

## University of Dayton eCommons

---

Physics Faculty Publications

Department of Physics

---


12-2012

# Logistic Curves, Extraction Costs and Peak Oil

Robert J. Brecha

*University of Dayton*, [rbrecha1@udayton.edu](mailto:rbrecha1@udayton.edu)

Follow this and additional works at: [https://ecommons.udayton.edu/phy\\_fac\\_pub](https://ecommons.udayton.edu/phy_fac_pub)

 Part of the [Earth Sciences Commons](#), [Engineering Physics Commons](#), [Environmental Health and Protection Commons](#), [Environmental Indicators and Impact Assessment Commons](#), [Environmental Monitoring Commons](#), [Natural Resource Economics Commons](#), [Natural Resources Management and Policy Commons](#), [Oil, Gas, and Energy Commons](#), [Optics Commons](#), [Other Physics Commons](#), [Quantum Physics Commons](#), and the [Sustainability Commons](#)

---

### eCommons Citation

Brecha, Robert J., "Logistic Curves, Extraction Costs and Peak Oil" (2012). *Physics Faculty Publications*. 17.  
[https://ecommons.udayton.edu/phy\\_fac\\_pub/17](https://ecommons.udayton.edu/phy_fac_pub/17)

This Article is brought to you for free and open access by the Department of Physics at eCommons. It has been accepted for inclusion in Physics Faculty Publications by an authorized administrator of eCommons. For more information, please contact [frice1@udayton.edu](mailto:frice1@udayton.edu), [mschlange1@udayton.edu](mailto:mschlange1@udayton.edu).

# Logistic Curves, Extraction Costs and Effective Peak Oil

Robert J. Brecha<sup>a,b,\*</sup>

<sup>a</sup>*Potsdam Institute for Climate Impact Research, Telegraphenberg, 14412 Potsdam, Germany*

<sup>b</sup>*Permanent address: Physics Department, Renewable and Clean Energy Program, and Sustainability, Energy and Environment (SEE) initiative, University of Dayton, Dayton, OH 45469-2314*

---

## Abstract

Debates about the possibility of a near-term maximum in world oil production have become increasingly prominent over the past decade, with the focus often being on the quantification of geologically available and technologically recoverable amounts of oil in the ground. Economically, the important parameter is not a physical limit to resources in the ground, but whether market price signals and costs of extraction will indicate the efficiency of extracting conventional or nonconventional resources as opposed to making substitutions over time for other fuels and (mainly, transportation) technologies. Here we present a hybrid approach to the peak-oil debate, using two models, in which the use of logistic curves for cumulative production are supplemented with data on projected extraction costs and historical rates of capacity increase to provide indicative evidence for a possible *effective* peak in world production of oil, while not denying the presence of large quantities of oil in the ground. Even with foresight, rates of production of new nonconventional resources are likely to not be sufficient to make up for declines in availability of conventional oil.

*Keywords:* peak oil, logistic curves, extraction costs

---

## 1. Introduction

A vigorous debate about the extent of the potential world petroleum resource has been carried out over the past several decades. On the one hand, concerns about lack of supplies have been expressed since the beginning of the petroleum age, whereas consumption has continued to increase (with few exceptions) for one and a half centuries; on the other hand, it is clear that petroleum represents a finite resource in the ground, and that there will necessarily be a production peak followed by a decline toward zero production at some point in

---

\*Corresponding author

*Email address:* [rbrecha1@udayton.edu](mailto:rbrecha1@udayton.edu) (Robert J. Brecha)

time. Most famously, petroleum geologist M. King Hubbert predicted in the late 1950s that production of oil in the continental United States would reach a maximum in about 1970 (Hubbert, 1956). In 1956, when Hubbert first made this startling prediction U.S. oil production had been increasing by roughly 7%/year for nearly a century, and indeed, for some time afterwards, continued to rise (Hubbert, 1981). Looking back, however, production of oil in the U.S. did reach a maximum in 1970, and even with the added production of oil from the Alaskan North Slope and from deepwater sources in the Gulf of Mexico, has never exceeded the level of production in that year. One might say that this prediction by Hubbert, due in equal parts to mathematical analysis and deep knowledge of petroleum geology, marked the beginning of the “peak oil” story. Although he was not taken very seriously at the time, with hindsight it is clear that Hubbert was correct, at least in the broad outlines of his projections.

The intensity of this debate increases at times of higher oil prices, such as during the mid- to late-1970s. Starting with an article in 1998 by Campbell and Laherrere (Campbell and Laherrere, 1998) and building over the past several years, serious effort has been directed toward answering several basic questions about oil supplies. First and foremost, is there a real danger of arriving at a supply-limited shortfall in either conventional petroleum itself, or more generally, in liquid fuels needed for (mainly) transportation? An interesting recent development has been an acknowledgement of the potential for a current peak in oil production on the part of respected mainstream sources (IEA, 2010; Kerr, 2011). A second question is whether there is enough solid data available to make a determination of the extent of *in situ* resources and the potential for economic conversion from resources to producible reserves (IEA, 2008). A related question is that of the efficacy of economic signals in ensuring both sufficient supply and in guiding markets toward substitutes for conventional petroleum, should that resource indeed be unable to fulfill worldwide demand. Also in the realm of economic analysis is the question of the feedback between potentially rising prices for liquid fuels and consumer demand.

It is our aim in this paper to examine the questions of petroleum reserves and resources and production using logistic curves. Although this is the general approach taken by Hubbert and many others since, we present here an analysis in terms of different regions and resource types, with the goal of modestly disaggregating conventional and non-conventional resources to the extent that we can construct a model that allows us to investigate some of the dynamics of production of different resources, both current and future. We extend the Hubbert-curve technique to include some basic economic information in the form of estimated extraction costs for different types of conventional and non-conventional oil resources. The specific questions addressed here, related to those posed above, are the following: Can the logistic function be used to project the ultimately recoverable resource (URR; symbol,  $Q_\infty$ ), and if so, under what circumstances (Section 2); what constraints are set on rates of extraction, given the finite resource size and historical precedent (Section 3); how might one expect average petroleum extraction costs to change with time, and how do average costs compare to marginal extraction costs (Section 4); and given

past and current patterns of production for various resources (either regional or type, as defined below), will there be enough flexibility going forward to meet growing world demand for liquid fuels (Section 5)? In Section 6 we will discuss our results and conclude.

The key results from our analysis are that, although non-conventional petroleum resources may be very large, historical evidence for growth rates in production indicate that there will likely be near-term supply-driven constraints to resource availability. Furthermore our construction of extraction cost curves as a function of time (or cumulative production) help explain the observation that the cost of the marginal (actually produced) barrel of oil, and therefore the price of oil, may be high although relatively large quantities of low-cost resources are still being produced. Put another way, the marginal cost curve differs qualitatively from those often presented in investigations that assume a particular functional form for the extraction costs, based on extraction of first lower-cost resources, followed sequentially by increasingly costly resources.

## 2. Logistic Function and Variations

It is difficult to make a projection of future oil production patterns based solely on the history of past production. Hubbert used a simple, logical curve-fitting technique to make his estimate, with one of the input parameters being an estimate of the ultimately recoverable resource (URR), which had to be obtained through other means. It should be noted here that Hubbert did *not* simply take the sum of past production and proven reserves to make his estimate of URR. It was perfectly clear to him that new fields would be discovered and that technological advances would lead to improved recovery in existing fields (Hubbert, 1956); we follow that same strategy in this work.

Hubbert (Hubbert, 1981) used a reasonable mathematical formulation that also has a useful physical interpretation. The logistic, or Verhulst, function is often used to describe growth in a system subject to a finite capacity or resource. The mathematical form of the logistic equation is given by

$$\frac{dQ}{dt} = bQ \left( 1 - \frac{Q}{Q_\infty} \right) \quad (1)$$

where  $Q$  is the cumulative production of oil up to a given date,  $Q_\infty$  is the ultimately recoverable amount of oil, and  $\frac{dQ}{dt}$  is the rate of extraction. The initial (exponential) rate of growth of cumulative production is described by the constant  $b$ . The logistic equation describes growth of a quantity that initially increases exponentially. As the ultimate limit to the resource becomes an important factor in production, extraction becomes more difficult, and the rate of extraction decreases. Relevant to the current work, since there is an ultimate limit to the amount of oil in the ground, at some point in time the rate of production will go to zero.

The solution to the Verhulst equation is given by

$$Q(t) = \frac{Q_\infty}{1 + \left(\frac{Q_\infty - Q(0)}{Q(0)}\right) e^{-bt}} = \frac{Q_\infty}{1 + ae^{-bt}} \quad (2)$$

with three undetermined constants that can in principle be found by comparison with actual data. An alternative formulation of the solution to the Verhulst equation is to write the cumulative and yearly production in terms of the time of peak production,

$$Q(t) = \frac{Q_\infty}{1 + e^{-b(t-t_p)}} \quad (3)$$

$$P(t) = \frac{bQ_\infty e^{-b(t-t_p)}}{(1 + e^{-b(t-t_p)})^2} \quad (4)$$

where  $t_p$  is the year of peak production; this form for the cumulative and yearly production functions will also be useful in what follows.

In reality, drawing conclusions from data fit to the logistic function is not trivial, especially because economic and political factors in the 1970s and 1980s led to a noticeable readjustment to trends in oil production in all areas of the world. In Fig. 1 are shown the data for cumulative oil production in the U.S. along with two logistic curves. By carrying out a nonlinear least-squares fit (NLSF) to the data from 1859 through 2010, one finds the upper curve, corresponding to parameters  $Q_\infty = 195 \pm 2$ ,  $b = 0.065 \pm 0.001$ ,  $t_p = 1971 \pm 1$ . However, if the data to be fit are limited to production from 1859-1970, the best fit parameters are  $Q_\infty = 152 \pm 5$ ,  $b = 0.072 \pm 0.001$ ,  $t_p = 1964$ .

Hubbert's 1981 paper (Hubbert, 1981) used data not available at the time of his original 1956 presentation. His data along with several other sets of fit parameters are summarized in Table 1. Hubbert predicted an ultimately recoverable reserve of about 170 Gb for the U.S. lower 48; it now appears that the trend will lead to an amount closer to 200 Gb, which was the upper limit of his probable range. The most striking feature of the data and fit parameters given in Table 1 is the strong increase in expected ultimately recoverable oil until the peak of production had actually passed in 1970. After the peak (or equivalently, after the inflection point in the logistic curve shown in Fig. 1) the value for  $Q_\infty$  changes only slightly with the length of data set considered. Of course, this point is problematic since one of the main reasons that one wishes to use the logistic equation approach is to help determine *when* the peak of production will be reached, and how much oil will ultimately be available.

Hubbert did not consider off-shore deep-water oil or some of the enhanced oil-recovery techniques that have been since applied, data that are included in the above. And of course, Alaskan North Slope oil is not included here, as that resource was not part of Hubbert's initial calculation. In fact, one argument used against the concept of "peak oil" is that there will *always* be another source of oil or replacement energy when the economic and technological conditions are ripe. However, as we shall see below, even these additional discoveries did not alter the fact that total U.S. production has never superseded the 1970 peak.

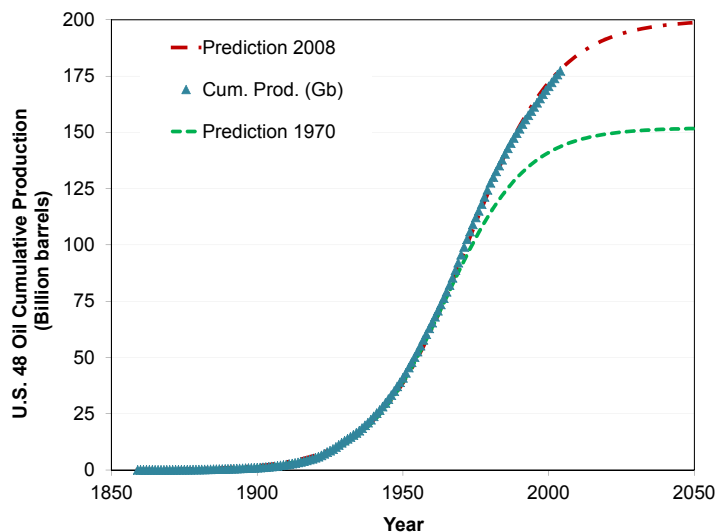


Figure 1: Cumulative U.S. oil production and predictions, in billions of barrels. The two logistic curves are based on predictions by M. King Hubbert (Ref. (Hubbert, 1981)) and from current trends (URR of 195 billion barrels).

The conclusion can be drawn that in the early, exponential growth stages of resource production, it is difficult to determine the final trajectory of the production curve based on the logistic cumulative production model. It is only when cumulative production reaches a significant fraction of the limiting URR that the two curves shown in Fig. 1 separate.

Another approach is to look at the yearly production as a function of time, effectively, the derivative of the logistic function. We can construct a similar table to that above, using the logistic function in terms of peak year,  $t_p$ . From the results in Table 2 we see that using the yearly production data appears to give more accurate predictions of both the URR, and especially, the date

Data through	$Q_\infty(\text{Gb})$	$t_p$	b
2010	$195 \pm 2$	$1971 \pm 1$	$0.065 \pm 0.001$
1990	$185 \pm 2$	$1969 \pm 1$	$0.068 \pm 0.001$
1980	$183 \pm 5$	$1969 \pm 1$	$0.068 \pm 0.001$
1962-1980 Ref. (Hubbert, 1981)	170	1500	0.069
1970	$152 \pm 5$	$1964 \pm 1$	$0.0724 \pm 0.001$
1960	$123 \pm 6$	$1959 \pm 1$	$0.077 \pm 0.001$

Table 1: Logistic function nonlinear least-squares fit parameters. Years represent the final year of the data set used for the fit, with the initial year being 1860.

of peak production. We note that although the estimates for URR tended to increase until the peak production year was passed, that increase was relatively small, on the order of 25% or less. Although both relations contain the same fundamental information, the fact that  $Q$  is an integral of production tends to hide year-to-year changes; therefore, we will mainly use fits to the derivative of the logistic function when looking at world oil production.

Data through	$Q_\infty$ (Gb)	$t_p$	b
2009	206 $\pm$ 5	1972 $\pm$ 1	0.059 $\pm$ 0.002
1990	190 $\pm$ 5	1970 $\pm$ 1	0.065 $\pm$ 0.002
1980	195 $\pm$ 8	1971 $\pm$ 1	0.064 $\pm$ 0.003
1970	206 $\pm$ 16	1972 $\pm$ 3	0.061 $\pm$ 0.003
1960	163 $\pm$ 19	1966 $\pm$ 3	0.068 $\pm$ 0.003

Table 2: Logistic function nonlinear least-squares fit parameters for yearly production. Years represent the final year of the data set used for the fit, with the initial year being 1860.

There are two further variants of the logistic curve approach. Both are obtained by eliminating time as a variable, writing the original differential equation as

$$\frac{P}{Q} = b \left( 1 - \frac{Q}{Q_\infty} \right) \quad (5)$$

where  $P = \frac{dQ}{dt}$ . If the ratio of yearly production to cumulative production is plotted as a function of cumulative production, the  $x$ -intercept is equal to  $Q_\infty$  and the  $y$ -intercept is  $b$ . Performing the same set of fits as with the yearly production curve yields very similar results, as shown in Table 3. Finally, it is possible to write yearly production as a nonlinear (quadratic) function of cumulative production,

$$P = bQ - b \frac{Q^2}{Q_\infty} \quad (6)$$

which can also be used to find the parameters  $b$  and  $Q_\infty$ . Reynolds (Reynolds, 2002; Reynolds and Baek, 2011) uses this approach, using in the latter paper an extension to allow for variation in the exponent.

Although the logistic approach has limitations, it does describe at least qualitatively the progress of oil production, especially as larger regions are aggregated, and is certain to offer a good approximation for the world as a whole. In fact, most oil-producing countries in the world have already passed a peak in output (Sorrell et al., 2010). With the above cautions in mind, we estimate parameters for total world production of crude oil using a nonlinear least-squares fit to historical production data, considering a modest disaggregation. We distinguish different types of oil that are of interest, using the example of current production, and data from the U.S. Energy Information Administration (<http://www.eia.doe.gov>). Total world oil supply (which would better be termed ‘‘total liquid fuel supply’’) in 2011 was 87,095,000 barrels per day

Data through	$Q_\infty(\text{Gb})$	intercept	slope
2009	$197 \pm 4$	$0.0638 \pm 0.0001$	$-(320 \pm 5) \times 10^{-6}$
2000	$193 \pm 4$	$0.0644 \pm 0.0001$	$-(330 \pm 6) \times 10^{-6}$
1990	$190 \pm 5$	$0.0648 \pm 0.0001$	$-(340 \pm 8) \times 10^{-6}$
1980	$190 \pm 7$	$0.0648 \pm 0.0001$	$-(340 \pm 12) \times 10^{-6}$
1970	$186 \pm 11$	$0.0650 \pm 0.0001$	$-(350 \pm 20) \times 10^{-6}$
1960	$172 \pm 17$	$0.0662 \pm 0.0001$	$-(380 \pm 36) \times 10^{-6}$
1950	$158 \pm 36$	$0.0669 \pm 0.0003$	$-(420 \pm 95) \times 10^{-6}$

Table 3: Logistic function linearization least-squares fit parameters. Years represent the final year of the data set used for the fit, with the initial year being 1860. The intercept represents the initial growth rate  $b$ .

(87,095 Mbpd). Of this amount, 8,570 Mbpd is accounted for by natural gas liquids; due to its different production source, we will consider natural gas liquids as part of the total mix of liquid fuels, but based solely on projections for future production increasing at an exponential rate. Another 2,300 Mbpd of the world oil supply comes from refinery processing gains; again, this will not be considered further. The category “other liquids” makes up 2100 Mbpd; this quantity consists of biofuels and liquids from coal; the latter are not included in our analysis, while biofuels will be treated in the same manner as natural gas liquids, based on published projections. In other words, we look at world petroleum supplies, broken into two regions, and then consider the influence of future and non-conventional petroleum sources such as unproduced Arctic and deep-sea oil, enhanced oil recovery (EOR), Canadian oil sands, and oil shales (kerogen, not to be confused with conventional oil recovered from shale source rock through hydraulic fracturing). Of the remaining 74,100 Mbpd (27.0 billion barrels per year, Gb/a) of production, 14.9 Gb/a comes from what we call “Rest of the world” (ROW), 11.6 Gb/a is from “OPEC” and 0.5 Gb/a comes from Canadian oil sands. These three regions/sources form the starting point for our further analysis.

The particular choice resource division is made to enable comparisons to one of the standard annual compilations of data on oil production, the World Energy Outlook (WEO) published by the International Energy Agency (IEA) (IEA, 2008). In that work supplies are divided into several categories of both current and future production; biofuels and natural gas liquids are not part of that mix. In Fig. 2 a schematic supply curve for both conventional and non-conventional oil is shown, adapted from Ref. (IEA, 2008). The presentation of resources in Fig. 2 might at first lead one to expect that oil resources such as tar sands, for example, would not be produced until roughly 4000 Gb of total cumulative production. At present, cumulative production is slightly more than 1150 Gb, and enhanced oil recovery (EOR), oil sands mining, coal-to-liquids (CTL), gas-to-liquids (GTL) and even oil shale (kerogen) are all either in production or have been very recently under serious consideration. We will return to this figure in Section 4.



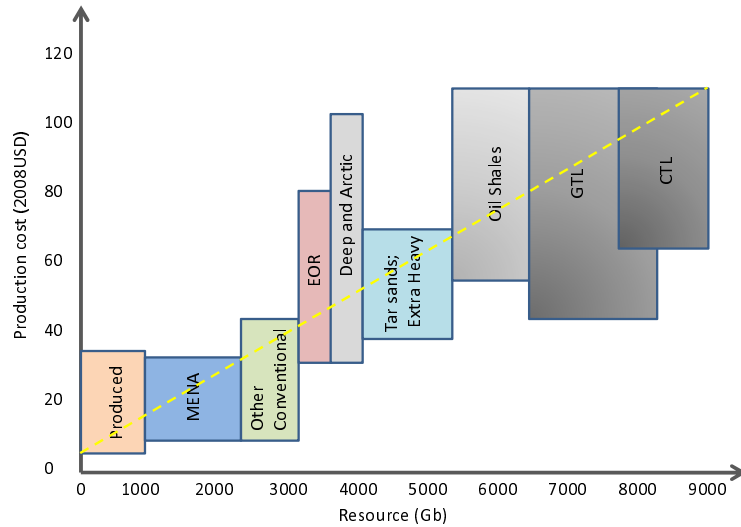


Figure 2: Estimated resource base of various types of conventional and non-conventional petroleum. For each separate resource, an estimate is shown of the range of extraction costs, in current dollars, along with the recoverable resource in billions of barrels. The abbreviations are: MENA - Middle East/North Africa; EOR - Enhanced Oil Recovery (with CO<sub>2</sub> and other methods); GTL - Gas-to-Liquids; CTL - Coal-to-liquids. (After Ref. (IEA, 2008))

We now make use of the logistic function description of past oil production to make projections about future production. Data for past production come from three principle sources: the American Petroleum Institute (API, 1959), Energy Information Administration (EIA, 2011), and BP (BP, 2010). We extract from past production data three separate categories of petroleum (OPEC conventional, Rest of the World conventional, Tar sands). Since there are no commercial mining projects for oil shales at present, this technology is considered as one for the future. In addition, for this model we treat Arctic and EOR as future technologies as well. Although the Alaskan North Slope has been producing oil for decades, and some EOR projects are in production, the potential referred to by IEA in the 2008 World Energy Outlook is currently classified as a potential resources; therefore, any current and past production for these resources are combined with the ROW category. Data for Canadian oil sands production is taken from data published by the Canadian Association of Petroleum Producers (CAPP, 2010). Finally, from EIA we obtain crude oil plus lease condensate data.

Oil production regions are disaggregated only minimally. Multiple logistic curves have been implemented previously (Nashawi et al., 2010) but without a clear delineation of how to determine the appropriate number of curves to use; the closest approach to the one taken here is found in (Gallagher, 2010) We take two country groupings into consideration: OPEC countries and the rest of the world (ROW). With this *ansatz* we proceed to use the logistic yearly-production function to fit ROW production. Resulting parameters for the fit

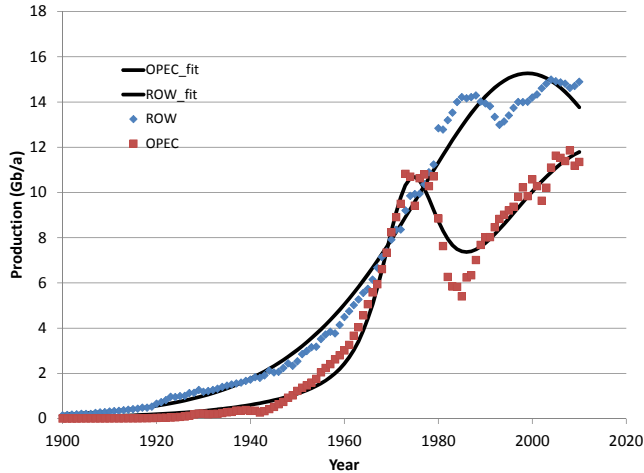


Figure 3: World conventional oil production, separated into two regions: Organization of Petroleum Exporting Countries (OPEC) and the Rest of the World (ROW). A logistic function is fit to the yearly production data for ROW (data: diamonds; fit: solid line), and a double-logistic function is used to describe the production of OPEC (data: squares; fit: solid line); parameters from the fits are given in the text.

are [ $Q_\infty = 1035 \pm 36$  Gb,  $b = 0.059 \pm 0.002$  yr $^{-1}$ ,  $t_p = 1999 \pm 1$ ;  $R^2 = 0.991$ .] The data and the fit are shown in Fig. 3. The production data for OPEC show a significant decline in the aftermath of the oil embargo and energy crises of the 1970s. To simulate the historical record, we will fit the data for OPEC oil production to a double logistic function, with results also shown in Fig. 3. For this fit to the data we find two values each for  $Q_\infty$ ,  $b$  and  $t_p$ , those values being ( $102 \pm 12$  Gb,  $0.27 \pm 0.03$  yr $^{-1}$ ,  $1974 \pm 1$ ) and ( $830 \pm 120$  Gb,  $0.058 \pm 0.005$  yr $^{-1}$ ,  $2015 \pm 4$ ), with  $R^2 = 0.986$ . It should be noted that the logistic curve cannot be expected *a priori* to reproduce the discontinuities seen in historical production due to political events. In effect the function will represent the available capacity for production, which might fall below its maximum level at any given time, as discussed in more detail by other authors (Reynolds and Baek, 2011). Here, we use a double-logistic as a means of improving the estimate of ultimately recoverable oil given past disruptions.

We also fit an exponential growth curve to production data for Canadian oil (Tar) sands, based on data from the Canadian Association of Petroleum Producers (CAPP, 2010). For oil sands, the logistic curve is not used, assuming that production is not near a peak, and therefore still in the early growth phase. For data covering the past 30 years, the growth rate is  $0.078$  yr $^{-1}$ , with projections from CAPP for slower growth rates in the future to 2030, which will be useful as a guideline in what follows.

With these data as a starting point, two separate analyses are carried out. In the first, a deterministic model is used to reproduce historical production patterns using a combination of logistic curves for conventional oil and for enhanced oil recovery (EOR), deep-sea and Arctic oil, heavy oil and tar sands, and

for shale oil. Industry and historical data (API, 1959; CAPP, 2010) show that early growth rates of new petroleum sources (aggregated by type or region) are below 10%; we use this stylized fact as an input to the logistic function model, along with estimates for resource amounts. The second analysis is an optimization problem for minimizing overall production costs during the lifetime of the production cycle, constrained by the actual production data history. The same regions and oil types are used as above. Production estimates for the next half-century are made from these models, based on the fact that the logistic curves, once the input parameters are determined from past production, provide a deterministic path going forward. In addition, we include as fixed quantities the production of natural gas liquids and of biofuels, as projected to 2030 (BP, 2012) and then extended at the same exponential rate to 2050. Under these assumptions, production of natural gas liquids increases from the current level of 3.1 Gb/a to 6.6 Gb/a in 2050 (2.0%/year); biofuels production increases from 0.8 Gb/a to 5.0 Gb/a in 2050 (5.0%/year).

Since it was determined above that fitting logistic curves to the yearly production data is marginally better than using other forms, this same procedure is carried out for world production, assuming the double-logistic function for OPEC production. For the rest of the world (ROW) the data are fit to a single logistic-derived function. The total URR of conventional oil, including EOR and Arctic resources, is approximately 2500 Gb, which lies between the USGS 95% and 50% probability estimates.

### 3. Deterministic Model

The first model uses the logistic fits obtained in the previous section, together with data for non-conventional and other resources, to project potential future production. Fig. 4 shows the two main resources, crude oil from “OPEC” and from “ROW” starting out individually with an exponentially growing yearly output, which then slows and begins to decline, in accord with their respective total resource amount. Other resources are included in Fig. 4 as well, each showing its own logistic production profile. Table 4 gives the data used for total resources, growth rates, and peak dates for the curves in this model, including the alternative cases to be discussed below. Current world production of conventional oil, including enhanced recovery, polar and deep water resources, has remained flat at 27 Gb/a from 2005-2011.

Two main points characterize the production curves shown in Fig. 4. First, if one assumes the logistic curve parameters found from the nonlinear least-squares fit, then future production is determined from the initial production pattern. For the curves shown in Fig. 4, the modeled yearly production quantities agree with actual data to within 10% over most of the past century. Second, given that new sources of petroleum, as evidenced by both past conventional production and current growth in oil sand production, do not grow during the exponential phase at more than 6%-8% per year, production curves such as the ones shown in Fig. 4 will result. Although there may be large conventional and non-conventional resources available “in the ground”, the logistic model indicates that rates of

Variable	ROW	OPEC1	OPEC2	Arctic	EOR	Tar	Shale
URR (Gb)	1035	102	830	200	400	1000	1000
Baseline: $b$ (%/yr)	0.059	0.27	0.058	0.06	0.06	0.06	0.06
$t_p$ (year)	1999	1974	2015	2100	2100	2085	2125
Fast growth, (%/yr)	0.059	0.27	0.058	0.10	0.10	0.10	0.10
$t_p$ (year)	1999	1974	2015	2075	2075	2060	2100
High URR (Gb)	1350	102	1070	200	400	1000	1000
$t_p$ (year)	2009	1974	2025	2100	2100	2085	2125
Cost <sub>min</sub> (2007\$)	5	5	5	30	30	35	50
Cost <sub>max</sub> (2007\$)	40	30	30	100	80	65	110

Table 4: Input parameters for the multiple logistic curves in the deterministic model. Not included in the table are the data for natural gas liquids and biofuels

production increase, combined with depletion of conventional reserves, are not sufficient to prevent a long period ( $\sim$  two decades) of supply constraints.

The logistic production function for non-conventional resources depends on all three parameters,  $Q_\infty$ ,  $b$ , and  $t_p$ . To test the sensitivity to both fitting and free parameters, we examine two additional cases. First, we keep the same URR in accord with IEA estimates and the logistic curve fits, but change the growth rates to 10%/yr for all future resources (EOR, Arctic, oil shale) and for oil sands. The peak year is chosen to ensure that production from these sources either matches current production (oil sands) or is negligible at present, corresponding to the fact that the other resources are not currently produced commercially. The results for the production curves are shown in Fig. 5a. Production does increase starting in about 2030, after two decades of either declining production or slow growth, but one would have to construct a scenario in which 20 years of flat or decreasing production would not lead to dramatic changes to the transportation sector away from just these liquid fuels. Furthermore, it must be realized that the scenario presented here results in yearly production of oil sands in 2035, for example, that is three to four times that projected by the industry itself (CAPP, 2012).

As a second sensitivity test, we increase the estimated URR for conventional resources by 30%, reverting to the growth rates for future resources used in the baseline case. Fig. 4b shows the production curves in this case - production of all liquid fuels remains essentially flat through out the time horizon to 2050, after which it will again increase. Given the close correlations between oil consumption and economic growth seen historically, a decrease in oil supplies over such a long period could imply potentially severe consequences for the world economy (Kaufmann et al., 2011; Hamilton, 2009).

Even in the “optimistic” case shown in Fig. 5a, the currently observed plateau in world oil production is reproduced EIA (2011). (Since 2005, world total liquids production has increased only slightly, and conventional crude oil

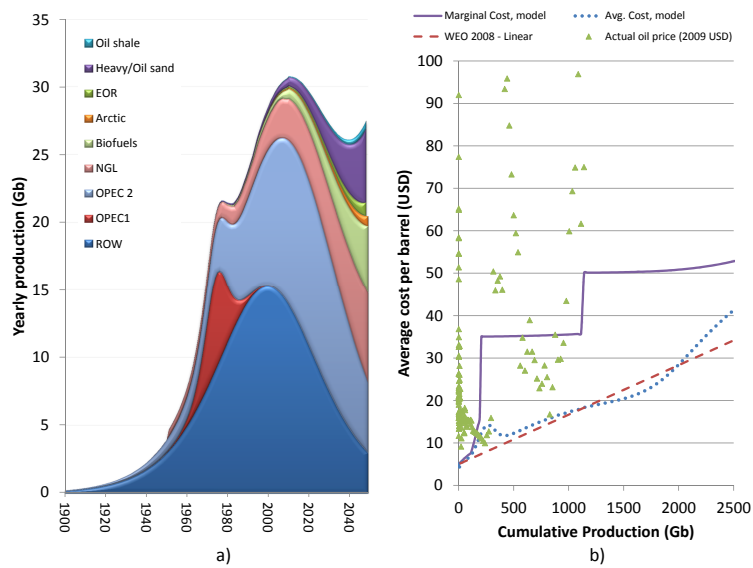


Figure 4: Yearly historical and potential future production (Gb/a) from multiple logistic curves for conventional and nonconventional oil resources. Logistic curves are generated to closely match historical data, then projected into the future. Parameters for resource amounts are given in the text and in Table 4. a) Rates of growth for all future resources are 6%/year for the baseline case. b) Marginal (solid line) and average (dotted line) cost of oil, together with oil price data (triangles) and a constant linear increase (dashed line) as described in the text.

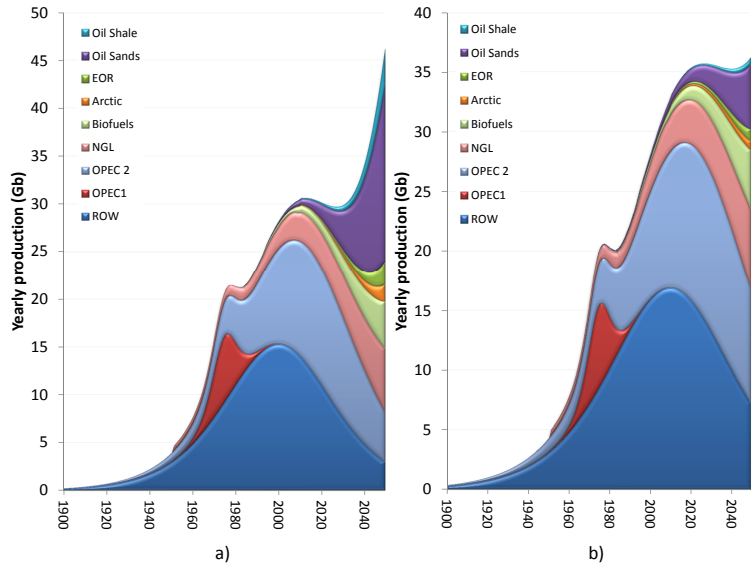


Figure 5: Yearly historical and potential future production (Gb/a) from multiple logistic curves for all resources a) Rates of growth for all future resources are 10%/year. b) Enhanced URR, growth rates for all future resources at 6%/year.

production has remained flat.) The declines or plateaus seen in total production extends for 20-40 years in these model result. Production weakness is not a sign of an absolute peak in resource availability, but rather one of rates of extraction of those resources. The question remains as to whether, after a long period of production decline and price increases, there would be substitution possibilities that would then obviate the need for the nonconventional resources in the long-term. In the slow-growth case, the decline, although once again not absolute, is even deeper and lasts longer.

We now turn to estimates of marginal and average extraction costs, again using this multiple-logistic curve model as a starting point.

#### 4. Marginal Extraction Costs

Referring to Fig. 2 with current cumulative production of conventional oil being 1100 Gb, one might expect a marginal barrel cost of under \$20. We can utilize the estimates made by IEA for the range of marginal extraction costs across a given resource type to construct a more realistic extraction cost curve for the production history observed thus far. We assume a linear increase between minimum and maximum extraction costs (given in constant 2008USD) from beginning to end of extraction for each individual resource type or region. Thus, for example, oil sands, with a total resource of 1000 Gb, are estimated to have extraction costs ranging from \$ 35/barrel to \$ 65/barrel. There is a strong overlap between resources with differing marginal extraction costs; if world oil markets were maximally efficient, and if the given data from IEA were known to be absolutely accurate, then the cheapest resources would be used first, followed by the next most expensive, etc. Furthermore, for this efficient path to be followed, it would be necessary for a producer to use a given, less-expensive, resource in a very unrealistic way, extracting at a nearly constant rate up until complete exhaustion of the resource. However, since none of these conditions hold, and most importantly, because we are postulating a logistic production trajectory for each resource type or region, it is clear that some more expensive resources will necessarily be tapped before the less-expensive ones are fully depleted. This dynamic leads to increases in marginal extraction costs at earlier times (or quantities of cumulative resource extraction) than might otherwise be expected.

In Fig. 4b we show the marginal extraction cost curve for the production curves in Fig. 4a. As described qualitatively above, there are discontinuous jumps in the marginal extraction cost each time a new, more expensive resource begins production. Since the logistic curve itself never goes absolutely to zero, we set a threshold of 0.1 Gb/yr at which the given resource contributes to the marginal cost curve. For example, one might consider the earlier, very low levels of production of a resource as a research phase, such that costs per barrel as related to actual market prices play a secondary role. With that caveat, to which changes the form of the extraction cost curve is insensitive, we see that the difference between the average cost per barrel produced and the extraction cost of the most expensive barrel produced, can be quite significant. The average

cost is calculated by taking the marginal cost per barrel for each type multiplied by the production of that type, and divided by the total production for the given year. For the range over which historical production is important, average costs per barrel follow rather closely the linear function shown in Fig. 2. As a point of reference, we have also included in Fig. 4b data points for actual *prices* of oil (BP, 2010) in real dollars. We see that some key features of the actual petroleum system are indeed captured here; for example, although average extraction costs remain relatively low, to satisfy demand some more expensive resources must be tapped even at relatively early stages of the production history. In Fig. 4b, although the extreme spikes in prices are not fully captured since there are many factors beyond extraction costs that contribute to final market prices, there is a rough correspondence to the historically observed jump to a higher cost level starting in the 1970s at about 500 Gb of cumulative extraction, with another jump occurring at around 1000 Gb, depending on whether slow or fast growth rates are considered.

To briefly summarize the results thus far, a model of logistic growth patterns for different conventional and non-conventional oil resources can reproduce the observed pattern of past oil consumption. Growth rates for new resources and technologies are observed to be less than 10%/year; using this observation of historical trends as a constraining input to the deterministic model, the growth in production of nonconventional resources is not fast enough to make up for a decline in conventional oil production. The net result is a (temporary) decline in overall production that might last for decades; it is difficult to imagine that a new surge in production illustrated in Fig. 5a would actually take place, since substitution possibilities would have been developed in the intervening period. Finally, marginal costs of extraction, and therefore market prices, can be shown to rise significantly more rapidly than average extraction costs due to the necessity for pulling forward more expensive resources, even if there are remaining reserves that are less expensive to extract.

## 5. Optimization Model

Our second model for investigating production limits in the petroleum system takes a different approach, by posing a question of optimization over time for production of different resources. A motivation for this approach, although idealized, is that the role of the IEA is exactly that of providing forward-looking information to member countries (and thus providing by default equivalent information to anyone accessing their reports) about the characteristics of the energy system. The procedure used for the results presented in this Section is based on the same data from the IEA used in Sec. 3 and posits an efficient use of what is taken to be reliable data. An optimization algorithm was programmed using the General Algebraic Modeling System (GAMS) to minimize the total cost of production over time (1860-2010) using different amounts of oil from various sources. A constraint is imposed, namely that of the logistic curve for cumulative production, such that the yearly production starts at an exponential rate, peaks, then falls toward zero again as the URR is reached. Again, since

the logistic curve has three free parameters, an examination of historical production records can help restrict the range of some parameters. As discussed at length in Sec. 2, logistic and exponential fits to data for conventional oil and oil sands production, respectively, give a robust starting point for understanding the dynamics of the system. For resources that have yet to be produced, we use IEA data as a guideline.

The general procedure used for the optimization was as follows: the URR for each resource was set to a fixed value, based on parameter fits to past production and IEA estimates. Sensitivity was tested by choosing different URR values. The parameter for initial exponential growth,  $b$ , was a variable to be determined through the optimization process. A delay was introduced for all resources except “ROW” to ensure that, as observed historically, resource extraction starts in the proper sequence. Using the marginal costs for each resource, based on the IEA estimates, the optimization searches for a path through time such that total costs of extraction over the whole time period of 1860 - 2010 (or 2030) are minimized. We also varied the functional dependence of marginal extraction cost on cumulative production, using either a linear increase or a quartic dependence to simulate a rather sudden increase in costs after a gradual initial increase.

One additional constraint imposed on the solution was that the production capacity (sum of all the Hubbert curves) was necessarily larger than the observed production for each year in the historical record. Effectively, this model aims to merge the ideas of the purely geological version of the Hubbert curve with a possible economic optimization scheme, using historical data as a guideline. The optimization finds the minimized extraction costs over time, and the resulting combination of Hubbert curves can be compared to historical data.

Results from a baseline case, along with a comparison to production curves and marginal extraction cost curves, will be the focus of this section. Additional questions may be addressed through this model as sensitivity analyses. The role played by a lack of foresight can be seen by looking at two cases, one for which projections of production by IEA into the future are taken as a “given”, and a second one for which only production up to the present is used in the optimization routine. We also investigate the effect on production if there is an overestimate of actual OPEC resources, as many in the “peak oil” community claim.

### *5.1. Large Resource Base*

We first present results from our baseline case, for which we assume URR and extraction-cost ranges as shown in Table 5. Values for the URR are taken from the least-squares fits and are identical to those used for the deterministic model in Sec. 3. For this trial we use both past production and projections for future production to 2030. This experiment represents an optimization for world oil producers of different resources who know not only past oil production patterns, but also plan as if they knew future demand (from IEA data and projections) and can estimate the extraction costs for both conventional and non-conventional reserves and resources. The minimization routine for total



extraction cost over the whole time range 1861 - 2030 yields the growth-rate parameter for each resource, with output parameters from the optimization shown in the last row of Table 5. In Fig. 6a are shown the yearly production curves for the period used for the optimization. Using the parameters from the optimization, the logistic curves are determined for the future as well; the extrapolation of the logistic production curves to 2100 is included in this figure.

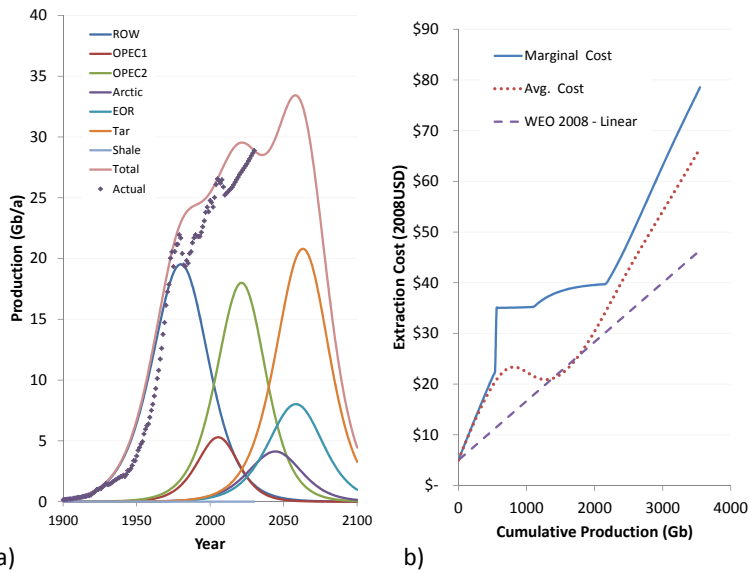
Variable	ROW	OPEC1	OPEC2	Arctic	EOR	Tar	Shale
URR (Gb)	1035	102	830	200	400	1000	1000
Cost <sub>min</sub> (2007\$)	5	5	5	20	30	35	50
Cost <sub>max</sub> (2007\$)	40	30	30	100	80	65	110
Growth rate	0.077	0.100	0.092	0.083	0.088	0.075	0

Table 5: Input and output parameters for the “Large OPEC resource base” experiment.

There are several interesting points to note in these results. It is clear that the heavy reliance on the relatively inexpensive ROW and OPEC resources leads to a delay in exploitation of the non-conventional and more expensive resources. The result is that total world oil production, while reaching a maximum around the middle of the century, shows a lower local maximum around 2020; even the growth of three different alternative sources of oil, i.e. from the Arctic, from Enhanced Oil Recovery and from oil sands, cannot completely and in a timely fashion compensate for the decline in conventional oil production. This dynamic is very reminiscent of the actual history of oil production in the United States, where even the rapid growth of Alaskan North Slope production in the 1970s could not do more than partially compensate for the decline in production from the continental U.S.

Three results from this optimization process reflect aspects of observed dynamics in the petroleum system. First, the optimized production rates are all between 6% and 10%, as seen in reality. There is no *a priori* reason that the optimization routine should have settled on values in this range, as the upper limit was not specified. Second, the delays that come about from optimizing overall system costs, together with the modest production growth rates, can lead to periods of time at which production actually decreases. This will be discussed further in Sec. 6. In addition, these curves also reflect the currently observed trend toward OPEC becoming ever more dominant in total oil production in the future. Finally, in Fig. 6a we note those points in time at which actual consumption meets the multi-logistic curve envelope; these are points in time at which very high oil prices were experienced. For an interpretation of this point, see Ref. (Reynolds and Baek, 2011)

Fig. 6b illustrates a marginal extraction cost curve corresponding to the production path shown in Fig. 6a. The dashed line is again an estimated curve based on Ref. (IEA, 2008). As described above in Sec. 2, we assume a linear increase of extraction cost across each resource, starting from cost<sub>min</sub> as given in Fig. 2, and increasing to cost<sub>max</sub>. By taking the maximum-cost resource



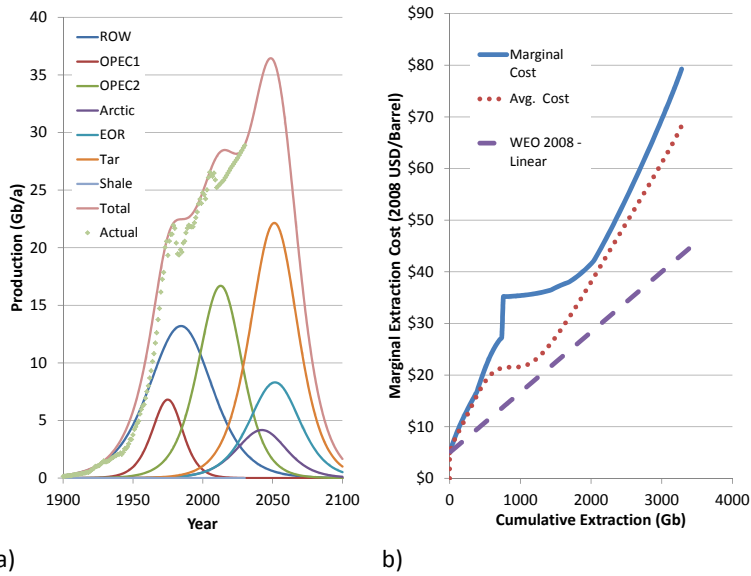
a) Yearly production of conventional and non-conventional sources of oil with minimized production costs based on imputed resource size from logistic fits. Parameters for the logistic curves are as given in Table 5. The time period for production is extended to 2100 using the logistic curve growth parameters found through the minimization routine, where the necessary supply is assumed known until 2030. b) Marginal and average extraction costs for conventional and non-conventional oil production, as a function of cumulative production in Gb. Upper curve is the marginal cost per barrel; discontinuities represent new, more expensive resources undergoing production. Lower, dashed curve is the IEA projected marginal cost, as constructed from the curve in Fig. 2. Dotted curve is the average extraction cost per barrel, calculated from the logistic-curve production amounts of each resource, together with their respective marginal extraction costs.

“produced” by the model at each time step, we construct the curve in Fig. 6b. From the total amount produced for each resource, along with the given marginal extraction cost at that time step, an average cost per barrel can be calculated as well. Once again we see that as a new, more expensive, resource is tapped, and given the pre-condition that this marginal barrel will be needed to satisfy demand, the implication is that one can expect to see prices rise, and not necessarily in a slow, smooth manner. Such discontinuities would likely be smoothed in reality, but the general feature supports our hypothesis that the average cost of extraction, or assurances that cheap deposits of oil are still available, will be an unreliable indicator of relative scarcity, since one would expect that the marginal barrel cost sets the market price, as determined by demand and supply availability from the various resources. In the marginal cost curve as shown in Fig. 6 we also observe the feature that there is a time period during which average costs of extraction actually decrease, whereas marginal costs are increasing. This dynamic is the result of the competition between increased production of relatively inexpensive “OPEC oil, while at the same time there is a need for additional smaller quantities of oil from more expensive sources.

The results of this trial, using the URR values found from fits to actual production curves, lead to some overall features that do not correspond well with historical observations. For example, OPEC production as a fraction of total world production is too low for the entire historical period to date. The interpretation of this result, within the framework of our modeled system, is that a minimal total extraction cost over time would have been to produce more of the ROW resource, then OPEC, followed by production of the more difficult and expensive resources such as EOR, Arctic and Tar. In this trial, and given the limited “knowledge” to 2030, the oil shale resource is not tapped at all; one would argue that, as the overall peak in production is neared sometime before 2070, that additional resource would come into production as well, all else being equal. Not surprisingly, the optimum over time was not followed, but part of the issue is that the world started out inefficiently, by not having discovered the cheapest Middle East resources first.

### 5.2. Lower Resource Base

Another experiment has the same  $\text{cost}_{\min}$  and  $\text{cost}_{\max}$  parameters as the previous one, but a lower URR for both ROW and OPEC, 800 Gb and 900 Gb, respectively. All parameters, along with the resulting logistic growth rates found from the minimization routine, are shown in Table 6. From Fig. 7a we see that the peak in total world production of oil occurs slightly earlier compared to the previous example, and occurs at a higher yearly production level. However, the smaller resource of less-expensive, ROW and OPEC conventional oil is depleted more quickly, and in an optimization routine with foresight, producers know that they will have to start extracting oil from more-expensive and non-conventional resources at an earlier point in time than they would have to do if they could rely on a large, inexpensive OPEC resource base.



a) b)

Figure 7: a) Yearly production of conventional and non-conventional sources of oil with minimized production costs necessary to ensure observed supply patterns in the past. Parameters as in Fig. 6, except that the URR for ROW and OPEC are assumed to be 800 Gb and 900 Gb, respectively. b) Marginal and average extraction costs for conventional and non-conventional oil production, as a function of cumulative production in Gb. Upper curve is the marginal cost per barrel; discontinuities represent new, more expensive resources undergoing production. Lower, dashed curve is the IEA projected marginal cost, as constructed from the curve in Fig. 2. Dotted curve is the average extraction cost per barrel, calculated from the logistic-curve production amounts of each resource, together with their respective marginal extraction costs.

Variable	ROW	OPEC1	OPEC2	Arctic	EOR	Tar	Shale
URR (Gb)	800	200	700	200	400	1000	1000
Cost <sub>min</sub> (2007\$)	5	5	5	20	30	35	50
Cost <sub>max</sub> (2007\$)	40	30	30	100	80	65	110
Growth rate	0.066	0.136	0.095	0.084	0.083	0.089	0

Table 6: Input and output parameters for the “Low Resource Base” experiment.

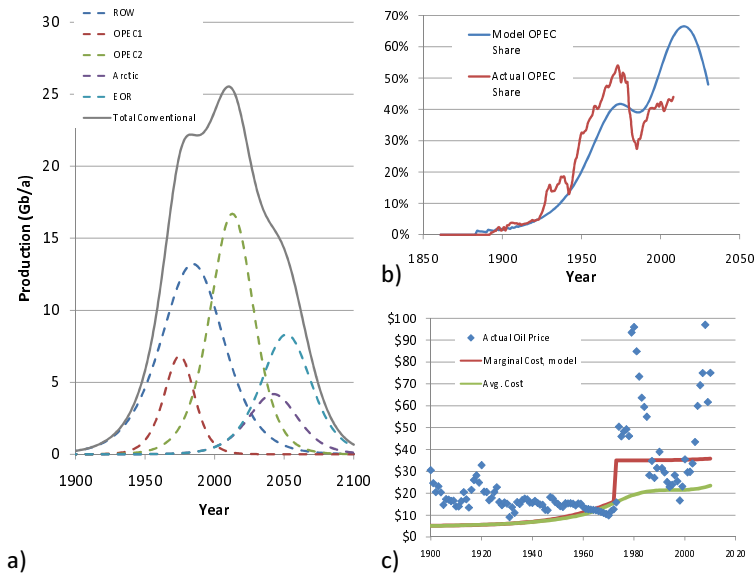


Figure 8: a) Plot of yearly production of conventional sources of oil only. Parameters as in Fig. 7, with the URR for ROW and OPEC are assumed to be 800 Gb and 900 Gb, respectively. b) Fraction of world oil production due to OPEC, as a function of time. The model with lower total URR reproduces the actual history in a broad sense. c) Actual oil price, as well as marginal and average costs, as calculated from the optimization model.

The extraction-cost curves shown in Fig. 7b show roughly the same characteristics discussed in Section 5.1, with overall higher costs determined from this minimization experiment. This result is not surprising, given both that the OPEC resource is smaller, and therefore reaches the higher end of its cost spectrum earlier, and that additional, higher-cost resources are tapped earlier.

As was the case for the deterministic model of production, a temporary decrease in production capacity is in evidence in the output from the optimization model. However, actual production does not necessarily decrease, as seen in Fig. 7. Note, however, that this scenario implies that oil producing entities take seriously the long-term projections for demand, that investments in infrastructure are made in a timely fashion, and that optimistic assumptions are made about the ability to scale-up production of non-conventional oil and of resources that require more advanced extraction technology. These are essentially the warn-

ings that have been given by the IEA over the past few years in various editions of the World Energy Outlook (IEA, 2009).

In Fig. 8a the production of conventional oil as a function of time is shown, including the Arctic and EOR components. The results of the optimization approach serve to illustrate one of the chief characteristics of the peak oil concept, namely, that the ability of technology and new discoveries to make up for current supplies of inexpensive oil is limited. Fig. 8a shows that future Arctic resources to be developed, along with advances in extraction technology as represented by Enhanced Oil Recovery, cannot make up for declines in other resources. This result holds although those new resources represent a 30% increase in the ultimate recoverable resource of conventional oil. The production of new resources proceeds at an exponential ramp-up rate, as matched by historical experience, that is too slow to make up for the late start of production. The late starting point is determined in turn by the relatively high abundance of inexpensive, first-choice resources.

Although this trial assumes a smaller resource base for conventional oil from ROW and OPEC than expected from our fit to data, results for the expected production history from the cost optimization routine match the historical record better than did the results from Sec. 5.1. If the totals from Arctic and EOR are included, the total of conventional oil resources still falls within the USGS estimated range of 2000 - 3000 Gb. In Fig. 8b we show the actual and model-calculated results for the fraction of OPEC production in total world oil production. This result reproduces, in broad outline, the increase, relative decrease, and projected future increase, of OPEC shares. The sharp decrease seen in the historical record due to the politically-induced disruptions of the 1970s, together with the resulting change in efficiency and consumption patterns among the largest oil-consumers, are not reproduced in complete detail, as might be expected from such a simple model.

Finally, Fig. 8c shows the development of the marginal and average extraction costs, calculated from the model, compared to actual oil price, as a function of time. It is interesting to note that world oil *prices* do appear to show almost discontinuous changes between price regimes, accompanied by a strong overshoot. Prior to the early 1970s, oil prices were consistently within a band between \$10 - \$20/barrel; during the 1990s, after the decrease from high prices, the range was \$20-\$30/barrel, and for the past four years, prices have been mainly within the range of \$80 - \$100/barrel.

### 5.3. Additional Parameters

Additional experiments were carried out using the same  $\text{cost}_{\min}$  and  $\text{cost}_{\max}$  parameters but changed the cost increase structure from a linear dependence on cumulative production relative to URR, to the fourth power or to a quadratic dependence. The results are very similar to those already shown. The effect of using a quartic dependence of extraction cost on cumulative production might be expected to mimic a lack of concern for future supplies, due to the initial slow rise of the function. This is equivalent to using increases in price as a sign of impending scarcity, but being mislead because a relatively flat price

trajectory over time gives way to a steep increase as the fraction extraction of the URR becomes significant. Overall, however, our experiments show that the URR itself has the biggest effect on changing output results; to a somewhat lesser extent, the  $\text{cost}_{\text{max}}$  and  $\text{cost}_{\text{min}}$  parameters also influence the timing and production amounts of various resources.

Another set of experiments examines how limited foresight might affect production histories into the future. Assuming the full IEA URR numbers and using only production data to the present (*i.e.* not the forecasts for future production to 2030), leads to an optimization that disregards entirely any need for EOR, oil sands, or shale oil, and has only a very minor contribution from Arctic oil. In other words, in the unrealistic case that producers were not looking to the future, reliance solely on current reserves would not be adequate to avoid a supply-side bottleneck in the relatively near future.

## 6. Discussion and Conclusion

The goal of this work is to combine a Hubbert/logistic curve analysis of oil production with some historically observed data indicating limits on flow rates and infrastructure build-up. Together with resource data for unconventional oil we determine that over a wide range of parameters there will likely be difficulty in maintaining oil supply growth, thereby leading to a potential mismatch between supply and expected demand. In addition, we show that a smooth, logistic-type extraction sequence of different resources necessarily implies early commencement of extraction of expensive resources, even when large amounts of the cheaper resource are still in the ground. This dynamic results in costs (and thus, prices) climbing more rapidly than might be indicated by a simple examination of the size of the cheap resource available.

We have not attempted a detailed economic model of supply and demand. Recently, Benes et al. combined a Hubbert linearization model with economic factors

A common opposing view to that of the logistic-curve analysis of peak oil is that economic forces will ensure that as prices for oil increase, there will be greater incentives to explore for and produce oil, be it from conventional or non-conventional sources. Effectively, the law of supply will automatically lead to sufficient production of conventional oil to satisfy demand at some market-clearing price. Alternatively, given the need for liquid fuels for the current configuration of the world's transportation system, as costs of production for conventional oil rise, substitutes will be found in the form of non-conventional resources such as tar sands, shale oil, biofuels and liquids from coal and natural gas. However, the critical point is that the time scales necessary for increasing production of alternative fuels to levels of consequence with respect to world demand leads to a potential for lack of supply in intermediate periods leading to high prices and then to significant economic disruptions that would induce further dynamics not present in the models presented here.

Two recent papers by Reynolds are of interest with respect to our own work. (Reynolds, 2002) uses the logistic curve to model a multi-variate, non-time-

series version of discovery to take into account economic and technology factors in oil discovery. His conclusion is that cumulative production is the key driving variable, and not oil price, *i.e.* that economic factors play a minor role. This result lends support to our approach.

In a second paper, Reynolds (Reynolds, 1999) constructs a stylized model of a mineral market economy that attempts to capture the basic features observed in real fossil-fuel markets. He finds a U-shaped price (or cost) trajectory, with decreasing costs as technology improves and while the finite nature of the resource is relatively unimportant, followed by sharply increasing costs when scarcity effects begin to dominate. Again, this type of analysis, carried out for one aggregated resource, forms a conceptual basis for our more detailed look at different resources.

One can, however, combine “geology”, in the form of the estimated resources for different regions and liquid fuel sources, along with “economics” in the form of estimated marginal costs of production for those resources through their respective production histories. With this combination one can construct an extension of the logistic curve story that provides more useful information about potential future production pathways, as we have demonstrated above. The minimization of production costs over time, taken as an input to the model, can reproduce observed production patterns reasonably well.

The marginal cost curves found here differ qualitatively from those that would result from extraction according to a Herfindahl sequence (Herfindahl, 1967). The logistic curve dynamic forces production from successively more expensive alternative resources earlier than one would expect from the box diagram shown in Fig. 2. This dynamic then necessarily leads to a much quicker rise in oil price as well, since the price will rise as demand keeps rising and the new, expensive resources are tapped, even as production continues from the older resource base. Rather than a concave-upward curve indicating slow cost increases into the future, the results here indicate concave downward marginal cost curves that often increase more quickly at the beginning, then more slowly later in the total production cycle. Not only does this reflect what is currently observed, it is what one would intuitively expect from the combined logistic growth pattern.

A comparison with recent data from the U.S. Energy Information Administration (EIA, 2010) indicates that the results presented here for the effects of marginal extraction costs may be very conservative. In that report the total upstream costs (finding and lifting) for oil and gas are seen to have increased dramatically in the past decade. Average worldwide costs for reporting companies have been around \$30/ barrel of oil equivalent; more importantly, marginal costs have been in the \$60-80 range, far higher than projected by the IEA data we have used.

Knowledge of the size of the resource base is a crucial input parameter to this model, as shown especially in the comparisons made in Sections 5.1 and 5.2. Again, those results show that knowledge of the size of the available inexpensive resource is a determining factor for the timing of a future peak in world oil production, as well as for the maximum total capacity of conventional plus non-



conventional sources of production. This conclusion is essentially independent of the size of the “backstop” resources; if the extraction of these alternatives is not begun soon enough, they will not be producing at quantities large enough to make up for the decline in conventional resources. It is essentially this point that has been made very recently by representatives of the IEA, most formally in Ref. (IEA, 2008). Paradoxically, when oil prices are relatively low, there is a disincentive for producers, private or state-owned, to develop more expensive resource bases. Price increases may come relatively suddenly, as has been seen in the recent past.

From the models presented in this paper one may conclude that i) although the total resource base for conventional and non-conventional oil is large, production patterns show that there will likely be a maximum ultimately recoverable quantity, or effective resource, that is significantly smaller than the resource-in-place; ii) limited production growth rates for alternative conventional and for non-conventional resources lead to the necessity of early commencement of production for these resources, well before the conventional resource is exhausted; iii) due to the need for early production of non-conventional resources with relatively high extraction costs, the overall marginal extraction cost curve for oil tends to rise much more rapidly than average production costs, with the consequence under an assumption of functioning markets that the price of oil rises in excess of what might be expected on the basis of data indicating very large remaining conventional resources. These results are consistent with the conclusions of other work, namely, that new liquid-fuel resources are unlikely to play a major role in supplanting conventional oil in the short-term (Hirsch et al., 2005; Soederbergh et al., 2007; Kaufmann and Shiers, 2008; de Castro et al., 2009). Because of the great importance of oil to our economic system, potential limits imposed by geological factors must be anticipated well in advance of becoming critical if we are to be able to react effectively to increasing scarcity, without experiencing economic distress (Hirsch, 2008; Hamilton, 2009; Kaufmann et al., 2011; Lutz et al., 2012).

Although the models presented here are very simple, they can reproduce the broad outlines of past oil production history and provide clues as to the future of oil production. By including some basic cost and economic optimization characteristics, these models help form a bridge between the “geology and “economic arguments about peak oil production. A comparison of model output to historical extraction data and technology development trends, along with an apparent ability of the models to roughly simulate the limited planning horizon of capacity expansion, leads to a conclusion that one may posit an “effective peak oil production in the near future, while realizing that the peak is not a measure of absolute scarcity of resources in the ground.

RJB would like to acknowledge support from the German Fulbright Foundation for the 2010-2011 academic year, as well as periodic support from the Potsdam Institute for Climate Impact Research.

API, 1959. Petroleum Facts and Figures. American Petroleum Institute.

BP, 2010. Statistical Review of World Energy. Technical Report. BP.

- BP, 2012. BP Energy Outlook 2030. Technical Report. BP.
- Campbell, C.J., Laherrere, J.H., 1998. The end of cheap oil. *Scientific American*, 78–83.
- CAPP, 2010. Crude Oil - Forecast, Markets and Pipelines. Technical Report. Canadian Association of Petroleum Producers.
- CAPP, 2012. Crude Oil - Forecast, Markets and Pipelines. Technical Report. Canadian Association of Petroleum Producers.
- de Castro, C., Miguel, L.J., Mediavilla, M., 2009. The role of non conventional oil in the attenuation of peak oil. *Energy Policy* 37, 1825–1833.
- EIA, 2010. Performance Profiles of Major Energy Producers 2009. Technical Report DOE/EIA-0206(09). Energy Information Administration.
- EIA, 2011. Eia historical data.
- Gallagher, B., 2010. Peak oil analyzed with a logistic function and idealized hubbert curve. *Energy Policy* 39, 790–802.
- Hamilton, J., 2009. Causes and Consequences of the Oil Shock of 2007-08. *Brookings Papers on Economic Activity*. Brookings Institute. <http://www.brookings.edu/economics/bpea.aspx>.
- Herfindahl, O.C., 1967. Extractive resources and taxation. Univ. of Wisconsin Press, Madison, WI. chapter Depletion and economic theory. p. 6390.
- Hirsch, R., 2008. Mitigation of maximum world oil production: Shortage scenarios. *Energy Policy* 36, 881–889.
- Hirsch, R., Bezdek, R., Wendling, R., 2005. Peaking of World Oil Production: Impacts, Mitigation, & Risk Management. Technical Report. U.S. Dept. of Energy.
- Hubbert, M.K., 1956. Nuclear energy and the fossil fuels, in: *Drilling and Production Practice*, Shell Development Company, San Antonio, TX.
- Hubbert, M.K., 1981. The world's evolving energy system. *American Journal of Physics* 49, 1007–1029.
- IEA, 2008. World Energy Outlook 2008. IEA International Energy Agency OECD, Paris. 2008 ed. edition.
- IEA, 2009. World Energy Outlook 2009. IEA International Energy Agency ;OECD, Paris.
- IEA, 2010. World Energy Outlook 2010. IEA International Energy Agency OECD.

- Kaufmann, R., Shiers, L., 2008. Alternatives to conventional crude oil: When, how quickly, and market driven? *Ecol. Econ.* 67, 405–411.
- Kaufmann, R.K., Gonzalez, N., Nickerson, T.A., Nesbit, T.S., 2011. Do household energy expenditures affect mortgage delinquency rates? *Energy Economics* 33, 188–194.
- Kerr, R., 2011. Peak oil production may already be here. *Science* 331, 1510–1511.
- Lutz, C., Lehr, U., Wiebe, K., 2012. Economic effects of peak oil. *Energy Policy* accepted for publication.
- Nashawi, I.S., Malallah, A., Al-Bisharah, M., 2010. Forecasting world crude oil production using multicyclic hubbert model. *Energy & Fuels* 24, 1788–1800.
- Reynolds, D., 1999. The mineral economy: how prices and costs can falsely signal decreasing scarcity. *Ecological Economics* 31, 155–166.
- Reynolds, D., 2002. Using non-time-series to determine supply elasticity: how far do prices change the hubbert curve? *OPEC Review* June, 147–167.
- Reynolds, D.B., Baek, J., 2011. Much ado about hotelling - beware the ides of hubbert. *Energy Economics* In Press.
- Soederbergh, B., Robelius, F., Aleklett, K., 2007. A crash programme scenario for the canadian oil sands industry. *Energy Policy* 35, 1931–1947.
- Sorrell, S., Speirs, J., Bentley, R., Brandt, A., Miller, R., 2010. Global oil depletion: A review of the evidence. *Energy Policy* 38, 5290–5295.