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Finite Time Interval Stablizability of Linear Continuous Descriptor Control System

Radhi.Ali Zaboon   Shimaa Mohammed Dawood
Department of mathematics, college of science, Al- Mustansiriyah, University, Baghdad, IRAQ

Abstract
In this paper, the finite time stable concept for a forced control system is modefied. A feedback controller has been designed with some necessary condition so that the solvability and the exponential finite time interval stablizability are guaranteed with computational algorithm and illustration.
Keywords: consistent initial condition, Drazin inverse, linear descriptor systems

I. INTRODUCTION

Singular control systems are those systems whose dynamic is governed by a mixture of differential and algebraic equations, and named as generalized system, descriptor systems, as well as semi-state systems. These systems include many real-world applications such as feedback system and robotics, chemical systems, biological systems etc. [Campbell et al., 1976].

The initial conditions of these systems should be designed based on the solvability of the algebraic equations which is not characteristic for the state space system where is no algebraic equation appeared. [Campbell et al., 1976], [Wilkinson, 1979], [Debeljković et al., 2002] [Sjoberg, 2005] some results on stabilization of some classes of descriptor system with some set of sufficient conditions are given in [Guoping, 2004], [Guoping and Daniel, (2006a)], [Batri and Murugesh, 2008].

The stability (robustness) of particular class of linear systems in the time domain using the Lyapunov approach is given in [Durovic et al., 1998]. Decomposition of unstructured impulse free perturbations was studied in [Zange and Lam, 2002].

II. Some Basic Concepts

Definition (2.1): [1]
A singular system of the form

$$E\dot{x}(t) = Ax(t) \tag{1}$$

Where $E, A \epsilon R^{n\times n}$, $x(t) \epsilon R^n$, $u(t) \epsilon R^r$ is called **Regular** if there exists a constant scalar $\gamma \epsilon \mathbb{C}$ such that $\det(\gamma E - A) \neq 0$

Definition (2.2): [2]
For E is $n \times n$ matrix the **index** of E denoted by $ind\,(E)$, is the smallest non-negative integer Z such that, $rank(A^Z) = rank(A^{Z+1})$ (2)

Definition (2.3): [2]
If $A\,\epsilon\, \mathbb{R}^{n\times n}$ with $ind(A) = z$, and if $A^D \epsilon\, \mathbb{R}^{n\times n}$ such that

$$AA^D = A^D A$$
$$A^D A A^D = A^D$$
$$A^D A^{Z+1} = A^Z \tag{3}$$

For $z \geq ind(A)$. then A^D is called **Drazin inverse**

Lemma (2.1): [3]
Let A be a real, symmetric positive- definite matrix. and $\lambda_{min}(A)$ and $\lambda_{max}(A)$ be the smallest and largest eigenvalues of A. respectively Then, for any $x\,\epsilon\, R^n$,

$$\lambda_{min}(A)\,\|x\|^2 \leq x^T A\, x \leq \lambda_{max}(A)\,\|x\|^2 \tag{4}$$

Where $\|x\|^2 = \sum_{i=1}^{n} |x_i|^2$, x_i is the i- th component of x.

Definition (2.4) (Consistent Initial Conditions): [2], [4]
Consider the singular differential equation $E\dot{x} + Ax = f$, an initial condition $x_0 \in R^n$ is said to be consistent if there exists a differentiable continuous solution of $E\dot{x} + Ax = f$.

Remarks (2.1): [4], [2]

1. if $EA = AE$ and $\aleph(E) \cap \aleph(A) = \{0\}$. Then $x = e^{-E^D At} EE^D q, q\,\epsilon\, C^n$ (5)
   is the general solution of $E\dot{x} + Ax = 0$.,and q may be obtained $EE^D q = x_0$
2. if $EA = AE$ and $\aleph(E) \cap \aleph(A) = \{0\}$ .Then $(I - EE^D)AA^D = (I - EE^D)$ (6)
3. Suppose that $EA = AE$ and $\aleph(E) \cap \aleph(A) = \{0\}$. Then there exists a solution to $E\dot{x} + Ax = f, x(0) = x_0$, if and only if x_0 is of the form $x_0 = EE^D q + (I - EE^D) \sum_{n=0}^{k-1} (-1)^n\, (EA^D)^n A^D f^{(n)}(0)$ (7)

For some vector q. Furthermore, the solution is unique

4. Suppose that $\hat{\lambda}$ such that $(\hat{\lambda}E + A)$ is invertible. Then $(\hat{\lambda}E + A)^{-1}E$ and $(\hat{\lambda}E + A)^{-1}A$ Commute since $\hat{\lambda}\left((\hat{\lambda}E + A)^{-1}E + (\hat{\lambda}E + A)^{-1}A\right) = I$ (8)
5. In particular, if f is identically zero, then $EE^D q = x_0$ characterizes consistent initial conditions.

III. Finite time stable

Definition (3.1): [7]

The linear homogeneous singular system of the form

$E\dot{x}(t) = Ax(t), x(t_0) = x_0 \in W_k$

Where $E, A \epsilon R^{n\times n}, x(t) \epsilon R^n$, with $|E| = 0$ and $ind(E) = z < n$ is finite time stable w.r.t. $\{J, \alpha, \beta, Q\}$, $\alpha < \beta$ if $\forall x(t_0) = x_0 \,\epsilon\, W_k$ satisfying $\|x_0\|_Q^2 < \alpha$ implies that $\|x(t)\|_Q^2 < \beta$

$t \epsilon J = \{t: t_0 \leq t + T\}$. Where W_k as the limit of the sub-space algorithm:

$W_0 = R^n, W_{j+1} = A^{-1}(EW_j), j \geq 0.$

Remarks (3.1)

Consider the linear singular system

$$E\dot{x}(t) = Ax(t) + Bu(t) \quad (9)$$

If the following conditions hold:

(1) The matrix pair (E, A) is regular and

(2) The matrix pair (E, A) is an impulse free and stable. Then for each $Q > 0$ there exist $P > 0$ is a solution of generalized Lyapunov Equation (G.L.E) satisfying
$(A^T PE) + (E^T PA) = -E^T QE$, see [8]

In the following theorem, a finite time stable concept for a forced control system is developed. A feedback controller has been designed with some necessary condition so that the solvability and the stablizability are guaranteed. A modified Lyapunov function approach is adapted for this theme.

Theorem (3.1)

Consider the singular linear control system:

$$E\dot{x} = Ax(t) + Bu(t) \quad (10)$$

Where $x(\cdot): R^+ \longrightarrow R^n, u(\cdot): R^n \longrightarrow R^m$ and $E, A, B \,\epsilon\, R^{n\times n}$ are constant Matrix with $|E| = 0$ and $ind(E) = z < n$ and $EA = AE$ and $\aleph(E) \cap \aleph(A) = \{0\}$

Let Lyapunov function defined by

$$V(x(t)) = x^T(t)\, E^T PEx(t) \triangleq \|x(t)\|_{E^T PE}^2 \quad (11)$$

If

1- $u = -kx$ is selected such that the matrix $(E, (A - Bk))$ is impulse free ,stable and regular $\det(\gamma E - (A - Bk)) \neq 0$,For $\gamma \in \mathbb{C}$ (12)

2- the consistent initial conditions is defined by $W_k = \{x_0(t) \,\epsilon\, R^n{}_{|\{0\}} | x_0 \in \aleph(A - Bk)^D\}$ where D is standing for Drazin inverse operator (13)

3- $\|x_0\|_{E^T PE}^2 = V(x_o(t)) < \alpha$, $\forall x_o(t) = x_0 \,\epsilon\, W_k$ (14)

4- The candidate Lyapunov function is defined to be $V(x, t) = x^T Px$ where the matrix $P = P^T$ is the unique solution of $(A - BK)^T PE + E^T P(A - BK) = -E^T QE$ for a given Q such that $E^T QE > 0$. (15)

5- $\ln\frac{\beta}{\alpha} > \frac{\lambda_{\min}(E^T QE)}{\lambda_{\max}(E^T PE)} t, \alpha < \beta, \forall\, t \epsilon J = \{t: t_0 \leq t \leq t + T\}$ (16)

Then $V(x(t)) \triangleq \|x(t)\| < \beta$, and hence the system is finite time stable w.r.t. $\{\alpha, \beta, K,, u(x), J, Q, P, W_k, V(x)\}$

Proof

Since $EA = AE$ and $\aleph(E) \cap \aleph(A) = \{0\}$, then the homogenous system solution is given by

$x(t) = e^{-AE^D t} q = E E^D e^{-AE^D t} q, q \epsilon R(E E^D)$, for some vector q

And the consistent initial condition is defined by W_k is defined by:-

$W_K \triangleq \{x_0 \in R^n | x(0) = (A - Bk)^D Aq , q \triangleq x(0)\}$ For arbitrary vector q.

$W_K \Leftrightarrow \{x_0 \in R^n | x(0) = (A - Bk)^D Ax_0, q \triangleq x(0)\}$

$W_K \Leftrightarrow \{x_0 \in R^n | ((A - Bk)^D A - I)x_0 = 0\}$

$W_K \Leftrightarrow \{x_0 \in R^n | \, x_0 \in \aleph(A - Bk)^D\}$

By Lyapunov function

$V(x(t)) \triangleq x^T(t) E^T PE\, x(t) \geq 0$, for $E^T PE \geq 0$ where the unknown P is designed as follows

$$\frac{d}{dt}V(x(t)) = \dot{V}(x(t))\Big|_{along\ the\ solution\ of\ system} = \frac{dv}{dx}\cdot\frac{dx}{dt}$$

$\Rightarrow \dot{V}(x(t)) = \dot{x}^T(t) E^T PEx(t) + x^T(t) E^T PE\, \dot{x}(t)$, hence

$\dot{V}(x(t)) = x^T[(A - BK)^T PE + E^T P(A - Bk)]x$ (17)

By condition (17) there exists a unique solution P for a given matrix $Q > 0$ so that $E^T QE > 0$ satisfying the following algebraic Lyapunov equation

$(A - BK)^T PE + E^T P(A - BK) = -E^T QE$ (18)

From (18) and (19), the following is obtained

$\dot{V}(x(t)) \triangleq -x^T(t)(E^T QE)x(t)$ (19)

Set

$\lambda_{max}(E^T PE) \triangleq \{\max(\lambda_1 \dots \lambda_n) \mid \lambda_i \in \delta(E^T PE), i = 1, \dots, n\}$

$\lambda_{min}(E^T QE) \triangleq \{\min(\lambda_1 \dots \lambda_n) \mid \lambda_i \in \delta(E^T QE), i = 1, \dots, n\}$, where $\delta(E^T PE)$ is standing for the spectrum value of $E^T PE$ (20)

$\Rightarrow \lambda_{min}(E^T QE) > 0 \;\&\; \lambda_{max}(E^T PE) > 0$, since $E^T PE > 0 \;\& P > 0$

By assumption it should be noted that on using (20) and (21), we have that

$\dot{V}(x(t)) = -x^T(t)(E^T QE)x(t) \leq -\lambda_{min}(E^T QE)\, x^T(t)x(t)$ (21)

$$\Leftrightarrow \dot{V}(x(t)) \leq -\lambda_{min}(E^T QE)\, x^T(t)x(t)\, \frac{\lambda_{max}(E^T PE)}{\lambda_{max}(E^T PE)}$$

$$\Leftrightarrow \dot{V}(x(t)) \leq \frac{-\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} x^T(t)x(t)\; \lambda_{max}(E^T PE)$$

$$\Leftrightarrow \dot{V}(x(t)) \leq \frac{-\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} x^T(t)E^T PE\, x(t)$$

$$\Leftrightarrow \dot{V}(x(t)) \leq \frac{-\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} V(x(t)), x \neq 0 \quad (22)$$

On using monotonicity of integration $\forall\, t \epsilon [0, J]$, and from (19) one gets

$$\int_0^t \frac{\dot{V}(x(t))}{V(x(t))} dt \leq \int_0^t \frac{-\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} dt, t\epsilon J \triangleq \{t: t_0 \leq t \leq t + T\}, T > 0$$

$$ln|V(x(t))| - ln|V(x(0))| \leq \frac{-\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} t, \quad x \neq 0, t\epsilon J \triangleq \{t: t_0 \leq t \leq t + T\}, T > 0$$

$$ln \frac{V(x(t))}{V(x(0))} \leq \frac{-\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} t, t\epsilon J \triangleq \{t: t_0 \leq t \leq t + T\}, T > 0$$

$$\Leftrightarrow V(x(t)) \leq e^{\frac{-\lambda_{min}(E^T QE)t}{\lambda_{max}(E^T PE)} t} . V(x(0)), t\epsilon J \triangleq \{t: t_0 \leq t \leq t + T\} \quad (23)$$

And from (15) and (20) we have that

$$V(x) = \|x(t)\|^2_{E^T PE} \leq e^{\frac{-\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} t} \|x_0(t)\|^2_{E^T PE}, \forall\, t\epsilon J \quad (24)$$

$$\|x(t)\|^2_{E^T PE} \leq e^{\frac{-\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} t} . \alpha, \forall t\epsilon J \triangleq \{t: t_0 \leq t \leq t + T\} \quad (25)$$

By setting $e^{\frac{-\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} t} \alpha < \beta \;\; \forall t\epsilon J \triangleq \{t: t_0 \leq t \leq t + T\}$

$$\Leftrightarrow \ln \frac{\beta}{\alpha} > \frac{\lambda_{min}(E^T QE)}{\lambda_{max}(E^T PE)} t \quad (26)$$

Then from (29) we have $\|x\|_{E^T PE} \leq \beta$ for all $\|x_0\|_{E^T PE} < \alpha$

And hence the system is finite time stable w.r.t. $\{J, \alpha, \beta, k, Q, P, V(x(t))\}$

Corollary (3.1)

the descriptor system $E\dot{x} = Ax(t) + Bu(t)$

Where $x(\cdot): R^+ \longrightarrow R^n, u(\cdot): R^n \longrightarrow R^m$ and $E, A, B\, \epsilon\, R^{n \times n}$ are constant matrix with $|E| = 0$ and $ind(E) = z < n$ and $EA \neq AE$ and $\aleph(E) \cap \aleph(A) \neq \{0\}$. One can choose and there exist $\hat{\lambda}$ such that $\hat{E} = (\hat{\lambda}E - A)^{-1}E, \hat{A} = (\hat{\lambda}E - A)^{-1}A, \hat{B} = (\hat{\lambda}E - A)^{-1}B$

If λ is chosen so that $\hat{E}, \hat{A}$, is invertible and

Let Lyapunov function defined by

$V(x(t)) = x^T(t)\, E^T PEx(t) \triangleq \|x(t)\|^2_{E^T PE}$ (27)

If

6- $\hat{u} = -\widehat{kx}$ is selected such that the matrix $\left(\hat{E}, (\hat{A} - \hat{B}\hat{k})\right)$ is impulse free ,stable and regular $\det(\gamma\hat{E} - (\hat{A} - \hat{B}\hat{k})) \neq 0$,For $\gamma \in \mathbb{C}$ (28)

7- the consistent initial conditions is defined by $W_k = \left\{x_0(t)\, \epsilon\, R^n{}_{|\{0\}} | x_0 \in \aleph(\hat{A} - \hat{B}\hat{k})^D\right\}$ where D is standing for Drazin inverse operator (29)

8- $\|x_0\|^2_{E^TPE} = V(x_o(t)) < \hat{\alpha}$, $\forall x_o(t) = x_0 \in W_k$ (30)

9- The candidate Lyapunov function is defined to be $V(x,t) = x^T\hat{P}x$ where the matrix $\hat{P} = \hat{P}^T$ is the unique solution of $(\hat{A} - \hat{B}\hat{k})^T\hat{P}\hat{E} + \hat{E}^T\hat{P}(\hat{A} - \hat{B}\hat{k}) = -E^TQE$ for a given Q such that $E^TQE > 0$. (31)

10- $\ln\frac{\beta}{\alpha} > \frac{\lambda_{\min}(E^TQE)}{\lambda_{\max}(E^TPE)}t$, $\alpha < \beta$, $\forall\, t\epsilon\, J = \{t: t_0 \le t \le t+T\}$ (32)

Then $V(x(t)) \triangleq \|x(t)\| < \beta$, and hence the system is finite time stable w.r.t. $\{\hat{\alpha}, \hat{\beta}, \hat{K}, , \hat{u}(x), \hat{J}, Q, P, W_k, V(x)\}$

Computational Algorithm

Input Singular Linear Control System

$E\dot{x} = Ax(t) + Bu(t)$

Where $x(\cdot): R^+ \longrightarrow R^n, u(\cdot): R^n \longrightarrow R^m$ and $E, A, B \,\epsilon\, R^{n\times n}$ are constant.

Output $J, \alpha, \beta, k, Q, P, v(x(t)$

Step (1):

If $EA = AE$ and $\aleph(E) \cap \aleph(A) = \{0\}$. Then go to step (2) and if $EA \neq AE$

Find λ s.t $|\lambda E + A| \neq 0$ is invertible and define

$\hat{A} = (\hat{\lambda}E - A)^{-1}A, \hat{B} = (\hat{\lambda}E - A)^{-1}B, \hat{E} = (\hat{\lambda}E - A)^{-1}E \Rightarrow \hat{E}\hat{A} = \hat{A}\hat{E}$ and

$\aleph(\hat{E}) \cap \aleph(\hat{A}) = \{0\}$ and hence replace $\hat{A}$ by A and $\hat{B}$ by B and the back to step (2)

Step (2): Define $u = -k$, Find k s.t $\det(\gamma E - (A - Bk)) \neq 0$, For $\gamma \in \mathbb{C}$

Step (3):

Define

$W_k = \{x_0(t) \,\epsilon\, R^n{}_{|\{0\}} | x_0 \in \aleph((A - Bk)^D)\}$ Where D is standing for Drazin inverse and

$\tilde{A} = (A - B * K)$

Step (4):

Find α such that $\|x_0\|^2_{E^TPE} = V(x_o(t)) < \alpha$, $\forall x_o(t) = x_0 \,\epsilon\, W_k$

Step (5):

Solve algebraic Ricatti equation we

$(A - BK)^TPE + E^TP(A - BK) = -E^TQE$, For a given matrix $Q, \forall E^TQE > 0$

Step (6):

Calculate $\lambda_{min}(E^TQE)$, $\lambda_{max}(E^TPE)$

Step (7):

Calculate $ln\frac{\beta}{\alpha} > \frac{\lambda_{min}(E^TQE)}{\lambda_{max}(E^TPE)}t$, $\alpha < \beta$, $\forall\, t\epsilon J = \{t: t_0 \le t \le t+T\}$

Step (6)

Define $v(x(t)) = x^T(t)\, E^TPEx(t) \triangleq \|x(t)\|^2_{E^TPE}$ where $E^TPE > 0$ s.t

$\|x_0\|^2_{E^TPE} = V(x_o(t)) < \alpha$

Then $V(x(t)) < \beta$, and the system is finite time stable w.r.t. $\{\alpha, \beta, k, J, Q, P, W_k, V(x)\}$

Illustrations (2.1): (E, A) is regular if $EA = AE$

Consider the singular linear control system:

$E\dot{x} = Ax(t) + Bu(t)$

Where

$$E = \begin{bmatrix} 6 & -1 & -3 \\ -4 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix} \quad A = \begin{bmatrix} -2 & 1 & 1 \\ 2 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} |E| = 0$$

step (1)

$|E| = 0$; $rank\ (E) = 2$ and $rank(\ E^2) = 2$, so $ind\ (E) = 1$

step (2)

The condition $EA = AE$ and $\aleph(E) \cap \aleph(A) = \{0\}$ are satisfying

Step (3)

(3.1) Define $u = -kX$, Find k s.t $\det(\gamma E - (A - Bk)) \neq 0$, For $\gamma \in \mathbb{C}$

$$\det\left(\gamma \begin{bmatrix} 6 & -1 & -3 \\ -4 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix} - \left[\begin{bmatrix} -2 & 1 & 1 \\ 2 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} [k_1 \quad k_2 \quad k_3]\right]\right)$$

$$\det\left(\begin{bmatrix} 6\gamma & -\gamma & -3\gamma \\ -4\gamma & \gamma & 2\gamma \\ 2\gamma & 0 & -\gamma \end{bmatrix} - \begin{bmatrix} -2-k_1 & 1-k_2 & 1-k_3 \\ 2-k_1 & -k_2 & -1-k_3 \\ -k_1 & 1-k_2 & -k_3 \end{bmatrix}\right)$$

$$\Rightarrow \begin{vmatrix} 6\gamma+2+k_1 & -\gamma-1+k_2 & -3\gamma-1+k_3 \\ -4\gamma-2+k_1 & \gamma+k_2 & 2\gamma+1+k_3 \\ 2\gamma+k_1 & -1+k_2 & -\gamma+k_3 \end{vmatrix} \neq$$

$$\Rightarrow (6\gamma+2+k_1)[(\gamma+k_2)(-\gamma+k_3)-(2\gamma+1+k_3)(-1+k_2)] - (-\gamma-1+k_2)[(-4\gamma-2+k_1)(-\gamma+k_3)-(2\gamma+1+k_3)(2\gamma+k_1)] + (-3\gamma-1+k_3)[(-4\gamma-2+k_1)(-1+k_2)-(\gamma+k_2)(2\gamma+k_1)]$$

$$\Rightarrow -k_1\gamma^2 - 2k_3\gamma^2 + 2k_1\gamma + 4k_3\gamma + k_1 + 2k_3 \neq 0$$

Step (3.1) on selection γ and k_1, k_2,as well $k_3$in different ways such that the necessary Condition (13) is satisfied and as follows

γ	Equation	k_1	k_2	k_3	Stable	Impulse free
$\gamma = 3$	$4k_1 - 4k_3 \neq 0$	2	3	1	yes	yes
$\gamma = -2$	$-7k_1 - 14k_3$	-1	-3	5	No	yes
$\gamma = 0$	$k_1 + 2k_3$	3	1	2	No	No
$\gamma = 1$	$2k_1 + 4k_3$	0	0	0	No	No
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Step (4)

The class of consistent initial conditions is found by:
$W_k = \{x_0(t) \epsilon R^3 \setminus \{0\} \,|x_0(t) \epsilon \aleph\, (A - Bk)^D\}$ Where D is standing for Drazin inverse and $K = [2 \quad 3 \quad 1]$

$$W_k = \left\{ x_0(t) \epsilon R^3 \setminus \{0\} |\, x_0(t) \epsilon \aleph \begin{bmatrix} -4 & -2 & 0 \\ 0 & -3 & -2 \\ -2 & -2 & -1 \end{bmatrix}^D \right\}$$

Step (4.1) on computing W_k the following algorithm is adopted; $F = (A - Bk)^D$ determine the Drazin inverse of the matrix of F by using $F^D = F(F^{2k+1})^{\dagger}F$ where $K = ind(A - Bk)$ and † is standing for pseudo inverse operator

Let $Y = F^{2k+1}$

$$Y = F^5 = \begin{bmatrix} -1800 & -2458 & -1104 \\ -1104 & -1675 & -802 \\ -1354 & -1906 & -873 \end{bmatrix}$$

First, the pseudo inverse of the matrix $Y(Y^T)$ is found by using singular value decomposition

$$YY^T = \begin{bmatrix} 10500580 & 6989758 & 8085940 \\ 6989758 & 4667645 & 5387512 \\ 8085940 & 5387512 & 622828 \end{bmatrix} \Rightarrow |YY^T| = 1.0486$$

$$Y^TY = \begin{bmatrix} 6292132 & 8854324 & 4054650 \\ 8854324 & 12480225 & 5720920 \\ 4054650 & 5720920 & 2624149 \end{bmatrix} \Rightarrow |Y^TY| = 1.0486$$

The Eigen values of $YY^T$and Y^TY are equals. The concept pseudo inverse of Y one can using the singular value decomposition (S.V.D) approach and as follows:-

$$U = \begin{bmatrix} -0.7006 & 0.5477 & -0.4573 \\ -0.4668 & -0.8366 & -0.2868 \\ -0.5397 & 0.0126 & 0.8418 \end{bmatrix}, S = \begin{bmatrix} 4.6245 & 0 & 0 \\ 0 & 0.1015 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 0.5421 & -0.7814 & -0.3089 \\ 0.7639 & 0.3052 & 0.5686 \\ 0.3501 & 0.5443 & -0.7624 \end{bmatrix}, H = \begin{bmatrix} 0.2162 & 0 & 0 \\ 0 & 9.8522 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The pseudo-inverse of the matrix Y is given by

$$Y^{\dagger}=\begin{bmatrix}-4.2990 & 6.3859 & -0.1599\\ 1.5311 & -2.5923 & -0.0514\\ 2.8840 & -4.5211 & 0.0265\end{bmatrix}$$

Therefore, the Drazin inverse of the matrix F is the computed as $F^D = F^k((F^{2k+1})^{\dagger}F^k$

$$F^D=\begin{bmatrix}-413.5433 & 81.9174 & 223.2671\\ 224.2230 & -228.4565 & -256.9608\\ -143.4064 & -35.8671 & 29.9601\end{bmatrix}$$

Let $(A-Bk)^D X_0(0)=\begin{bmatrix}-413.5433 & 81.9174 & 223.2671\\ 224.2230 & -228.4565 & -256.9608\\ -143.4064 & -35.8671 & 29.9601\end{bmatrix}\begin{bmatrix}X_1(0)\\ X_2(0)\\ X_3(0)\end{bmatrix}=0$

Since the Elementary Row Operations does not change the null space, convert $(A-Bk)^D$ to reduce row echelon form, to get:

$$\begin{bmatrix}1 & 0 & -0.3936\\ 0 & 1 & 0.7385\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}X_1(0)\\ X_2(0)\\ X_3(0)\end{bmatrix}=0$$

$X_1(0)-0.3936X_3(0)=0 \Rightarrow X_1(0)=0.3936X_3(0)$

$X_2(0)+0.0394X_3(0)=0 \Rightarrow X_2(0)=-\,0.7385X_3(0)$

Let $S_1=X_3(0)$

$(X_1(0),X_2(0),X_3(0))=(0.3936S_1,-\,0.7385S_1,S_1)$

The class of consistent initial conditions is found to be:

$W_k=\{X_0(0)\epsilon\, R^3\setminus\{0\}\,|(X_1(0),X_2(0),X_3(0))=(0.3936S_1,-\,0.7385S_1,S_1)\}$

Step (5)

Since $E^TQE\geq 0$ and the system is regular and impulse free the unique solution of (19) is guaranteed solving on algebraic ricatti equation (20) we have that

$(A-BK)^T PE+E^T P(A-BK)=-E^T QE$, For a given matrix Q s.t $E^TQE\geq 0$

Where $Q=\begin{bmatrix}1 & -1 & 4\\ -1 & 3 & 5\\ 4 & 5 & 4\end{bmatrix}$, $E^TQE=\begin{bmatrix}164 & -26 & -82\\ -26 & 6 & 13\\ -82 & 13 & 41\end{bmatrix}$ and $eig(E^TQE)=\begin{bmatrix}6.9976\\ 1.8407\\ 209.1593\end{bmatrix}$

$$\begin{bmatrix}-4 & 0 & -2\\ -2 & -3 & -2\\ 0 & -2 & -1\end{bmatrix}\begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{21} & P_{22} & P_{23}\\ P_{31} & P_{32} & P_{33}\end{bmatrix}\begin{bmatrix}6 & -1 & -3\\ -4 & 1 & 2\\ 2 & 0 & -1\end{bmatrix}+\begin{bmatrix}6 & -4 & 2\\ -1 & 1 & 0\\ -3 & 2 & -1\end{bmatrix}\begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{21} & P_{22} & P_{23}\\ P_{31} & P_{32} & P_{33}\end{bmatrix}\begin{bmatrix}-4 & -2 & 0\\ 0 & -3 & -2\\ -2 & -2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}164 & -26 & -82\\ -26 & 6 & 13\\ -82 & 13 & 41\end{bmatrix}$$

$\Rightarrow\begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{21} & P_{22} & P_{23}\\ P_{31} & P_{32} & P_{33}\end{bmatrix}=\begin{bmatrix}0.1615 & -0.3635 & 0.4978\\ -0.3635 & 0.8304 & -1.1406\\ 0.4978 & -1.1406 & 1.5693\end{bmatrix}$ and $eig(P)=\begin{bmatrix}0.0004\\ 0.0035\\ 2.5574\end{bmatrix}$

Step (6):

Find α such that $\|x_0\|^2_{E^TPE}=x_o{}^T\,E^TPEx_o\triangleq V\big(x_o(t)\big)<\alpha$, $\forall x_o(t)=x_0\,\epsilon\,W_k$

$$[0.3936\quad -0.7385\quad 1]\begin{bmatrix}6 & -4 & 2\\ -1 & 1 & 0\\ -3 & 2 & -1\end{bmatrix}\begin{bmatrix}0.1615 & -0.3635 & 0.4978\\ -0.3635 & 0.8304 & -1.1406\\ 0.4978 & -1.1406 & 1.5693\end{bmatrix}\begin{bmatrix}6 & -1 & -3\\ -4 & 1 & 2\\ 2 & 0 & -1\end{bmatrix}\begin{bmatrix}0.3936\\ -0.7385\\ 1\end{bmatrix}$$

$\|x_0\|^2_{E^TPE}=0.037<\alpha$ Choose $\alpha=1$

Step (7)

Calculate

$$ln\frac{\boldsymbol{\beta}}{\boldsymbol{\alpha}}>\frac{-\lambda_{min}(E^TQE)}{\lambda_{max}(E^TPE)}t\,,\alpha<\beta\,,t\epsilon J\triangleq\{t\colon t_0\leq t\leq t_0+T\}\Leftrightarrow\beta>e^{\frac{-\lambda_{min}(E^TQE)}{\lambda_{max}(E^TPE)}t}.\alpha$$

$\lambda_{max}(E^TPE)=2.5574$ and $\lambda_{min}(E^TQE)=1.8407$ and Choose $\alpha=1$

t_0	$t_0 \le t \le t+T$	$e^{\frac{-\lambda_{min}(E^TQE)}{\lambda_{max}(E^TPE)}t}.\alpha$	β
0	$0 \le t \le 1$	1	6
0.1	$0.1 \le t \le 1.1$	0.9306	6
0.2	$0.2 \le t \le 1.2$	0.8659	6
0.3	$0.3 \le t \le 1.3$	0.8058	6
0.4	$0.4 \le t \le 1.4$	0.7498	6
0.5	$0.5 \le t \le 1.5$	0.6978	6
0.6	$0.6 \le t \le 1.6$	0.6493	6
0.7	$0.7 \le t \le 1.7$	0.6042	6
0.8	$0.8 \le t \le 1.8$	0.5623	6
0.9	$0.9 \le t \le 1.9$	0.5232	6
1	$1 \le t \le 2$	0.4869	6

The value of T is then selected to be $T = 1$ for $, t\epsilon J \triangleq \{t: 0 \le t \le 1\}$

Step (7)

Define $V(x(t)) = x^T(t)\, E^T PEx(t) \triangleq \|x(t)\|^2_{E^TPE}$ where $E^TPE > 0$

s.t $\|x\|^2_{E^TPE} = V(x(t)) < \beta$

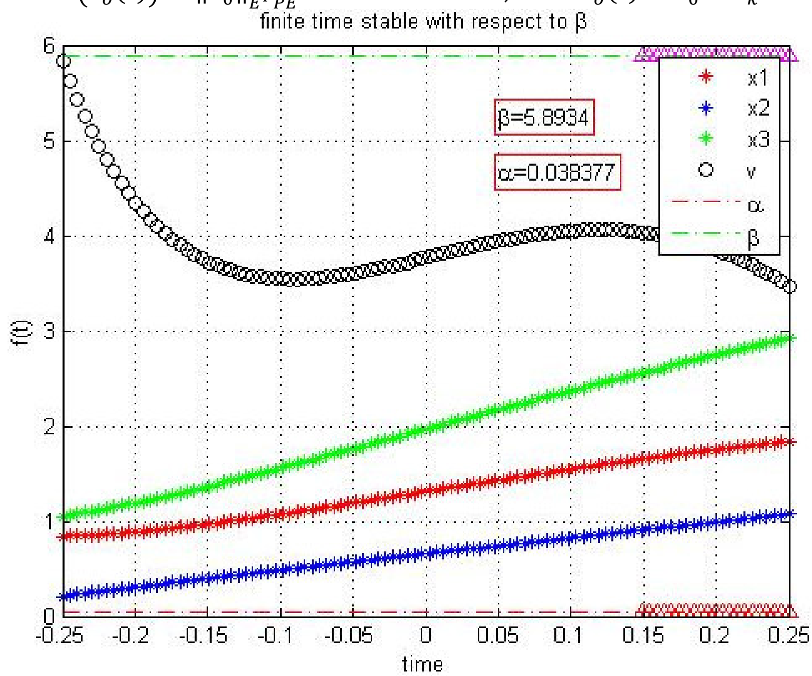
$$V(x(t)) = [x_1(t) \quad x_2(t) \quad x_3(t)]^T \begin{bmatrix} 7.3020 & -1.1202 & -3.6510 \\ -1.1202 & 0.1719 & 0.5601 \\ -3.6510 & 0.5601 & 1.8255 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$\Rightarrow V(x(t)) = x_1(t)(7.3020x_1(t) - 1.1202x_2(t) - 3.6510x_3(t)) + x_2(t)(-1.1202x_1(t) + 0.1719x_2(t) + 0.5601x_3(t)) + x_3(t)(-3.6510x_1(t) + 0.5601x_2(t) + 1.8255x_3(t))$$

$$\Rightarrow V(x(t)) = 7.3020x_1^2(t) - 1.1202x_1(t)x_2(t) - 3.6510x_1(t)x_3(t) - 1.1202x_2(t)x_1(t) + 0.1719\, x_2^2(t) + 0.5601x_2(t)x_3(t) - 3.6510x_1(t)x_3(t) + 0.5601x_2(t)x_3(t) + 1.8255\, x_3^2(t)$$

Hence

$$V(x_o(t)) = \|x_0\|^2_{E^TPE} = 0.038377 < 1 \ , \quad \forall x_o(t) = x_0 \, \epsilon \, W_k$$



From the figure (1) the value of $V(x(t)) \triangleq \|x(t)\|^2_{E^TPE} < \beta = 6$ $, \alpha < \beta$ and that completes the assertion

CONCLUSION

Based on the previses results a finite time stablizability of forced control system is guaranteed

A computational algorithm have been developed for computing the feedback gain matrix and Lyapunov function for this propose with illustration

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