Chapter 4

From light to baryonic mass: the effect of stellar mass–to–light ratio on the Tully–Fisher relation
Abstract

In this chapter we investigate the statistical properties of the Baryonic Tully-Fisher relations for a sample of 32 galaxies with measured distances from the Cepheid period–luminosity relation and/or TRGB stars. We study the effect of the stellar mass–to–light ratio on the statistical properties of the BTFr by estimating the stellar masses of our sample galaxies with four different methods. We take advantage of resolved H\text{I} kinematics in order to investigate the statistical properties of the BTFr, based on three different kinematic measures ($W_{50}$, $V_{\text{max}}$ and $V_{\text{flat}}$). We find the intrinsic perpendicular scatter of our BTFr $\sigma_\perp = 0.026$ dex to be consistent with the intrinsic perpendicular scatter of the 3.6 $\mu$m luminosity–based TFr. However, we find a shallower slope of $\sim 3$ for the BTFr, in comparison with the slope equal to $\sim 4$ for the 3.6 $\mu$m luminosity–based TFr. We present comparisons of our BTFr with theoretical predictions as our BTFr is intended to be a reliable tool to put observational constraints on theories of galaxy formation and evolution.


4.1 Introduction

The empirical scaling relations of galaxies are a clear demonstration of the underlying physical processes of the formation and evolution of galaxies. Therefore, the main quest of any particular theory of galaxy formation and evolution is to explain their origin and intrinsic properties such as their slope, scatter and zero point. One of the most versatile and well-studied scaling relations is the relation between the width of the neutral hydrogen line and the luminosity of a galaxy (Tully & Fisher 1977), as known as the Tully-Fisher relation (TFr). Originally established as a tool to measure distances to galaxies, it became one of the most widely used relations to constrain theories of galaxy formation and evolution (Navarro & Steinmetz 2000; Vogelsberger et al. 2014; Schaye et al. 2015; Macciò et al. 2016). Even though the TFr has been extensively studied and explored during the past decades (Papastergis et al. 2016; Sorce et al. 2013; Tully & Courtois 2012; McGaugh 2005; Verheijen 2001), many open questions still remain, especially, those relating to the physical origin and the underlying physical mechanisms which maintain the TFr as galaxies evolve (McGaugh & de Blok 1998; Courteau & Rix 1999; van den Bosch 2000). Finding answers to the questions of the origin and nature of the TFr is crucial for our comprehension of galaxies and how they form and evolve.

To date, the physical principle of the TFr is widely considered to be a relation between the baryonic mass of a galaxy and the mass of the host dark matter (DM) halo. (McGaugh 2005; Freeman 1999; Milgrom & Braun 1988). This explanation is based on the fact that the TFr links the baryonic content of a galaxy (characterised by its luminosity) to a dynamical property, which is related to the host dark matter halo (characterised by the rotational velocity). Therefore, if a galaxy’s luminosity is a proxy for a certain baryonic mass fraction, a relation between its rotational velocity and its total baryonic mass should exist. Indeed, McGaugh et al. (2000) have shown that such a relation not only exists, but its scatter becomes significantly smaller if both stellar and gas masses are considered. This relation between the rotational velocity of a spiral galaxy and its baryonic mass is widely known as the Baryonic Tully-Fisher relation (BTFr).

Subsequently, the BTFr was broadly studied (Bell & de Jong 2001; Zaritsky et al. 2014; Papastergis et al. 2016; Lelli et al. 2016) as it has a great potential to put quantitative constraints on models of galaxy formation and evolution. Moreover, it clearly offers some challenges to the ΛCDM
cosmology model. First, it is measured to follow just a single power–law
over a broad range of galaxy masses, which is contrary to the expected
correlation in the ΛCDM paradigm of galaxy formation, where the BTFr
“curves” at the low velocity range (Papastergis et al. 2016; Trujillo-Gomez
et al. 2011; Desmond 2012). Second, the BTFr appeared to be extremely
tight, suggesting a zero intrinsic scatter (Verheijen 2001; McGaugh 2012).
For instance, Lelli et al. (2016) have found an intrinsic scatter of ∼ 0.1 dex,
while Dutton (2012) predicts a minimum intrinsic scatter of ∼ 0.15 dex,
using a semi-analytic galaxy formation model. It is difficult to explain
such a small observational scatter in the BTFr, as various theoretical
prescriptions in simulations, such as the mass–concentration relation of dark
matter halos or the baryon–to–halo mass ratio, contribute to a significant
scatter. However, Papastergis et al. (2016) have shown that theoretical
results seem to reproduce the observed BTFr better if hydrodynamic
simulations are considered instead of semi-analytical models (Governato
et al. 2012; Brooks & Zolotov 2014; Christensen et al. 2014). This suggests
that the mechanisms which could cause an intrinsic scatter (halo spin,
halo concentration, baryon fraction) are not completely independent from
each other. Moreover, the BTFr is also used to test alternative theories
of gravity. Hence, various studies argue that the observed properties of
the BTFr can be better explained by modification of the gravity law (e.g.
MOND, Milgrom 1983) than by a theory in which the dynamical mass of
galaxies is dominated by the DM, such as ΛCDM.

Certainly, the degree to which the BTFr can be considered as a
reliable tool to test galaxy formation and evolution models depends on
how accurately the statistical properties of the BTFr can be measured,
both observationally and from simulations. So far, various observational
results differ in details, even though they find similar results in general.
For instance, the slope of the observed relation varies from 3.5 (Zaritsky
et al. 2014; Bell & de Jong 2001) to ∼ 4.0 (Papastergis et al. 2016;
Lelli et al. 2016; McGaugh et al. 2000). Therefore, it is important to
address the observational limitations when studying the BTFr because the
measurements of the rotational velocity and of the baryonic mass of galaxies
are rather difficult. The baryonic mass of a galaxy is usually measured as
the sum of the stellar and gaseous components. While the atomic gas
mass can be measured straightforward from 21-cm line observations and
the molecular mass contribution is often negligible, the biggest contributor
to the uncertainty in the BTFr is the stellar mass measurement. Even
though various prescriptions to determine the stellar mass are available, the uncertainty in the stellar mass derived from photometric imaging usually ranges between 60-100 % (Pforr et al. 2012). Moreover, various recipes for deriving the stellar mass-to-light ratio depend on a number of parameters, such as the adopted initial stellar mass function (IMF), the star formation history (SFH) and uncertainties in modelling the advanced phases of stellar evolution, such as AGB stars (Conroy et al. 2009; Maraston et al. 2006).

There are alternative ways to measure the stellar mass of galaxies, for example by measuring the vertical velocity dispersion of stars in nearly face–on disk galaxies (Bershady et al. 2010; Aniyan et al. 2016). However, such methods are observationally expensive and have systematic limitations as well (Bershady et al. 2010).

Next, it requires to accurately measure the rotational velocity of galaxies. There are several methods to estimate the rotational velocity of spiral galaxies: from the width of the global HI profile or/and from spatially resolved HI kinematics. It was shown by Verheijen (2001) that the scatter in the luminosity–based TFr can be decreased if the velocity of the outer (flat) part ($V_{flat}$) of the rotation curve is used as a measure of the rotational velocity, instead of the corrected width of the global HI profile $W_{50}$ (see also Chapter 3). As was shown in Chapter 2 (Fig 2.6) the rotational velocity derived from the width of the global HI profile and as measured from the flat part of the rotation curve, may differ, especially for galaxies which have either rising or declining rotation curves (see Chapter 2). These issues should be taken into account when studying the statistical properties of the BTFr.

In order to avoid the uncertainties mentioned above and to establish a definitive study of the BTFr, we consider in detail four methods to estimate the stellar mass of galaxies (see Section 4.5). This allows us to study the dependence of the statistical properties of the BTFr as a function of the method used to determine the stellar mass. Furthermore, we consider the BTFr based on three velocity measures: $W_{50}$ from the corrected width of the global HI profile, and $V_{flat}$ and $V_{max}$ from the rotation curve. This allows us to study how the slope, scatter and tightness of the BTFr change if the relation is based on a different definition of the rotational velocity.
4.2 The sample

In order to study the statistical properties of the BTFr and to be able to compare our results with the luminosity-based TFr, we adopt the sample of 32 galaxies from Ponomareva et al. (2016) (see Chapter 2 & Chapter 3). As mentioned above, we intend to minimize the observational uncertainties when studying the statistical properties of the BTFr. These uncertainties are usually associated with: 1. poorly known distances; 2. converting light into stellar mass; 3. the lack of high-quality HI rotation curves. First, good-quality, independent distance measurements are crucial for computing distance-dependent stellar and gaseous masses of galaxies. Advantageously, the galaxies in our sample have independently measured distances, either from the Cepheid period–luminosity relation (Freedman et al. 2001) or/and from the tip of the red giant branch (Rizzi et al. 2007). Distance uncertainties might contribute up to 0.4 mag to the observed scatter of the luminosity–based TFr if simple Hubble flow distances are used for the nearby galaxies in our sample. In contrast, the distance uncertainty contribution to the observed scatter in the TFr is only 0.07 mag if independently measured distances are adopted (see Chapter 3). Next, the adopted sample benefits from homogeneously analysed photometric data and flux measurements over a broad wavelength range (from FUV to 4.5 μm) (see Chapter 3). This allows us to perform full spectral energy distribution (SED) fitting to derive the stellar masses of the sample galaxies based on stellar population modelling. Finally, all galaxies from our sample have HI synthesis imaging data and high-quality rotation curves available, from which $V_{\text{max}}$ and $V_{\text{flat}}$ were derived (Ponomareva et al. 2016), see Chapter 2.

Galaxies in our sample were selected according to the following criteria: 1) Sa or later in type, (see Fig 2.1), 2) inclination above 45°, 3) HI profiles with adequate S/N and without obvious distortions or contributions from possible companions to the flux (please see Chapter 2 and Chapter 3 for more details). The main properties of the sample are summarised in Table 2.1.
4.3 Data sources

To derive the main ingredients for the BTFr such as stellar mass, molecular and atomic gas masses and rotational velocities, we use the following data sources and techniques.

4.3.1 21–cm aperture synthesis imaging

For our study we collected 21–cm aperture synthesis imaging data from the literature, since many of our galaxies were already observed as part of several large HI surveys (see Chapter 2 for the details). Moreover, we observed ourselves three more galaxies with the GMRT (Ponomareva et al. 2016). All data cubes were analysed in the same manner and various data products were derived, including global HI profiles, surface density profiles and high–quality rotation curves (Fig. B.1 – B.32). The rotational velocities of galaxies were measured in three ways: from the corrected width of the global HI profile \( \left( V_{\text{circ}} = W_{50}^{R,i} / 2 \right) \), as the maximal rotational velocity \( V_{\text{max}} \) from the rotation curve and the velocity of the outer “flat” part of the rotation curve \( V_{\text{flat}} \), for more details see Chapter 2 and Chapter 3. The atomic gas masses were measured from the global HI profiles, assuming the gas to be optically thin (see Chapter 2 & Section 4.4).

4.3.2 Photometry

To study the wavelength dependence of the slope, scatter and tightness of the luminosity–based TFr, in Chapter 3 we derived the main photometric properties of our sample galaxies in 12 photometric bands from FUV to 4.5 \( \mu \)m. First, we calculated aperture magnitudes in every passband and then extrapolated the surface brightness profiles to obtain the total magnitudes. Subsequently, the total magnitudes were corrected for internal and Galactic extinction. We use the photometric measurements as derived in Chapter 3 to calculate the stellar masses of the sample galaxies for our current study (Section 4.5).

Furthermore, we collected and analysed Wide-field Infrared Survey Explorer (WISE, Wright et al. 2010) imaging data at 12 \( \mu \)m and 22 \( \mu \)m, following the prescriptions described in Chapter 3. These measurements allow us to account for the thermal emission from warm and hot dust while performing the SED fitting (Section 4.5.1). Besides, we use the 22 \( \mu \)m
photometric imaging to estimate the mass of the molecular gas component. This approach is motivated by the tight correlation between CO and infrared emission due to warm dust (Young & Scoville 1991; Paladino et al. 2006) (Section 4.4.2).

4.4 Gas mass

Gas is an important contributor to the baryonic mass of a spiral galaxy and plays a crucial role in the study of the BTFr. For instance, if the adopted stellar mass-to-light ratio used to calculate the stellar mass of a galaxy is the same for all galaxies, then only the gas mass would be responsible for any difference in the slope or tightness of the BTFr compared to the luminosity–based TFr.

In this section we describe how the masses of the atomic and molecular gas were derived. The H\textsubscript{i} mass can be directly measured from the 21-cm radio observations, while the H\textsubscript{2} mass can only be obtained indirectly using either CO or hot dust observations (Leroy et al. 2009; Westfall et al. 2011; Martinsson et al. 2013). Although generally the atomic gas mass dominates over the molecular component, there are several known cases where the estimated mass of the molecular gas is similar to or exceeds the mass of the atomic gas (Leroy et al. 2009; Saintonge et al. 2011; Martinsson et al. 2013). Therefore, it is important to take both constituents into account when studying the BTFr.

4.4.1 H\textsubscript{i} mass

We calculate the H\textsubscript{i} masses of our sample galaxies using the integrated H\textsubscript{i}–line flux density \( S_\nu dv \) [Jy kms\textsuperscript{-1}] derived during the analysis of the 21-cm radio synthesis observations (Chapter 2, Table 2.6), according to the relation:

\[
M_{HI} [M_\odot] = 2.36 \times 10^5 \cdot D^2 [Mpc] \int S_\nu dv [Jykms^{-1}],
\]

where \( D \) is the distance to the galaxy, as listed in Table 2.1. We derive the error on the H\textsubscript{i} mass by following a full error propagation calculation, taking into account the measurement error on the flux density as listed in Table 2.6 and the error on the distance modulus. Furthermore, we calculate
4.4. Gas mass

the total neutral atomic gas mass as

\[ M_{\text{atom}} = 1.4 \times M_{\text{HI}}, \]  

(4.2)

where 1.4 is a factor accounting for the primordial abundance of helium and metals. The mass of the neutral atomic gas component is listed in Table 4.2. It is important to note that we estimate the H\(_{\text{i}}\) mass under the assumption that all of the 21cm emission is optically thin.

4.4.2 H\(_{2}\) mass

Unfortunately, the distribution of the molecular hydrogen (H\(_{2}\)) in galaxies can not be directly observed. Therefore, indirect methods are required to estimate the mass of the H\(_{2}\) (M\(_{\text{H}_{2}}\)). The most straightforwardly and widely studied tracer of the H\(_{2}\) gas is the CO emission line, which can be directly observed (Leroy et al. 2009; Saintonge et al. 2011; Young & Scoville 1991). The M\(_{\text{H}_{2}}\) can be estimated, using the \(^{12}\text{CO}(J = 1 \rightarrow 0)\) column-density (I\(_{\text{CO}}\Delta V\)) and the \(^{12}\text{CO}(J = 1 \rightarrow 0)\)-to-H\(_{2}\) conversion factor X\(_{\text{CO}}\) – the ratio of the H\(_{2}\) column density to the CO emission line. However, only 5 out
of 32 galaxies in our sample have direct $CO$ measurements. Therefore, we use WISE $22 \, \mu m$ imaging to estimate the $CO$ column–density distribution. Indeed, various studies demonstrate a tight correlation between the infrared luminosity of spiral galaxies, associated with the thermal dust emission, and their molecular gas content as traced by the $CO$ emission (Westfall et al. 2011; Bendo et al. 2007; Paladino et al. 2006). For our study we use the following relation from Westfall et al. (2011) to derive $I_{CO\Delta V}$:

$$\log(I_{CO\Delta V}) = 1.08 \cdot \log(I_{22\mu m}) + 0.15,$$

(4.3)

where $I_{CO\Delta V}$ is in K kms$^{-1}$ and $I_{22\mu m}$ is the $22 \, \mu m$ surface brightness in MJy sr$^{-1}$. Note that Westfall et al. (2011) used $24 \, \mu m$ fluxes in their study. However, the $24 \, \mu m$ and $22 \, \mu m$ bands trace dust of the same temperature and therefore we can proceed our study using the $22 \, \mu m$ flux.

Next, we calculate the $M_{H_2}$, using the $X_{CO}$ conversion factor (Westfall et al. 2011; Martinsson et al. 2013):

$$\Sigma M_{H_2}[M_{\odot}pc^{-2}] = 1.6I_{CO\Delta V} \times X_{CO} \cdot \cos(i),$$

(4.4)

where $i$ is the kinematic inclination angle, listed in Table 2.5. Even though the use of $X_{CO}$ is a standard procedure to convert $CO$ column density into molecular hydrogen gas mass, different studies offer various derivations of $X_{CO}$. Here, we adopt $X_{CO} = 2.7(\pm0.9) \times 10^{20} cm^{-2}(K\text{ kms}^{-1})^{-1}$ from Westfall et al. (2011). In that study, they use a mean value of the Galactic measurement of $X_{CO}$ from Dame et al. (2001) and the measurements for M31 and M33 from Bolatto et al. (2008). In Figure 4.1 we compare our $H_2$ masses with those derived from the direct $CO$ measurements from the HERACLES survey (Leroy et al. 2009) for five galaxies in our sample. It is clear that our estimates are in good agreement. To account for the molecular fraction of helium and heavier elements we calculate the mass of the molecular gas component as:

$$M_{mol} = 1.4 \times M_{H_2}.$$  

(4.5)

It is important to note that, despite a good agreement with the HERACLES measurements, the method to estimate the $CO$ column density from the $22 \, \mu m$ surface brightness has its limitations which result in a significant estimated error on the molecular gas mass of $\sim 42\%$ (Westfall et al. 2011; Martinsson et al. 2013).
4.4. Gas mass

Figure 4.2 – $M_{\text{atom}}$ vs. $M_{\text{mol}}$ for our sample galaxies. The solid line indicates the fit from Saintonge et al. (2011). The dashed lines represent the scatter in the $M_{\text{atom}}$ vs. $M_{\text{mol}}$ relation ($\sigma = 0.41$ dex) also from Saintonge et al. (2011).

Figure 4.3 – Correlations between $R_{\text{mol}}$ and global galaxy properties.
**4.4.3 $M_{\text{atom}}$ vs. $M_{\text{mol}}$**

Presuming that the molecular gas forms out of collapsing clouds of atomic gas, it seems reasonable to expect a tight correlation between the masses of the atomic and molecular components. However, recent studies of the gas content of large galaxy samples have shown that this is not the case. A large scatter is present in the $M_{\text{atom}}$–$M_{\text{mol}}$ relation (Leroy et al. 2009; Saintonge et al. 2011; Martinsson et al. 2013). Figure 4.2 demonstrates the $M_{\text{atom}}$–$M_{\text{mol}}$ relation for our sample galaxies. Even though the majority of our galaxies follow the relation from Saintonge et al. (2011) with a similar scatter, we have eight outliers with a somewhat smaller molecular gas ratio ($R_{\text{mol}} = M_{\text{mol}}/M_{\text{atom}}$). In Figure 4.3 we present correlations between $R_{\text{mol}}$ and global galaxy properties such as absolute magnitude, colour, central surface brightness and star formation rate. Even though there are some hints that more luminous, redder galaxies with higher star formation rate tend to have a larger fraction of $M_{\text{mol}}$, the scatter in these correlations is very large. In general, $R_{\text{mol}}$ for individual galaxies ranges greatly from 0.001 to 3.97 with a mean value of $<R_{\text{mol}}>=0.38$, which is in good agreement with previous studies (Leroy et al. 2009; Saintonge et al. 2011; Martinsson et al. 2013). We find one extreme case, NGC 3637, with $R_{\text{mol}} = 3.97$, comparable to UGC 463 with $R_{\text{mol}} = 2.98$ (Martinsson et al. 2013), NGC 4736 with $R_{\text{mol}} = 1.13$ (Leroy et al. 2009) and G38462 with $R_{\text{mol}} = 4.09$ (Saintonge et al. 2011).

It is important to mention that in this section we deliberately do not compare masses of the gaseous components with the estimated masses of the stars in our sample galaxies, because the measurement of the stellar masses is not straightforward and the contribution of the stellar mass to the baryonic mass budget can vary, depending on the method used to estimate stellar masses. We discuss this subject in the following section.

**4.5 Stellar masses**

The stellar masses of galaxies, unlike the light, cannot be measured directly and, therefore, the estimation of the stellar masses is a very tricky process with various assumptions and uncertainties. The most common method of estimating the stellar mass of a galaxy is to convert the measured light into mass using a relevant mass–to–light ratio. However, deciding which mass–to–light ratio to use is not straightforward. It can be derived either
from stellar population synthesis models or by measuring the dynamical mass (surface) density of a galaxy. Of course, every method of estimating the mass–to–light ratio has its uncertainties and limitations. Therefore, we refrain from adopting any unique method of estimating the stellar masses and consider, instead, various methods so as to investigate the effect of the different stellar mass estimates on the statistical properties of the BTFRr.

4.5.1 Full SED modeling

The light that comes from stars of different ages dominates the flux in different photometric bands. Thus, for example, young hot stars dominate the flux in the UV bands while old stellar populations are more dominant in the infrared bands. Moreover, mid- and far-infrared bands can trace the galactic dust at different temperatures. The difference between magnitudes in these photometric bands (galactic colours) contains information on various properties of the stars in a galaxy like their age or metallicity. Therefore, stellar population models aim to create a mix of stellar populations that is able to simultaneously reproduce a wide range of observed colours. Thus, modelling of the spectral energy distribution allows us to estimate the total stellar mass of the composite stellar population. This process is called spectral energy distribution (SED) fitting. It is important to measure the luminosity of a galaxy at as many wavelengths as possible in order to provide more constraints on the various physical parameters of a model. Nonetheless, the SED–fitting has its limitations. Some of the parameters of stellar evolution are known and come to the modelling from empirical stellar libraries. However, there are various assumptions that are prescribed analytically and are very uncertain, such as the star formation history (SFH) or the initial mass function (IMF).

To calculate the stellar masses of our sample galaxies using SED–fitting, we derived fluxes in 14 photometric bands from $FUV$ to $22\,\mu m$ (see Section 3.2). Moreover, we collected from the literature far–infrared fluxes at 60 $\mu m$ and 100 $\mu m$ as measured by IRAS, and at 70 $\mu m$ and 160 $\mu m$, as measured with Hershel/MIPS. Consequently, we ended up with measured fluxes in 18 photometric bands for every galaxy (except for 10 galaxies that lack SDSS data, see Chapter 3). Then, we performed the fitting of the spectral energy distribution of every galaxy, using the SED–fitting code “MAGPHYS”, following the approach described in da Cunha et al. (2008). The advantage of this code is its ability to interpret the mid- and far-
Figure 4.4 – An example of the best–fit model, performed with MAPGPHYS (in black) over the observed spectral energy distribution of NGC 3031. The blue curve shows the unattenuated stellar population spectrum. The bottom plot shows the residuals for each measurement $((L_{\text{obs}} - L_{\text{mod}})/L_{\text{obs}})$.

infrared spectra of galaxies consistently with the UV, optical and near-infrared wavelengths. To interpret stellar evolution it uses the Bruzual & Charlot (2003) synthesis stellar population model. This model predicts the spectral evolution of stellar populations at ages between $1 \times 10^5$ and $2 \times 10^{10}$ yr. In this model, the stellar populations of a galaxy are described with a series of instantaneous bursts, so called “simple stellar populations”. The code adopts the Chabrier (2003) Galactic disk IMF. The code also takes into account a new prescription for the evolution of low and intermediate mass stars on the thermally pulsating asymptotic giant branch (Marigo & Girardi 2007). This prescription helps to improve the prediction of the near-infrared colours of an intermediate age stellar population, which is important in the context of spiral galaxies. To describe the attenuation of the stellar light by the dust, the code uses the two–component model of Charlot & Fall (2000). It calculates the emission from the dust in giant molecular clouds and in the diffuse ISM, and then distributes the luminosity over wavelengths to compute the infrared spectral energy distribution. The ability of the SED–fitting code to take a dusty component into account while performing the stellar mass estimate is very important for our study.
4.5. Stellar masses

because we deal with star forming spirals in which the amount of dust is not negligible.

The example of the best–fit SED model for NGC 3031 is shown in Figure 4.4. From the model we derive a stellar mass estimate for each galaxy in our sample. Thereby we obtain the stellar mass–to–light ratio ($\Upsilon_\star$) for the light in several photometric bands. We present the $\Upsilon_\star$ for the $K$ and 3.6 $\mu$m bands in Table 4.1 together with the other parameters obtained from the SED modelling. Notably, we will refer to the stellar mass–to–light ratio, measured from the SED-fitting as $\Upsilon_\star^{\text{SED},\lambda}$, where $\lambda$ is the luminosity either in the $K$ or in the 3.6 $\mu$m band. Curiously, we do not find any correlation between $\Upsilon_\star^{\text{SED},3.6}$ and the $[3.6] - [4.5]$ colour (Figure 4.5), while this correlation exists in case $\Upsilon_\star^{[3.6]}$ is measured with other methods (see below).

We assign an error to the SED–based stellar mass–to–light ratio ($\Upsilon_\star^{\text{SED},\lambda}$) equal to $\epsilon_{\Upsilon_\star^{\text{SED},\lambda}} = 0.1$ dex motivated by the test by Roediger & Courteau (2015), who performed SED–fitting with “MAGPHYS” on a sample of mock galaxies. They could recover the known stellar masses with a scatter of 0.1 dex for various samples using a different number of observational bands. Finally, we calculate a fractional error on the stellar mass as follows:

$$\epsilon_{M_\star^{\text{SED}}}^2 = (10^{\epsilon_m/2.5})^2 + (\epsilon_{\Upsilon_\star})^2 - 1,$$  \hspace{1cm} (4.6)
Table 4.1 – Results of the SED-fitting performed with MAGPHYS

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<th>name</th>
<th>log(sSFR)</th>
<th>log(SFR)</th>
<th>log(M_⋆_{SED})</th>
<th>log(M_{dust})</th>
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Notes. Column (1): name; Column (2): log of the specific star formation rate; Column (3): log of the star formation rate; Column (4): log of the stellar mass; Column (5): log of the dust mass; Column (6): stellar mass-to-light ratio for the stellar masses from Column (4) and light in the 3.6µm band; Column (7): stellar mass-to-light ratio for the stellar masses from Column (4) and light in the K– band;
4.5. Stellar masses

where $\epsilon_m$ is the mean error in the apparent magnitude over all bands equal to $\epsilon_m = 0.15$ mag. Note that the distance uncertainty is already included in this error on the magnitude. The global parameters of our sample galaxies derived with the SED–fitting method are summarised in Table 4.1.

### 4.5.2 Dynamical $\Upsilon_*$ calibration

Another way to estimate the stellar masses of spiral galaxies is by measuring the dynamical masses. The strategy for disk galaxies is to measure the vertical stellar velocity dispersion ($\sigma_z$), which can be used to obtain the dynamical mass surface density profile of a collisionless stellar disk in equilibrium:

$$
\Sigma_{dyn} = \frac{\sigma_z^2}{\pi G \kappa h_z \mu},
$$

where $\mu$ is the surface brightness, $G$ is the gravitational constant, $h_z$ is the disk scale height and $\kappa$ is the vertical mass distribution parameter (van der Kruit & Searle 1981; Bahcall & Casertano 1984). While $\mu$ can be easily measured from photometric studies, and there is a well–calibrated relation between the disk scale length $h_r$ and disk scale height $h_z$ (de Grijs & van der Kruit 1996; Kregel et al. 2002), $\sigma_z$ is very difficult to measure. Here, we take advantage of the DiskMass Survey (DMS) (Bershady et al. 2010) in order to calibrate the dynamical stellar mass-to-light ratios, which were obtained by measuring $\sigma_z$ for a sample of 30 spiral galaxies (Martinsson et al. 2013).
that study the line-of-sight stellar velocity dispersion ($\sigma_{LOS}$) was measured and then converted into $\sigma_z$. To minimize errors on $\sigma_z$, which significantly affect $\Sigma_{dyn}$, spiral galaxies close to face-on were observed. Consequently, the stellar mass surface density was calculated following:

$$\Sigma_* = \Sigma_{dyn} - \Sigma_{mol} - \Sigma_{atom},$$

(4.8)

where $\Sigma_{mol}$ and $\Sigma_{atom}$ are the mass surface densities of the molecular and atomic hydrogen, see Section 4.4. Then, the stellar mass-to-light ratio $\Upsilon_*$ can be expressed as follows:

$$\Upsilon_* = \frac{\Sigma_*}{\mu},$$

(4.9)

where $\mu$ is the K-band surface brightness (Martinsson et al. 2013). We refer to this stellar mass-to-light ratio as $\Upsilon_ {Dyn,K}$. Then, we use $\Upsilon_ {Dyn,K}$ from the DMS and check if those values correlate with a colour term, which can be measured directly from the photometry. If such a correlation exists, we would be able to adopt the $\Upsilon_ {Dyn,K}$ as a function of colour for our sample. However, we did not find any correlation (see Figure 4.6). Therefore, we adopt a median value for $\Upsilon_ {Dyn,K}$ from Martinsson et al. (2013), equal to $<\Upsilon_ {Dyn,K}> = 0.29$ and we use it with the K–band magnitudes to derive stellar masses for our sample galaxies:

$$M_*^{Dyn} = <\Upsilon_ {Dyn,K} > \cdot L_K(L_\odot),$$

(4.10)

where the absolute luminosity of the Sun in the K–band is equal to 3.27 mag. For the error on $<\Upsilon_ {Dyn,K} >$ we adopt the median error from Martinsson et al. (2013) equal to $\epsilon_{<\Upsilon_ {Dyn,K} >} = 0.19$ dex and then we calculate the fractional error on the stellar mass according to Equation 4.6. We estimate the error on the magnitude as the mean error of the K–band apparent magnitude, equal to $\epsilon_m = 0.17$ mag.

### 4.5.3 $\Upsilon_*^{[3.6]}$ as a function of [3.6]-[4.5] colour

The flux in the 3.6 $\mu$m band is considered to trace well the old stellar population of galaxies which is the main contributor to the total stellar mass, especially in early–type galaxies (ETGs). Therefore, in recent years much attention has been given to finding the best way to convert the 3.6 $\mu$m flux into stellar mass (Eskew et al. 2012; Querejeta et al. 2015; Röck...
et al. (2015, Meidt et al. 2012). Moreover, many of these studies found a correlation between $\Upsilon_{[3.6]}^*$ and the $[3.6] - [4.5]$ colour.

For instance, Eskew et al. (2012) used measurements of the resolved Large Magellanic Cloud (LMC) star formation history (SFH) (Harris & Zaritsky 2009) to calibrate $\Upsilon_{[3.6]}^*$ by linking the mass in various regions of the LMC to the 3.6 $\mu$m flux. They found that the stellar mass can be traced well by the 3.6 $\mu$m flux if a bottom-heavy initial mass function (IMF), such as Salpeter, or heavier was assumed. They estimated the stellar mass-to-light ratio to be $\Upsilon_{[3.6]}^* = 0.54$ with a 30% uncertainty. Subsequently, they found that $\Upsilon_{[3.6]}^*$ in each region of the LMC correlates with the local $[3.6] - [4.5]$ colour, according to:

$$\log \Upsilon_{[3.6]}^* = -0.74([3.6] - [4.5]) - 0.23.$$  \hspace{1cm} (4.11)

Thus, Equation 4.11 can be applied to calculate the stellar masses of our galaxies, if the fluxes at 3.6 $\mu$m and 4.5 $\mu$m are known.

However, it was demonstrated by Meidt et al. (2012) that the flux in the 3.6 $\mu$m band can be contaminated by non-stellar emission from warm dust and from PAHs (Shapiro et al. 2010). Therefore, they applied an Independent Component Analysis (ICA) to separate the 3.6 $\mu$m flux into contributions from the old stellar population and from non-stellar sources. Thus, according to Meidt et al. (2014) and Norris et al. (2014), a single $\Upsilon_{[3.6]}^* = 0.6$ can be used to convert the 3.6 $\mu$m flux into stellar mass, with an uncertainty of only 0.1 dex, provided the observed flux is corrected for non-stellar contamination. Remarkably, a constant $\Upsilon_{[3.6]}^* = 0.6$ was also found by stellar population synthesis models in the infrared wavelength range (2.5–5 $\mu$m), using empirical stellar spectra (Röck et al. 2015). In addition, Querejeta et al. (2015) presented an empirical calibration of $\Upsilon_{[3.6]}^*$ as a function of $[3.6] - [4.5]$ colour for galaxies for which the correction for non-stellar contamination was applied. Thus, they expressed the corrected stellar mass-to-light ratio as

$$\Upsilon_{[3.6], corr}^* = (\Upsilon_{[3.6]}^* = 0.6) \times \frac{F_{[3.6], cor}}{F_{[3.6], uncor}},$$  \hspace{1cm} (4.12)

where $F_{[3.6], cor}$ is the total 3.6 $\mu$m flux corrected for non-stellar contamination and $F_{[3.6], uncor}$ is the observed total flux. Hence, a constant $\Upsilon_{[3.6]}^* = 0.6$ is applicable to observed galaxies without any non-stellar contamination.
such as ETGs, while $\Upsilon^{[3.6]}_\star$ will decrease for those galaxies which suffer the most from contamination, such as star–forming spirals. Furthermore, they expressed $\Upsilon^{[3.6],\text{cor}}_\star$ as a function of the $[3.6] - [4.5]$ colour according to:

$$\log \Upsilon^{[3.6]}_\star = -0.339(\pm 0.057)([3.6] - [4.5]) - 0.336(\pm 0.002). \quad (4.13)$$

As shown in Section 3.6.3, the scatter in the luminosity–based TFr can be reduced if the corrected 3.6 $\mu$m luminosities are used. Therefore, we prefer Eq. 4.13 for the calibration of $\Upsilon^{[3.6]}_\star$ as a function of $[3.6] - [4.5]$ colour. In the remainder of this text, we refer to this mass–to–light ratio as $\Upsilon^{\text{cor},[3.6]}_\star$. We assign an error to $\Upsilon^{\text{cor},[3.6]}_\star$ equal to:

$$\epsilon^2_{\Upsilon^{[3.6]}_\star} = \epsilon^2_{\Upsilon^{[3.6],\text{cor}}_\star} + \epsilon^2_{F^{[3.6],\text{cor}}_\star/F^{[3.6],\text{uncor}}_\star}, \quad (4.14)$$

where $\epsilon_{\Upsilon^{[3.6],\text{cor}}_\star} = 0.6$ is equal to 0.1 dex (Meidt et al. 2014) and $\epsilon_{F^{[3.6],\text{cor}}_\star/F^{[3.6],\text{uncor}}_\star}$ is an averaged error on the flux ratios at 3.6 $\mu$m, equal to 0.1 dex. Furthermore, we calculate the fractional error on the stellar mass according to Eq. 4.6, using the error on the magnitude as the mean error on the 3.6 $\mu$m apparent magnitude, equal to $\epsilon_m = 0.08$ mag.

4.5.4 Constant $\Upsilon^{[3.6]}_\star$

Despite all previously listed motivations to assign different stellar mass–to–light ratios to disk galaxies, various studies advocate the use of a single mass-to-light ratio for the 3.6 $\mu$m flux. Different stellar population modelling results estimate $\Upsilon^{[3.6]}_\star$ in the range between 0.42 (McGaugh 2012; Schombert & McGaugh 2014) and 0.6 (Röck et al. 2015; Meidt et al. 2014; Norris et al. 2014), pointing out that it is metallicity–dependent. McGaugh et al. (2016) argue that assigning a universal $\Upsilon^{[3.6]}_\star$ allows for a direct representation of the data with minimum assumptions, while other methods introduce many more uncertainties.

Furthermore, Lelli et al. (2016) studied the statistical properties of the BTFr with resolved H\textsc{i} kinematics for a different sample of galaxies, using a single value of $\Upsilon^{[3.6]}_\star = 0.5$ (Schombert & McGaugh 2014). They found an extremely small vertical scatter in the BTFr of $\sigma = 0.1$ dex. This motivated us to adopt a single mass–to–light ratio of $\Upsilon^{[3.6]}_\star = 0.5$ as one of the methods for estimating the stellar mass of our sample galaxies. We adopt an error on the stellar mass-to-light ratio equal to $\epsilon_{\Upsilon^{[3.6]}_\star} = 0.07$ dex as reported
4.5. Stellar masses

Stellar masses of spirals can not be estimated easily and straightforward, as the four different methods from the previous subsections have demonstrated different mass–to–light ratios and various ways of estimating it. Here we consider these four different methods of estimating the stellar mass for our sample galaxies:

- We calculate stellar masses by performing SED–fitting, using 18 photometric bands for each galaxy (except for the 10 galaxies without SDSS data). Using these stellar masses we obtain stellar mass–to–light ratios for the $K$– and 3.6 $\mu$m bands ($\Upsilon_{*}^{SED,K}/[3.6]$). We find that the stellar mass–to–light ratios obtained with the SED–fitting, cover a wide range of values between 0.04 and 0.67 for the $K$–band and from 0.03 to 0.52 in the 3.6 $\mu$m band. The difference between $\Upsilon_{*}^{SED,K}$ and $\Upsilon_{*}^{SED,[3.6]}$ is shown in Figure 4.7 and it clearly illustrates the scatter in $\Upsilon_{*}^{SED}$ between the two bands, that is mostly dominated by uncertainties in the $K$–band luminosities and the $K - [3.6]$ colour term.

Figure 4.7 – The difference between the stellar mass-to-light ratio, obtained with SED–fitting in 3.6 $\mu$m band $\Upsilon_{*}^{SED,[3.6]}$ and in K–band $\Upsilon_{*}^{SED,[K]}$ by Schombert & McGaugh (2014), and calculate the fractional error on the stellar mass according to Eq. 4.6., with the magnitude error to be the mean error in the 3.6 $\mu$m apparent magnitudes, equal to $\epsilon_m = 0.08$ mag.
Table 4.2 – Total masses of the baryonic components

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<th>Name</th>
<th>$M_{*,1}$</th>
<th>$M_{*,2}$</th>
<th>$M_{*,3}$</th>
<th>$M_{*,4}$</th>
<th>$M_{\text{atom}}$</th>
<th>$M_{\text{mol}}$</th>
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</tbody>
</table>

Notes. Column (1): galaxy name; Column (2–5): stellar mass, estimated with different methods: 1– using SED–fitting, 2– using dynamical $M_{*,\text{Dyn}}$, 3– using SED–fitting $M_{*,\text{SED}}$, 4– using dynamical $M_{*,\text{Dyn}}$ as a function of [3.6]–[4.5] colour, 4– using constant $M_{*,\text{Dyn}}$ = 0.5; Column (6): total mass of the atomic gas, including molecular gas fraction Column (7): total mass of the molecular gas, including molecular gas fraction a) – we remind that there is no data available for the NGC 0224 in K–band, therefore it lacks the stellar mass estimation, based on the second method.
Figure 4.8 – Comparison of the distribution of stellar mass-to-light ratios for the $K$-band from the DiskMass Survey for a sample of 30 face-on galaxies (dark shade) and from the SED-fitting for our sample (Section 4.5.1), shown with a light shade. Distributions have almost the same median with a difference of only 0.01.

Figure 4.9 – Comparison of the distribution of stellar mass-to-light ratios at 3.6 $\mu$m as a function of colour (Method 3) and from the SED-fitting (Method 1). The distribution of $\Upsilon_{* \, SED, \, \lambda=3.6}$ is much broader than the distribution of $\Upsilon_{* \, Cor, \, \lambda=3.6}$. 
Obviously, such low mass–to–light ratios are not realistic and presumably the large scatter is driven by the measurement errors and model uncertainties. Indeed, it is very complicated to assign a single mass–to–light ratio even within a galaxy, as spirals tend to have various components, such as a bulge, disk and spiral arms. Therefore, a gradient in the mass–to–light ratio should be present within a galaxy, indicating the differences in IMF and in star formation histories. However, in our analysis we ignore radial trends in mass–to–light ratios which may also be a reason for the large scatter and uncertainties in $\Upsilon^{SED,K}/[3.6]$.

- We adopt a single $\Upsilon^\star$ for the $K$–band, using the median value of the dynamical mass–to–light ratios from The DiskMass Survey (Martinsson et al., 2013) $<\Upsilon^\star_{Dyn,K}> = 0.29$. The values of the dynamical mass–to–light ratios are also spread over a wide range (see Figure 4.8) and this large scatter is partly intrinsic and partly the result of measurement errors and model assumptions.

- We calculate $\Upsilon^\star$ for the 3.6 $\mu$m band as a function of the $[3.6]–[4.5]$ colour (Querejeta et al., 2015). With this method, $\Upsilon^\star_{[3.6]}$ covers a limited range from 0.44 to 0.49.

- We adopt a single mass–to–light ratio for the 3.6 $\mu$m band equal to $\Upsilon^\star_{[3.6]} = 0.5$, motivated by an empirical minimisation of the vertical scatter in the BTFr by (Lelli et al., 2016)

The resulting stellar masses derived with these different methods are summarised in Table 4.2.

As was mentioned above, the values of mass–to–light ratios from the SED–fitting and from the DMS independently spread with a large scatter which is most likely driven by the measurement and model uncertainties. Figure 4.8 demonstrates the comparison between the distribution of $\Upsilon^K$ from the DMS and from the SED–fitting for different but representative samples of spiral galaxies. Remarkably, these distributions are very similar with a difference in the median of only 0.01, even though the values are measured using different methods for different samples. Furthermore, we perform a comparison between $\Upsilon^*_{[3.6]}$ from the SED–fitting and $\Upsilon^*_{[3.6]}$ as a function of the $[3.6]–[4.5]$ colour, as shown in Figure 4.9. While the $\Upsilon^*_{[3.6]}$ is ranging from 0.03 to 0.52, the $\Upsilon^*_{[3.6]}$ is spread over a
4.6. A comparison of Baryonic Tully–Fisher relations

In this section we present the BTFrs based on different rotational velocity measures ($W_{50}$, $V_{\text{max}}$ and $V_{\text{flat}}$) and using different stellar mass estimates (Section 4.5). This allows us to study how the slope, scatter and tightness of the BTFr depend on these parameters.

We calculate the baryonic mass of a galaxy as the sum of the individual baryonic components: stellar mass, atomic gas mass and molecular gas mass, as listed in Table 4.2:

$$M_{\text{bar,m}} = M_{\star,m} + M_{\text{atom}} + M_{\text{mol}},$$

(4.15)

where $M_{\star,m}$ it is one of four stellar masses, estimated with the four different methods ($m = 1, 2, 3, 4$) (see Section 4.5). We further calculate the error on the baryonic mass by applying a full error propagation calculation:

$$\Delta M_{\text{bar,m}} = \sqrt{\Delta M_{\star,m}^2 + \Delta M_{\text{atom}}^2 + \Delta M_{\text{mol}}^2},$$

(4.16)

the derivation of $\Delta M_{\star,m}$, $\Delta M_{\text{atom}}$ and $\Delta M_{\text{mol}}$ is described in the previous sections (Section 4.4 & Section 4.5).

Consequently, we obtain 12 BTFrs for which we measure slope, scatter and tightness. To be able to perform a fair comparison with the statistical properties of the luminosity–based TFr, we calculate the above mentioned values of scatter and tightness in the BTFrs in the same manner as described in Chapter 3. All 12 relations are shown in Figure 4.10 with solid lines the best-fit models are demonstrated.

First, we perform an orthogonal fit to the data points, where the best–fit model minimises the orthogonal distances from the data points to the model. We apply the python implementation of the BCES fitting method (Akritas & Bershady 1996; Nemmen et al. 2012), which takes correlated errors in both directions into account. Moreover, this method assigns less
Figure 4.10 – The BTFs based on the different rotational velocity measures and using different stellar mass estimates. From top to bottom: 1. using SED–fitting; 2. using dynamical mass–to–light ratio calibration $\Upsilon_{\text{Dyn,K}} = 0.29$; 3. using $\Upsilon_{\text{cor},[3.6]}$ as a function of $[3.6] – [4.5]$ colour; 4. using constant $\Upsilon_{[3.6]} = 0.5$. With solid lines the best-fit models are shown.
4.6. A comparison of Baryonic Tully–Fisher relations

Figure 4.11 – The slope, vertical scatter and tightness of the BTFRs. Black symbols indicate the values for the relation based on $W_{50}$ as a rotational velocity measure, green on $V_{\text{max}}$ and red on $V_{\text{flat}}$. The values are presented for the BTFRs, using different stellar mass estimates: 1. SED–fitting; 2. dynamical $<\Upsilon^\text{Dyn,K}_\star> = 0.29$; 3. $\Upsilon_\star^\text{cor,[3.6]}$ as a function of [3.6] – [4.5] colour; 4. constant $\Upsilon_\star^\text{[3.6]} = 0.5$. The solid lines show the slope, scatter and tightness of the 3.6 $\mu$m luminosity–based TFr based on the different rotational velocity measures.
weight to outliers and to data points with large errorbars, while it permits errors in both directions to be dependent. Subsequently, we calculate the vertical scatter $\sigma$ and the perpendicular tightness $\sigma_\perp$ of each relation, following Eq. 3.11 and Eq. 3.12 respectively. Figure 4.11 shows the slope, scatter and tightness of the BTFs for different rotational velocity measures and using different stellar mass estimates. We find that the BTF with the stellar mass estimated from the SED–fitting ($M_{\text{bar},1}$) shows the largest observed scatter and worst tightness. Next, the BTF with the stellar mass based on the DMS–motivated dynamical mass–to–light ratio estimate of $<\Upsilon_{\ast}^{\text{Dyn,K}}>$ = 0.29 ($M_{\text{bar},2}$), demonstrates a bit less scatter and appears to be tighter. However, the tightest BTFs with the smallest vertical scatter are those BTFs with a stellar mass estimated either by using $\Upsilon_{\ast}\text{cor,}[3.6]$ as a function of the [3.6] – [4.5] colour ($M_{\text{bar},3}$) or by using a constant mass–to–light ratio of $\Upsilon_{\ast}[3.6] = 0.5$ ($M_{\text{bar},4}$). This suggests that the vertical scatter and tightness of the BTF strongly depend on the fraction of the stellar mass that contributes to the baryonic mass budget of a galaxy. Indeed, with an increasing stellar mass fraction the BTF becomes tighter and demonstrates a smaller scatter (Figure 4.12). Moreover, all the BTFs demonstrate a shallower slope, larger scatter and are less tight compared to the 3.6 $\mu$m luminosity–based TFr (Chapter 3). This result is contrary to

### Table 4.3 – The statistical properties of the BTFs

<table>
<thead>
<tr>
<th>$M_{\text{bar}}$</th>
<th>Slope</th>
<th>Zero point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W_{50}$</td>
<td>$V_{\text{max}}$</td>
</tr>
<tr>
<td>$M_{\text{bar},1}$</td>
<td>3.11±0.19</td>
<td>3.07±0.27</td>
</tr>
<tr>
<td>$M_{\text{bar},2}$</td>
<td>2.82±0.14</td>
<td>2.78±0.20</td>
</tr>
<tr>
<td>$M_{\text{bar},3}$</td>
<td>2.94±0.11</td>
<td>2.89±0.19</td>
</tr>
<tr>
<td>$M_{\text{bar},4}$</td>
<td>2.95±0.11</td>
<td>2.91±0.18</td>
</tr>
</tbody>
</table>

**Notes.** Column (1): Baryonic mass of a galaxy with different stellar mass estimations; Column (2)-Column(4): slopes of the BTFs based on $W_{50}$, $V_{\text{max}}$ and $V_{\text{flat}}$; Column (5)-Column(7): zero points of the TFs based on $W_{50}$, $V_{\text{max}}$ and $V_{\text{flat}}$.

### Table 4.3 – The statistical properties of the BTFs (continued)

<table>
<thead>
<tr>
<th>$M_{\text{bar}}$</th>
<th>Vertical scatter ($\sigma$)</th>
<th>Tightness ($\sigma_\perp$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W_{50}$</td>
<td>$V_{\text{max}}$</td>
</tr>
<tr>
<td>$M_{\text{bar},1}$</td>
<td>0.21±0.01</td>
<td>0.23±0.02</td>
</tr>
<tr>
<td>$M_{\text{bar},2}$</td>
<td>0.15±0.01</td>
<td>0.17±0.01</td>
</tr>
<tr>
<td>$M_{\text{bar},3}$</td>
<td>0.13±0.01</td>
<td>0.16±0.01</td>
</tr>
<tr>
<td>$M_{\text{bar},4}$</td>
<td>0.14±0.01</td>
<td>0.16±0.01</td>
</tr>
</tbody>
</table>

**Notes.** Column (2)-Column(4): scatters of the BTFs based on $W_{50}$, $V_{\text{max}}$ and $V_{\text{flat}}$; Column (5)-Column(7): tightnesses of the BTFs based on $W_{50}$, $V_{\text{max}}$ and $V_{\text{flat}}$. 
Figure 4.12 – The slope, vertical scatter and tightness of the BTFRs as a function of the mass–to-light ratio. Black lines indicate the values for the relation based on $W_{50}$ as a rotational velocity measure, green on $V_{\text{max}}$ and red on $V_{\text{flat}}$. The solid lines show the trends for the BTFr ($M_* + M_{\text{atom}} + M_{\text{mol}}$). The dashed lines show the trends for $M_* + M_{\text{atom}}$. The solid horizontal lines show the slope, scatter and tightness of the 3.6 μm luminosity–based TFr based on the different rotational velocity measures.
previous studies (McGaugh et al. 2000; McGaugh 2005), since it suggests that inclusion of the gas mass does not help to tighten the TFr. Instead, it introduces additional scatter, especially for the low mass-to-light ratios. We performed a test by assigning different mass–to–light ratios for our sample. We vary mass–to–light ratios from 0.1 to 10, but assign the same value to all galaxies. From Figure 4.12 it is clear, that increasing the mass–to–light ratio helps to reduce the vertical scatter and improve the tightness of the BTFr, suggesting that the scatter in the BTFr is introduced by the gaseous component. From Figure 4.12 it is also clear that the contribution of the molecular gas component does not largely affect the statistical properties of the BTFr.

The other important result from our study is that, independent of the stellar mass estimate, each BTFr shows a smaller scatter and improved tightness when based on $W_{50}$ as a rotational velocity measure. This result is also in contradiction with theoretical hypotheses concerning the origin of the TFr, being a relation between the baryonic mass of a galaxy and that of its host dark matter halo. Only $V_{\text{flat}}$ can properly trace the gravitational potential of a dark matter halo, because it is measured in the outskirts of the extended H$\text{I}$ disk where the potential is dominated by the dark matter halo. However, it is also important to note that the scatter and tightness of the BTFr based on $W_{50}$ and on $V_{\text{flat}}$ are consistent within their error. Table 4.3 summarises the statistical properties of the BTFrs.

4.7 Our adopted Baryonic Tully–Fisher relation

As was described in the previous sections, the choice of the mass–to–light ratio is not straightforward and requires estimates from different methods, e.g. from stellar population modelling, or from dynamical modelling. Interestingly, from our SED–fitting and from the dynamical estimate of $\Upsilon^K_\star$ from the DMS, we get the same median of $<\Upsilon^K_\star> \sim 0.3$ (Figure 4.8) for galaxies that cover similar morphological types. However, the method of estimating the mass–to–light ratio as a function of the [3.6]–[4.5] colour gives a somewhat larger mass–to–light ratios with a much smaller scatter; $\Upsilon^{\text{cor.}[3.6]}_\star$ lies in the range between 0.44 and 0.49.

From Section 4.6 we conclude that the individual mass–to–light ratios from the SED–fitting are not applicable for our galaxies, as their values show large scatter and demonstrate unrealistically low mass–to–light ratios for many galaxies. We also can not draw certain conclusions regarding
Our adopted Baryonic Tully–Fisher relation

\[ M_{\text{bar}, \text{fin}} = (2.99 \pm 0.22) \cdot \log(2V_{\text{flat}}) + 2.88 \pm 0.56. \] (4.17)

Eq. 4.17 describes the relation with an observed vertical scatter of \( \sigma = 0.16 \pm 0.1 \) dex and a tightness of \( \sigma_\perp = 0.052 \pm 0.013 \) dex. These results are consistent with recent studies of the vertical (Lelli et al. 2016) and perpendicular (Papastergis et al. 2016) scatter of the BTFr, but are somewhat larger compared to the 3.6 \( \mu \text{m} \) luminosity–based TFr.

The contributions from the stellar and gaseous components to the BTFr separately are shown in Figure 4.14.
Figure 4.14 – The final choice BTFr based on $V_{\text{flat}}$ is shown with the black symbols. Stellar component is shown with the red symbols and gaseous component with the blue symbols.
4.7. Our adopted Baryonic Tully–Fisher relation

because the tightness is a slope independent measure and should be used as a possible constraint on theories of galaxy formation and evolution (Section 3.6.2). We compare the perpendicular distances \(d_{\perp,i}\) of the data point to the line, with the projected measurement errors \(\epsilon_i\) based on the error on the baryonic mass (\(\Delta M_{\text{bar},i}\)) and the error on the rotational velocity (\(\epsilon_{V_{\text{flat},i}}\)) (See Section 3.6.3 for more details). Figure 4.15 demonstrates the histogram of \(d_{\perp,i}/\epsilon_i\). In case of a zero intrinsic perpendicular scatter \(\sigma_{\perp}\), this histogram would follow the standard normal distribution, shown with the black Gaussian in Figure 4.15. However, it is clear that the distribution of \(d_{\perp,i}/\epsilon_i\) (shown with the dashed Gaussian) is broader with the standard deviation of the Gaussian equal to 1.53. Consequently, we can estimate the best guess value of the intrinsic \(\sigma_{\perp,\text{int}}\) as follows:

\[
\sigma_{\perp,\text{int}} = \sqrt{\sigma_{\perp,\text{obs}}^2 - \sigma_{\perp,\text{err}}^2},
\]

where \(\sigma_{\perp,\text{err}} = 0.045\) dex is the perpendicular scatter due to the measurement uncertainties only. Hence, we estimate the \(\sigma_{\perp,\text{int}} = 0.026\)
Figure 4.16 – Residuals of the $M_{\text{bar,fin}}-2V_\text{flat}$ relation as a function of global galactic properties. $r$ is the Pearson’s correlation coefficient.

dex, which happens to be similar to the $\sigma_{\perp,\text{int}}$ of the 3.6 $\mu$m luminosity–based TFr (Section 3.6.3). Therefore, we can conclude that even if the BTFr has a larger observed perpendicular scatter compared to the 3.6 $\mu$m luminosity–based TFr, it is only due to the measurement uncertainties because both relations have the identical best guess intrinsic perpendicular scatter $\sigma_{\perp,\text{int}} = 0.026$ dex.

### 4.7.1 Search for a 2nd parameter

As was suggested by various authors [Aaronson & Mould 1983, Rubin et al. 1985], the vertical scatter in the luminosity–based TFr can be decreased by invoking a second parameter. However, we demonstrated in Section 3.6.4 that the residuals of the 3.6 $\mu$m TFr do not correlate significantly with any of the galactic properties and, therefore, we could not find any second parameter of use. In this section we repeat this exercise for the $M_{\text{bar,fin}}-V_\text{flat}$ relation and examine the nature of the residuals along the
fitted model line described with Eq. 4.17. Figure 4.16 demonstrates the residuals of the BTFr \((\log \Delta M_{\text{bar}})\) as a function of global galactic properties, such as star formation rate, outer slope of the rotation curve, central surface brightness, \(i - [3.6]\) colour, and gas fraction. We calculate \(\Delta M_{\text{bar}}\) as \(\Delta M_{\text{bar}} = M_{\text{bar}}/M_{\text{bar, model}}\), where \(M_{\text{bar, model}}\) is described by Eq. 4.17.

To quantitatively describe the strength of the correlations we calculate the Pearson’s correlation coefficients \(r\) for each of the relations. We find the largest \(r = 0.47\) for the correlation between \(\Delta M_{\text{bar}}\) and the total gas fraction \(f_{\text{gas}} = (M_{\text{atom}} + M_{\text{mol}})/M_{\text{bar}}\) and the smallest \(r = 0.02\) for the correlation between \(\Delta M_{\text{bar}}\) and \(i - [3.6]\) colour. However, the strength of neither correlations is sufficient to identify a possible second parameter.

Finally, we consider the residuals of the relation in order to determine which stellar mass–to–light ratio needs to be assigned to every galaxy, so as to bring all galaxies to the relation with a zero scatter. Figure 4.17 demonstrates the spread of these modelled mass–to–light ratios \(\Upsilon_{\star}^{[3.6, \text{mod}]}\). We again find a large scatter in the \(\Upsilon_{\star}^{[3.6, \text{mod}]}\) values. However the mean of the spread is equal to 0.37 and, as expected, is very similar to the mass–to–light ratio \(\Upsilon_{\text{star}}^{[3.6, \text{fin}]} = 0.35\), adopted for this study.
4.8 Comparison with previous observational studies and theoretical results

Comparing any observed BTFr with previous observational and theoretical studies is a challenging process (Bradford et al. 2016). In this section we compare our results with previous studies and with the theoretical predictions, following the methodology described in Papastergis et al. (2016). First, we compare our results with other observational studies. Second, we test theoretical models of galaxy formation and evolution within the ΛCDM framework and alternative theories against our fiducial BTFr.

4.8.1 Previous studies

The biggest challenge in comparing measurements of the statistical properties of the BTFr with other studies is posed by the different methods used to derive the main properties such as the baryonic mass and the rotational velocity. For instance, the galaxy sample, the mass range of galaxies, the applied corrections and the choice of the fitting method contributes significantly to the measurement uncertainties. Moreover, it is critical in the comparison that the rotational velocities are similarly defined. Therefore, it is important to note that we consider our $M_{\text{bar}, \text{fin}} - 2V_{\text{flat}}$ relation for
4.8. Comparison with previous observational studies and theoretical results

Table 4.4 – Statistical properties of the BTFr from different studies

<table>
<thead>
<tr>
<th></th>
<th>This work</th>
<th>Verheijen01</th>
<th>Noordermeer+07</th>
<th>Papastergis+16</th>
</tr>
</thead>
<tbody>
<tr>
<td>slope</td>
<td>2.99/3.18</td>
<td>2.98</td>
<td>3.51</td>
<td>4.56</td>
</tr>
<tr>
<td>zero point</td>
<td>2.82/2.88</td>
<td>2.82</td>
<td>1.4</td>
<td>-0.66</td>
</tr>
<tr>
<td>σ₀</td>
<td>0.16/0.15</td>
<td>0.12</td>
<td>0.20</td>
<td>0.23</td>
</tr>
<tr>
<td>σ⊥</td>
<td>0.053/0.045</td>
<td>0.045</td>
<td>0.058</td>
<td>0.050</td>
</tr>
</tbody>
</table>

Notes. Slope, zero point, scatter and tightness of the BTFrs from different studies. Column (1): name of the parameter; Column (2): slope, zero point, scatter and tightness from this work. First parameter is with taking low mass outlier into the fit and second, excluding the outlier.; Column (3): slope, zero point, scatter and tightness from Verheijen (2001); Column (4): slope, zero point, scatter and tightness from Noordermeer & Verheijen (2007); Column (5): slope, zero point, scatter and tightness from Papastergis et al. (2016).

These comparisons. However, it is not always the case that previous studies of the BTFr are based on $2V_{flat}$ as a rotational velocity measure. In the literature, it is more common that the global H\textsc{i} profile widths are used to derive the circular velocity, therefore we further refer to the rotational velocity as $V_{circ}$.

First, we compare our results with those by Lelli et al. (2016) and McGaugh (2012) as both these studies use $V_{circ} = V_{flat}$. In general, our results are in a good agreement: we obtain the same observed vertical scatter $σ = 0.18$ dex as reported by Lelli et al. (2016) and our tightness $σ⊥ = 0.056$ dex is consistent with the total tightness $σ⊥ = 0.06$ dex found by McGaugh (2012). However, the largest difference we find is the slope of the BTFr. The slope of the BTFr reported by both Lelli et al. (2016) and McGaugh (2012) is measured to be $a ≈ 4$, while we find the slope of the BTFr equal to $a ≈ 3$. Our slope is more consistent with the result by Zaritsky et al. (2014), who find the slope of the BTFr to be in the range from $a = 3.3$ to $a = 3.5$ but they, however, used the corrected width of the global H\textsc{i} profile ($V_{circ} = W_{50}/2$).

The low mass end of the BTFr is not well presented in our sample. Moreover, the low–mass galaxies tend to have rising rotation curves and, therefore, can not be considered for the $M_{bar,fin} − 2V_{flat}$ relation. Furthermore, the only low–mass galaxy in our sample appears to be an outlier. If we remove this galaxy from the fit, the statistical properties of our BTFr slightly change, as the slope of the relation increases from $a = 2.99$ to $a = 3.18$ and the observed tightness changes from $σ⊥ = 0.058$ to $σ⊥ = 0.045$ dex, see Table 4.4. However, the slope does not reach the
value of $a = 4$, which is contrary to results reported by Lelli et al. (2016) and McGaugh (2012). This can be understood as they applied a higher stellar mass-to-light ratio which reduces the relative contribution of the gas to $M_{\text{bar}}$ such that their BTFr approaches our $L_{\text{[3.6]}} - 2V_{\text{flat}}$ relation. They also neglect the presence of the molecular gas and do not take it into account in their study.

For a more detailed comparison with previous studies we present the comparison analysis of the statistical properties of our BTFr with the BTFr from Papastergis et al. (2016); Verheijen (2001) and Noordermeer & Verheijen (2007). The sample from Papastergis et al. (2016) is a sample of heavily gas-dominated ($M_{\text{gas}}/M_* \geq 2.7$) galaxies which cover the low-mass end of the BTFr ($M_*$ in a range from 1.25 to 31.6 $10^9M_\odot$), where $V_{\text{circ}}$ was measured as $V_{\text{circ}} = W_{\text{50}}/2$. Therefore, we adopt only those 68 galaxies from their sample that have large negative kurtosis ($h^4 < -1.2$) of the global HI profile, indicating that this profile is double-peaked and, therefore, in these galaxies $W_{\text{50}}/2$ is most likely a good approximation of $V_{\text{flat}}$. The sample from Verheijen (2001) is the Ursa Major sample of intermediate mass galaxies ($M_*$ in a range from 1.58 to 39.8 $10^9M_\odot$) where $V_{\text{circ}} = V_{\text{flat}}$ was measured in the same manner as in our study. The Noordermeer & Verheijen (2007) sample is a sample of the high-mass galaxies ($M_*$ in a range from 1.99 to 199.5 $10^9M_\odot$) where $V_{\text{circ}} = V_{\text{flat}}$ as well. It is critical for the comparison of the BTFr studies that the fitting algorithm is defined similarly (Bradford et al. 2016). Therefore, we perform our fitting routine for the above-mentioned samples from Papastergis et al. (2016); Verheijen (2001) and Noordermeer & Verheijen (2007), in order to derive the statistical properties of the BTFr in a homogeneous way with our studies. In Figure 4.18 we present the comparison between our sample and those three from the literature. The low-mass end and the high-mass end samples show different results in comparison to our BTFr. For instance, the slope of the high-mass end sample (Noordermeer & Verheijen 2007) is equal to $a = 3.51$, while the slope of the low-mass sample (Papastergis et al. 2016) has the extreme value of $a = 4.56$. Meanwhile, the statistical properties of the Verheijen (2001) Ursa Major BTFr and our BTFr are in excellent agreement, as the data for both these samples have been treated in the same way (see Figure 4.18 and Table 4.4).

The other important issue to keep in mind is that the baryonic masses of galaxies were measured in different ways in various studies. This also contributes to the uncertainties while performing these comparisons. For
4.8. Comparison with previous observational studies and theoretical results

Figure 4.19 – Left panel: The comparison between our BTFr with the SAMs from Trujillo-Gomez et al. (2011) (with adiabatic contraction (solid green line) and without (dashed green line)) and with the semi–analytic model from Desmond (2012) (red line). With dashed red lines the 2σ intrinsic scatter is indicated. Right panel: the same comparisons but of the combined sample with Noordermeer & Verheijen (2007) (red circles) and Verheijen (2001) (blue stars).

example, Papastergis et al. (2016) use the average value of the stellar mass as derived with five different methods for each galaxy in the sample. Zaritsky et al. (2014) adopt a stellar mass as measured from the ratio of the 3.6 and 4.5 µm fluxes. Lelli et al. (2016) and McGaugh (2012) adopt a single mass–to–light ratio for all galaxies equal to \( \Upsilon_{3.6}^* = 0.5 \) (see Section 4.5 for more details). The results for our BTFr with the adopted stellar mass–to–light ratio equal to \( \Upsilon_{3.6}^* = 0.5 \) are presented in Section 4.6, and we recall that this choice of \( \Upsilon_{3.6}^* = 0.5 \) for our sample also results in a shallower slope of \( a = 3.15 \pm 0.21 \) for the BTFr.

4.8.2 Semi–analytical models

In the ΛCDM cosmological model, the BTFr is supposed to follow a single power–law of the form \( M_{\text{bar}} \propto V_{\text{rot}}^3 \). This form follows from the tight correlation between the mass of the dark matter halo and its maximal
rotational velocity $M_h \propto V_{h,\text{max}}^3$, measured from the DM-only simulations \citep{Klypin_2011}. A further study of the BTFr in the ΛCDM context can be done by using semi-analytical models (SAMs) of galaxy formation. Semi-analytical models assign observationally motivated masses of stars and gas to the host dark matter halo as they are typically calibrated to reproduce some global observational properties of galaxy populations, such as the stellar mass function. In SAMs the stellar masses are usually assigned to the haloes using an abundance matching (AM) technique \citep{Moster_2010} while the gas masses are assigned based on observational scaling relations between the stellar mass and the gas fraction. $V_{\text{circ}}$ is usually calculated by computing the rotation curve, which takes into account the addition of the baryonic components to the rotation curve of the DM halo only \citep{Trujillo-Gomez_2011} and assigning $V_{\text{circ}}$ either by its value at a particular radius or assuming it to be the rotational velocity of the peak of the simulated rotation curve $V_{\text{circ}} = V_{\text{max}}$.

We compare our BTFr with two SAMs from \cite{Trujillo-Gomez_2011} and from \cite{Desmond_2012}. From \cite{Trujillo-Gomez_2011} we consider two models: one in which DM halos experience adiabatic contraction due to the infall of the baryons to the halo centres, and one without adiabatic contraction. The rotation velocity in \cite{Trujillo-Gomez_2011} is calculated at a fixed radius of 10 kpc. The semi-analytical model of \cite{Desmond_2012} uses the $V_{\text{circ}} = V_{\text{max}}$ estimation of the rotational velocity. Moreover, the \cite{Desmond_2012} model also calculates the intrinsic scatter of the BTFr, expected in a ΛCDM cosmology. This scatter is caused by various mechanisms, such as scatter in the concentration parameter of the DM halos, scatter in the halo spin parameter and scatter in the baryon fraction of the halo.

Figure \ref{fig:4.19} demonstrates the comparison of our sample BTFr (left panel) and of the combined sample with those from \cite{Verheijen_2001} and \cite{Noordermeer_2007} (right panel) with these two models. Note that for the observational samples we use $V_{\text{circ}} = V_{\text{max}}$ for the fair comparison. Both models introduce a slight curvature on the BTFr, which is a general prediction of the ΛCDM cosmological model \citep{Papastergis_2012}. The \cite{Trujillo-Gomez_2011} model can reproduce our sample relatively well, especially the model in which DM halos do not experience adiabatic contraction. The \cite{Desmond_2012} model is systematically offset from the observed data points of all three samples and can reproduce the
4.8. Comparison with previous observational studies and theoretical results

Figure 4.20 – The comparison of our BTFr combined with Noordermeer & Verheijen (2007) (red circles) and Verheijen (2001) (blue stars) samples, with the galaxies produced by the hydrodynamical simulations from Piontek & Steinmetz (2011) (red triangles), and by Governato et al. (2012), Brooks & Zolotov (2014) and Christensen et al. (2014) (blue triangles).
relations only within the $2\sigma$ uncertainty, as indicated with the dashed red lines in Figure 4.19.

### 4.8.3 Hydrodynamical simulations

In Figure 4.20 we compare our BTFr with individual galaxies, produced by hydrodynamical simulations in the context of a ΛCDM cosmological model. For the high–mass end of the BTFr we consider eight galaxies from Piontek & Steinmetz (2011) and for the intermediate and low–mass end we consider twelve galaxies, produced in a set of hydrodynamical simulations by Governato et al. (2012); Brooks & Zolotov (2014) and Christensen et al. (2014). In Figure 4.20 it is shown that these sets of hydrodynamical simulations are successful at reproducing the observed BTFr. The rotational velocities of galaxies in these simulations are measured from the width of the global H\textsc{i} profiles, therefore, for the fair comparison we also use $W$ as a rotational velocity measure for the observational samples.

### 4.8.4 MOND

Modified newtonian dynamics (MOND) is an alternative to the ΛCDM model of galaxy formation and evolution, which does not require the presence of dark matter, and where gravitational forces are entirely defined by the amount and distribution of the baryonic matter (Milgrom & Braun 1988). Therefore, it predicts that the BTFr with $V_{\text{circ}} = V_{\text{flat}}$ can be described with a single power–law with zero intrinsic scatter. Moreover, it predicts that the relation has a slope of exactly 4 when the relation is based on $V_{\text{flat}}$ as a rotational velocity measure, while the normalisation of the relation depends only on the acceleration parameter $\alpha_0$ (McGaugh 2012).

The advantage of our sample is that it is based on $V_{\text{flat}}$ as a rotational velocity measure and therefore allows us to directly compare our results with the predictions from MOND. Figure 4.21 demonstrates the results for our sample combined with the Verheijen (2001) and Noordermeer & Verheijen (2007) samples where $V_{\text{circ}} = V_{\text{flat}}$. While the slope and scatter of the BTFr from the MOND prediction are fixed, the normalisation of the relation can vary due to the uncertainty in the observational determination of the acceleration parameter $\alpha_0$ (Begeman et al. 1991). In Figure 4.21 we consider two values for $\alpha_0$, which are consistent with Begeman et al. (1991).
Figure 4.21 – Our BTFr combined with the samples from Noordermeer & Verheijen (2007) and Verheijen (2001) and in comparison with the BTFr predicted by MOND.
From Figure [4.21] it is clear that, while the MOND normalisation works well for the intermediate mass galaxies, it fails to reproduce the high–mass end of the BTFr (represented by our, Verheijen (2001) and Noordermeer & Verheijen (2007) samples) by introducing a slope that is too steep. In conclusion, neither the non–zero intrinsic scatter nor the slope of our BTFr are consistent with the relation predicted by MOND. This inconsistency can be mitigated, however, by assigning significantly higher mass–to–light ratios to our observed galaxies.

4.9 Conclusions

In this chapter we perform a detailed study of the Baryonic Tully–Fisher relations based on different stellar mass estimates and taking advantage of resolved HI kinematics. The aim of our study is to investigate how the various stellar mass estimation methods affect the statistical properties of the BTFr. For this, we estimate the stellar masses of our sample galaxies following four different prescriptions. First, we measured stellar masses by performing a full SED–fitting using 18 photometric bands (from FUV to far infrared). Second, we adopt a median value of the dynamical mass–to–light ratio from the DiskMass survey $\Upsilon_{\text{Dyn,K}}^\star = 0.3$. Then, we calculate the stellar mass–to–light ratio for the 3.6 $\mu$m band as a function of $[3.6]–[4.5]$ colour and, finally, we adopt a single mass–to–light ration equal to $\Upsilon_{\star}^{[3.6]} = 0.5$ from Lelli et al. (2016), which is motivated by an empirical minimisation of the vertical scatter in the BTFr.

Using each stellar mass estimate, we construct the Baryonic Tully–Fisher relations. Each of the relations is based on three different velocity measures: $W_{50}$ from the global HI profile, $V_{\text{max}}$ and $V_{\text{flat}}$ from the rotation curve. For each of the relations we measure the slope, vertical scatter and tightness. We find that the tightest BTFrs with the smallest vertical scatter are those based on larger mass–to–light ratios (method 3 and method 4) and based on $W_{50}$ as a rotational velocity measure. However, none of the relations demonstrates as small a vertical and perpendicular scatter as the 3.6 $\mu$m luminosity–based TFr. This allows us to conclude that mostly the gas component is responsible for the increase of the scatter (vertical and perpendicular) in the BTFr. Hence, increasing the mass–to–light ratio reduces the vertical and perpendicular scatter of the BTFr, as it makes the gas contribution negligible.
4.9. Conclusions

We consider in detail our BTFr of choice, which is based on $V_{\text{flat}}$ and a stellar mass computed with a single mass–to–light ratio $\Upsilon_{3.6}^{*} = 0.35$. This choice of the mass–to–light ratio is motivated by adopting the mean value between the median mass–to–light ratios of method 1 (SED–fitting) and method 3 ([3.6]-[4.5] colour). We measure the slope, vertical scatter and tightness of our BTFr. We find the slope equal to $3.18 \pm 0.22$, which is shallower in comparison with previous studies (Lelli et al. 2016; McGaugh 2012) (Section 4.8.1). We measure the vertical scatter $\sigma = 0.18$ dex, which is consistent with the previous study by Lelli et al. (2016). We find the observed tightness $\sigma_{\perp} = 0.045$ dex to be smaller than the ones found by Papastergis et al. (2016) and McGaugh (2012). This observed perpendicular scatter is larger than the perpendicular scatter of the 3.6 $\mu$m luminosity–based TFr. However, the estimated intrinsic perpendicular scatter is shown to be similar.

Furthermore, we compare the results of our BTFr with various theoretical predictions from ΛCDM and MOND theories of galaxy formation. We find that the semi–analytic model of Trujillo-Gomez et al. (2011) represents our relation well when the DM halos do not undergo the process of adiabatic contraction. Moreover, various hydrodynamical simulations of galaxies with different masses (Governato et al. 2012; Brooks & Zolotov 2014; Christensen et al. 2014; Piontek & Steinmetz 2011) also tend to fall well on our observed BTFr. However, the predictions from MOND do not show quite as good results since they fail to reproduce the observed BTFr at the high–mass end, where the observed galaxies of high masses tend to lie below the MOND predicted relation as the slope predicted by MOND is steeper.

In conclusion, it is important to point out that there is no unique solution to measure the stellar masses of spiral galaxies. Various methods lead to estimates which may differ significantly from each other. Therefore, the statistical properties of the Baryonic Tully–Fisher relation remain uncertain as different stellar mass–to–light ratios can lead to the different interpretations of the relation in the framework of theoretical models of galaxy formation and evolution.

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Appendix 4.A  SED best–fit models for our sample galaxies

In Figures A.1–A.32 we present the best–fit models performed with MAPGPHYS (in black) over the observed spectral energy distribution of our sample galaxies. The blue curve shows the unattenuated stellar population spectrum. The bottom plot shows the residuals for each measurement ($(L_{obs} - L_{mod})/L_{obs}$).
4.A. SED best-fit models for our sample galaxies

Figure A.1

Figure A.2
Figure A.3

Figure A.4
4.A. SED best-fit models for our sample galaxies

Figure A.5

Figure A.6
Figure A.7

Figure A.8
4.A. SED best-fit models for our sample galaxies

Figure A.9

Figure A.10
4.A. SED best–fit models for our sample galaxies

Figure A.13

Figure A.14
4.A. SED best–fit models for our sample galaxies

Figure A.17

Figure A.18
4.A. SED best-fit models for our sample galaxies

Figure A.21

Figure A.22
4.A. SED best–fit models for our sample galaxies

Figure A.25

Figure A.26
4.A. SED best–fit models for our sample galaxies

Figure A.29

Figure A.30
Figure A.31

Figure A.32
References

Desmond, H. 2012, ArXiv e-prints
References

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