is discrete.

3.5: Optimal Order–Up–To Quantity When Demand Is A Continuous Random Variable.

Let $g(x)$ be the probability density function of *D,* and $G(x)=P[d\le x]=\int_0^x g(y)dy$ be the cumulative distribution function (*CDF*) of D. We assume that g(*x*) is continuous in $[o,\infty)$ in the following proof. From the model, the leftover (overage) tons of cocoa are given by $(q-d)^{-}$ with expected value as: $E\left[(q-D)^{+}\right]=\int_0^{\infty}(q-x)^{+}g(x)dx=\int_0^{\infty}(q-x)g(x)dx.$ The markdown (underage) tons $(D-q)^{+}$ when d > *q* is given by $\min(D,q)^{+}$ and the expected markdown demand as

$$E[\min(D,q)]=\int_0^{\infty}\min(x,q)^{+}g(x)dx=\int_0^{q}xg(x)dx+\int_q^{\infty}qg(x)dx$$

On setting $a=p-c$ to be the profit margin and $b=v-c$ to be the cost margin we have the profit function as

$$e(q)=\mathrm{E}_D\left[Z(q,d)\right]=a\left(\int_o^q x\,g(x)dx+q\int_q^{\infty}g(x)dx\right)+b\int_o^q(q-x)\,g(x)dx$$

$$=a\int_o^q x\,g(x)\,dx+a\,q\int_q^{\infty}g(x)\,dx+b\,q\int_0^q g(x)\,dx-b\int_o^q x\,g(x)\,dx$$

For maximum profit, a necessary condition is to find $e'(q)=\frac{d}{dq}e(q)=0$

From fundamental theorem of calculus, we have that $\frac{d}{dq}e(q)=a\int_q^{\infty}g(x)dx+b\int_o^q g(x)dx$

For $e'(q)=0,$ then $a\int_q^{\infty}g(x)dx+b\int_o^q g(x)=a\left(1-G(q)\right)+bG(q)=0$

$$\text{Hence } G(q^*)=\frac{a}{a-b}=\frac{p-c}{p+v} \qquad \ldots \qquad (2)$$

So, *q* optimum is given by $q^*=G^{-1}\left(\frac{p-c}{p-v}\right)$

Proposition: A twice differentiable function g of a single variable defined on the interval I is concave if and only if $g''(q)\le 0.$

To show that *g*(*q*) has unique maximum , we apply the second derivative test as:

$e''(q)=bg(q)-ag(q)=(b-a)g(q)$, since $b-a=v-c<0$

$$\therefore\ e''(q) \le 0$$

Since g is unique, it must have a global maximum. Hence g has a unique maximum on $[0, \infty)$. The optimal order-up-to quantity using single critical number policy is $G(q^*) = \dfrac{a}{a-b} = \dfrac{p-c}{p-v}$. From the above condition, the value of q* is selected such that the probability $x \le q^* = \dfrac{p-c}{p-v}$

The optimal ordering policy given x is on hand before an order is placed is given by

$$\begin{cases} if\ \ q^* > x, \text{ order } q^* - x \\ if\ \ q^* \le x, \text{ do not order} \end{cases}$$

This model holds when D is a general continuous variable.

4.0: ANALYTICAL APPLICATION

Table showing the demand of cocoa per ton

Years	**Demand of Cocoa per ton (*x*)**	***Quantity Per ton.***
2007	70,000	120,000
2008	90,000	115,000
2009	80,000	40,000
2010	120,000	95,000
2011	100,000	105,000

Source: Cocoa collecting centre Ilaro, Egbado south Local Government Area, Ogun state.

e fixed selling price per ton of cocoa is P = N50000 and the cost price per ton is C = N10000 The random demand is $\mathrm{d}[40000, 120000]$ and the lead time is zero.

4.0: MAXIMIZING THE EXPECTED PROFIT

The expected profit function is given by $g(q,d) = -c\,q + \begin{cases} p\,d \text{ if } d \le q \\ p\,\mathrm{q} \text{ if } d > q \end{cases}$

From the profit function $g(q,d) = -c\,q + \begin{cases} p\,d \text{ if } d \le q \\ p\,\mathrm{q} \text{ if } d > q \end{cases}$

$$g(q,d) = -10000\,q + \begin{cases} 50000\,d \text{ if } d \le q \\ 50000\,\text{q if } d > q \end{cases}$$

We calculate the expected profit: $E_D\left[z(q,d)\right] = \int_{40,000}^{120,00} z(q,d)\, f(d)\,dd$

$$= -cq + \int_{40000}^{q} pd\ f(d)dd + \int_{q}^{120000} pq\ f(d)dd$$

$$= -10000q + \int_{40000}^{q} 50000d\,\frac{1}{80000}\,dd + \int_{q}^{120000} 50000q\,\frac{1}{80000}\,dd$$

$$= -10000q + \frac{50000}{80000}\left[\frac{1}{2}d^2\right]_{40000}^{q} + \frac{50000}{80000}q[d]_{q}^{120000}$$

$$\frac{d}{dq}(e(q)) = 0 \text{ i.e., } e^{'}(q) = 0$$

Thus $-10000 + \frac{50000}{80000}q - \frac{50000}{80000}q + \frac{50}{80}(120000 - q) = 0$

i.e., $\dfrac{120000 - q}{80000} = \dfrac{10000}{50000}$

$$q^* = 104000$$

Comment: This is the optimum order-up-to quantity for cocoa.

$q^* = G^{-1}\left(\dfrac{p-c}{p-v}\right) = G^{-1}\left(\dfrac{50,000 - 10,000}{50,0000}\right) = G^{-1}(0.8)$ is the critical ratio

5.1: CONCLUSION

Thus g has a unique maximum on [0, ∞). So the optimal order-up-to quantity q^* is $G(q^*) = \dfrac{a}{a-b} = \dfrac{p-c}{p-v}$. This model holds when D is a general continuous variable. The expression

$$a(1 - G(q)) + bG(q)$$

is a non–increasing function as G *is* non-decreasing. The critical ratio given by $\dfrac{P-C}{P-V}$ maximized the profit at q^*

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5.3: REFERENCES

David E. Bell (2001): *Incorporating the customers perspective into the newsboy p*roblem.Technical report, Graduate School of Business Administration, Harvard University, Boston, Massachusetts 02163.

Gerchak, Y. and D. Massman. (1992): On the effect of demand randomness on inventories and costs. Operations Research. 40(4), 804-807.

Julian Mostard and Ruud Teunter (2002): *The newsboy problem with resalable returns.* Technical report, Erasmus University Rotterdam School of management and economic institute. P.O. Box 1738 3000 Dr Rotterdam, the Netherlands.

Ridder A., E. Van der Lean, and M. Solomon (1998): *How larger demand variability may lead to lower costs in the newsvendor problem.* Operation Research **46** 934-936.

Erwin Kalvelagen (3003): The Newsboy Problem. GAMS DEVELOPMENT CORP., WASHINGTON D.C. E-mail address: Erwin@gams.com

Zipkin. P. H (2000): Foundation of inventory management MC Graw – Hill