How To Measure Oscillator's Short-Term Stability Using Frequency Counter

Ivica Milanović, Snežana Renovica, Ivan Župunski, Mladen Banović, and Predrag Rakonjac

***Abstract*—In this paper a few methods of how to use frequency counter in time-domain frequency stability analysis are described. Three implemented methods are presented. As an experiment, a comparison of the realized methods in the Technical Test Center (TOC) and the "references" obtained in the Directorate of Measures and Precious Metals (DMDM) in Belgrade are accomplished. The measurement uncertainty estimation for time interval measurement with one frequency counter is presented as well.**

***Index Terms*—Frequency, Frequency Counter, Oscillator, Short-Term Stability.**

I. Introduction

Frequency stability is one of the most important specifications of an oscillator. Stability does not specify how much frequency is accurate, but how much it is stable during observed time interval. If considered time intervals up to 100 seconds (10 ms, 100 ms, 1 s, etc.), then we talk about so-called short-term stability. Otherwise, there is analysis of long-term stability, and then we specify the oscillator's stability for an hour, and more often for a day, a month, or a year [1].

Stability is defined as the statistical estimation of the frequency or time fluctuations of a signal over a given time interval. Statistical estimations can be presented in the frequency or, more often, in the time domain [2]. To achieve frequency stability in the time domain a set of a frequency offset measurements have to be carried out, along with the calculation of the collected data scattering.

II. Theory Of Frequency Stability Measurements

Sine wave signal can be presented as:

$$V(t) = [V_0 + \varepsilon(t)]\sin[2\pi\nu_0 t + \varphi(t)] \tag{1}$$

where V_0 is nominal voltage, $\varepsilon(t)$ is amplitude deviation, ν_0 is nominal frequency and $\varphi(t)$ is phase deviation.

In order to simplify further analysis, nominal voltage and nominal frequency will be assumed as being constant. Also, it is assumed that the amplitude deviation is negligible in comparison with nominal voltage [3]. Due to that, the instantaneous frequency is equal to:

$$\nu(t) = \nu_0 + \frac{1}{2\pi}\frac{d\varphi}{dt} = \nu_0 + \nu_\nu \tag{2}$$

and it is the sum of a constant nominal value ν_0 and variable term $\nu_\upsilon(t)$.

We are not interested in large frequency deviations because we are talking about reasonably stable oscillators. Therefore, another restriction is:

$$|\nu_\nu(t)| << \nu_0 \tag{3}$$

The objective of the frequency stability analysis is to characterize the phase and frequency oscillator fluctuations with time [4]. In spite of that, we are primarily concerned with the $\varphi(t)$ term.

The aim is to determine the fractional frequency offset of oscillator (device) under test (DUT) and reference oscillator:

$$\frac{\Delta f}{f} = \frac{\nu(t) - \nu_0}{\nu_0} = \frac{1}{2\pi\nu_0}\frac{d\varphi}{dt} = y(t) \tag{4}$$

Measuring of frequency stability is a process which can be divided into a few steps [3]:

- Preprocessing
- Collecting and storing data
- Outliers removal
- Noise type determination
- Data analysis (statistics)
- Results interpreting - reporting

A. Preprocessing

Oscillator's characteristics are highly dependent on environment conditions, like a temperature change. Preprocessing includes preparing and monitoring those conditions, and monitoring the electrical power quality [3].

B. Acquisition

Frequency stability is observed over some period of time. To determine it, we have to realize a set of frequency offset measurements equally-spaced in time. The essential data is an array of equally-spaced phase or frequency values taken at particular measurement interval. Phase data are preferred, because they can be used to obtain frequency data. This is not

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always true if we want reverse analysis – absolute phase cannot be reconstructed from frequency data, and all gaps in frequency data will lead to loosing phase continuity [1]. In the literature the sampling time or the measurement interval is usually marked as τ_0 [4]. The averaging time (τ) is a multiple of the measurement interval (τ_0):

$$\tau = m \cdot \tau_0 \tag{5}$$

where m presents the averaging factor.

C. Outliers removal

System imperfection or some other external influences can produce abnormalities in collected and stored data - some values will significantly exceed expected quantities. Those data are called outliers, and they have to be removed from the collected array of data, before further analysis is carried out.

The median absolute deviation (*MAD*) is a robust way to set the criteria for an outlier [3]. It is the median of the absolute deviations of the data points from their median value (scaled), and is defined as:

$$MAD = Median\left\{\frac{|y_i - m|}{0.6745}\right\} \tag{6}$$

where m is equal to Median$\{y(i)\}$. The factor 0.6745 makes the *MAD* equal to the standard deviation for normally distributed data.

An outlier criteria of $5 \cdot MAD$ [3] is usually a good choice.

Another, maybe more common way, is to use next criteria:

$$m + 3s < x(j) < m - 3s \tag{7}$$

where x are data, j is number of data points, m is the mean value of x, and s is the classical standard deviation of x.

D. Statistics – data evaluating

Frequency stability is a result of data taken in some period of time, yet the independent variable is not the running time t, but the averaging time τ. Regarding that, the experimental data cannot be accurately described as a stationary process, so the usual variances are not good way to express frequency stability – the stationary concept means that observed process has its beginning and its end. Limited time intervals of observation are the main reason for inventing a new statistical tool called Allan Variance [2].

It is developed in order to solve the problem that the standard variance doesn't converge to a single value for the non-white FM noises as the number of measurements is increased [1]. It is described as:

$$\sigma_y^2(\tau) = \frac{1}{2(M-1)} \sum_{i=1}^{M-1} \left(\overline{y_{i+1}} - \overline{y_i}\right)^2 \tag{8}$$

where σ is Allan Variation, τ is averaging time, M is number of fractional frequency values, and y_i is i^{th} of M fractional frequency data averaged over the τ.

While standard deviation subtracts the mean from each measurement before squaring their summation, the Allan deviation subtracts the previous data point. This differencing of successive data points removes the time dependent noise contributed by the frequency offset.

The stability is being improved as the averaging time (τ) gets longer, because, in some cases, noises can be removed by averaging [1]. However, on some level further averaging no longer improves the results – that level is called the "noise floor".

The non-overlapping Allan, or two-sample variance, is the standard time domain measure of frequency stability [3].

But, this kind of calculation can be performed by utilizing all possible combinations of data sets. This is so-called overlapping method [5]. It can be performed over the Standard Allan Variation in order to improve the confidence of a stability estimate:

$$\sigma_y^2(\tau) = \frac{1}{2m^2(M-2m+1)} \sum_{j=1}^{M-2m+1} \sum_{i=j}^{j+m-1} (\overline{y_{i+m}} - \overline{y_i}) \tag{9}$$

where σ is Overlapped Allan Variation, and m is averaging factor.

Allan Variance can be described both tabular or in log-log sigma-tau (σ-τ) diagrams. Those diagrams describe how much we need to average in order to get rid of the noise contributed by the reference and the measurement system.

There are several other variances which can be used, like Modified Allan, Hadamard, Total, Time Variance etc. However, Overlapping Allan Variance should be used as the first choice [3].

E. Confidence Intervals

Sample variances are distributed according to:

$$\chi^2 = \frac{edf \cdot s^2}{\sigma^2} \tag{10}$$

where χ^2 is Chi-square probability, *edf* is Equivalent number of Degrees of Freedom, s^2 is the sample variance, and σ^2 is the true variance.

The *edf* depends of number on data samples and the noise type. The lower and the upper bound of the sample variance are:

$$\sigma_{\min}^2 = \frac{edf}{\chi^2(p, edf)}, (0 < p < 1) \tag{11}$$

$$\sigma_{\max}^2 = \frac{edf}{\chi^2(1-p, edf)}, (0 < p < 1) \tag{12}$$

where p is desired confidence factor.

F. Noise type determination

The instability of the most frequency sources can be modeled by a combination of their frequency fluctuations $S_y(f)$. Measure of frequency stability versus the time over which the frequency is averaged can be presented as:

$$S_y(f) \approx f^{\alpha} \quad or \quad \sigma_y(\tau) \approx \tau^{\mu/2} \tag{13}$$

where $S_y(f)$ is power spectral density, α is the parameter that defines the noise model in a frequency domain, $\sigma_y(\tau)$ is frequency stability vs averaging time, and μ is the parameter that defines the noise model in a time domain, and it is equal to $\mu = -\alpha-1$.

Some typical noises with α parameter values are shown in Fig.1.

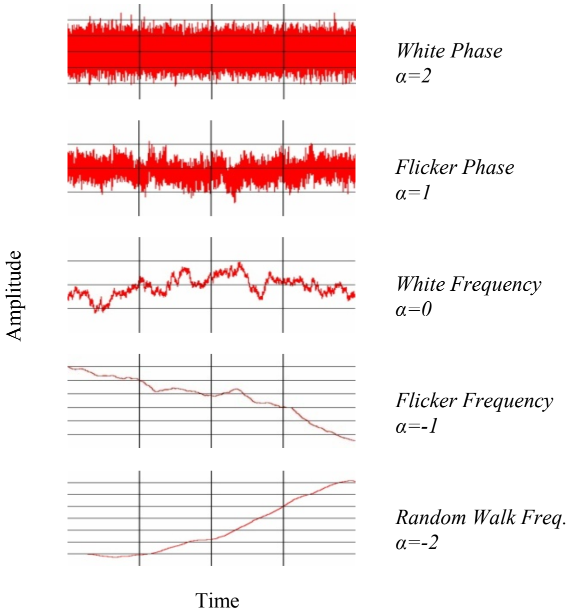

Fig. 1. Simulated Noises in the Time Domain.

White PN – usually exists as a result of signal amplifying, and has no relation with resonance mechanism.

Flicker PN – it is related to resonance mechanism, and it is usually made by noisy electronics.

White FN – it is a common type for passive resonator frequency standards (cesium or rubidium). They contain slave (usually quartz) oscillators whose frequency is "locked" to a resonance feature of another device.

Flicker FN – its physical cause is typically related to the physical resonance mechanism of an active oscillator, electronics parts, or environmental properties. It is common in high-quality oscillators, but it can be masked by white FN or flicker PN in lower-quality oscillators.

Random Walk FN – it usually exists very close to the carrier [6], and it is related to an oscillator's physical environment – mechanical shock, vibration, temperature, etc.

In a σ-τ diagram those simulations can be presented as in Fig.2.

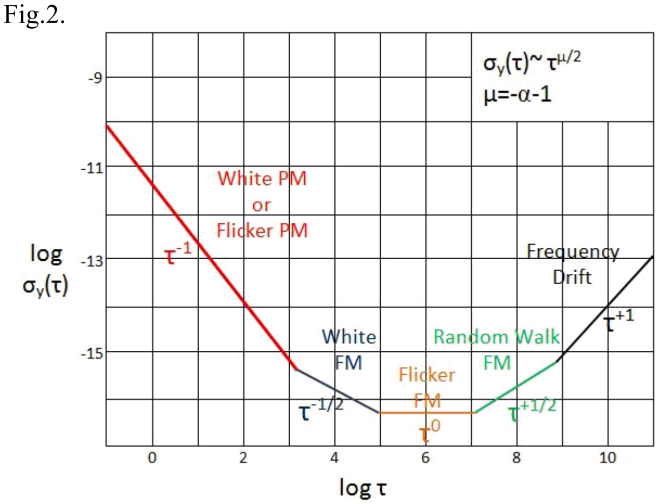

Fig. 2. Noises in the Time-Domain – σ-τ diagram.

So, if we calculate the slope of the derived Allan variation curve, we can determine the dominant noise type of the measured oscillator.

III. Electronic Counters In Time Interval Measurements

Frequency difference measurements can be carried out with time interval counters – devices with two inputs (signal in one input starts the measurement, and signal in second input stops it). In this case we have a comparison between two signals: the output signal from the reference oscillator and the output signal from the oscillator under calibration (device under test – DUT).

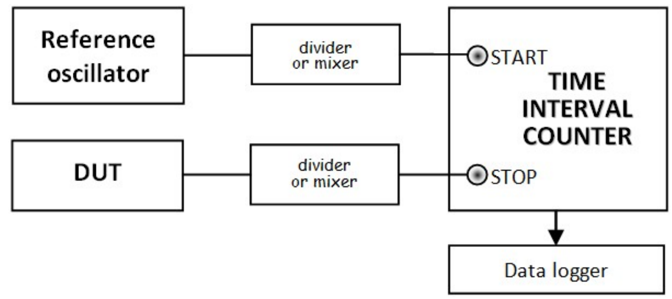

Fig. 3. Using the time interval (or frequency) counter for frequency stability measurements – basic idea.

This is the scheme which presents the basic idea of the stability measurements [7]. A few methods are realized according to that principle.

A. Reference oscillator

A measurement compares the DUT to a reference or standard. The standard should have better short-term characteristic – the test uncertainty ratio (TUR) should be 10:1, or even higher. When we talk about short-term stability, the most common types of oscillators can be arranged in ascending order: like the best quartz, then rubidium and then the cesium commercial oscillators. Nowadays, the best standards for short-term stability are so-called BVA quartz oscillators ("*Boitier a Vieillissement Ameliore*"). For example, Oscilloquartz BVA OCXO, type 8607 with option 15 (short term option) has $\sigma(\tau)$ better than $1.5 \cdot 10^{-13}$, for τ from 1 to 30 seconds.

B. Frequency counters

The frequency difference between DUT and standard is detected by a time interval counter. Frequency counters are most commonly used instruments with capabilities of the time interval measuring. There are conventional counters, reciprocal counters, counters with digital interpolation scheme, etc. [8]. However, when we are talking about the time interval measurements, a few characteristics of the counters are dominant:

– Single-shot time interval resolution. It represents the number of digits that counter can display. This characteristic limits counter's ability to measure frequency offset, and determines the smallest frequency change that can be detected without averaging,

– Accuracy in the time interval measurements,
– Dead time. It represents instrumentation delay between successive measurements,
– Trigger level timing error,
– Trigger offset,
– Internal noises,
– Aging of the oscillator and its temperature changes,
– Asymmetry between channels (mismatch),
– Averaging capabilities, etc.

Some of those characteristics can be suppressed or even overcome, and some of them cannot. The measuring uncertainty calculation will show their effects on the short-term stability measurements. This will be discussed later on in a chapter VI.

C. Frequency dividers or frequency mixers

Most common output frequencies of oscillators are 5 MHz or 10 MHz. Since they are not practical to measure with frequency counters, frequency dividers or frequency mixers are used to convert them to lower frequencies.

Despite the greater simplicity of the frequency dividers, frequency mixers are more used [1]. They are more expensive, require more hardware and additional oscillator, but they have a much higher signal-to-noise ratio, and this is the main reason for their usage.

D. Data logging

As a result, frequency counters give a set of frequency offset measurements. Those data can be written to a paper, or stored into some external memory space, in order to be analyzed later.

In order to avoid extra usage of counter hardware resources, and to suppress the measuring uncertainty and the dead time, some external accumulators can be used. At the beginning, analog plotters were used. They are changed with the accumulators – while the dead time data are sent to the accumulator's memory. After the measurement, the data can be read out later on. The main disadvantage is memory space.

Nowadays, the interfaces between the counters and personal computers extend the capability to store data directly to PC memory.

IV. Realized Methods For Short-Term Stability Measurements

Two methods are realized in Technical Test Center laboratory: Direct time interval measurement and dual mixer time difference method.

A. Direct time interval measurement method

In this method Hewlett Packard 5370A universal time interval counter is used. It has good time interval resolution – 20 ps in a single-shot. Also, it has capability to work in a binary mode of operation for time interval measurements – minimum time between measurements is 165 µs instead of 330 µs (like in a normal mode). This way counter does not perform any type of statistical measurement (mean, standard deviation, etc.). Instead, counter outputs raw data – five binary data form one decimal data (information of time interval value).

HP 5370A is connected to a PC USB port with Agilent 82357A GPIB/USB interface. Short-time stability measurement is automated with Agilent VEE Pro 7.0 software. Minimum sample time is 100 ms.

When the measurement is finished program transforms binary data into decimal values, calculates Allan deviation and draws σ-τ diagram. Frequency offset is calculated as:

$$\frac{\Delta f}{f} = -\frac{\Delta t}{t} \quad (14)$$

where Δt is time interval between two successive measurements, t is averaging time, and $\Delta f/f$ is the fractional frequency offset.

Further data analysis is carried out with the AlaVar 5.2 software. It removes outliers (according to (7)), gives Allan variation table results and charts, and can determine dominant noise type (five noise types, as is described in the chapter II-F).

B. Dual mixer time difference method

This method is realized with: HP 5345A frequency counter, two HP 10830A frequency mixers, HP 5358A accumulator (as an additional plug in the HP 5345A), and HP 59308A timing generator for HP 5345A external arming.

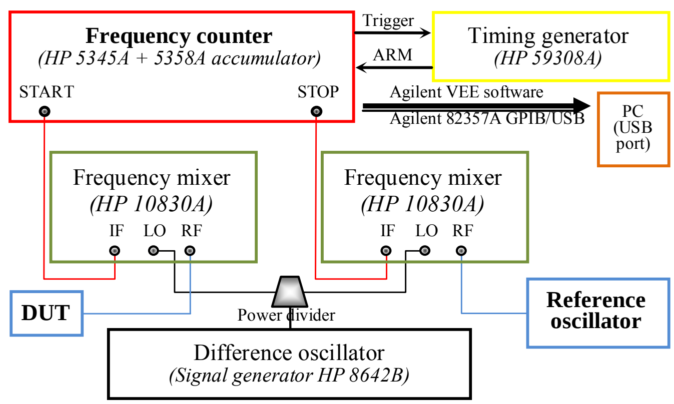

Fig. 4. Dual Mixer Time Difference Method with HP 5345A.

As a difference oscillator HP 8642B signal generator is used. It provides ν_b= 6 kHz beat frequency (6 kHz sine wave signal). External arming of HP 5345A is with 200 µs pulses from HP 59308A.

The difference oscillator's output ($\nu_0 \pm \nu_b$) is split by a power divider, and applied to each mixer. Further measurement is taken over 6 kHz IF signals.

To calculate $\sigma(\tau)$ formulas (15) and (16) are used:

$$\Delta f_i = \frac{\nu_b}{\tau}\left[\left(t_{i+1} - t_i\right) - \left(t_i - t_{i-1}\right)\right] \quad (15)$$

$$\sigma_y(\tau) = \sqrt{\frac{1}{2N}\sum_{j=1}^{N}\frac{(\Delta f_i)^2}{\nu_0^2}} \quad (16)$$

where t_i is time interval between 2 successive measurements,

ν_b is beat frequency, τ is averaging time, ν_0 is nominal (carrier) frequency, and N is number of samples.

This measuring method is also automated with Agilent VEE Pro 7.0.

Because of the HP 5345A poor time interval measurement resolution (2 ns), this method is also configured with HP 5370A frequency counter. The configuration is the same, but timing generator was not used, because HP 5370A has no capability of the external arming.

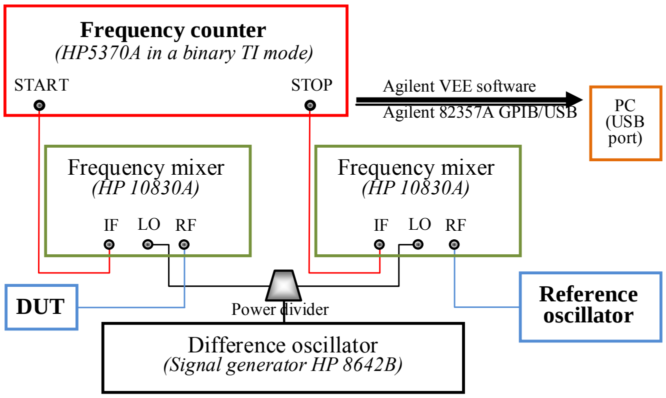

Fig. 5. Dual Mixer Time Difference Method with HP 5370A.

Three of these methods were compared with the "reference" method realized in the Directorate of Measures and Precious Metals. DMDM uses specially designed for time interval analysis, TSC 5110A Time Interval Analyzer. It is designed to measure the phase difference between two signals, to measure frequency, to determine and draw Allan deviation and to draw phase and frequency plots. Optionally, it can determine SSB (single-sideband) phase noise.

This time interval analyzer is based on the heterodyne method (method with two mixers), and uses intermediate frequency (IF) of approximately 100 Hz (when equal frequency oscillators are compared). The smallest sampling interval is one period of the IF or 10 ms.

For the frequency standard DMDM uses Oscilloquartz BVA OCXO 8607 (described in the chapter III-A), which is one of the best commercial short-term stability standards.

V. Experimental Results

Three different types of oscillators are used for short-term stability measurements: HP 105B and HP 5061A quartz oscillators, Racal Dana 9475 rubidium oscillator and Oscilloquartz 3210 cesium frequency standard. They are compared with three realized methods: with HP 5370A universal time interval counter - direct method (in binary mode of operation) and dual mixer method, and dual mixer method with HP 5345A frequency counter.

The environmental conditions were 23°C±1°C, and humidity 50%±10%.

In order to compare them, measurements are also taken with TSC 5110A and frequency references BVA 8607 and the Symmetricom 5071A high performance cesium standard, in DMDM`s time and frequency laboratory. This method was assumed "reference", which will be shown in the charts below.

In Fig.6 the σ-τ diagram (Allan variance chart) for rubidium oscillator stability is shown. It was compared with two different quartz oscillators for 5 MHz outputs, measured with HP 5370A (direct measurement). In the range from 0.1 to 4 seconds the difference between DMDM and TOC results is significant. For τ=1s DMDM result is $5\cdot10^{-12}$, and with direct measurement $2\cdot10^{-11}$. Nevertheless, as manufacturer specifies Allan variance better than $5\cdot10^{-11}$, the conclusion for the averaging time 1s will not be wrong.

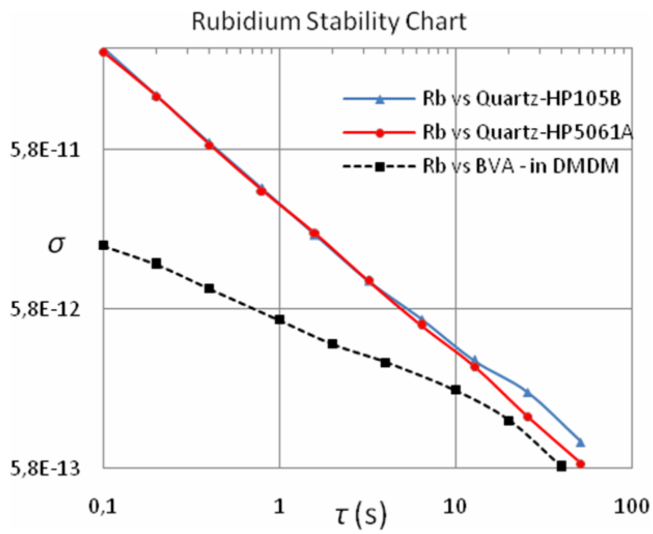

Fig. 6. Direct method with HP5370A in a binary mode.

The same measurement was made using the cesium 3210. The results are given in Fig.7. For τ = 1 s they are practically the same. Still, under 1s difference is bigger as τ gets smaller.

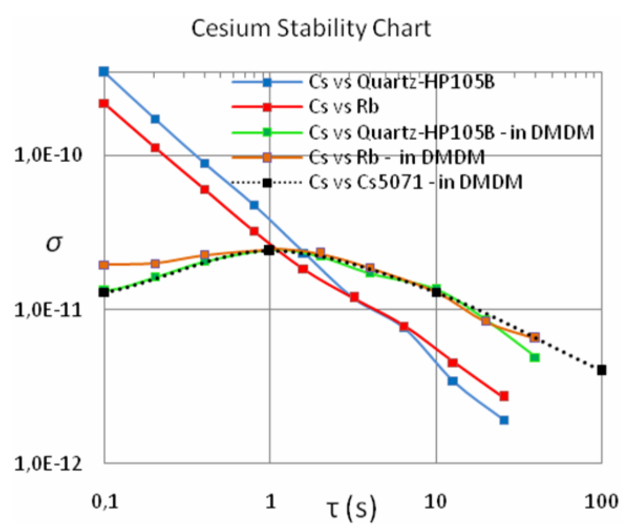

Fig. 7. Direct method with HP5370A in binary mode.

As we introduced mixers in the measurements, the results became better (Fig.8).

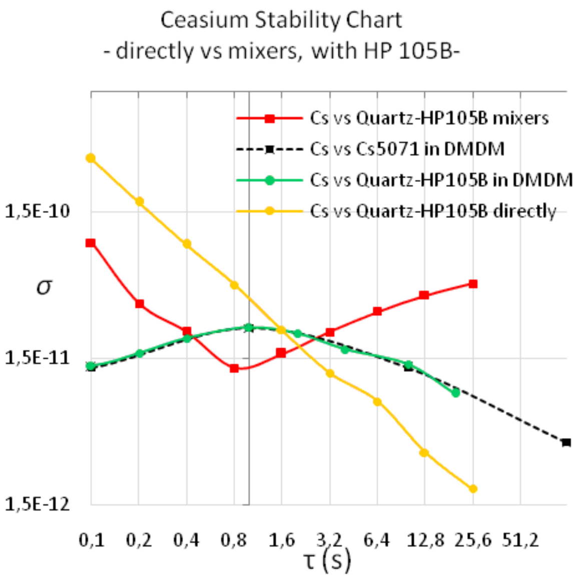

Fig. 8. Use of HP5370A in binary mode – directly and with mixers.

Mixing 10 MHz with 6 kHz signal improves system capabilities, especially in a band below 1 second.

In Fig. 9 we can clearly see the improvements made by using time difference method, and with use of good standards.

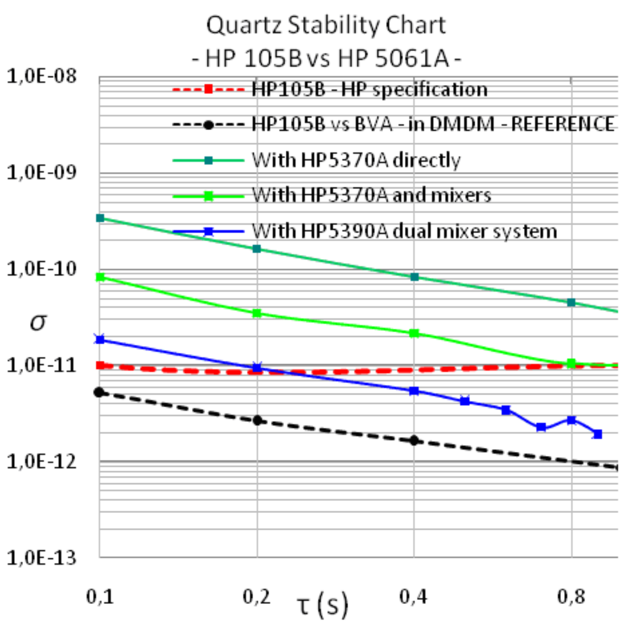

Fig. 9. HP 105B quartz oscillator stability measured with direct and dual mixers methods – comparison.

Two quartz oscillators were compared (HP 105B and HP 5061A). The "worst" results were achieved with the direct method.

Even the better counter was used (20 ps HP 5370A), because of the lack of the timing generator, dual time difference method realized with HP 5345A gave better results. That shows the importance of the counter's accurate arming.

The conclusion is that for oscillators with high short-term stability, only dual time difference method can produce reliable results.

VI. Measuring Uncertainty

Regarding previous chapters it is obvious that counter's specifications are dominant in measuring uncertainty contribution. All of those methods are based on time interval measurements. Like an example, the measuring uncertainty estimation for direct measurement with HP 5370A time interval counter will be discussed.

A. Random Effects – Uncertainty Type A

The random effects vary in an unpredictable way each time you make a measurement. They produce an unstable reading on the counter's display. This uncertainty is often assumed to have an approximately normal distribution.

1. Resolution or Quantization uncertainty

This is uncertainty due to single-shot time interval resolution of a counter [9]. For HP 5370A this resolution is 20 ps, and producer defines this uncertainty like:

$$\delta_r = \frac{\pm 20\,ps}{\sqrt{sample\,size}} \pm 2\,ps \qquad (17)$$

If we choose sample size 1, this contribution is ±22 ps.

2. Accuracy of a time interval measurement

Because of a great influence of jitter Hewlett Packard defines uncertainty due to time interval measurement accuracy as:

$$\delta_a = jitter \qquad (18)$$

HP 5370A has typical jitter of 100 ps, so, total amount of this contribution is ±100 ps.

3. Start/stop trigger point uncertainty due to noise

This uncertainty occurs when a time interval measurement starts or stops too early or too late because of noise on the input signal [9], as shown in Fig.10.

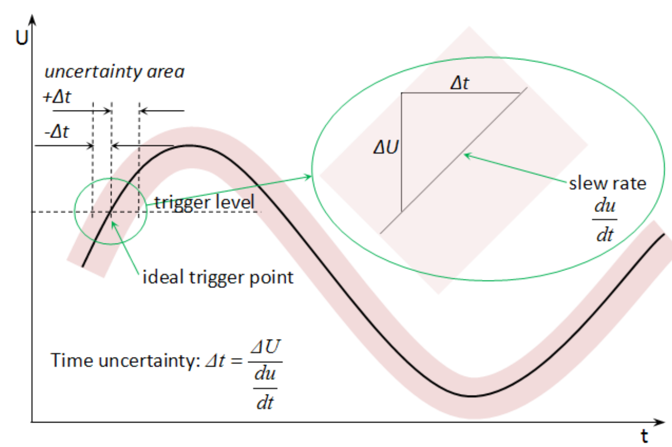

Fig. 10. Start trigger points uncertainty due to noise.

There are two sources of the noise: noise on the signal being measured and noise added to this signal by the counter's input circuitry:

$$\delta_{tn} = \frac{\sqrt{V_{ni}^2 + V_{ne}^2}}{\frac{du}{dt}} \quad (19)$$

where V_{ni} is internal noise, V_{ne} is external noise, and du/dt is signal slew rate at trigger point.

The slew rate (du/dt) for sine-wave signal at the zero-crossing is:

$$\frac{du}{dt} = u \cdot \omega \cdot \cos(0) = 2 \cdot \pi \cdot f \cdot U_{RMS} \cdot \sqrt{2} \quad (20)$$

The internal noise for HP 5370A is 150 µV. If we assume that input is 10 MHz sine-wave signal, with U_{RMS} =1 V, and signal to noise ratio (*SNR*) 60 dB, formula (20) now is:

$$\delta_{tn} = \frac{\sqrt{\left(\frac{150\mu V}{U_{RMS}}\right)^2 + \left(\frac{1}{SNR}\right)^2}}{2 \cdot \pi \cdot f \cdot U_{RMS} \cdot \sqrt{2}} = \frac{\sqrt{\left(\frac{150\mu V}{1V}\right)^2 + (0.001)^2}}{2 \cdot \pi \cdot 10MHz \cdot 1V \cdot \sqrt{2}} \quad (21)$$

Finally, for start trigger point, measuring contribution is ±11 ps.

If we assume the same U_{RMS} for both signals at the counter inputs, the uncertainty for stop trigger point will be the same as for start trigger point, so we have $\delta_{tn\text{-}start}=\delta_{tn\text{-}stop}= \pm 11$ ps.

B. *Systematic Effects – Uncertainty Type B*

Uncertainty type B is unchanged when a measurement is repeated under the same conditions. Instead, those effects cause an offset of the measurement result from the true value.

4. Start/stop trigger points uncertainty due to trigger level offset or Trigger level timing uncertainty

This measuring uncertainty results from trigger level setting uncertainty due to deviation of the actual trigger level from the indicated, and from input amplifier hysteresis if the input signals do not have equal slew rates [9][10].

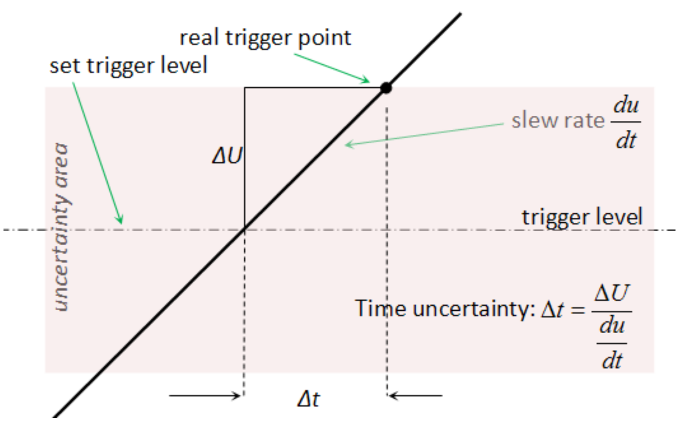

Fig. 11. Start trigger point uncertainty due to trigger level offset.

This uncertainty can be presented as:

$$\delta_{to} = \frac{\Delta U}{\frac{du}{dt}} \quad (22)$$

where ΔU is offset from zero.

For HP 5370A this offset from zero is less than 2.5 mV, so:

$$\delta_{to} = \frac{2.5mV}{2 \cdot \pi \cdot 10MHz \cdot 1V \cdot \sqrt{2}} \quad (23)$$

As the input signals are equal we have:

$\delta_{to\text{-}start}=\delta_{to\text{-}stop}= \pm 28$ ps

If we assume a rectangular distribution [10], the corresponding standard uncertainty can be calculated by dividing by $\sqrt{3}$, so we will have:

$\delta_{to\text{-}start}=\delta_{to\text{-}stop}= \pm 16$ ps

5. Channel asymmetry uncertainty or Channel mismatch uncertainty

This uncertainty is a result of unequal propagation delays in the two counter's inputs, and differences in rise times of the input amplifiers.

Hewlett Packard for 5370A defines asymmetry better than 700 ps. Assuming a rectangular distribution this measuring uncertainty contribution is $\delta_{asymm}= \pm 404$ ps.

6. Timebase uncertainty

This uncertainty is frequency deviation from it's nominal value (10 MHz in this example). HP 5370A uses external reference from cesium frequency standard, so, the uncertainty is the result of two main sources:

– aging of the oscillator (stability) – ε_a

(for Oscilloquartz 3210 Allan deviation for 0.1 s is: $1.3\cdot10^{-11}$ for 2σ, or $0.65\cdot10^{-11}$ for 1σ)

– temperature changes – ε_t

(for Oscilloquartz 3210 temperature changes are defined as: $2\cdot10^{-12}$ in the range -5 °C to 55 °C)

The timebase uncertainty [9] is defined by:

$$\delta_{TB} = TI \cdot \sqrt{\frac{\varepsilon_a^2 + \varepsilon_t^2}{3}} \quad (24)$$

where TI is measured time interval (in this example it is sampling interval of 100 ms), so, this contribution is: $\delta_{TB} = \pm 0.4$ ps.

The influence of type A measuring uncertainty can be reduced by averaging [5]. Experimental measurements are realized within 90 s, 100 s, 180 s and 360 s, or 900, 1000, 1800, 3600 or 9000 samples, respectively. The summary contribution of type A measuring uncertainty is calculated using formula 25:

$$u_{RAND} = \frac{\sqrt{\delta_r^2 + \delta_a^2 + \delta_{tn-start}^2 + \delta_{tn-stop}^2}}{\sqrt{N}} = 3\,ps \quad (25)$$

where N stands for the number of samples.

Finally, the expanded uncertainty for $k=2$ is equal to:

$$U(k=2) = 2 \cdot \sqrt{u_{RAND}^2 + \delta_{to-start}^2 + \delta_{to-stop}^2 + \delta_{asymm}^2 + \delta_{TB}^2} \quad (26)$$

$U(k=2)$ = 0.809 ns

The influence of channel asymmetry is dominant one, and it is shown that bigger averaging time does not result in reducing measuring uncertainty.

In deciding which counter should be used, the user has to pay particular attention on: channel asymmetry, accuracy of the time interval measuring, and time base uncertainty of a counter.

This way of estimating measuring uncertainty can be used for all time interval measurements which are carried out with a frequency counters.

VII. The Conclusion

In this paper the oscillator's short-term stability and procedure of it's measurement in the time domain was described, in short. Particular phases of measurement, and the way for a data analysis are given, too.

Three methods realized with frequency counter HP 5345A, and time interval counter HP 5370A are described. The results of real measurements are given in graphs. They are compared with the results acquired in DMDM which are considered referent ones. In the DMDM measurement was carried out using the time interval analyzer with two standards: BVA quartz oscillator and ultra stable cesium frequency standard. The shot-term stability was measured for quartz oscillator HP 105B, rubidium frequency standard Racal Dana 9475 and cesium frequency standard Oscilloquartz 3210.

The comparisons are presented in graphs. It is shown that methods based upon the frequency counters can be used to determine short-term stability for averaging time of one second, or more. For smaller averaging intervals, more reliable is method realized with time interval analyzers in accordance with ultra stable oscillators like a references.

Direct measurements with counters are possible, but if we want to improve measuring system capabilities, it is better to compare IF frequencies, rather than their nominal values. These methods can be realized either with dividers or frequency mixers. Advantages and disadvantages are shown in this paper.

The measurement of short-term stability is, basically, measurement of time interval between two sinusoidal signals. According to that, the measurement uncertainty estimation for time interval measurement using frequency counter is given. The analysis shows that the mismatch between counter channels has the greatest influence to total measuring uncertainty. For counters which are going to be used in frequency stability measurements, this analysis shows what are the most important characteristics we have to pay attention on.

References

[1] M.A.Lombardi, "Fundamentals of Time and Frequency", NIST, 2002.

[2] P.Kartashoff, "Frequency and Time", Monographs in Physical Measurement, 1978.

[3] W.J.Riley, "Handbook of Frequency Stability Analysis", Hamilton Technical Services, July 2007.

[4] Rohde&Schwarz, "Time Domain Oscillator Stability Measurement Allan Variance", Application Note, Feb.2009.

[5] W.J.Riley, "The Averaging of Phase and Frequency Data", Hamilton Technical Services, Nov.2010.

[6] D.A.Howe, D.W.Allan, J.A.Barnes, "Properties of Oscillator Signals and Measurement Method", NIST

[7] Hewlett Packard, "Fundamentals of Time Interval Measurements", Application Note 200-3, June1997.

[8] Hewlett Packard, "Fundamentals of the Electronic Counters", Application Note 200, Mar.1997.

[9] Fluke, "Sources of Error in Time Interval Measurements", Application Note

[10] Pendulum Instruments, "Accurate Phase Calibration", Application Note CNT-81, AN-07