# A Comparison of Confidence Interval Techniques for Dependent Correlations 

## By

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To my family, friends, and advisor, who saw the culmination of this endeavor well before I could.

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## CHAPTER 1

## INTRODUCTION

### 1.1 The Ubiquity of Correlation Matrices

A correlation is a measure of the relationship between two variables. The term "correlation" is used colloquially and interchangeable with "relationship," however, the correlation that statistical users are most familiar with is the Pearson product-moment correlation, which was conceived by Sir Francis Galton and articulated in its modern form by Karl Pearson over one hundred years ago (Pearson, 1896; Stigler, 1989). The Pearson productmoment correlation coefficient (or the "correlation coefficient" or "correlation" for short) quantifies a linear relationship between two quantitative variables, although it may be used for binary or ordinal variables as well.

Correlations are prolific in fields that use quantitative data. Despite early discussion by Galton and Pearson positing the application of correlations only for genetic data, correlations rapidly spread to serve as the basis of many statistical methods, including those used in psychology, economics, medicine, biology, even physics and the other hard sciences. This popularity is in part due to the multiple formulations and interpretations of the correlation coefficient across disciplines (Rodgers \& Nicewander, 1988). The ubiquity of correlations also in part may be due to misinterpretations as well. Carroll (1961) wrote that the correlation coefficient is "one of the most frequently used tools... and perhaps also one of the most frequently misused."

However, few researchers investigate simply bivariate phenomena. Theories about natural phenomena and behavior are generally more complex than two variables may capture. Rather, multivariate data analysis is the standard in quantitative analyses, and so often researchers deal with correlation matrices rather than single correlations.

A correlation matrix captures the correlations between many variables in a row-bycolumn arrangement. A correlation matrix is more than a collection of correlation coefficients organized into a table. The general properties of a correlation matrix are understood by introductory scholars and users of statistics: a correlation matrix, $\boldsymbol{R}$, is a symmetric matrix that has unity (ones) as diagonal elements and correlation coefficients (that range from $[-1,+1]$ ) as off-diagonal elements. The final property of all real correlation matrices is positive semidefiniteness (PSD) or positive definiteness (PD), which ensures that all of the underlying variances of latent components for the correlation matrix are nonnegative. In practice the eigenvalues of the correlation matrix are often checked to ensure the matrix is PSD or PD; if all of the eigenvalues are nonnegative, the matrix is PSD, and if all are positive, the matrix is PD. If any of the eigenvalues are negative, the matrix is indefinite, and cannot be a true correlation matrix.

For many statistical analyses (e.g., factor analysis, structural equation modeling), a correlation matrix - or its unstandardized counterpart, the covariance matrix - can be the data input rather than the raw data. Correlation matrices are also frequently presented as descriptive information in other types of analyses. Therefore, correlation matrices are generally calculated as a part of the analytic strategy to support more advanced statistical techniques, and correlation matrices are often reported with other descriptive statistics in substantive articles.

### 1.2 Reporting Correlation Matrices

For psychological research, reporting the correlation or covariance matrix, along with means and standard deviations, of study variables is considered best practice in studies that use structural equation modeling (SEM) and its special cases (Appelbaum et al., 2018). Reporting covariance information is helpful for two reasons. First, if the author fully specifies the SEM used in the study (including estimation method, identification method, statistical package, etc.), readers may fully replicate the authors' results or may fit competing models. Second, reporting correlation matrices may help future research, both by informing effect sizes for future studies in that substantive domain and by informing meta-analyses, such that correlation matrices from multiple studies may be statistically analyzed or pooled for further analysis (e.g., Cheung \& Chan, 2005).

Concurrently, substantive researchers face growing pressure from peers, methodologists, and editors to move beyond reporting point estimates and significance tests (i.e., p-values) and to instead report effect sizes and parameter variability - that is, confidence intervals (CIs; e.g., Greenland et al., 2016; Thompson, 2002; Psychological Science, 2018). This push for reporting CIs has occurred because confidence intervals often provide more information than significance tests, as detailed in the next section. Indeed, a whole "NHST revolution" occurred in the late 1990's, signaling a transition toward effect sizes and CI's to enhance (or even replace) the standard NHST (null hypothesis significance test) approach, including p-values (see Harlow, Mulaik, \& Steiger, 1997; Wilkinson, 1999).

Taken together, these two trends in psychology research reporting indicate that best practice for many studies that depend on covariance matrices may be to report complete correlation matrices, including CIs for the coefficients, as well as the means and standard
deviations that are typically reported in practice. For studies with relatively few variables, reporting correlation matrices with CIs can often be easily implemented. For studies with many variables, reporting correlation matrices may be a challenge given space and page width limitations, but online supplemental options offer recourse to thorough researchers seeking to implement best practices.

In reality, best practice reporting standards are rarely achieved. To investigate current practices in reporting correlation matrices, I conducted a systematic review of SEM applications published between 2016-2018 in five top psychology journals: Journal of Applied Psychology, Journal of Experimental Psychology: General, Developmental Psychology, Journal of Family Psychology, and Journal of Abnormal Psychology. Included articles utilized SEM, path analysis, or factor analysis as part of their primary analyses. I randomly selected 20 articles from each journal within the three-year span for a total of 100 articles.

Of the 100 reviewed articles, 44 reported the full sample correlation matrix in the article or in online supplemental materials. No article reported the covariance matrix, but 41 reported standard deviations of study variables in addition to the correlation matrix (thus providing sufficient information for the covariance matrix to be constructed). Thirty-one additional articles reported partial correlation matrices, including correlations between some of the manifest variables, between total scores, or between latent variables. Of the 75 articles reporting full or partial correlation matrices, 66 ( $88 \%$ ) reported significance (p-values) of the individual correlations in the matrix (only two used family-wise error correction); the other nine articles reported correlations without standard errors, CIs, or p-values.

From this review, it is evident that there is interest among psychological researchers in reporting correlation matrices, as well as some measure of uncertainty associated with those
correlations. However, fidelity in implementing best practices is imperfect. There are likely several reasons for this. First, as mentioned, reporting large correlation matrices may be limited by journal space considerations, although online supplement options alleviate many of these concerns. Second, a researcher may be uncertain how best to report a correlation matrix if they have missing data, as is often the case. Third, there are barriers to reporting parameter uncertainty for correlation coefficients, foremost the lack of software that readily reports standard errors or CIs for correlations. Therefore, researchers often resort to reporting p-values for individual correlations, most often denoted with asterisks or bolded font; this practice was noted and discouraged twenty years ago by Wilkinson et al. (1999).

However, there are issues with reporting significance of correlations in a matrix, most of which stem from the rote use of p -values supplied as default output from statistical software. First, these p-values are generally calculated for each correlation coefficient independently (this problem is discussed in detail in Section 1.5). Second, these $p$-values are often based on a $z$-test or $t$-test, the assumptions of which may be violated in small or skewed samples. Third, p-values supplied as default from software are typically associated with a nil-null test of population correlations, $\rho$; that is, they test if $\rho=0$ and provide no information for other interesting nullhypothesized values. And finally, even well-constructed p-values cannot provide an interval estimate for $\rho$. To capture more information on parameter variability, researchers can report confidence intervals instead of p -values.

### 1.3 Confidence Intervals for Correlation Coefficients

### 1.3.1 Confidence intervals

Confidence intervals (CIs) often simultaneously can give an estimate of parameter variability and provide information on null-hypothesis testing. Confidence intervals are interval estimates formed around a given point estimate (such as a correlation) and are characterized by an upper bound and a lower bound. For a given confidence level, $C$, over repeated sampling from the same population under the same sampling and model assumptions, the C\% CI should contain the true value of the parameter, $\rho, C \%$ of the time. In relation to NHST, the $C \% \mathrm{CI}$ contains all the null-hypothesized values, $\rho_{0}$, that would not be rejected at the two-tailed $\alpha$ level such that $(1-\alpha) \times 100=C$. As such, CIs are preferable to reporting a p-value for $\rho$ because the result of a significance test for any $\rho_{0}$ can be inferred from a CI. There are many techniques for calculating CIs for correlation coefficients, including the parametric $z$ CI technique, the Spearman rank-order CI technique, and bootstrap CI techniques.

### 1.3.2 Parametric z CI technique (z CI)

The parametric $z$ confidence interval (referred to as $z$ CI here for brevity) for the correlation coefficient relies on transforming the theoretical $r$ distribution using Fisher's $z$ transformation to an approximately standard normal distribution and using this distribution to create the bounds of the CI (Fisher, 1915). Fisher's transformation converts the naturally-skewed correlation, $r$, to a $z$-score using

$$
\begin{equation*}
z=\frac{1}{2} \ln \left(\frac{1+r}{1-r}\right) \tag{1}
\end{equation*}
$$

The standard error of the $z$-transformed variable is

$$
\begin{equation*}
S E_{z}=\sqrt{\frac{1}{N-3}} \tag{2}
\end{equation*}
$$

where $N$ is the sample size, i.e., the number of pairs of values. The $z$-transformed $(1-\alpha) \times$ $100 \% \mathrm{CI}$ for the population value of the correlation, $\rho$, is

$$
\begin{equation*}
z \pm z_{a / 2} \times S E_{z} \tag{3}
\end{equation*}
$$

where $z_{a / 2}$ is the critical value associated with the two-tailed $\alpha$. The limits of the $z$-transformed CI are then converted back into $r$ to create the limits of the CI for $\rho$. The $z$ CI has good coverage when $N$ is large and when the data are approximately bivariately normally distributed. However, when data are markedly non-normal, the $z$ CI has poor coverage (Bishara \& Hittner, 2017). When non-normality of data is suspected, the Spearman rank-order correlation coefficient, and the accompanying Spearman rank-order CI, may be computed.

### 1.3.3 Spearman rank-order CI technique (Sp CI)

The formula for the Spearman correlation is a special case of the Pearson correlation formula where data reflect rank-orders rather than quantitative measures. Alternatively, intervalor ratio-level quantitative data may be converted into ranks, and the Spearman correlation calculated, if robustness to non-normality in the correlation estimate is desired. The Spearman rank-order $\mathrm{CI}(S p \mathrm{CI})$ can similarly be constructed by converting the raw data into rank scores
within variable and proceeding to calculate the parametric CI as shown above. The equation for the standard error (Equation 2) is modified, however; several modifications have been proposed, but the modification such that

$$
\begin{equation*}
S E_{z(s)}=\sqrt{\frac{1+\frac{r^{2}}{2}}{N-3}} \tag{4}
\end{equation*}
$$

performs well under a variety of simulation techniques (Bonett and Wright, 2000).

### 1.3.4 Bootstrap CI techniques

Alternatively, non-normality may be dealt with by bootstrapping the data rather than transforming the data. Bootstrapping is a resampling method that offers an approach to CI construction that does not require reference to a theoretical distribution (Efron, 1979). Rather, the observed sample of $N$ observations is treated as a proxy for the population and $B$ new samples of size $N$ are drawn from the observed sample with replacement. The estimate of interest (in this context the correlation from the bootstrapped sample, $r^{*}$ ) is calculated for each of the $B$ bootstrapped samples. The $C \%$ "percentile bootstrap" CI is calculated using the values of $r^{*}$ that mark the middle $C \%$ of the distribution of $r^{*}$ values (of which there are $B$ total). Bootstrapping is hardcoded into many software programs and performs well under a variety of data-generating structures. Often, bias-correction and acceleration ( BCa ) adjustments are made to the bootstrap to account for possible bias (bias-correction) and skewness (acceleration) in the bootstrapped statistic distribution, thereby improving the performance of the bootstrap (Efron, 1987). BCa
correction is considered default by many researchers now over the naïve percentile bootstrap (Puth, Neuhäuser, \& Ruxton, 2015b).

There are two general approaches for constructing the bootstrapped CI for $\rho$ between two variables, $X_{1}$ and $X_{2}$. The first is called the bivariate bootstrap, or more generally the multivariate bootstrap. The multivariate bootstrap retains the pairing of $X_{1}$ and $X_{2}$ values - that is, $N$ cases are drawn with replacement from the $N$ cases in the observed sample. This approach is straightforward and has been used for decades (e.g., Diaconis \& Efron, 1983). However, in small samples, this approach can lead to invalid bootstraps - bootstrapped samples where only a single case is resampled or is substantially oversampled so as to cause the correlation to be incalculable.

The second bootstrapping approach, the univariate bootstrap, samples observations on each variable univariately, thereby unlinking paired scores on $X_{1}$ and $X_{2}$ within cases. This approach was first proposed as a method of null hypothesis significance testing (Lee \& Rodgers, 1998) and was expanded to construct confidence intervals for $\rho$ (Beasley et al. 2007; observedimposed univariate bootstrap). The univariate bootstrap allows for a larger sampling frame than the multivariate bootstrap; rather than sampling $N$ cases with replacement from $N$ cases, $N$ "cases" (constructed by sampling observations on each variable independently) are sampled from the $N^{2}$ possible pairings of $X_{1}$ and $X_{2}$ scores. The $N^{2}$ pairs necessarily have correlation $r_{12}=0$ (the sampling frame is a rectangle in the two-dimensional Cartesian plane); to reintroduce the original observed correlation back into the sampling frame, the $N^{2}$ cases in the sampling frame are rotated - diagonalized - by multiplying the sampling frame by the Cholesky factoring (or some other factoring) of the $2 \times 2$ correlation matrix $\boldsymbol{R}$ (Kaiser \& Dickman, 1962). $N$ cases are then sampled with replacement from the rotated $N^{2}$ cases in the sampling frame to create a bootstrapped sample, $\boldsymbol{X}_{b}^{\text {case }}$. The correlation, $r^{\text {case }}$ is computed from $\boldsymbol{X}_{b}^{\text {case }}$, and the process is
repeated $B$ times to create a distribution of $r^{c a s e}$ values. The $C \% \mathrm{CI}$ is constructed by using the values of $r^{\text {case }}$ that mark the middle $C \%$ of the distribution of $r^{\text {case }}$ values, with BCa adjustments if desired. The univariate bootstrap approach is less likely to create invalid bootstraps than the multivariate bootstrap approach and performs better than the multivariate bootstrap under a variety of simulation conditions (Beasley et al., 2007; Bishara \& Hittner, 2017).

### 1.4 Performance of CI Techniques for Correlations

Confidence interval performance is typically indicated by CI coverage, width, and balance of CI errors. Coverage is the percentage that a CI technique tends to include the true parameter value (i.e., $\rho$ ) over repeated sampling. The coverage should be equal or very close to the nominal rate; that is, a CI technique that nominally claims to include $\rho 95 \%$ of the time (the nominal rate) should actually include $\rho 95 \%$ of the time over repeated sampling (coverage probability). CIs with coverage that is higher than the nominal rate are considered too conservative, and CIs with coverage lower than the nominal rate are considered too liberal.

In addition to CI coverage, researchers indicate CI performance in terms of CI width. If two CI techniques have coverage close to the nominal rate, the CI with the shortest length tends to be preferred. Narrower CI widths allow the researcher to entertain a narrower window of plausible values for $\rho$. Additionally, researchers tend to prefer CI techniques that generally have a balance of CI errors (i.e., CIs that do not contain $\rho$ ), or an equal percentage of CI "misses" to the left and right of $\rho$, as this means the CI technique neither under- nor overcover the parameter on average.

Much of the literature that has reviewed the performance of statistical techniques for correlations has focused on null hypothesis significance testing, with emphasis on comparing power and Type I error rates of comparable statistical tests in the null-hypothesis significance testing (NHST) framework, as opposed to focusing specifically on confidence interval performance. For example, Beasley et al. (2007) did not explicitly report CI coverage and width for the bootstrap techniques they investigated, as their focus was primarily on NHST, but they did report Type I errors rates. However, studies on NHST can provide some indication of how CI techniques may perform. Coverage percentages can be extrapolated from reported Type I error percentages, assuming two-tailed tests are used: Coverage $=100-$ Type 1 error. Generally, conservative (too low) Type I error rates are associated with conservative (too high) CI coverage, and vice versa for liberal Type I error and CI coverage. Reported standard errors are related to CI widths, with smaller standard errors associated with narrower CI widths. For example, Beasley et al. (2007) found nominal Type I error rates across the conditions examined (including $N=5$ to $N$ $=60$ and a variety of non-normal distributions) for the bivariate and univariate bootstraps, indicating coverage rates under these data conditions would be near nominal levels for these bootstrap procedures. However, Type I errors rates were somewhat liberal for the bivariate bootstrap versus the univariate bootstrap and too liberal for non-normal versus normal conditions. Although the univariate bootstrap was very close to nominal coverage, there was a small trend of increased Type I error as $\rho$ increased for all $N \geq 10$ for the normal condition.

Recent work has focused directly on empirically testing and reporting CI performance among multiple CI techniques. This work allows applied researchers to feel more confident reporting and interpreting confidence intervals rather than point estimates, and it helps inform appropriate choices of CI techniques for correlations depending on the data conditions.

Particularly, Bishara and Hittner (2017) provided a recent and comprehensive review of CI techniques for correlation coefficients. In their study they investigated how non-normality affected CI performance for a range of CI techniques, including the $z \mathrm{CI}$, the $S p \mathrm{CI}$, the univariate bootstrap CI technique (uni CI), and the bivariate bootstrap (mult CI) across a range of smaller sample sizes ( $N=0$ to $N=160$ ) and two values of $\rho(0$ and .5). In this study, the $z \mathrm{CI}$ performed well under normal conditions, with worse performance under non-normality and as $\rho$ increased. There was no notable effect of sample size when $\rho=0$, but the $z$ CI performed worse as $N$ increased for $\rho=.5$. The $S p$ CI had much better coverage when distributions were nonnormal, but there was still a small trend of decreased performance as $N$ increased. The uni CI performed well for normal, but had slightly conservative coverage when $N$ was very small for non-normal conditions and when $\rho=0$. The mult CI typically was slightly liberal, with worse coverage for non-normal conditions.

### 1.5 Extending to Multiple Dependent Correlations

Although there is some guidance in the literature as to how CIs perform empirically for single correlations, there is almost no discussion in the literature on how different CI techniques perform when extended to multivariate settings. That is, the performance of CI techniques may differ when constructed simultaneously for multiple dependent correlations in a correlation matrix. There do not yet exist extensions of many CI techniques to multivariate settings, and therefore the analytic performance of these CIs is unknown.

Two mechanisms may cause CIs to perform differently in a multivariate setting: dependency among correlations and construction of multiple CIs simultaneously. First, as previously stated, a correlation matrix is not a mere assemblage of correlations, but rather a unit
with dependence among its constituent correlations. The value of a correlation in one part of the correlation matrix affects the permissible values for many other correlations in the matrix, socalled restriction of range (Hubert, 1972; Glass and Collins, 1970). This interdependence may cause CI performance to differ from independent correlation CI performance because the point estimates of correlations in the matrix may differ from those generated independently; for example, if one correlation in the matrix overestimates $\rho$, the population correlation, other correlations in the matrix may also be overestimated.

Second, it is well known that confidence intervals constructed individually differ from those constructed simultaneously (e.g., Durand, 1954). This inconsistency occurs because, although each confidence interval may have coverage at the nominal level (say, $95 \%$ ), the joint or family-wise coverage probability may be well below the nominal level. This problem has been examined extensively in terms of Type I error, such that conducting multiple NHSTs at a nominal $\alpha$ level can dramatically increase the actual Type I error rate (e.g., Bender \& Lange, 2001; Keselman, Cribbie, \& Holland, 1999). In terms of confidence intervals, this means that family-wise coverage may be significantly below the nominal rate, however, the precise rate of Type I error inflation or coverage deflation is difficult to analytically derive. An upper bound on Type I error inflation can be derived, for which the Bonferroni correction may be applied, but in practice Bonferroni correction produces overly conservative tests and CIs (Bender \& Lange, 2001). More research is needed to investigate the effect of simultaneous CI construction in multivariate settings generally, and for correlation matrices specifically.

### 1.6 Present Study and Hypotheses

Because the performance of many correlation CI techniques has not been evaluated analytically or asymptotically, I chose to evaluate several CI techniques using simulation. In this study, I compared the relative performance of several CI techniques while systematically varying the following data conditions.

### 1.6.1 Population distributions

Based on previous research, I hypothesized that the $z \mathrm{CI}$ and the bootstrap CIs would perform better when population distributions were normal compared to non-normal; however, I expected bootstrap CIs to outperform the $z \mathrm{CI}$ under non-normality. I also expected that the $S p$ CI would perform better under non-normality than normality. These hypotheses were consistent with findings from Bishara and Hittner (2017) and were consistent with the statistical theory of the parametric $z$ CI, Spearman CI, and bootstrap techniques generally. Previous literature has not directly investigated the effect of population distribution on CI width for CIs with similar coverage; this was an open empirical question.

### 1.6.2 Population correlation magnitude ( $\rho$ )

Theory and previous research indicated that the $z$ CI would have worse coverage when $\rho$ was of higher magnitude when population distributions depart from normality. Therefore, I hypothesized an interaction between $\rho$ magnitude and population distribution such that $\rho$ would not affect $z$ CI coverage when the distribution was normal, but higher magnitude $\rho$ would result in lower $z$ CI coverage when the distribution was non-normal.

Bishara and Hitner (2017) observed a minor decrease in coverage of the $S p \mathrm{CI}$ as $\rho$ increased in magnitude, although the $S p$ CI still had near-nominal coverage for both $\rho=0$ and $\rho=.5$; a similar trend was observed in coverage of the uni CI when distributions were normal, and an opposite trend was found when distributions were non-normal. Given the minor effects of $\rho$ magnitude observed in previous studies, I expected $\rho$ magnitude may have a minor effect on the coverage of the $S p$ CI and bootstrap CIs, and this effect may be moderated by the population distribution.

For all CI techniques, I hypothesized a main effect of $\rho$ magnitude on CI width. I expect narrower CIs when the magnitude of $\rho$ is large, compared to when $\rho$ is closer to zero, due to the boundedness of the correlation coefficient.

### 1.6.3 Sample size ( $N$ )

I hypothesized a main effect of $N$ on CI widths for all CI techniques, with narrower CIs expected as $N$ increased. This is readily deduced from CI formulas, because large N necessarily reduces the standard error, and has been demonstrated empirically. The CI techniques investigated in Bishara and Hittner (2017) did not show a strong main effect of $N$ on coverage probability, however, the uni CI had overly conservative coverage for small $N$ that reached closer-to-nominal coverage when $N$ increased. Therefore, I expected negligible effects of $N$ in the range of sample sizes that Bishara and Hittner (2017) observed, with perhaps stronger effects for larger $N$.

### 1.6.4 Number of variables (K)

According to the multiple testing literature, as the number of tests increase, the Type I error rate increases. As the number of variables $(K)$ increases in a correlation matrix, there are more CIs to construct, therefore I hypothesized coverage would be higher and CI widths wider as $K$ increased for any CIs that are constructed simultaneously, and coverage and CI widths would not be affected by $K$ for CIs constructed independently.

### 1.6.5 Matrix structure

CI performance for different types of correlation matrices of the same size have not been studied either analytically or empirically. Therefore, how the structure of the correlation matrix overall affects the performance of CI techniques was an empirical question. Correlation matrices with different eigenstructure or variability of correlations in the matrix may cause CI performance to differ.

### 1.6.6 Confidence interval construction

Given the literature on multiple CI construction, I expected that confidence intervals implemented simultaneously ("matrixwise", as indicated in Section 2.3) would have higher, closer-to-nominal coverage, and would be wider in width than those implemented independently ("pairwise", as indicated in Section 2.3). These findings would be consistent with previous research on Type I error rate correction.

## CHAPTER 2

## METHODS

### 2.1 Simulation Study Conditions

### 2.1.1 Population correlation matrix structure ( $\mathbf{P}$ )

In the systematic review conducted for SEM applications, the median reported maximum correlation (using the absolute value) was .695 , with maximum correlation magnitudes reported ranging from .20 to .95 (middle $80 \%$ of reported maximum correlations: .44 to .87 ). Studies vary in the maximum correlation reported, the average correlation reported, and the variability of correlations reported. Therefore, I varied the structure of the population correlation matrices, $\mathbf{P}$, used to generate samples of data.

To develop different $\mathbf{P}$ structures, I varied the maximum population correlation $(\rho)$ in $\mathbf{P}$ and the variability of correlations in $\mathbf{P}$, which had the effect of also varying the variability in eigenvalues of $\mathbf{P}$ and the relative size of the first eigenvalue of $\mathbf{P}$. To do this, I considered correlations that would be observed from different underlying factor and loading patterns. Variables were related by either one underlying factor $(F=1)$, two underlying factors (with a factor correlation of $0 ; F=2$ ), or three underlying factors (with factor correlations of $\{0, .25, .5\}$; $F=3$ ). As $F$ increased, the variability of correlations in $\mathbf{P}$ increased and the eigenstructure of $\mathbf{P}$ became more complex. Each $\mathbf{P}$ represented by $F=1$ had no variability in correlations (i.e., one unique value of $\rho$ for all off-diagonal elements of $\mathbf{P}$ ), and their eigenstructure was dominated by a single largest eigenvalue. Each $\mathbf{P}$ represented by $F=3$ had the highest variability in
correlations (i.e., four unique values of $\rho$ for the off-diagonal elements of $\mathbf{P}$ ), and their eigenstructure had three dominant eigenvalues of differing sizes.

Each of the $K$ variables loaded onto a single factor (i.e., there were no cross-loadings), and all variables within a particular $\mathbf{P}$ matrix had factor loadings that were $\sqrt{0.2}(\cong 0.447)$, $\sqrt{0.5}(\cong 0.707)$, or $\sqrt{0.8}(\cong 0.894)$. For ease of presentation and without loss of generality, I considered only nonnegative correlations for $\mathbf{P}$; this space corresponds to the positive manifold, as discussed in the intelligence literature and first noted by Spearman over a century ago (e.g., Van Der Maas, 2006). Varying the magnitude of the factor loadings changed the average and maximum $\rho$ in $\mathbf{P}$ and the relative size of the first eigenvalue within each $\mathbf{P}$. Matrices with $\sqrt{0.2}$ factor loadings had the lowest maximum $\rho=.2$ and the smallest first eigenvalue, and matrices with $\sqrt{0.8}$ loadings had the highest maximum $\rho=.8$ and the largest first eigenvalue.

In total, 9 correlation matrix structures for $\mathbf{P}$ (3 factor patterns $(F) \times 3$ loading patterns $(\rho))$ were investigated. Figures 1-3 summarize the correlation matrix structures and demonstrate how the factor and loading patterns produced different $6 \times 6 \mathbf{P}$. Table 1 summarizes the eigenstructure for the corresponding $6 \times 6 \mathbf{P}$. To develop $\mathbf{P}$ for larger $K$ (see below), I increased the number of variables that load onto each latent variable. This kept the eigenstructures indicated in Table 1 consistent across $\mathbf{P}$ of different sizes.


Loadings $=\sqrt{0.2}$
$\left[\begin{array}{cccccc}1 & & & & & \\ .20 & 1 & & & & \\ .20 & .20 & 1 & & & \\ .20 & .20 & .20 & 1 & & \\ .20 & .20 & .20 & .20 & 1 & \\ 20 & .20 & .20 & .20 & .20 & 1\end{array}\right]$
$\left.\begin{array}{c}\text { Loadings }=\sqrt{0.5} \\ {\left[\begin{array}{ccccc}1 & & & & \\ .50 & 1 & & & \\ .50 & .50 & 1 & & \\ .50 & .50 & .50 & 1 & \\ .50 & \\ .50 & .50 & .50 & .50 & 1 \\ .50 & .50 & .50 & .50 & .50 \\ \hline\end{array}\right]}\end{array} \begin{array}{ccccc}1 & & & & \\ .80 & 1 & & & \\ .80 & .80 & 1 & & \\ .80 & .80 & .80 & 1 & \\ .80 & .80 & .80 & .80 & 1 \\ .80 & .80 & .80 & .80 & .80 \\ \hline\end{array}\right]$

Figure 1. Correlation structures generated from one underlying factor $(F=1)$. Example population correlation matrices show the 6 -variable condition for $\mathbf{P}$ with loadings $\sqrt{0.2}, \sqrt{0.5}$, or $\sqrt{0.8}$.

Loadings $=\sqrt{0.2}$
$\left[\begin{array}{cccccc}1 & & & & & \\ .20 & 1 & & & & \\ .20 & .20 & 1 & & & \\ .00 & .00 & .00 & 1 & & \\ .00 & .00 & .00 & .20 & 1 & \\ .00 & .00 & .00 & .20 & .20 & 1\end{array}\right]$
Loadings $=\sqrt{0.5}$
Loadings $=\sqrt{0.8}$

$$
\left[\begin{array}{cccccc}
1 & & & & & \\
.50 & 1 & & & & \\
.50 & .50 & 1 & & & \\
.00 & .00 & .00 & 1 & & \\
.00 & .00 & .00 & .50 & 1 & \\
.00 & .00 & .00 & .50 & .50 & 1
\end{array}\right]
$$

$$
\left[\begin{array}{cccccc}
1 & & & & & \\
.80 & 1 & & & & \\
.80 & .80 & 1 & & & \\
.00 & .00 & .00 & 1 & & \\
.00 & .00 & .00 & .80 & 1 & \\
.00 & .00 & .00 & .80 & .80 & 1
\end{array}\right]
$$

Figure 2. Correlation structures generated from two underlying factors $(F=2)$. Example correlation matrices show the 6-variable condition for $\mathbf{P}$ with loadings $=\sqrt{0.2}, \sqrt{0.5}$, or $\sqrt{0.8}$.


Loadings $=\sqrt{0.2}$
$\left[\begin{array}{cccccc}1 & & & & & \\ .20 & 1 & & & & \\ .00 & .00 & 1 & & & \\ .00 & .00 & .20 & 1 & & \\ .10 & .10 & .05 & .05 & 1 & \\ .10 & .10 & .05 & .05 & .20 & 1\end{array}\right]$

$$
\begin{gathered}
\text { Loadings }=\sqrt{0.5} \\
{\left[\begin{array}{ccccc}
1 & & & & \\
.50 & 1 & & & \\
.0 & \text { Loadings }=\sqrt{0.8} \\
.00 & .00 & 1 & & \\
.00 & .00 & .50 & 1 & \\
.25 & .25 & .125 & .125 & 1 \\
.25 & \\
25 & .25 & .125 & .125 & .50 \\
.
\end{array}\right]\left[\begin{array}{ccccc}
1 & & & & \\
.80 & 1 & & & \\
.00 & .00 & 1 & & \\
.00 & .00 & .80 & 1 & \\
.40 & .40 & .20 & .20 & 1 \\
.40 & .40 & .20 & .20 & .80 \\
\hline
\end{array}\right]}
\end{gathered}
$$

Figure 3. Correlation structures generated from three underlying factors ( $F=3$ ). Example correlation matrices show the 6 -variable condition for $\mathbf{P}$ with loadings $=\sqrt{0.2}, \sqrt{0.5}$, or $\sqrt{0.8}$.

| $\rho$ |  | $F=1$ | $F=2$ |  | $F=3$ |
| ---: | ---: | :--- | :--- | :--- | :--- |
| .2 | Eigenvalues | $2.0,0.8,0.8,0.8,0.8,0.8$ | $1.4,1.4,0.8,0.8,0.8,0.8$ | $1.42,1.2,0.98,0.8,0.8,0.8$ |  |
|  | Relative \% | 33.3 | 23.3 | 23.7 |  |
| .5 | Eigenvalues | $3.5,0.5,0.5,0.5,0.5,0.5$ | $2.0,2.0,0.5,0.5,0.5,0.5$ | $2.06,1.5,0.94,0.5,0.5,0.5$ |  |
|  | Relative \% | 58.3 | 33.3 | 34.3 |  |
| .8 | Eigenvalues | $5.0,0.2,0.2,0.2,0.2,0.2$ | $2.6,2.6,0.2,0.2,0.2,0.2$ | $2.69,1.8,0.91,0.2,0.2,0.2$ |  |
|  | Relative \% | 83.3 | 43.3 | 44.9 |  |

Table 1. Eigenstructure of $6 \times 6 \mathbf{P} . F=$ number of underlying factors for $\mathbf{P}, \rho=$ largest population correlation in $\mathbf{P}$. Relative $\%=$ percent of first eigenvalue relative to the sum of the eigenvalues. Eigenvalues and relative \% are similar for $\mathbf{P}$ with different size $K$.

### 2.1.2 Number of variables (K)

According to the systematic review conducted, the median number of study variables reported was 20 variables, with reported number of variables ranging from 3 to 164 (middle 80\%: 8 and 54 variables). For this study, I investigated correlation matrices of order $K=6,18$,

30, and 42. Correlation structures for the 6-variable condition for $\mathbf{P}$ are shown in Figure 1.a-c. $\mathbf{P}$ generated using $F=1$ had all variables $(6,18,30$, or 42 total) load onto a single factor. $\mathbf{P}$ generated using $F=2$ had one-half of the variables $(3,9,15$, or 21$)$ as indicators of each factor. $\mathbf{P}$ generated using $F=3$ had one-third of the variables ( $2,6,10$, or 14 ) as indicators of each factor.

### 2.1.3 Variable population distributions

To investigate the performance of these CI techniques under different variable distributions, I used the mvrnonnorm function in the semTools R package, to generate samples of observations from normal and non-normal populations with $\mathbf{P}$ specified above. The mvrnonnorm function implements Vale and Maurelli's (1983) power constants approach for using polynomial transformations of standard normal data to generate multivariate non-normal data with specified mean, covariance, skew, and kurtosis. Extensions to the power constants approach exist: Headrick (2002) extended this approach to allow specification of the first six moments of the distribution, rather than the first four moments that can be specified by the Vale and Maurelli (1983) approach. I chose the Vale and Maurelli (1983) method because 1) the function is hardcoded into a popular R package, and 2) this method is fast, which was necessary to reduce computing time.

I varied the degree of non-normality for variables in the population by varying skew and kurtosis. Samples were drawn from populations with 1) skew=0 and kurtosis=0 (i.e., normally distributed, "Normal"); 2) skew=1 and kurtosis=1 ("Low Skew"); or 3) skew=2 and kurtosis=6 ("High Skew"). All variables were sampled from populations with means of 0 and variance of 1. Figure 4 demonstrates univariate and bivariate plots with 10,000 randomly generated
observations to visualize the extent of non-normality investigated across the simulation conditions.

### 2.1.4 Sample size ( $N$ )

According to the systematic review conducted, the median sample size $(N)$ reported was 447 observations, with sample sizes ranging from 80 to 50,712 observations (middle 80\%: 161 and 2,243 observations). Therefore, for this study, I examined samples of size $N=50,100,500$, and 2000.


Figure 4. Bivariate and univariate plots representing sample distributions for population correlation $\rho=$ .8 and $N=10,000$. Rows represent the normal, low skew (skew $=1$, kurtosis $=1$ ), and high skew (skew=2, kurtosis $=6$ ) conditions respectively. Columns represent the scatterplots (with both variables drawn from the same distribution), and the frequency plots respectively. Observed correlations are shown on the scatterplots. Standard normal distributions are superimposed in blue on the relative frequency plots to assist in visualizing degree of non-normality.

### 2.2 Summary of Simulation Conditions

In total, this simulation considered $3(F) \times 3(\rho) \times 4(K) \times 3$ (distribution) $\times 4(N)=432$ conditions for the population. For each condition, I generated a large number of samples (i.e., replications, $M$ ), and for each sample I calculated $95 \%$ confidence intervals for each correlation in the matrix using four techniques: the parametric $z \mathrm{CI}(z \mathrm{CI})$, the Spearman rank-order $\mathrm{CI}(S p$ $C I$ ), the univariate bootstrap CI (uni CI), and the multivariate bootstrap CI (mult CI). To summarize results for the many correlations in each correlation matrix, I present here only the results for the largest $\rho$ in each correlation matrix: either $\rho=.2, \rho=.5$, or $\rho=.8$. All correlations in a matrix that share the same value of $\rho$ are treated as exchangeable, and their results are presented together.

Due to the many duplicate values of $\rho$ for conditions with large $K$, and due to the high computation time for these conditions, I decreased the number of $M$ samples generated for conditions with larger $K$. For $K=6$ and $K=18$, I generated $M=1000$ samples; for $K=30$, I generated $M=750$ samples; and for $K=42$, I generated $M=500$ samples per condition. The number of factors $(F)$ also affected the number of total replications as different $F$ were associated with different numbers of each $\rho$ appearing in $\mathbf{P}$. Therefore, the total replications for results differed depending on $K$ and $F$; the number of replications by condition are reported in Table 2.

|  |  | $F=1$ | $F=2$ | $F=3$ |
| :--- | :--- | ---: | ---: | ---: |
| $K=6$ | $M=1000$ | 15,000 | 6,000 | 3,000 |
| $K=18$ | $M=1000$ | 153,000 | 72,000 | 45,000 |
| $K=30$ | $M=750$ | 326,250 | 157,500 | 101,250 |
| $K=42$ | $M=500$ | 430,500 | 210,000 | 136,500 |

Table 2. Number of total replications generated for each $\rho$ by correlation matrix size $(K)$ and underlying factors $(F) . M=$ number of Monte Carlo samples generated.

### 2.3 Confidence Interval Techniques

In this study I compared four confidence interval techniques: two parametric confidence interval techniques ( $z \mathrm{CI}$ and $S p \mathrm{CI}$ ) and two bootstrap techniques (mult CI and uni CI ). To investigate the effect of implementing these techniques on correlations in the context of a correlation matrix, I applied the two bootstrap CI techniques on pairs of variables ("pairwise", denoted with the subscript $P$ notation) and on all variables simultaneously ("matrixwise", denoted with the subscript $M$ notation). Therefore, the CI techniques were investigated using an incomplete two-factor design: CI technique ( $z \mathrm{CI}, S p \mathrm{CI}$, uni CI , and mult CI ) by CI application (pairwise and matrixwise, for the bootstrap techniques only).

### 2.3.1 Parametric z (z CI)

The $95 \% z$ CI for each correlation in the matrix was calculated using the Fisher $z$ transformation, as described previously. There is currently no extension of the $z$ CI technique to multivariate research settings, therefore this technique was applied only pairwise, and not matrixwise.

### 2.3.2 Spearman rank-order ( Sp CI)

The $95 \% S p$ CI for each correlation in the matrix was calculated by converting raw scores to ranks within each variable and calculating the CI, as described previously. There is currently no extension of the Spearman rank-order technique to multivariate research settings, therefore this technique was applied only pairwise, and not matrixwise.

### 2.3.3 Multivariate bootstrap (mult CI)

The mult CI was implemented by bootstrapping cases, as described previously with $B=$ 1000 bootstapped samples. The mult CI was implemented pairwise ( mult $_{P} \mathrm{CI}$ ) and matrixwise ( mult $_{M} \mathrm{CI}$ ). To implement mult $_{M} \mathrm{CI}$, the bootstrapped sample $\boldsymbol{X}_{b}^{\text {multM }}$ was created by sampling $N$ cases from the original $N$ cases with replacement, leaving all $K$ observations for a particular case linked. Stated another way, rows were sampled with replacement from the entire, original data frame, as in Bradley (1978). The bootstrapped correlation matrix $\boldsymbol{R}_{b}^{m u l t M}$ was calculated on $\boldsymbol{X}_{b}^{\text {multM }} . B=1000$ bootstrapped samples were used to create a distribution of $\boldsymbol{R}_{b}^{\text {multM }} .95 \%$ CIs for each correlation $\rho$ in $\mathbf{P}$ were calculated marginally by using the percentile bootstrap method with BCa adjustments for the distribution of $r_{b}^{\text {multM }}$ values for each $\rho$ respectively. To implement mult $_{P}$ CI, the multivariate bootstrap procedure as just described was implemented separately for each of the $\frac{1}{2} K(K-1)$ pairs of variables in the matrix, thereby distributions of $r_{b}^{\text {multP }}$ values for each $\rho$ are independent of the other variables in the matrix. Stated another way, each pair of variables was treated in turn as the entire data frame, and rows were sampled with replacement from this smaller data frame. BCa adjustments were then applied to calculate $95 \%$ multp CIs.

### 2.3.4 Univariate bootstrap

The univariate bootstrap was implemented in both a pairwise (uniP CI ) and matrixwise (uni ${ }_{\mathbf{M}} \mathrm{CI}$ ) manner. The uni procedure was implemented as developed by Beasley et al. (2007) separately for each of the $\frac{1}{2} K(K-1)$ pairs of variables in the matrix. $B=1000$ bootstrap iterations were used for each pair of variables to create a distribution of $r_{b}^{\text {uniP }}$ values. BCa
adjustments were then applied to each distribution of $r_{b}^{\text {uniP }}$ values separately to calculate $95 \%$ CIs.

The observed-imposed univariate bootstrap proposed by Beasley et al. (2007) posed an issue for direct extension to matrixwise implementation. The original procedure requires creating the entire null-sampling frame ( $N^{2}$ cases large when $K=2$ ), rotating the frame with Cholesky factoring of $\boldsymbol{R}$, and drawing $N$ cases with replacement from this frame to create $\boldsymbol{X}_{b}^{u n i}$. In the more general case, where $K>2$, the null-sampling frame has $N^{K}$ cases, each with observations on $K$ variables; this becomes unwieldy quickly. For instance, the null-sampling frame for $N=25$ people measured on $K=10$ variables would contain 9,765,625 total cases from which to draw samples of 25 with replacement to construct $\boldsymbol{X}_{b}^{u n i m}$; the sampling frame for $N=2,000$ and $K=$ 42, the largest dataset investigated in this simulation, is much larger. Therefore, the uni ${ }_{M}$ procedure instead created each bootstrapped sample, $\boldsymbol{X}_{b}^{\text {unim }}$ by 1) bootstrapping $N$ observations with replacement from each of the $K$ variables independently to create a "null" bootstrapped sample of $N$ "cases" observed on $K$ variables, 2) re-standardizing each variable, then 3) rotating this null bootstrapped sample with Cholesky factoring of the observed $\boldsymbol{R} . \boldsymbol{R}_{b}^{\text {unim }}$ was then calculated on $\boldsymbol{X}_{b}^{u n i m}$.

Each null bootstrapped sample (i.e., $\boldsymbol{X}_{b}^{u n i M}$ before Cholesky rotation) has bivariate correlations near zero; any correlation between independently sampled variables is spurious. This is in contrast to the correlation matrix of the null sampling frame in the Beasley et al. (2007) approach, which is necessarily equal to an identity matrix. After applying the Cholesky transformation to $\boldsymbol{X}_{b}^{u n i M}, \boldsymbol{R}_{b}^{u n i m}$ is close to $\boldsymbol{R}$, but not exactly equal to $\boldsymbol{R}$. $B=1000$ bootstrap samples were used to create distributions of $\boldsymbol{R}_{b}^{\text {unim }} .95 \%$ CIs for each correlation $\rho$ in $\mathbf{P}$ were
calculated marginally by using the percentile bootstrap method with BCa adjustments for the distribution of $r_{b}^{\text {unim }}$ values for each $\rho$ respectively.

### 2.4 Summary of CI Techniques Investigated

The performance of four CI techniques was investigated in this study: the $z \mathrm{CI}$, the $S p \mathrm{CI}$, the mult CI, and the uni CI. The two bootstrap CI techniques were implemented in two manners: pairwise, which calculates a CI for each pair of variables while ignoring relationships among other variables, and matrixwise, which calculates CIs for each correlation simultaneously. The $z$ and $S p$ CIs do not have extensions to multivariate settings and so were only implemented pairwise. Therefore, the CI techniques investigated represent an incomplete 2-factor investigation: type of CI (parametric, Spearman, multivariate, univariate) by implementation method (pairwise and matrixwise).

### 2.5 Outcomes of Interest

To assess the performance of the four CI techniques across simulation conditions, I assessed coverage, with CI techniques including $\rho 95 \%$ of the time performing better than those that include $\rho$ more or less frequently. I considered successful coverage performance to be any CI coverage between $92.5 \%-97.5 \%$; this meets the criterion set forth by Bradley (1978) as the most liberal, but still entertainable, quantitative definition of successful performance. I also assessed the width of the CI ; assuming equal coverage, CI techniques that tended to be narrower were considered more successful. Finally, I investigated misses to the left and right of the CI. Assuming 95\% coverage, CI techniques that tended to produce equal numbers of misses to the
left ( $2.5 \%, \rho$ is less than the CI) and to the right $(2.5 \%, \rho$ is greater than CI) were considered more successful.

## CHAPTER 3

## RESULTS

### 3.1 Simulation Check

Before investigating the performance of the CI techniques, I first ensured that simulated samples were able to recover the value of $\rho$ on average in each condition. Table 3 reports the median generated values of $\rho$. There are no apparent systematic patterns in median observed correlations among variables due to sample size, correlation magnitude, matrix variability, or distribution type.

### 3.2 Pairwise CI Coverage Results

The CI coverage for the pairwise-implemented CI techniques are reported in Appendix A. Because the pairwise CI techniques were expected to perform consistently across the $K$ and $F$ factors (see Discussion), the coverage results below for the pairwise CI techniques are collapsed across the $K$ and $F$ factors, and only results for the $K=42, F=1$ conditions are shown below in Table 4. Below, results are discussed by CI technique.

|  |  |  | $N=50$ |  |  | $N=100$ |  |  | $N=500$ |  |  | $N=2000$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | K | $F$ | N | L | H | N | L | H | N | L | H | N | L | H |
| $\begin{aligned} & \text { پ } \\ & \text { ॥ } \\ & \text { e } \end{aligned}$ | 6 | 1 | . 199 | . 205 | . 195 | . 199 | . 201 | . 200 | . 200 | . 199 | . 200 | . 200 | . 200 | . 200 |
|  |  | 2 | . 201 | . 204 | . 199 | . 199 | . 202 | . 195 | . 200 | . 201 | . 200 | . 200 | . 200 | . 199 |
|  |  | 3 | . 199 | . 203 | . 190 | . 204 | . 199 | . 200 | . 201 | . 200 | . 199 | . 200 | . 200 | . 200 |
|  | 18 | 1 | . 202 | . 200 | . 195 | . 201 | . 200 | . 197 | . 200 | . 200 | . 200 | . 200 | . 200 | . 199 |
|  |  | 2 | . 202 | . 200 | . 197 | . 202 | . 199 | . 198 | . 200 | . 200 | . 199 | . 200 | . 200 | . 200 |
|  |  | 3 | . 201 | . 202 | . 196 | . 200 | . 197 | . 197 | . 201 | . 200 | . 199 | . 200 | . 200 | . 200 |
|  | 30 | 1 | . 204 | . 201 | . 198 | . 202 | . 200 | . 196 | . 200 | . 199 | . 200 | . 201 | . 200 | . 200 |
|  |  | 2 | . 201 | . 201 | . 195 | . 203 | . 199 | . 199 | . 201 | . 200 | . 199 | . 200 | . 200 | . 200 |
|  |  | 3 | . 201 | . 201 | . 194 | . 201 | . 199 | . 198 | . 200 | . 200 | . 199 | . 200 | . 200 | . 200 |
|  | 42 | 1 | . 202 | . 199 | . 195 | . 202 | . 203 | . 199 | . 200 | . 200 | . 200 | . 200 | . 200 | . 200 |
|  |  | 2 | . 202 | . 199 | . 198 | . 200 | . 201 | . 198 | . 200 | . 200 | . 199 | . 200 | . 200 | . 200 |
|  |  | 3 | . 201 | . 199 | . 197 | . 200 | . 200 | . 200 | . 200 | . 200 | . 199 | . 200 | . 200 | . 200 |
|  | 6 | 1 | . 506 | . 500 | . 500 | . 502 | . 501 | . 500 | . 501 | . 502 | . 501 | . 500 | . 500 | . 500 |
|  |  | 2 | . 505 | . 506 | . 502 | . 501 | . 501 | . 501 | . 501 | . 501 | . 501 | . 500 | . 500 | . 500 |
|  |  | 3 | . 504 | . 506 | . 507 | . 500 | . 502 | . 502 | . 501 | . 500 | . 500 | . 500 | . 500 | . 500 |
|  | 18 | 1 | . 502 | . 504 | . 502 | . 503 | . 502 | . 502 | . 501 | . 501 | . 501 | . 500 | . 500 | . 500 |
|  |  | 2 | . 501 | . 506 | . 506 | . 503 | . 503 | . 502 | . 500 | . 500 | . 500 | . 500 | . 500 | . 500 |
| ! |  | 3 | . 504 | . 503 | . 504 | . 502 | . 502 | . 503 | . 500 | . 501 | . 499 | . 500 | . 500 | . 500 |
| $\begin{aligned} & \\| \\ & 2 \end{aligned}$ | 30 | 1 | . 507 | . 502 | . 504 | . 503 | . 504 | . 503 | . 501 | . 501 | . 501 | . 500 | . 500 | . 500 |
|  |  | 2 | . 505 | . 504 | . 504 | . 501 | . 500 | . 502 | . 501 | . 500 | . 501 | . 501 | . 500 | . 500 |
|  |  | 3 | . 504 | . 505 | . 502 | . 503 | . 502 | . 503 | . 502 | . 500 | . 499 | . 500 | . 500 | . 500 |
|  | 42 | 1 | . 505 | . 502 | . 502 | . 503 | . 502 | . 503 | . 499 | . 501 | . 499 | . 501 | . 499 | . 500 |
|  |  | 2 | . 502 | . 503 | . 504 | . 502 | . 503 | . 503 | . 501 | . 500 | . 502 | . 500 | . 500 | . 500 |
|  |  | 3 | . 506 | . 501 | . 505 | . 502 | . 504 | . 500 | . 501 | . 501 | . 499 | . 500 | . 500 | . 501 |
| $\infty$ | 6 | 1 | . 802 | . 804 | . 805 | . 801 | . 802 | . 803 | . 800 | . 801 | . 801 | . 800 | . 800 | . 800 |
|  |  | 2 | . 804 | . 802 | . 805 | . 803 | . 802 | . 803 | . 800 | . 800 | . 799 | . 800 | . 800 | . 800 |
|  |  | 3 | . 802 | . 802 | . 804 | . 801 | . 803 | . 802 | . 800 | . 800 | . 801 | . 800 | . 801 | . 800 |
|  | 18 | 1 | . 804 | . 803 | . 804 | . 801 | . 802 | . 802 | . 800 | . 800 | . 801 | . 800 | . 800 | . 800 |
|  |  | 2 | . 804 | . 804 | . 805 | . 802 | . 802 | . 802 | . 800 | . 801 | . 800 | . 800 | . 800 | . 800 |
|  |  | 3 | . 804 | . 802 | . 806 | . 802 | . 801 | . 803 | . 800 | . 801 | . 801 | . 800 | . 800 | . 800 |
| $\begin{aligned} & \\| \\ & Q \end{aligned}$ | 30 | 1 | . 801 | . 806 | . 808 | . 802 | . 801 | . 803 | . 801 | . 800 | . 801 | . 800 | . 800 | . 800 |
|  |  | 2 | . 804 | . 805 | . 806 | . 801 | . 802 | . 803 | . 800 | . 800 | . 801 | . 800 | . 800 | . 800 |
|  |  | 3 | . 803 | . 804 | . 806 | . 802 | . 802 | . 804 | . 800 | . 800 | . 801 | . 800 | . 800 | . 800 |
|  | 42 | 1 | . 802 | . 804 | . 807 | . 803 | . 803 | . 801 | . 801 | . 800 | . 799 | . 800 | . 800 | . 800 |
|  |  | 2 | . 803 | . 803 | . 805 | . 802 | . 802 | . 804 | . 800 | . 801 | . 800 | . 800 | . 800 | . 800 |
|  |  | 3 | . 803 | . 802 | . 806 | . 802 | . 803 | . 803 | . 800 | . 800 | . 800 | . 800 | . 800 | . 800 |

Table 3. Median correlations generated across the 432 simulation conditions. $K=$ number of variables, $F$ $=$ variability of correlation matrices ( $1=$ no variability, $2=$ medium variability, $3=$ high variability $), N=$ sample size, $\mathrm{N}=$ normal distribution, $\mathrm{L}=$ low skew, $\mathrm{H}=$ high skew.

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | uni ${ }_{P}$ |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\text {ni }}{ }_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dis | $\rho$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| N | . 2 | . 9503 | . 0258 | . 9537 | . 0183 | . 9373 | . 0321 | . 9483 | . 0266 | . 9506 | . 0250 | . 9470 | . 0143 | . 9470 | . 0265 | . 9486 | . 0257 |
|  | . 5 | . 9512 | . 0270 | . 9521 | . 0133 | . 9379 | . 0332 | . 9477 | . 0285 | . 9494 | . 0229 | . 9166 | . 0111 | . 9459 | . 0241 | . 9470 | . 0235 |
|  | . 8 | . 9540 | . 0256 | . 9491 | . 0081 | . 9401 | . 0308 | . 9457 | . 0282 | . 9507 | . 0252 | . 8901 | . 0391 | . 9467 | . 0263 | . 9475 | . 0259 |
| L | . 2 | . 9401 | . 0342 | . 9541 | . 0214 | . 9310 | . 0314 | . 9402 | . 0379 | . 9395 | . 0323 | . 9509 | . 0271 | . 9448 | . 0276 | . 9395 | . 0344 |
|  | . 5 | . 9305 | . 0399 | . 9571 | . 0165 | . 9274 | . 0343 | . 9362 | . 0348 | . 9271 | . 0398 | . 9554 | . 0205 | . 9432 | . 0290 | . 9388 | . 0327 |
|  | . 8 | . 9251 | . 0485 | . 9603 | . 0127 | . 9268 | . 0395 | . 9354 | . 0341 | . 9165 | . 0412 | . 9451 | . 0105 | . 9427 | . 0263 | . 9476 | . 0229 |
| H | . 2 | . 9177 | . 0554 | . 9523 | . 0289 | . 9149 | . 0316 | . 9295 | . 0569 | . 9108 | . 0510 | . 9328 | . 0586 | . 9349 | . 0304 | . 9217 | . 0513 |
|  | . 5 | . 8737 | . 0810 | . 9592 | . 0241 | . 9045 | . 0384 | . 9237 | . 0429 | . 8595 | . 0733 | . 9314 | . 0616 | . 9299 | . 0304 | . 9355 | . 0322 |
|  | . 8 | . 8458 | . 1044 | . 9675 | . 0155 | . 9035 | . 0466 | . 9412 | . 0273 | . 8290 | . 0851 | . 9631 | . 0285 | . 9303 | . 0288 | . 9661 | . 0120 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| N | . 2 | . 9508 | . 0260 | . 9524 | . 0185 | . 9427 | . 0298 | . 9490 | . 0266 | . 9505 | . 0251 | . 9322 | . 0093 | . 9481 | . 0262 | . 9482 | . 0262 |
|  | . 5 | . 9502 | . 0272 | . 9484 | . 0121 | . 9423 | . 0305 | . 9477 | . 0278 | . 9490 | . 0268 | . 8428 | . 0740 | . 9460 | . 0280 | . 9467 | . 0275 |
|  | . 8 | . 9539 | . 0270 | . 9422 | . 0083 | . 9455 | . 0302 | . 9491 | . 0285 | . 9467 | . 0246 | . 6772 | . 3034 | . 9437 | . 0255 | . 9441 | . 0253 |
| L | . 2 | . 9394 | . 0356 | . 9526 | . 0248 | . 9362 | . 0317 | . 9387 | . 0390 | . 9400 | . 0306 | . 9507 | . 0289 | . 9473 | . 0260 | . 9402 | . 0314 |
|  | . 5 | . 9239 | . 0443 | . 9519 | . 0204 | . 9299 | . 0355 | . 9332 | . 0375 | . 9287 | . 0335 | . 9540 | . 0163 | . 9474 | . 0238 | . 9414 | . 0271 |
|  | . 8 | . 9102 | . 0539 | . 9579 | . 0126 | . 9247 | . 0383 | . 9326 | . 0320 | . 9183 | . 0439 | . 9448 | . 0083 | . 9475 | . 0272 | . 9518 | . 0243 |
| H | . 2 | . 9142 | . 0554 | . 9501 | . 0340 | . 9204 | . 0323 | . 9247 | . 0575 | . 9082 | . 0491 | . 8734 | . 1240 | . 9413 | . 0286 | . 9211 | . 0462 |
|  | . 5 | . 8692 | . 0810 | . 9530 | . 0346 | . 9124 | . 0385 | . 9265 | . 0415 | . 8603 | . 0752 | . 8420 | . 1570 | . 9417 | . 0294 | . 9431 | . 0307 |
|  | . 8 | . 8429 | . 0924 | . 9654 | . 0206 | . 9088 | . 0402 | . 9477 | . 0209 | . 8226 | . 0980 | . 9291 | . 0677 | . 9397 | . 0312 | . 9726 | . 0129 |

Table 4. CI coverage for pairwise-implemented techniques. Results reported for the $K=42, F=1$, which represents coverage percentages from CIs for 430,500 sample correlations in each cell. $N=$ sample size. Cov = coverage probabilities across the 430,500 replications for each technique. Left = probability of $\rho$ lower than CI boundary (i.e., miss to the left).

### 3.2.1 Parametric z

The $z$ CI was only implemented pairwise. Coverage for the normal conditions was around the nominal $95 \%$ coverage rate for all $N$ and for all values of $\rho$ : for $\rho=.2$, coverage for the four sample size conditions was $.9503, .9508, .9506$, and .9505 for $N=50,100,500$, and 2000 respectively; for $\rho=.5$, coverage was $.9512, .9502, .9494$, and .9490 respectively; and for $\rho=.8$, coverage was $.9540, .9539, .9507$, and .9467 respectively. Coverage for the low skew conditions was lower (liberal) compared to the normal conditions; this was true for virtually all values of $\rho$. In addition, coverage decreased further as $\rho$ increased: for $\rho=.2$, coverage was $.9540, .9539$, .9507 , and .9467 respectively; for $\rho=.5$, coverage was $.9305, .9239, .9271$, and .9287 respectively; and for $\rho=.8$, coverage was $.9251, .9102, .9165$, and .9183 . Coverage for the high skew conditions was overall lower (liberal) than the low skew condition; with coverage also decreasing as $\rho$ increased. In addition, there was a notable trend of performance due to sample size, with larger $N$ associated with more liberal coverage: for $\rho=.2$, coverage was $.9177, .9142$, .9108 , and .9082 respectively for $N=50,100,500$, and 2000 ; for $\rho=.5$, coverage was .8737 , $.8692, .8595$, and .8603 ; and for $\rho=.8$, coverage was $.8458, .8429, .8290$, and .8226 .

### 3.2.2 Spearman rank-order

The $S p$ CI was only implemented pairwise. Coverage was dependent on distribution type, $N$, and $\rho$ magnitude. Generally, the $S p$ CI performed better when $N$ was smaller and distributions were skewed.

For the normal distribution conditions, coverage was best at smaller sample sizes and with smaller $\rho$ : for $N=50$, coverage was $.9537, .9521$, and .9491 for $\rho=.2, \rho=.5$, and $\rho=.8$ respectively; for $N=100$, coverage was $.9524, .9484$, and .9422 ; for $N=500, .9470, .9166$, and
.8901 ; and for $N=2000$, coverage was $.9322, .8428$, and .6772 . For the low skew distribution conditions, coverage was close to the nominal rate for all values of $N$ and $\rho$, and unlike with the normal conditions there were no strong trends in terms of $N$ or $\rho$ : for $N=50$, coverage was $.9541, .9571$, and 9603 ; for $N=100$, coverage was $.9526, .9519$, and .9579 ; for $N=500$, coverage was $.9509, .9554$, and .9451 ; and for $N=2000$, coverage was $.9507, .9540$, and .9448 . For the high skew distribution conditions, coverage was best for small $N$ and small $\rho$, with large $\rho$ associated with conservative intervals for small $N$ and large $\rho$ associated with liberal intervals for large N : for $N=50$, coverage was .9523 , .9592 , and .9675 ; for $N=100, .9501, .9530$, and .9654 ; for $N=500$, coverage was $.9328, .9314$, and .9631 ; and for $N=2000$, coverage was .8734, .8420, and .9291 .

### 3.2.3 Multivariate (pairwise)

The mult ${ }_{P}$ CI performed adequately (i.e., within the 92.5-97.5 range) under most conditions, although tended to be slightly below $95 \%$ coverage. Although the mult ${ }_{P}$ CI had more consistent performance under different distributions relative to the $z$ CI and $S p$ CI, performance was closest to nominal level when the distribution was normal. Coverage was also best when $N$ was small, and coverage was not largely influenced by $\rho$.

Coverage for the normal conditions was close to $95 \%$ coverage rate for all $N$ and for all values of $\rho$, with slightly better performance as $N$ increased: for $N=50$, coverage was .9373 , .9379 , and .9401 for $\rho=.2, \rho=.5$, and $\rho=.8$ respectively; for $N=100$, coverage was .9427 , .9423 , and .9455 ; for $N=500$, coverage was $.9470, .9459$, and .9467 ; and for $N=2000$, coverage was $.9481, .9460$, and .9437 . A similar trend was observed for the low skew conditions, but overall results for the low skew conditions were slightly more liberal than the normal
conditions. There was also a small effect of $\rho$, such that larger $\rho$ tended to have slightly worse coverage: for $N=50$, coverage was $.9310, .9274$, and .9268 ; for $N=100$, coverage was .9362 , .9299 , and .9247 ; for $N=500$, coverage was $.9448, .9432$, and .9427 ; and for $N=2000$, coverage was $.9473, .9474$, and .9475 . The pattern of results was augmented for the high skew conditions; overall performance was more liberal for the high skew conditions relative to the low skew conditions, with more liberal coverage associated with smaller $N$ and larger $\rho$ : for $N=50$, coverage was $.9149, .9045$, and .9035 ; for $N=100$, coverage was $.9204, .9124$, and .9088 ; for $N$ $=500$, coverage was $.9349, .9299$, and .9303 ; and for $N=2000$, coverage was $.9413, .9417$, and . 9397.

### 3.2.4 Univariate (pairwise)

The uniP CI performed adequately (i.e., within the 92.5-97.5 range) under most simulation conditions. Generally, the $u n i_{P} \mathrm{CI}$ had higher coverage than the mult $_{P} \mathrm{CI}$; that is, the coverage was less liberal than the mult $_{P} \mathrm{CI}$, and in some cases was slightly conservative. Like the uni $i_{P} \mathrm{CI}$, coverage was less affected by distribution than the $z \mathrm{CI}$ and the $S p \mathrm{CI}$, but coverage was still best when the distribution was normal. When distributions were skewed, there were more notable effects of $\rho$ and $N$.

Coverage for the normal condition was very close to $95 \%$ coverage rate for conditions, with little trend evident due to $N$ and $\rho$ : for $N=50$, coverage was $.9483, .9477$, and .9457 for $\rho$ $=.2, \rho=.5$, and $\rho=.8$ respectively; for $N=100$, coverage was $.9490, .9477$, and .9491 ; for $N=$ 500 , coverage was $.9486, .9470$, and .9475 ; and for $N=2000$, coverage was $.9482, .9467$, and .9441. For the low skew distribution conditions, coverage decreased with larger $\rho$ when $N$ was small, but when $N$ was larger the trend in coverage reversed: $N=50$, coverage was $.9402, .9362$,
and .9354 ; for $N=100$, coverage was $.9387, .9332$, and .9326 ; for $N=500$, coverage was .9395 , .9388 , and .9476 ; and for $N=2000$, coverage was $.9402, .9414$, and .9518 . For the high skew conditions, larger $\rho$ was associated with increased coverage, with stronger effects as $N$ increased: $N=50$, coverage was. $9295, .9237$, and .9412 ; for $N=100$, coverage was .9247 , .9265 , and .9477 ; for $N=500$, coverage was $.9217, .9355$, and .9661 ; and for $N=2000$, coverage was $.9211, .9431$, and .9726 .

### 3.3 Pairwise CI Width Results

Median, first, and third quartile CI widths across replications for all pairwiseimplemented CI techniques are reported in Appendix B, but a summary of results for the $K=42$, $F=1$ conditions is presented in Table 5. For all CI techniques, larger $N$ and $\rho$ were associated with narrower CI intervals. Across all conditions, the median CI width for the z CI was narrower than the $S p \mathrm{CI}$, with larger differences in width for small $N$ and large $\rho$ conditions compared to the large $N$ and small $\rho$ conditions. Results for the two bootstrap CIs were mixed. The mult CI tended to be narrower than the uni CI , but in conditions for which coverage was comparable between the two CI techniques, the widths of the CIs were also comparable.

|  |  |  |  |  | Sp |  |  | mult $^{\text {P }}$ |  |  | uni ${ }_{P}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dis | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| $N=50$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| N | . 2 | . 5090 | . 5370 | . 5516 | . 5189 | . 5384 | . 5507 | 4658 | 5277 | 6005 | . 5280 | . 5466 | 5640 |
|  | . 5 | . 3709 | . 4247 | . 4712 | . 4075 | . 4444 | . 4776 | . 4029 | . 4617 | . 5261 | . 4391 | . 4857 | . 5259 |
|  | . 8 | . 1636 | . 2045 | . 2490 | . 2078 | . 2418 | . 2786 | . 1991 | . 2441 | . 2957 | . 2311 | . 2796 | . 3344 |
| L | . 2 | . 5113 | . 5360 | . 5506 | . 5229 | . 5411 | . 5520 | . 4752 | . 5218 | . 5726 | . 5111 | . 5339 | . 5512 |
|  | . 5 | . 3803 | . 4245 | . 4649 | . 4165 | . 4531 | . 4856 | . 3783 | . 4301 | . 4841 | . 3924 | . 4362 | . 4765 |
|  | . 8 | . 1736 | . 2069 | . 2431 | . 2145 | . 2501 | . 2880 | . 1790 | . 2160 | . 2582 | . 1857 | . 2217 | . 2614 |
| H | . 2 | . 5120 | . 5354 | . 5501 | . 5248 | . 5423 | . 5525 | . 4678 | . 5135 | . 5613 | . 5073 | . 5315 | . 5496 |
|  | . 5 | . 3831 | . 4232 | . 4607 | . 4232 | . 4593 | . 4908 | . 3590 | . 4091 | . 4618 | . 3786 | . 4207 | . 4604 |
|  | . 8 | . 1793 | . 2093 | . 2423 | . 2268 | . 2635 | . 3023 | . 1708 | . 2039 | . 2420 | . 1750 | . 2063 | . 2408 |
| $N=100$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| N | . 2 | . 3642 | . 3779 | . 3869 | . 3696 | . 3792 | . 3862 | . 3463 | . 3866 | . 4360 | . 3789 | . 3899 | . 4018 |
|  | . 5 | . 2694 | . 2963 | . 3205 | . 2901 | . 3089 | . 3264 | . 3021 | . 3387 | . 3804 | . 3267 | . 3520 | . 3769 |
|  | . 8 | . 1235 | . 1444 | . 1665 | . 1469 | . 1637 | . 1815 | . 1519 | . 1771 | . 2062 | . 1776 | . 2045 | . 2347 |
| L | . 2 | . 3651 | . 3773 | . 3860 | . 3719 | . 3810 | . 3875 | . 3484 | . 3752 | . 4051 | . 3646 | . 3769 | . 3878 |
|  | . 5 | . 2744 | . 2966 | . 3177 | . 2978 | . 3165 | . 3337 | . 2796 | . 3086 | . 3393 | . 2858 | . 3082 | . 3294 |
|  | . 8 | . 1257 | . 1429 | . 1615 | . 1530 | . 1708 | . 1898 | . 1342 | . 1541 | . 1761 | . 1388 | . 1576 | . 1777 |
| H | . 2 | . 3660 | . 3774 | . 3858 | . 3737 | . 3823 | . 3883 | . 3423 | . 3672 | . 3933 | . 3615 | . 3747 | . 3860 |
|  | . 5 | . 2763 | . 2964 | . 3158 | . 3031 | . 3212 | . 3378 | . 2638 | . 2900 | . 3177 | . 2733 | . 2947 | . 3152 |
|  | . 8 | . 1286 | . 1433 | . 1592 | . 1595 | . 1780 | . 1973 | . 1243 | . 1411 | . 1598 | . 1262 | . 1421 | . 1592 |
| $N=500$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| N | . 2 | . 1659 | . 1684 | . 1706 | . 1673 | . 1691 | . 1707 | . 1718 | . 1849 | . 2010 | . 1724 | . 1765 | . 1810 |
|  | . 5 | . 1266 | . 1320 | . 1371 | . 1333 | . 1371 | . 1407 | . 1530 | . 1646 | . 1785 | . 1590 | . 1665 | . 1753 |
|  | . 8 | . 0594 | . 0637 | . 0682 | . 0682 | . 0716 | . 0750 | . 0794 | . 0867 | . 0952 | . 0934 | . 1011 | . 1102 |
| L | . 2 | . 1661 | . 1684 | . 1704 | . 1683 | . 1700 | . 1714 | . 1654 | . 1727 | . 1807 | . 1651 | . 1689 | . 1727 |
|  | . 5 | . 1273 | . 1316 | . 1359 | . 1362 | . 1399 | . 1434 | . 1341 | . 1412 | . 1487 | . 1331 | . 1382 | . 1433 |
|  | . 8 | . 0600 | . 0635 | . 0671 | . 0707 | . 0744 | . 0783 | . 0658 | . 0704 | . 0753 | . 0673 | . 0717 | . 0763 |
| H | . 2 | . 1663 | . 1684 | . 1703 | . 1689 | . 1705 | . 1719 | . 1605 | . 1666 | . 1730 | . 1634 | . 1673 | . 1713 |
|  | . 5 | . 1280 | . 1320 | . 1359 | . 1391 | . 1427 | . 1462 | . 1249 | . 1309 | . 1372 | . 1264 | . 1313 | . 1362 |
|  | . 8 | . 0603 | . 0633 | . 0664 | . 0735 | . 0772 | . 0810 | . 0591 | . 0628 | . 0667 | . 0595 | . 0629 | . 0665 |
| $N=2000$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| N | . 2 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0908 | . 0953 | . 1006 | . 0866 | . 0885 | . 0905 |
|  | . 5 | . 0644 | . 0658 | . 0670 | . 0674 | . 0684 | . 0693 | . 0811 | . 0851 | . 0898 | . 0824 | . 0853 | . 0884 |
|  | . 8 | . 0304 | . 0315 | . 0327 | . 0346 | . 0355 | . 0363 | . 0423 | . 0447 | . 0474 | . 0498 | . 0525 | . 0555 |
| L | . 2 | . 0836 | . 0842 | . 0847 | . 0845 | . 0850 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |
|  | . 5 | . 0648 | . 0658 | . 0669 | . 0690 | . 0699 | . 0708 | . 0691 | . 0713 | . 0737 | . 0676 | . 0693 | . 0711 |
|  | . 8 | . 0307 | . 0316 | . 0324 | . 0360 | . 0369 | . 0378 | . 0340 | . 0353 | . 0367 | . 0347 | . 0359 | . 0373 |
| H | . 2 | . 0836 | . 0842 | . 0846 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0820 | . 0837 | . 0855 |
|  | . 5 | . 0647 | . 0657 | . 0667 | . 0702 | . 0711 | . 0719 | . 0634 | . 0653 | . 0673 | . 0637 | . 0654 | . 0671 |
|  | . 8 | . 0308 | . 0316 | . 0324 | . 0375 | . 0384 | . 0394 | . 0303 | . 0314 | . 0325 | . 0304 | . 0314 | . 0325 |

Table 5. CI widths for pairwise-implemented techniques. CI widths calculated as the upper bound of the CI minus the lower bound of the CI. Results reported for the $K=42, F=1$ conditions, which represents replications from 430,500 sample correlations in each cell. Q1 = first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile. $N=$ sample size, $\mathrm{N}=$ normal distribution, $\mathrm{L}=$ low skew distribution, $\mathrm{H}=$ high skew distribution.

### 3.4 Matrixwise versus Pairwise Performance

### 3.4.1 Multivariate bootstrap CI

There was little to no difference in CI coverage between mult $_{P}$ and mult $_{M} \mathrm{CI}$ techniques in terms of $K$ or $F$. Table 6 shows CI coverage for all conditions where $\rho=.8$, with the higher CI coverage of mult $_{P}$ and mult $_{M}$ highlighted in blue. The only conditions for which CI coverage seems systematically different were for the $N=2000$ and normal conditions, for which mult $_{M}$ tended to have very slightly higher CI coverage; however, this pattern was not evident for other distribution types or sample sizes, therefore, this pattern may be spurious. This lack of pattern was consistent for the two smaller $\rho$ conditions as well. Figure 5 shows coverage rates for mult $_{P}$ and mult $_{M}$ separated by distribution type, $F, \rho$, and $N$; there are no meaningful differences in performance between the two techniques.

Likewise, there was little difference in CI width between mult $_{P}$ and mult $_{M}$, as shown in Table 7 for the $\rho=.8$ conditions. Although the results showed a possible, slight tendency of the mult $_{M} \mathrm{CI}$ to be wider for the normal and high skew conditions, these results were not practically relevant, as results between the two CI techniques were nearly equal up to four decimal places; this tendency was also not observed for the low skew condition. Results for $\rho=.2$ and $\rho=.5$ were similar to the $\rho=.8$ condition.

|  |  |  | Normal |  |  |  | Low Skew |  |  |  | High Skew |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | K | $F$ | mult $_{P}$ | mult $_{M}$ | $u^{\prime \prime} i_{P}$ | uni $_{M}$ | mult $_{P}$ | mult $_{\text {M }}$ | $u^{\text {uni }}$ | uni $_{M}$ | mult $_{P}$ | mult $_{\text {M }}$ | uni $_{P}$ | uni $_{M}$ |
|  | 6 | 1 | . 9359 | . 9369 | . 9411 | . 8357 | . 9271 | . 9275 | . 9365 | . 7828 | . 9033 | . 9029 | . 9406 | . 6735 |
|  |  | 2 | . 9315 | . 9322 | . 9343 | . 8003 | . 9170 | . 9208 | . 9277 | . 7480 | . 8978 | . 8967 | . 9382 | . 6502 |
|  |  | 3 | . 9367 | . 9370 | . 9407 | . 7993 | . 9267 | . 9280 | . 9360 | . 7417 | . 9047 | . 9043 | . 9383 | . 6457 |
|  | 18 | 1 | . 9395 | . 9399 | . 9449 | . 8691 | . 9221 | . 9215 | . 9307 | . 8137 | . 9015 | . 9011 | . 9379 | . 7144 |
|  |  | 2 | . 9382 | . 9375 | . 9425 | . 8744 | . 9220 | . 9225 | . 9311 | . 8180 | . 8981 | . 8982 | . 9353 | . 7133 |
| i |  | 3 | . 9364 | . 9364 | . 9429 | . 8787 | . 9243 | . 9242 | . 9324 | . 8289 | . 9002 | . 8998 | . 9374 | . 7252 |
| $\begin{aligned} & 11 \\ & z \end{aligned}$ | 30 | 1 | . 9348 | . 9351 | . 9406 | . 8767 | . 9249 | . 9248 | . 9344 | . 8268 | . 8963 | . 8963 | . 9342 | . 7081 |
|  |  | 2 | . 9349 | . 9352 | . 9403 | . 8886 | . 9201 | . 9203 | . 9294 | . 8313 | . 9008 | . 9010 | . 9388 | . 7389 |
|  |  | 3 | . 9352 | . 9352 | . 9411 | . 8952 | . 9204 | . 9203 | . 9289 | . 8408 | . 8995 | . 8994 | . 9378 | . 7439 |
|  | 42 | 1 | . 9401 | . 9405 | . 9457 | . 8886 | . 9268 | . 9265 | . 9354 | . 8401 | . 9035 | . 9038 | . 9412 | . 7318 |
|  |  | 2 | . 9326 | . 9322 | . 9377 | . 8946 | . 9248 | . 9251 | . 9330 | . 8543 | . 8988 | . 8984 | . 9369 | . 7437 |
|  |  | 3 | . 9386 | . 9386 | . 9444 | . 9074 | . 9251 | . 9250 | . 9334 | . 8599 | . 9018 | . 9021 | . 9378 | . 7555 |
|  | 6 | 1 | . 9455 | . 9458 | . 9470 | . 8452 | . 9329 | . 9341 | . 9403 | . 7873 | . 9087 | . 9073 | . 9501 | . 6703 |
|  |  | 2 | . 9377 | . 9368 | . 9415 | . 8032 | . 9265 | . 9270 | . 9357 | . 7458 | . 9143 | . 9163 | . 9568 | . 6537 |
|  |  | 3 | . 9467 | . 9497 | . 9493 | . 8000 | . 9263 | . 9257 | . 9323 | . 7233 | . 9063 | . 9080 | . 9447 | . 6330 |
|  | 18 | 1 | . 9392 | . 9383 | . 9424 | . 8580 | . 9328 | . 9325 | . 9395 | . 8137 | . 9088 | . 9085 | . 9478 | . 6999 |
|  |  | 2 | . 9429 | . 9430 | . 9459 | . 8672 | . 9288 | . 9290 | . 9369 | . 8046 | . 9102 | . 9098 | . 9492 | . 7019 |
| 8 |  | 3 | . 9389 | . 9392 | . 9422 | . 8681 | . 9329 | . 9322 | . 9394 | . 8197 | . 9043 | . 9044 | . 9458 | . 7039 |
| II | 30 | 1 | . 9362 | . 9366 | . 9391 | . 8625 | . 9333 | . 9333 | . 9405 | . 8252 | . 9101 | . 9094 | . 9502 | . 7108 |
| $\geq$ |  | 2 | . 9438 | . 9433 | . 9469 | . 8833 | . 9308 | . 9308 | . 9378 | . 8263 | . 9078 | . 9078 | . 9477 | . 7167 |
|  |  | 3 | . 9423 | . 9415 | . 9452 | . 8863 | . 9309 | . 9312 | . 9390 | . 8344 | . 9129 | . 9129 | . 9518 | . 7284 |
|  | 42 | 1 | . 9455 | . 9460 | . 9491 | . 8801 | . 9247 | . 9245 | . 9326 | . 8096 | . 9088 | . 9083 | . 9477 | . 7291 |
|  |  | 2 | . 9410 | . 9407 | . 9441 | . 8871 | . 9322 | . 9325 | . 9401 | . 8382 | . 9077 | . 9077 | . 9482 | . 7231 |
|  |  | 3 | . 9391 | . 9391 | . 9414 | . 8906 | . 9295 | . 9291 | . 9369 | . 8399 | . 9116 | . 9126 | . 9512 | . 7385 |
|  | 6 | 1 | . 9438 | . 9425 | . 9426 | . 8359 | . 9393 | . 9394 | . 9438 | . 7825 | . 9300 | . 9299 | . 9671 | . 6693 |
|  |  | 2 | . 9452 | . 9445 | . 9453 | . 8050 | . 9400 | . 9425 | . 9452 | . 7478 | . 9308 | . 9337 | . 9710 | . 6445 |
|  |  | 3 | . 9530 | . 9517 | . 9540 | . 7987 | . 9433 | . 9453 | . 9450 | . 7433 | . 9240 | . 9237 | . 9593 | . 6130 |
|  | 18 | 1 | . 9476 | . 9473 | . 9487 | . 8619 | . 9437 | . 9444 | . 9491 | . 8077 | . 9304 | . 9303 | . 9663 | . 6904 |
|  |  | 2 | . 9491 | . 9490 | . 9500 | . 8610 | . 9408 | . 9402 | . 9454 | . 7979 | . 9297 | . 9295 | . 9654 | . 6914 |
| $\begin{aligned} & 8 \\ & i n \end{aligned}$ |  | 3 | . 9470 | . 9459 | . 9473 | . 8622 | . 9426 | . 9432 | . 9482 | . 8086 | . 9302 | . 9301 | . 9674 | . 6923 |
| $\stackrel{11}{ }$ | 30 | 1 | . 9465 | . 9464 | . 9474 | . 8592 | . 9414 | . 9413 | . 9464 | . 8077 | . 9301 | . 9300 | . 9675 | . 6906 |
| z |  | 2 | . 9439 | . 9441 | . 9451 | . 8572 | . 9426 | . 9430 | . 9471 | . 8110 | . 9326 | . 9326 | . 9688 | . 6967 |
|  |  | 3 | . 9420 | . 9422 | . 9430 | . 8656 | . 9414 | . 9410 | . 9453 | . 8192 | . 9280 | . 9280 | . 9651 | . 7027 |
|  | 42 | 1 | . 9467 | . 9470 | . 9475 | . 8610 | . 9427 | . 9429 | . 9476 | . 8067 | . 9303 | . 9301 | . 9661 | . 6996 |
|  |  | 2 | . 9461 | . 9461 | . 9466 | . 8655 | . 9420 | . 9427 | . 9480 | . 8128 | . 9268 | . 9262 | . 9652 | . 6917 |
|  |  | 3 | . 9465 | . 9466 | . 9469 | . 8756 | . 9435 | . 9436 | . 9481 | . 8295 | . 9274 | . 9276 | . 9652 | . 7123 |
|  | 6 | 1 | . 9409 | . 9423 | . 9425 | . 8337 | . 9467 | . 9486 | . 9515 | . 7944 | . 9409 | . 9391 | . 9734 | . 6770 |
|  |  | 2 | . 9437 | . 9438 | . 9440 | . 8108 | . 9472 | . 9473 | . 9535 | . 7470 | . 9382 | . 9373 | . 9745 | . 6287 |
|  |  | 3 | . 9420 | . 9437 | . 9393 | . 7877 | . 9457 | . 9473 | . 9470 | . 7313 | . 9460 | . 9443 | . 9760 | . 6310 |
|  | 18 | 1 | . 9467 | . 9471 | . 9471 | . 8566 | . 9460 | . 9459 | . 9502 | . 8053 | . 9389 | . 9390 | . 9725 | . 6850 |
|  |  | 2 | . 9476 | . 9478 | . 9476 | . 8505 | . 9426 | . 9428 | . 9466 | . 7972 | . 9390 | . 9390 | . 9721 | . 6820 |
| ৪ion |  | 3 | . 9451 | . 9458 | . 9450 | . 8569 | . 9431 | . 9439 | . 9491 | . 8040 | . 9400 | . 9410 | . 9738 | . 6948 |
| II | 30 | 1 | . 9524 | . 9520 | . 9524 | . 8650 | . 9465 | . 9466 | . 9507 | . 8065 | . 9379 | . 9380 | . 9718 | . 6863 |
| $z$ |  | 2 | . 9444 | . 9437 | . 9441 | . 8557 | . 9437 | . 9436 | . 9475 | . 8005 | . 9411 | . 9410 | . 9737 | . 6912 |
|  |  | 3 | . 9497 | . 9508 | . 9513 | . 8716 | . 9456 | . 9456 | . 9499 | . 8178 | . 9417 | . 9426 | . 9740 | . 7056 |
|  | 42 | 1 | . 9437 | . 9439 | . 9441 | . 8509 | . 9475 | . 9474 | . 9518 | . 8062 | . 9397 | . 9395 | . 9726 | . 6868 |
|  |  | 2 | . 9460 | . 9456 | . 9461 | . 8572 | . 9427 | . 9428 | . 9469 | . 8017 | . 9363 | . 9364 | . 9707 | . 6857 |
|  |  | 3 | . 9477 | . 9480 | . 9476 | . 8704 | . 9479 | . 9477 | . 9522 | . 8269 | . 9415 | . 9413 | . 9743 | . 7069 |

Table 6. CI coverage probabilities for pairwise and matrixwise implementations of mult CI and uni CI for $\rho=0.8$. The CI technique (pairwise or matrixwise) which had the highest coverage for each of mult CI and uni CI is shaded in blue. $K=$ number of variables, $F=$ variability of correlation matrices ( $1=$ no variability, $2=$ medium variability, $3=$ high variability), $N=$ sample size. Additional results for matrixwise CI coverage are presented in Appendix C.


Figure 5. Comparison of pairwise and matrixwise bootstrap CI coverage. Colors represent $\rho:$ red $=.2$, blue $=.5$, green $=.8$. Shape of point represents distribution type: circle $=$ normal, square $=$ low skew, diamond $=$ high skew. Point fill and position within CI technique represents $F$ : left, unshaded $=1$; middle, crossed $=2$, filled, right $=3$.

|  |  |  | Normal |  |  |  | Low Skew |  |  |  | High Skew |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | K | $F$ | mult $_{P}$ | mult $_{\text {M }}$ | uni $_{P}$ | uni $_{M}$ | mult $_{P}$ | mult $_{\text {M }}$ | uni $_{P}$ | uni $_{M}$ | mult $_{P}$ | mult $_{\text {M }}$ | uni $_{P}$ | uni $_{M}$ |
|  | 618 | 1 | . 2034 | . 2030 | . 2058 | . 1488 | . 2180 | . 2184 | . 2233 | . 1477 | . 2457 | . 2459 | . 2826 | . 1467 |
|  |  | 2 | . 2004 | . 2012 | . 2039 | . 1379 | . 2194 | . 2184 | . 2232 | . 1395 | . 2466 | . 2471 | . 2813 | . 1373 |
|  |  | 3 | . 2028 | . 2024 | . 2054 | . 1343 | . 2188 | . 2179 | . 2253 | . 1352 | . 2459 | . 2457 | . 2813 | . 1339 |
| $\begin{aligned} & \text { on } \\ & \text { II } \\ & z \end{aligned}$ |  | 1 | . 2013 | . 2014 | . 2041 | . 1581 | . 2183 | . 2183 | . 2246 | . 1598 | . 2467 | . 2469 | . 2830 | . 1598 |
|  |  | 2 | . 2018 | . 2016 | . 2042 | . 1624 | . 2171 | . 2173 | . 2227 | . 1625 | . 2452 | . 2454 | . 2802 | . 1616 |
|  | 30 | 3 | . 2022 | . 2023 | . 2041 | . 1660 | . 2187 | . 2187 | . 2249 | . 1668 | . 2450 | . 2446 | . 2795 | . 1632 |
|  |  | 1 | . 2037 | . 2038 | . 2063 | . 1652 | . 2167 | . 2166 | . 2226 | . 1623 | . 2441 | . 2441 | . 2790 | . 1614 |
|  |  | 2 | . 2007 | . 2008 | . 2033 | . 1702 | . 2178 | . 2175 | . 2234 | . 1708 | . 2457 | . 2456 | . 2812 | . 1696 |
|  | 42 | 3 | . 2017 | . 2017 | . 2046 | . 1748 | . 2172 | . 2172 | . 2225 | . 1740 | . 2465 | . 2460 | . 2823 | . 1726 |
|  |  | 1 | . 2039 | . 2040 | . 2063 | . 2063 | . 2160 | . 2158 | . 2217 | . 1665 | . 2441 | . 2441 | . 2796 | . 1646 |
|  |  | 2 | . 2010 | . 2010 | . 2038 | . 1766 | . 2189 | . 2188 | . 2249 | . 1783 | . 2448 | . 2451 | . 2804 | . 1752 |
|  |  | 3 | . 2020 | . 2019 | . 2044 | . 1803 | . 2193 | . 2193 | . 2251 | . 1814 | . 2455 | . 2457 | . 2813 | . 1775 |
|  | 6 | 1 | . 1421 | . 1422 | . 1430 | . 1042 | . 1548 | . 1548 | . 1581 | . 1036 | . 1792 | . 1789 | . 2084 | . 1033 |
|  | 18 | 2 | . 1409 | . 1408 | . 1427 | . 0959 | . 1549 | . 1544 | . 1584 | . 0962 | . 1790 | . 1790 | . 2070 | . 0956 |
|  |  | 3 | . 1411 | . 1413 | . 1421 | . 0926 | . 1543 | . 1545 | . 1578 | . 0923 | . 1790 | . 1793 | . 2084 | . 0935 |
| $\frac{8}{11}$ |  | 1 | . 1418 | . 1418 | . 1427 | . 1097 | . 1545 | . 1546 | . 1581 | . 1094 | . 1803 | . 1804 | . 2086 | . 1105 |
|  |  | 2 | . 1409 | . 1409 | . 1417 | . 1099 | . 1551 | . 1550 | . 1586 | . 1098 | . 1802 | . 1802 | . 2085 | . 1104 |
|  | 30 | 3 | . 1412 | . 1411 | . 1422 | . 1123 | . 1545 | . 1543 | . 1582 | . 1123 | . 1791 | . 1791 | . 2075 | . 1119 |
|  |  | 1 | . 1406 | . 1407 | . 1416 | . 1105 | . 1547 | . 1546 | . 1583 | . 1119 | . 1791 | . 1790 | . 2070 | . 1125 |
|  |  | 2 | . 1418 | . 1418 | . 1428 | . 1143 | . 1544 | . 1544 | . 1580 | . 1138 | . 1792 | . 1791 | . 2068 | . 1140 |
|  | 42 | 3 | . 1408 | . 1408 | . 1421 | . 1169 | . 1543 | . 1543 | . 1576 | . 1165 | . 1798 | . 1798 | . 2080 | . 1160 |
|  |  | 1 | . 1411 | . 1412 | . 1421 | . 1118 | . 1541 | . 1542 | . 1576 | . 1124 | . 1771 | . 1768 | . 2045 | . 1154 |
|  |  | 2 | . 1413 | . 1413 | . 1423 | . 1167 | . 1548 | . 1549 | . 1584 | . 1167 | . 1789 | . 1790 | . 2070 | . 1161 |
|  |  | 3 | . 1416 | . 1417 | . 1428 | . 1196 | . 1550 | . 1550 | . 1586 | . 1191 | . 1799 | . 1798 | . 2081 | . 1192 |
| $\begin{aligned} & 8 \\ & \text { n } \\ & \text { II } \\ & z \end{aligned}$ | 6 | 1 | . 0628 | . 0628 | . 0629 | . 0463 | . 0704 | . 0704 | . 0717 | . 0462 | . 0864 | . 0865 | . 1010 | . 0462 |
|  | 18 | 2 | . 0630 | . 0631 | . 0631 | . 0424 | . 0703 | . 0703 | . 0716 | . 0422 | . 0865 | . 0863 | . 1009 | . 0428 |
|  |  | 3 | . 0630 | . 0630 | . 0631 | . 0402 | . 0702 | . 0701 | . 0717 | . 0404 | . 0860 | . 0861 | . 1004 | . 0407 |
|  |  | 1 | . 0630 | . 0630 | . 0630 | . 0478 | . 0704 | . 0704 | . 0717 | . 0476 | . 0867 | . 0867 | . 1015 | . 0478 |
|  |  | 2 | . 0629 | . 0629 | . 0630 | . 0475 | . 0703 | . 0703 | . 0716 | . 0473 | . 0862 | . 0862 | . 1009 | . 0476 |
|  | 30 | 3 | . 0629 | . 0629 | . 0630 | . 0486 | . 0702 | . 0702 | . 0714 | . 0483 | . 0864 | . 0865 | . 1012 | . 0484 |
|  |  | 1 | . 0628 | . 0628 | . 0629 | . 0476 | . 0702 | . 0702 | . 0715 | . 0479 | . 0862 | . 0862 | . 1010 | . 0480 |
|  |  | 2 | . 0628 | . 0628 | . 0629 | . 0481 | . 0703 | . 0703 | . 0716 | . 0482 | . 0864 | . 0863 | . 1011 | . 0482 |
|  | 42 | 3 | . 0629 | . 0629 | . 0630 | . 0496 | . 0702 | . 0701 | . 0715 | . 0495 | . 0864 | . 0864 | . 1012 | . 0497 |
|  |  | 1 | . 0628 | . 0628 | . 0629 | . 0478 | . 0704 | . 0704 | . 0717 | . 0481 | . 0867 | . 0867 | . 1011 | . 0489 |
|  |  | 2 | . 0628 | . 0629 | . 0630 | . 0485 | . 0702 | . 0702 | . 0715 | . 0484 | . 0864 | . 0864 | . 1011 | . 0488 |
|  |  | 3 | . 0628 | . 0629 | . 0629 | . 0499 | . 0703 | . 0703 | . 0716 | . 0501 | . 0865 | . 0865 | . 1010 | . 0504 |
| $\begin{aligned} & 8 \\ & 8 \\ & \text { N} \\ & 11 \\ & z \end{aligned}$ | 6 | 1 | . 0314 | . 0314 | . 0314 | . 0232 | . 0354 | . 0354 | . 0360 | . 0232 | . 0446 | . 0446 | . 0523 | . 0232 |
|  |  | 2 | . 0314 | . 0314 | . 0314 | . 0212 | . 0354 | . 0354 | . 0360 | . 0212 | . 0445 | . 0445 | . 0522 | . 0212 |
|  | 18 | 3 | . 0314 | . 0314 | . 0314 | . 0196 | . 0353 | . 0354 | . 0360 | . 0196 | . 0446 | . 0448 | . 0523 | . 0199 |
|  |  | 1 | . 0314 | . 0314 | . 0314 | . 0237 | . 0354 | . 0354 | . 0360 | . 0237 | . 0446 | . 0446 | . 0524 | . 0237 |
|  | 30 | 2 | . 0314 | . 0314 | . 0314 | . 0236 | . 0354 | . 0354 | . 0360 | . 0236 | . 0446 | . 0446 | . 0523 | . 0236 |
|  |  | 3 | . 0314 | . 0314 | . 0314 | . 0240 | . 0354 | . 0354 | . 0360 | . 0240 | . 0447 | . 0447 | . 0525 | . 0241 |
|  |  | 1 | . 0314 | . 0314 | . 0314 | . 0237 | . 0354 | . 0354 | . 0360 | . 0237 | . 0446 | . 0446 | . 0523 | . 0238 |
|  |  | 2 | . 0314 | . 0314 | . 0314 | . 0238 | . 0354 | . 0354 | . 0360 | . 0238 | . 0446 | . 0446 | . 0523 | . 0238 |
|  | 42 | 3 | . 0314 | . 0314 | . 0314 | . 0245 | . 0354 | . 0354 | . 0360 | . 0245 | . 0447 | . 0447 | . 0524 | . 0245 |
|  |  | 1 | . 0314 | . 0314 | . 0314 | . 0237 | . 0353 | . 0353 | . 0359 | . 0237 | . 0447 | . 0447 | . 0525 | . 0237 |
|  |  | 2 | . 0314 | . 0314 | . 0315 | . 0238 | . 0353 | . 0353 | . 0359 | . 0238 | . 0446 | . 0446 | . 0523 | . 0239 |
|  |  | 3 | . 0314 | . 0314 | . 0314 | . 0246 | . 0353 | . 0353 | . 0360 | . 0245 | . 0447 | . 0447 | . 0524 | . 0247 |

Table 7. Median CI widths for pairwise- and matrixwise-implemented mult CI and uni CI for $\rho=0.8$. Widths are calculated as the CI upper bound minus the CI lower bound. The CI technique (pairwise or matrixwise) which had the largest width for each of mult CI and uni CI is shaded in blue. $K=$ number of variables, $F=$ variability of correlation matrices $(1=$ no variability, $2=$ medium variability, $3=$ high variability), $N=$ sample size. Additional results for matrixwise CI widths are presented in Appendix D.

### 3.4.2 Univariate bootstrap CI

The uni ${ }_{P}$ and $\boldsymbol{u n i}_{M}$ CI techniques had very different performance from each other in terms of both coverage and CI width because the procedure implemented between the two was slightly different. The uni $_{M}$ CI had far worse (lower) coverage than the uni ${ }_{P}$ CI (see Table 6 for $\rho=.8$ results), such that the $\boldsymbol{u n i}_{M} \mathrm{CI}$ had acceptable coverage for only certain conditions. The uni CI performed best when $\rho=.2$ or $\rho=.5$ and when the variable distributions were normal, however, as $N$ increased performance of the $u n i_{M} \mathrm{CI}$ improved such that some low skew conditions reached acceptable coverage (see Figure 5). The widths of the uni $_{M} \mathrm{CI}$ were much narrower than the unip CI (Table 7); often the width for the uni $_{M} \mathrm{CI}$ was approximately $60 \%$ of the width of the uni ${ }_{P} \mathrm{CI}$ technique.

Although the $\boldsymbol{u n i}_{M} \mathrm{CI}$ technique performed poorly across many simulation conditions, the method showed differential performance due to $K$ and $F$, unlike the mult $_{M} \mathrm{CI}$. In general, as $K$ increased, the coverage of $\boldsymbol{u n i}_{M}$ CI tended to increase; this trend was observed for all distributions and all $N$, but was most pronounced for $N<2000$. Further, for $K>6$ conditions, as $F$ increased, coverage tended to be slightly higher (the opposite pattern was observed for $K=$ 6). Taken together, these results indicate that as the population correlation matrix became larger and more complex, the $\operatorname{uni}_{M}$ CI tended to get a boost in performance.

## CHAPTER 4

## DISCUSSION

In this study I investigated how several confidence interval techniques performed in a multivariate setting. I compared the coverage and width of the parametric $z(z \mathrm{CI})$, Spearman rank-order ( Sp CI ), multivariate bootstrap (mult CI ), and univariate bootstrap (uni CI) across a range of multivariate conditions. Whereas these techniques have been investigated in singlecorrelation settings, they have not been explored in multivariate settings. Further, the two bootstrap procedures were implemented in two ways: simultaneously (matrixwise) and independently (pairwise). The performance of these CI techniques in matrixwise versus pairwise settings has not been investigated previously. Further, I explored an extension to the univariate bootstrap CI algorithm to extend this technique more readily to multivariate applications.

### 4.1 Pairwise CI Techniques

None of the four CI techniques implemented pairwise performed differentially in terms of both coverage (reported in Appendix A) and width (reported in Appendix B) when $F$ and $K$ varied but other factors were held constant. This result was expected, as the pairwise CI implementation treats each bivariate relationship among variables as independent of the other variables in the sample.

However, the multivariate CI techniques did perform differentially due to variable distribution, $N$, and magnitude of $\rho$. The $z$ CI performed at nominal coverage rate when the distribution was normal, with decreased coverage as the variable distributions became more
skewed. The decreasing coverage for larger $\rho$ in the nonnormal conditions is also expected and is consistent with performance observed in Bishara and Hittner (2017). The $S p$ CI had adequate coverage across a large range of condition, with wider CI widths than $z \mathrm{CI}$ when the distribution was normal. Coverage was poor under certain combinations of conditions, particularly when $N=2000$. For example, when the distribution is normal, Spearman covers poorly for large correlations, and when the distribution is highly skewed Spearman covers poorly for small correlations. In Bishara and Hitner (2017), coverage for the Spearman rank-order CI with the Bonett and Wright (2000) correction was found to be at nominal level for most simulation conditions; however, as $N$ increased there was a slight dip in coverage. A similar but less noticeable pattern was observed in Bonett and Wright (2000). Lower coverage as $\rho$ increased was also consistent with previous findings (e.g., Puth, Neuhäuser, \& Ruxton, 2015a). The likeliest reason for differences in my results compared to previous studies was the range of conditions investigated: I considered larger $\rho$ (. 8 vs . 5 investigated in Bishara and Hittner (2017)) and far larger $N$ (2000 as the largest level vs typical maximum sample sizes of 200).

Overall, mult $_{P}$ and unip tended to have near nominal, but slightly libera,l coverage. This is consistent with previous findings (e.g., Puth et al., 2015b; Bishara \& Hittner, 2017) and has been noted as a commonality among bootstrap CI techniques more generally. Across the range of conditions investigated, the multp CI performed more consistently than the unip CI despite its consistent (slightly) lower-than-nominal coverage. For example, when $\rho$ was large and when the distribution was nonnormal, the $u n i_{P}$ tended to have higher-than-nominal coverage, but when $\rho$ was small and when the distribution was nonnormal, the unip tended to have lower-than-nominal coverage. These patterns were more pronounced as $N$ increased. This interaction was not observed with mult $_{P}$.

Both bootstrap CI techniques outperformed $z$ CI under nonnormality, and moreso when nonormality and $\rho$ were greater. Under normality, the two bootstrap techniques performed nearly as well as the $z \mathrm{CI}$, with unip outperforming mult ${ }_{P}$ at smaller $N$; relatedly, both CI techniques had narrower widths than the $z \mathrm{CI}$. In fact, the multp tended to have the narrowest width across all conditions for the pairwise techniques, with differences in width being present but negligible at larger $N$. The $S p$ CI tended to have the widest intervals.

### 4.2 Pairwise versus Matrixwise Multivariate CI

There was little difference in coverage or width between the mult $_{P}$ and mult $_{M}$ techniques for different $F$ and $K$ conditions. While mult $_{M}$ was hypothesized to have higher (more conservative) coverage and wider intervals due to the dependence among correlations in the correlation matrix, in retrospect, this dependence would be more likely to affect CI performance for different $K$ rather than CIs calculated by the multp or multp procedure. However, an effect of $K$ on CI performance was also not observed; this may be because, although correlations in a correlation matrix are constrained by other correlations in the matrix, those constraints differ between sample correlation matrices, as all correlation estimates vary across samples. Therefore, the distribution of a given $r$ in a correlation matrix may be less constrained than hypothesized for different $K$ across samples. In addition, this study did not focus on familywise coverage, which is likely to decrease as $K$ increases, consistent with the literature on familywise Type I correction.

### 4.3 Pairwise versus Matrixwise Univariate CI

The $u n i_{P}$ and $\operatorname{uni}_{M}$ CIs were slightly different techniques due to the issue of scalability of the univariate procedure proposed by Beasley et al. (2007) to large $K$ conditions. The procedure
used for $u n i_{M}$ should approximate $u n i_{P}$ as $N$ and $K$ increase, and indeed, the coverage of $u n i_{M}$ improved as $N$ and $K$ increased. However, the performance of $u n i_{M}$ was still well below nominal rate and the coverage of $u n i_{P}$ in most conditions, and the $u n i_{M}$ intervals tended to be much narrower than unip (often about two-thirds the width). Therefore, the results of the two univariate implementations are not directly comparable. The unim $_{\text {m }}$ did show differential performance for different matrix structures, with generally better coverage and wider intervals as $K$ increased and for larger $F$ (i.e., more matrix variability) when $K>6$. In light of the performance of the $u n i_{M}$, improvements are suggested in the future directions below.

### 4.4 Recommendations

The results of the simulation study and literature review suggest three recommendations to researchers using multivariate data. First, researchers should report correlations among all study variables, either in the body of the work or in an online appendix; in addition, researchers should include confidence intervals for those correlations rather than or in addition to p -values, if characterizing parameter uncertainty is desired. This recommendation is in line with recent standards of quantitative reporting (e.g., Appelbaum et al., 2018) and less recent appeals for changes applying statistics (Harlow et al., 1997). Because researchers often report correlations in a lower triangular table ( $70 \%$ of studies in the systematic review), the upper triangular of the table may be used to report correlation CI widths; although reporting CI width is not as informative as reporting the upper and lower bounds, the width would be more informative than only reporting p-values and would make use of space that is often left blank anyway.

Second, in terms of the CI techniques investigated, both bootstrap CIs tended to have good coverage over a wide range of conditions. The univariate CI (implemented pairwise)
performed better than the multivariate bootstrap for $N<500$, with less liberal CIs. The univariate bootstrap also had generally good coverage for $N \geq 500$, but its performance varied from liberal to conservative for the high skew conditions depending on $\rho$, whereas the multivariate bootstrap had consistent - albeit slightly liberal - coverage for all $N \geq 500$ conditions, regardless of $\rho$ or variable distribution. The multivariate CI also tended to have narrower intervals than other CI techniques, including the univariate CI. Because there was no difference in performance between the matrixwise and pairwise implementations, the matrixwise implementation of the multivariate bootstrap is recommended, as the procedure is faster to implement than the pairwise implementation. Overall, the results of the simulation suggest that the univariate CI performed consistently well for smaller samples, while the multivariate procedure performed consistently well for larger samples.

Third, and related to my recommendation of the multivariate CI technique, is a recommendation for researchers to clearly report what type of bootstrap technique they use, including any corrections (e.g., for bias or acceleration). This recommendation has been made in the literature (e.g., Puthet al., 2015b), but reports still often lack sufficient details on the bootstrap technique to replicate the procedure. Both bootstrap techniques implemented in this simulation included BCa correction, as BCa corrected CIs tends to outperform percentile and bias-only corrected bootstrap CIs. Researchers should report specific bootstrap techniques and corrections they make or software makes on their behalf.

### 4.5 Future Directions

Future work should be done to improve the $\boldsymbol{u n i}_{M}$ technique investigated in this study. One possible avenue of improvement may be to adjust the standardization done in the technique ( P .

O’Keefe, personal communication, May 02, 2019). Before applying the Cholesky rotation, each variable is standardized for both the $u n i_{P}$ and ипі $_{M}$ procedures; however, the mean and standard deviation used to standardize the same variables are different between the two methods because the $u n i_{P}$ procedure standardizes the entire null sampling frame (of which there are $N^{2}$ cases), whereas the $u_{n i}$ procedure standardizes the $N$ cases from independently resampled variables. The $u n i_{P}$ necessarily has a larger standard deviation. This difference in standardization is likely to affect to Cholesky rotation and, in turn, the distribution of $r_{b}^{\text {unim }}$.

Additional work should be done to investigate the data conditions under which the multivariate bootstrap may perform differently, if any. One potential extension may be to investigate different types of correlation matrix structures than those investigated here; while the correlations in each matrix varied depending on $F$, the eigenstructure of matrices (Table 1) was not systematically varied. In retrospect, inclusion of a population matrix $\mathbf{P}=\mathbf{I}$ for each of the $K$ conditions would allowed me to investigate both more diversity in eigenstructure and $\rho=0$ across different matrix conditions.

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APPENDIX A: CI COVERAGE TABLES FOR PAIRWISE TECHNIQUES

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | , |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\prime} i_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Lef | Cov | Left | Cov | Left | Cov | Le | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9230 | 540 | . 9537 | . 0288 | . 9206 | . 0297 | . 9305 | . 0565 | . 9077 | . 05 | . 9342 | . 0575 | . 9317 | 030 | 919 | 523 |
|  | 2 | . 9210 | 532 | . 9527 | . 0285 | . 9125 | . 0325 | . 9288 | . 0572 | . 9143 | . 0523 | . 9335 | . 0570 | . 9382 | . 0307 | . 9232 | . 0538 |
|  | 3 | . 9127 | 543 | . 9520 | . 0243 | . 9093 | . 0287 | . 9300 | . 0543 | . 9097 | . 0547 | . 9330 | . 0603 | . 9353 | . 0317 | . 9200 | 0530 |
| 18 | 1 | . 9174 | 557 | . 9543 | . 0277 | . 9141 | . 0320 | . 9292 | . 0571 | . 9136 | . 0493 | . 9351 | . 0566 | . 9378 | . 0287 | . 9240 | 0499 |
|  | 2 | . 9175 | . 0567 | . 9537 | . 0286 | . 9151 | . 0324 | . 9283 | . 0586 | . 9120 | . 0500 | . 9351 | . 0569 | . 9358 | . 0293 | . 9232 | 0501 |
|  | 3 | . 9187 | . 0555 | . 9558 | . 0270 | . 9160 | . 0319 | . 9297 | . 0581 | . 9102 | . 0497 | . 9335 | . 0578 | . 9345 | . 0298 | . 9223 | . 0492 |
| 30 | 1 | . 9165 | . 0577 | . 9530 | . 0297 | . 9145 | . 0334 | . 9280 | . 0591 | . 9118 | . 0516 | . 9337 | . 0578 | . 9364 | . 0305 | . 9221 | . 0519 |
|  | 2 | . 9177 | . 0557 | . 9541 | . 0274 | . 9150 | . 0310 | . 9299 | . 0568 | . 9115 | . 0492 | . 9358 | . 0557 | . 9358 | . 0290 | . 9237 | . 0492 |
|  | 3 | . 9186 | . 0541 | . 9543 | . 0271 | . 9148 | . 0304 | . 9311 | . 0557 | . 9092 | . 0512 | . 9340 | . 0571 | . 9345 | . 0298 | . 9204 | . 0514 |
| 42 | 1 | . 9177 | . 0554 | . 9523 | . 0289 | . 9149 | . 0316 | . 9295 | . 0569 | . 9108 | . 0510 | . 9328 | . 0586 | . 9349 | . 0304 | . 9217 | . 0513 |
|  | 2 | . 9173 | . 0570 | . 9534 | . 0291 | . 9149 | . 0325 | . 9289 | . 0584 | . 9099 | . 0503 | . 9336 | . 0571 | . 9347 | . 0293 | . 9216 | . 0504 |
|  | 3 | . 918 | . 055 | 53 | . 0283 | . 9163 | . 03 | . 92 | . 057 | . 9091 | . 05 | . 93 | . 0582 | . 9334 | . 0303 | . 920 | . 051 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9155 | . 0543 | . 9503 | . 0351 | . 9223 | . 0318 | . 9253 | . 0567 | . 9115 | . 0475 | 778 | . 1196 | . 9439 | . 0271 | 92 | . 0453 |
|  | 2 | . 9170 | . 0518 | . 9520 | . 0320 | . 9208 | . 0317 | . 9262 | . 0555 | . 9112 | . 0435 | . 8818 | . 1160 | . 9445 | . 0252 | . 9263 | . 0415 |
|  | 3 | . 9090 | . 0590 | . 9437 | . 0407 | . 9197 | . 0333 | . 9210 | . 0600 | . 9133 | . 0467 | . 8793 | . 1183 | . 9460 | . 0260 | . 9247 | . 0440 |
| 18 | 1 | . 9147 | . 0540 | . 9506 | . 0336 | . 9198 | . 0314 | . 9249 | . 0566 | . 9101 | . 0459 | . 8743 | . 1230 | . 9424 | . 0266 | . 9236 | . 0430 |
|  | 2 | . 9135 | . 0555 | . 9496 | . 0336 | . 9194 | . 0325 | . 9247 | . 0573 | . 9079 | . 0491 | . 8747 | . 1226 | . 9418 | . 0290 | . 9213 | . 0460 |
|  | 3 | . 9162 | . 0531 | . 9513 | . 0333 | . 9209 | . 0312 | . 9276 | . 0548 | . 9103 | . 0474 | . 8771 | . 1204 | . 9434 | . 0271 | . 9234 | . 0445 |
| 30 | 1 | . 9160 | . 0519 | . 9494 | . 0340 | . 9204 | . 0303 | . 9267 | . 0546 | . 9085 | . 0495 | . 8716 | . 1257 | . 9420 | . 0286 | . 9214 | . 0467 |
|  | 2 | . 9136 | . 0558 | . 9500 | . 0346 | . 9206 | . 0323 | . 9239 | . 0584 | . 9098 | . 0479 | . 8753 | . 1221 | . 9426 | . 0276 | . 9225 | . 0449 |
|  | 3 | . 9131 | . 0552 | . 9501 | . 0339 | . 9191 | . 0321 | . 9240 | . 0573 | . 9085 | . 0502 | . 8719 | . 1256 | . 9417 | . 0291 | . 9210 | . 0477 |
| 42 | 1 | . 9142 | . 0554 | . 9501 | . 0340 | . 9204 | . 0323 | . 9247 | . 0575 | . 9082 | . 0491 | . 8734 | . 1240 | . 9413 | . 0286 | . 9211 | . 0462 |
|  | 2 | . 9151 | . 0546 | . 9516 | . 0331 | . 9215 | . 0325 | . 9257 | . 0567 | . 9095 | . 0475 | . 8746 | . 1226 | . 9427 | . 0272 | . 9230 | . 0444 |
|  | 3 | . 9137 | . 0565 | . 9493 | . 0349 | . 9204 | . 0332 | . 9240 | . 0584 | . 9087 | . 0483 | . 8716 | . 1257 | . 9423 | . 0279 | . 9222 | . 0453 |

Table A. 1. CI coverage for pairwise techniques: High skew distribution and $\rho=.2$.

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | uni ${ }_{P}$ |  | $z$ |  | Sp |  | mult $_{P}$ |  | uni ${ }_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| 6 | 1 | . 8770 | . 0774 | . 9568 | . 0248 | . 9078 | . 0357 | . 9258 | . 0400 | 8601 | . 0781 | . 9305 | . 0625 | . 9356 | . 0309 | . 9395 | . 0315 |
|  | 2 | . 8765 | . 0773 | . 9558 | . 0270 | . 9067 | . 0370 | . 9262 | . 0412 | . 8673 | . 0752 | . 9282 | . 0673 | . 9345 | . 0327 | . 9367 | . 0347 |
|  | 3 | . 8883 | . 0727 | . 9633 | . 0237 | . 9147 | . 0337 | . 9283 | . 0417 | . 8610 | . 0793 | . 9260 | . 0670 | . 9303 | . 0343 | . 9397 | . 0343 |
| 18 | 1 | . 8742 | . 0816 | . 9583 | . 0261 | . 9036 | . 0400 | . 9230 | . 0447 | . 8619 | . 0766 | . 9343 | . 0592 | . 9327 | . 0317 | . 9388 | . 0333 |
|  | 2 | . 8703 | . 0855 | . 9566 | . 0267 | . 9034 | . 0409 | . 9223 | . 0453 | . 8655 | . 0722 | . 9332 | . 0613 | . 9333 | . 0304 | . 9382 | . 0325 |
|  | 3 | . 8759 | . 0808 | . 9574 | . 0256 | . 9082 | . 0377 | . 9241 | . 0441 | . 8616 | . 0740 | . 9347 | . 0595 | . 9309 | . 0310 | . 9378 | . 0322 |
| 30 | 1 | . 8754 | . 0800 | . 9553 | . 0269 | . 9057 | . 0387 | . 9243 | . 0427 | . 8654 | . 0769 | . 9352 | . 0595 | . 9351 | . 0311 | . 9397 | . 0335 |
|  | 2 | . 8728 | . 0830 | . 9586 | . 0250 | . 9047 | . 0395 | . 9230 | . 0449 | . 8607 | . 0772 | . 9301 | . 0644 | . 9316 | . 0319 | . 9369 | . 0338 |
|  | 3 | . 8771 | . 0780 | . 9571 | . 0254 | . 9065 | . 0375 | . 9265 | . 0415 | . 8648 | . 0720 | . 9339 | . 0594 | . 9318 | . 0303 | . 9376 | . 0323 |
| 42 | 1 | . 8737 | . 0810 | . 9592 | . 0241 | . 9045 | . 0384 | . 9237 | . 0429 | . 8595 | . 0733 | . 9314 | . 0616 | . 9299 | . 0304 | . 9355 | . 0322 |
|  | 2 | . 8737 | . 0817 | . 9582 | . 0244 | . 9045 | . 0393 | . 9232 | . 0442 | . 8625 | . 0804 | . 9312 | . 0635 | . 9328 | . 0341 | . 9383 | . 0357 |
|  | 3 | . 8767 | . 0806 | . 9574 | . 0252 | . 9073 | . 0397 | . 9250 | . 0442 | . 8605 | . 0723 | . 9365 | . 0572 | . 9301 | . 0298 | . 9365 | . 0312 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| 6 | 1 | . 8678 | . 0774 | . 9550 | . 0317 | . 9122 | . 0360 | . 9277 | . 0381 | . 8624 | . 0733 | . 8423 | . 1563 | . 9417 | . 0281 | . 9415 | . 0298 |
|  | 2 | . 8710 | . 0790 | . 9573 | . 0292 | . 9147 | . 0357 | . 9243 | . 0415 | . 8603 | . 0755 | . 8472 | . 1518 | . 9468 | . 0250 | . 9487 | . 0268 |
|  | 3 | . 8637 | . 0813 | . 9547 | . 0310 | . 9137 | . 0327 | . 9243 | . 0407 | . 8587 | . 0743 | . 8440 | . 1550 | . 9357 | . 0310 | . 9383 | . 0320 |
| 18 | 1 | . 8714 | . 0802 | . 9559 | . 0310 | . 9155 | . 0368 | . 9304 | . 0391 | . 8595 | . 0747 | . 8434 | . 1556 | . 9413 | . 0293 | . 9438 | . 0301 |
|  | 2 | . 8654 | . 0819 | . 9534 | . 0327 | . 9110 | . 0378 | . 9246 | . 0425 | . 8571 | . 0754 | . 8420 | . 1566 | . 9388 | . 0297 | . 9399 | . 0314 |
|  | 3 | . 8651 | . 0830 | . 9541 | . 0330 | . 9109 | . 0396 | . 9243 | . 0426 | . 8624 | . 0732 | . 8401 | . 1586 | . 9441 | . 0271 | . 9452 | . 0283 |
| 30 | 1 | . 8724 | . 0799 | . 9547 | . 0324 | . 9170 | . 0369 | . 9299 | . 0403 | . 8582 | . 0731 | . 8435 | . 1553 | . 9406 | . 0282 | . 9417 | . 0298 |
|  | 2 | . 8700 | . 0788 | . 9548 | . 0315 | . 9134 | . 0366 | . 9272 | . 0401 | . 8561 | . 0729 | . 8516 | . 1470 | . 9402 | . 0274 | . 9411 | . 0292 |
|  | 3 | . 8661 | . 0830 | . 9552 | . 0308 | . 9120 | . 0386 | . 9253 | . 0422 | . 8596 | . 0759 | . 8438 | . 1550 | . 9408 | . 0298 | . 9417 | . 0312 |
| 42 | 1 | . 8692 | . 0810 | . 9530 | . 0346 | . 9124 | . 0385 | . 9265 | . 0415 | . 8603 | . 0752 | . 8420 | . 1570 | . 9417 | . 0294 | . 9431 | . 0307 |
|  | 2 | . 8695 | . 0801 | . 9556 | . 0314 | . 9133 | . 0379 | . 9270 | . 0412 | . 8587 | . 0737 | . 8390 | . 1600 | . 9405 | . 0285 | . 9426 | . 0293 |
|  | 3 | . 8689 | . 0761 | . 9560 | . 0306 | . 9115 | . 0349 | . 9269 | . 0376 | . 8555 | . 0796 | . 8346 | . 1643 | . 9390 | . 0314 | . 9406 | . 0325 |

Table A. 2. CI coverage for pairwise techniques: High skew distribution and $\rho=.5$.

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\text {un }}{ }_{P}$ |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\prime} i_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | F | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| 6 | 1 | . 8423 | . 1008 | . 9671 | . 0135 | . 9033 | . 0409 | . 9406 | . 022 | . 8235 | . 0981 | . 9569 | . 0364 | . 9300 | . 0341 | . 9671 | . 0135 |
|  | 2 | . 8425 | . 1022 | . 9623 | . 0167 | . 8978 | . 0460 | . 9382 | . 0265 | . 8317 | . 0860 | . 9638 | . 0262 | . 9308 | . 0305 | . 9710 | . 0095 |
|  | 3 | . 8483 | . 0983 | . 9637 | . 0137 | . 9047 | . 0407 | . 9383 | . 0260 | . 8287 | . 0903 | . 9620 | . 0313 | . 9240 | . 0343 | . 9593 | . 0167 |
| 18 | 1 | . 8471 | . 0971 | . 9658 | . 0153 | . 9015 | . 0436 | . 9379 | . 0260 | . 8278 | . 0933 | . 9568 | . 0334 | . 9304 | . 0317 | . 9663 | . 0132 |
|  | 2 | . 8400 | . 1032 | . 9684 | . 0156 | . 8981 | . 0456 | . 9353 | . 0275 | . 8300 | . 0907 | . 9600 | . 0326 | . 9297 | . 0312 | . 9654 | . 0139 |
|  | 3 | . 8441 | . 1006 | . 9669 | . 0147 | . 9002 | . 0443 | . 9374 | . 0259 | . 8248 | . 0996 | . 9608 | . 0320 | . 9302 | . 0342 | . 9674 | . 0144 |
| 30 | 1 | . 8347 | . 1105 | . 9646 | . 0169 | . 8963 | . 0481 | . 9342 | . 0283 | . 8254 | . 1010 | . 9610 | . 0320 | . 9301 | . 0355 | . 9675 | . 0152 |
|  | 2 | . 8468 | . 0997 | . 9670 | . 0150 | . 9008 | . 0447 | . 9388 | . 0262 | . 8301 | . 0958 | . 9583 | . 0335 | . 9326 | . 0334 | . 9688 | . 0139 |
|  | 3 | . 8424 | . 1015 | . 9664 | . 0141 | . 8995 | . 0447 | . 9378 | . 0264 | . 8244 | . 0949 | . 9594 | . 0326 | . 9280 | . 0329 | . 9651 | . 0140 |
| 42 | 1 | . 8458 | . 1044 | . 9675 | . 0155 | . 9035 | . 0466 | . 9412 | . 0273 | . 8290 | . 0851 | . 9631 | . 0285 | . 9303 | . 0288 | . 9661 | . 0120 |
|  | 2 | . 8408 | . 1033 | . 9673 | . 0134 | . 8988 | . 0440 | . 9369 | . 0259 | . 8206 | . 0989 | . 9603 | . 0315 | . 9268 | . 0349 | . 9652 | . 0150 |
|  | 3 | . 8449 | . 1001 | . 9659 | . 0151 | . 9018 | . 0431 | . 9378 | . 025 | . 8250 | . 0919 | . 9570 | . 0341 | . 9274 | . 0314 | . 9652 | . 0131 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| 6 | 1 | . 8345 | . 1027 | . 9666 | . 0174 | . 9087 | . 0425 | . 9501 | . 0212 | . 8233 | . 0844 | . 9246 | . 0721 | . 9409 | . 0246 | 9734 | . 0091 |
|  | 2 | . 8495 | . 0923 | . 9667 | . 0208 | . 9143 | . 0415 | . 9568 | . 0192 | . 8222 | . 0952 | . 9208 | . 0758 | . 9382 | . 0305 | . 9745 | . 0108 |
|  | 3 | . 8297 | . 1013 | . 9627 | . 0210 | . 9063 | . 0423 | . 9447 | . 0217 | . 8333 | . 0853 | . 9293 | . 0697 | . 9460 | . 0240 | . 9760 | . 0093 |
| 18 | 1 | . 8349 | . 0990 | . 9670 | . 0161 | . 9088 | . 0393 | . 9478 | . 0202 | . 8208 | . 0969 | . 9257 | . 0711 | . 9389 | . 0316 | . 9725 | . 0134 |
|  | 2 | . 8367 | . 0967 | . 9657 | . 0169 | . 9102 | . 0383 | . 9492 | . 0194 | . 8221 | . 0915 | . 9238 | . 0733 | . 9390 | . 0295 | . 9721 | . 0125 |
|  | 3 | . 8325 | . 1011 | . 9641 | . 0194 | . 9043 | . 0430 | . 9458 | . 0223 | . 8255 | . 0921 | . 9258 | . 0721 | . 9400 | . 0295 | . 9738 | . 0112 |
| 30 | 1 | . 8358 | . 1005 | . 9667 | . 0179 | . 9101 | . 0404 | . 9502 | . 0208 | . 8213 | . 0891 | . 9311 | . 0657 | . 9379 | . 0288 | . 9718 | . 0117 |
|  | 2 | . 8363 | . 0996 | . 9666 | . 0191 | . 9078 | . 0414 | . 9477 | . 0213 | . 8259 | . 0902 | . 9237 | . 0737 | . 9411 | . 0278 | . 9737 | . 0112 |
|  | 3 | . 8390 | . 1008 | . 9662 | . 0182 | . 9129 | . 0400 | . 9518 | . 0204 | . 8258 | . 0893 | . 9268 | . 0704 | . 9417 | . 0278 | . 9740 | . 0113 |
| 42 | 1 | . 8429 | . 0924 | . 9654 | . 0206 | . 9088 | . 0402 | . 9477 | . 0209 | . 8226 | . 0980 | . 9291 | . 0677 | . 9397 | . 0312 | . 9726 | . 0129 |
|  | 2 | . 8327 | . 1060 | . 9648 | . 0194 | . 9077 | . 0448 | . 9482 | . 0234 | . 8191 | . 0894 | . 9225 | . 0747 | . 9363 | . 0280 | . 9707 | . 0115 |
|  | 3 | . 8366 | . 1010 | . 9672 | . 0179 | . 9116 | . 0408 | . 9512 | . 0207 | . 8239 | . 0879 | . 9277 | . 0695 | . 9415 | . 0259 | . 9743 | . 0101 |

Table A. 3. CI coverage for pairwise techniques: High skew distribution and $\rho=.8$.

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\prime} i_{P}$ |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\text {u }} i_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9379 | . 0368 | . 9548 | . 0239 | . 9328 | . 0322 | . 9385 | . 040 | . 9389 | . 0319 | . 9540 | . 0241 | . 9435 | . 0271 | . 9385 | . 0337 |
| 6 | 2 | . 9420 | . 0358 | . 9587 | . 0217 | . 9347 | . 0320 | . 9423 | . 0385 | . 9420 | . 0297 | . 9570 | . 0233 | . 9472 | . 0240 | . 9435 | . 0308 |
|  | 3 | . 9397 | . 0367 | . 9520 | . 0237 | . 9330 | . 0323 | . 9423 | . 0380 | . 9383 | . 0310 | . 9483 | . 0260 | . 9443 | . 0263 | . 9350 | . 0347 |
| 18 | 1 | . 9384 | . 0359 | . 9534 | . 0220 | . 9297 | . 0329 | . 9384 | . 0398 | . 9389 | . 0322 | . 9509 | . 0264 | . 9443 | . 0276 | . 9391 | . 0342 |
|  | 2 | . 9395 | . 0354 | . 9537 | . 0219 | . 9305 | . 0327 | . 9393 | . 0397 | . 9380 | . 0330 | . 9511 | . 0267 | . 9435 | . 0281 | . 9378 | . 0350 |
|  | 3 | . 9407 | . 0341 | . 9551 | . 0211 | . 9308 | . 0315 | . 9399 | . 0387 | . 9390 | . 0323 | . 9497 | . 0270 | . 9453 | . 0266 | . 9396 | . 0342 |
| 30 | 1 | . 9394 | . 0361 | . 9549 | . 0215 | . 9313 | . 0325 | . 9394 | . 0399 | . 9381 | . 0314 | . 9512 | . 0250 | . 9432 | . 0270 | . 9384 | . 0333 |
|  | 2 | . 9400 | . 0344 | . 9545 | . 0213 | . 9317 | . 0311 | . 9405 | . 0379 | . 9388 | . 0318 | . 9516 | . 0257 | . 9436 | . 0272 | . 9388 | . 0338 |
|  | 3 | . 9390 | . 0355 | . 9536 | . 0224 | . 9303 | . 0330 | . 9379 | . 0402 | . 9395 | . 0324 | . 9504 | . 0268 | . 9438 | . 0282 | . 9389 | . 0349 |
| 42 | 1 | . 9401 | . 0342 | . 9541 | . 0214 | . 9310 | . 0314 | . 9402 | . 0379 | . 9395 | . 0323 | . 9509 | . 0271 | . 9448 | . 0276 | . 9395 | . 0344 |
|  | 2 | . 9394 | . 0350 | . 9538 | . 0218 | . 9299 | . 0322 | . 9392 | . 0386 | . 9392 | . 0321 | . 9511 | . 0266 | . 9444 | . 0276 | . 9393 | . 0344 |
|  | 3 | . 9398 | . 0349 | . 9537 | . 0222 | . 9304 | . 0322 | . 9400 | . 0386 | . 9389 | . 0318 | . 9507 | . 0264 | . 9441 | . 0275 | . 9387 | . 0338 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9393 | . 0327 | . 9497 | . 0263 | . 9353 | . 0299 | . 9411 | . 0355 | . 9377 | . 0317 | . 9518 | . 0283 | . 9465 | . 0269 | . 9393 | . 0327 |
|  | 2 | . 9393 | . 0368 | . 9547 | . 0240 | . 9403 | . 0297 | . 9397 | . 0400 | . 9383 | . 0312 | . 9518 | . 0265 | . 9455 | . 0268 | . 9367 | . 0338 |
|  | 3 | . 9393 | . 3658 | . 9510 | . 0217 | . 9397 | . 0270 | . 9377 | . 0340 | . 9423 | . 0303 | . 9510 | . 0297 | . 9460 | . 0257 | . 9383 | . 0323 |
| 18 | 1 | . 9411 | . 0330 | . 9538 | . 0228 | . 9373 | . 0295 | . 9405 | . 0365 | . 9390 | . 0314 | . 9491 | . 0301 | . 9460 | . 0272 | . 9387 | . 0326 |
|  | 2 | . 9385 | . 0334 | . 9526 | . 0225 | . 9367 | . 0290 | . 9378 | . 0372 | . 9393 | . 0316 | . 9499 | . 0299 | . 9468 | . 0269 | . 9389 | . 0325 |
|  | 3 | . 9395 | . 0331 | . 9536 | . 0224 | . 9363 | . 0293 | . 9403 | . 0362 | . 9408 | . 0289 | . 9523 | . 0271 | . 9478 | . 0241 | . 9401 | . 0299 |
| 30 | 1 | . 9398 | . 0338 | . 9531 | . 0230 | . 9365 | . 0300 | . 9392 | . 0376 | . 9394 | . 0307 | . 9505 | . 0292 | . 9470 | . 0260 | . 9395 | . 0318 |
|  | 2 | . 9404 | . 0332 | . 9536 | . 0225 | . 9360 | . 0295 | . 9401 | . 0365 | . 9387 | . 0309 | . 9506 | . 0286 | . 9465 | . 0264 | . 9387 | . 0321 |
|  | 3 | . 9410 | . 0328 | . 9540 | . 0224 | . 9373 | . 0291 | . 9403 | . 0364 | . 9399 | . 0303 | . 9500 | . 0288 | . 9467 | . 0261 | . 9402 | . 0312 |
| 42 | 1 | . 9394 | . 0356 | . 9526 | . 0248 | . 9362 | . 0317 | . 9387 | . 0390 | . 9400 | . 0306 | . 9507 | . 0289 | . 9473 | . 0260 | . 9402 | . 0314 |
|  | 2 | . 9404 | . 0334 | . 9533 | . 0234 | . 9369 | . 0303 | . 9399 | . 0372 | . 9398 | . 0308 | . 9512 | . 0293 | . 9466 | . 0267 | . 9396 | . 0321 |
|  | 3 | . 9402 | . 0337 | . 9516 | . 0243 | . 9357 | . 0306 | . 9405 | . 0366 | . 9388 | . 0311 | . 9501 | . 0293 | . 9460 | . 0266 | . 9390 | . 0321 |

Table A. 4. CI coverage for pairwise techniques: Low skew distribution and $\rho=.2$.

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\prime} i_{P}$ |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\text {u }}{ }_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9295 | . 0428 | . 9587 | . 0152 | . 9283 | . 0349 | . 9357 | 037 | 287 | . 0382 | . 9570 | . 0187 | . 9446 | . 0275 | . 9407 | . 0313 |
|  | 2 | . 9248 | . 0458 | . 9597 | . 0153 | . 9263 | . 0348 | . 9300 | . 0400 | . 9280 | . 0393 | . 9565 | . 0200 | . 9447 | . 0298 | . 9378 | . 0340 |
|  | 3 | . 9287 | . 0423 | . 9553 | . 0183 | . 9273 | . 0340 | . 9333 | . 0370 | . 9273 | . 0347 | . 9597 | . 0130 | . 9463 | . 0250 | . 9390 | . 0283 |
| 18 | 1 | . 9314 | . 0410 | . 9578 | . 0168 | . 9296 | . 0340 | . 9373 | . 0354 | . 9288 | . 0385 | . 9560 | . 0185 | . 9449 | . 0278 | . 9406 | . 0316 |
|  | 2 | . 9291 | . 0425 | . 9563 | . 0179 | . 9284 | . 0350 | . 9358 | . 0363 | . 9257 | . 0390 | . 9555 | . 0192 | . 9424 | . 0281 | . 9383 | . 0319 |
|  | 3 | . 9272 | . 0426 | . 9546 | . 0181 | . 9255 | . 0360 | . 9341 | . 0367 | . 9260 | . 0402 | . 9548 | . 0186 | . 9436 | . 0287 | . 9387 | . 0332 |
| 30 | 1 | . 9306 | . 0402 | . 9564 | . 0175 | . 9272 | . 0345 | . 9361 | . 0353 | . 9245 | . 0404 | . 9537 | . 0180 | . 9414 | . 0293 | . 9368 | . 0330 |
|  | 2 | . 9272 | . 0435 | . 9554 | . 0184 | . 9255 | . 0366 | . 9335 | . 0379 | . 9269 | . 0378 | . 9544 | . 0180 | . 9433 | . 0274 | . 9392 | . 0307 |
|  | 3 | . 9270 | . 0456 | . 9569 | . 0182 | . 9260 | . 0377 | . 9338 | . 0391 | . 9268 | . 0373 | . 9536 | . 0180 | . 9432 | . 0268 | . 9385 | . 0306 |
| 42 | 1 | . 9305 | . 0399 | . 9571 | . 0165 | . 9274 | . 0343 | . 9362 | . 0348 | . 9271 | . 0398 | . 9554 | . 0205 | . 9432 | . 0290 | . 9388 | . 0327 |
|  | 2 | . 9285 | . 0420 | . 9561 | . 0181 | . 9263 | . 0357 | . 9340 | . 0368 | . 9280 | . 0365 | . 9553 | . 0171 | . 9447 | . 0259 | . 9404 | . 0294 |
|  | 3 | . 9295 | . 0402 | . 9561 | . 0179 | . 9258 | . 0345 | . 9346 | . 0354 | . 9262 | . 0397 | . 9557 | . 0189 | . 9436 | . 0281 | . 9382 | . 0322 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9275 | . 0401 | . 9523 | . 0182 | . 9344 | . 0309 | . 9361 | . 0343 | 9256 | . 0369 | . 9527 | . 0176 | . 9469 | . 0251 | . 9383 | . 0306 |
|  | 2 | . 9290 | . 0393 | . 9553 | . 0180 | . 9338 | . 0312 | . 9362 | . 0328 | . 9302 | . 0332 | . 9575 | . 0142 | . 9487 | . 0233 | . 9435 | . 0273 |
|  | 3 | . 9240 | . 0467 | . 9537 | . 0240 | . 9320 | . 0360 | . 9323 | . 0417 | . 9307 | . 0370 | . 9563 | . 0160 | . 9483 | . 0267 | . 9433 | . 0300 |
| 18 | 1 | . 9275 | . 0417 | . 9557 | . 0190 | . 9332 | . 0330 | . 9354 | . 0356 | . 9296 | . 0359 | . 9562 | . 0168 | . 9483 | . 0255 | . 9422 | . 0291 |
|  | 2 | . 9242 | . 0437 | . 9553 | . 0197 | . 9300 | . 0344 | . 9328 | . 0367 | . 9248 | . 0384 | . 9540 | . 0170 | . 9447 | . 0274 | . 9385 | . 0312 |
|  | 3 | . 9278 | . 0422 | . 9570 | . 0185 | . 9342 | . 0331 | . 9366 | . 0352 | . 9305 | . 0358 | . 9570 | . 0149 | . 9502 | . 0249 | . 9432 | . 0285 |
| 30 | 1 | . 9249 | . 0435 | . 9557 | . 0187 | . 9308 | . 0339 | . 9340 | . 0364 | . 9285 | . 0365 | . 9568 | . 0165 | . 9477 | . 0257 | . 9413 | . 0296 |
|  | 2 | . 9268 | . 0397 | . 9542 | . 0181 | . 9324 | . 0311 | . 9350 | . 0337 | . 9226 | . 0397 | . 9525 | . 0183 | . 9431 | . 0283 | . 9363 | . 0323 |
|  | 3 | . 9264 | . 0410 | . 9540 | . 0191 | . 9324 | . 0326 | . 9351 | . 0346 | . 9271 | . 0372 | . 9550 | . 0168 | . 9461 | . 0265 | . 9402 | . 0301 |
| 42 | 1 | . 9239 | . 0443 | . 9519 | . 0204 | . 9299 | . 0355 | . 9332 | . 0375 | . 9287 | . 0335 | . 9540 | . 0163 | . 9474 | . 0238 | . 9414 | . 0271 |
|  | 2 | . 9281 | . 0424 | . 9577 | . 0190 | . 9338 | . 0333 | . 9372 | . 0353 | . 9260 | . 0379 | . 9537 | . 0167 | . 9460 | . 0267 | . 9390 | . 0307 |
|  | 3 | . 9259 | . 0439 | . 9560 | . 0190 | . 9333 | . 0341 | . 9353 | . 0367 | . 9280 | . 0388 | . 9553 | . 0179 | . 9481 | . 0272 | . 9410 | . 0316 |

Table A. 5. CI coverage for pairwise techniques: Low skew distribution and $\rho=.5$.

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\prime} i_{P}$ |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\text {un }}{ }_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9219 | . 0487 | . 9593 | . 0115 | . 9271 | . 0376 | . 9365 | . 030 | . 91 | . 0479 | . 9491 | . 0109 | . 9393 | . 0305 | . 9438 | . 0275 |
|  | 2 | . 9183 | . 0472 | . 9592 | . 0110 | . 9170 | . 0398 | . 9277 | . 0332 | . 9170 | . 0448 | . 9533 | . 0102 | . 9400 | . 0302 | . 9452 | . 0265 |
|  | 3 | . 9213 | . 0487 | . 9607 | . 0100 | . 9267 | . 0350 | . 9360 | . 0290 | . 9197 | . 0413 | . 9453 | . 0137 | . 9433 | . 0260 | . 9450 | . 0237 |
| 18 | 1 | . 9189 | . 0485 | . 9557 | . 0112 | . 9221 | . 0388 | . 9307 | . 0332 | . 9191 | . 0440 | . 9543 | . 0115 | . 9437 | . 0287 | . 9491 | . 0245 |
|  | 2 | . 9205 | . 0475 | . 9587 | . 0118 | . 9220 | . 0391 | . 9311 | . 0326 | . 9149 | . 0460 | . 9545 | . 0096 | . 9408 | . 0295 | . 9454 | . 0259 |
|  | 3 | . 9218 | . 0457 | . 9601 | . 0107 | . 9243 | . 0366 | . 9324 | . 0311 | . 9182 | . 0467 | . 9572 | . 0111 | . 9426 | . 0304 | . 9482 | . 0263 |
| 30 | 1 | . 9206 | . 0516 | . 9566 | . 0134 | . 9249 | . 0406 | . 9344 | . 0343 | . 9165 | . 0451 | . 9518 | . 0119 | . 9414 | . 0297 | . 9464 | . 0261 |
|  | 2 | . 9137 | . 0539 | . 9555 | . 0119 | . 9201 | . 0415 | . 9294 | . 0349 | . 9174 | . 0422 | . 9516 | . 0111 | . 9426 | . 0270 | . 9471 | . 0238 |
|  | 3 | . 9162 | . 0517 | . 9573 | . 0123 | . 9204 | . 0408 | . 9289 | . 0345 | . 9153 | . 0446 | . 9529 | . 0104 | . 9414 | . 0286 | . 9453 | . 0251 |
| 42 | 1 | . 9251 | . 0485 | . 9603 | . 0127 | . 9268 | . 0395 | . 9354 | . 0341 | . 9165 | . 0412 | . 9451 | . 0105 | . 9427 | . 0263 | . 9476 | . 0229 |
|  | 2 | . 9215 | . 0452 | . 9565 | . 0115 | . 9248 | . 0356 | . 9330 | . 0300 | . 9166 | . 0469 | . 9548 | . 0117 | . 9420 | . 0304 | . 9480 | . 0258 |
|  | 3 | . 9231 | . 0456 | . 9577 | . 0107 | . 9251 | . 0362 | . 9334 | . 0301 | . 9192 | . 0422 | . 9553 | . 0096 | . 9435 | . 0265 | . 9481 | . 0231 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9202 | . 0475 | . 9593 | . 0121 | . 9329 | . 0339 | . 9403 | . 0283 | . 9191 | . 0416 | . 9375 | . 0062 | . 9467 | . 0273 | . 9515 | . 0243 |
|  | 2 | . 9170 | . 0498 | . 9573 | . 0120 | . 9265 | . 0387 | . 9357 | . 0328 | . 9183 | . 0407 | . 9413 | . 0048 | . 9472 | . 0248 | . 9535 | . 0210 |
|  | 3 | . 9140 | . 0567 | . 9607 | . 0087 | . 9263 | . 0417 | . 9323 | . 0370 | . 9150 | . 0443 | . 9407 | . 0053 | . 9457 | . 0277 | . 9470 | . 0263 |
| 18 | 1 | . 9202 | . 0458 | . 9583 | . 0117 | . 9328 | . 0328 | . 9395 | . 0280 | . 9177 | . 0417 | . 9408 | . 0062 | . 9460 | . 0263 | . 9502 | . 0235 |
|  | 2 | . 9154 | . 0502 | . 9592 | . 0114 | . 9288 | . 0362 | . 9369 | . 0303 | . 9133 | . 0430 | . 9361 | . 0065 | . 9426 | . 0270 | . 9466 | . 0242 |
|  | 3 | . 9202 | . 0475 | . 9573 | . 0123 | . 9329 | . 0350 | . 9394 | . 0296 | . 9138 | . 0447 | . 9385 | . 0062 | . 9431 | . 0286 | . 9491 | . 0244 |
| 30 | 1 | . 9230 | . 0428 | . 9600 | . 0109 | . 9333 | . 0318 | . 9405 | . 0267 | . 9175 | . 0434 | . 9371 | . 0071 | . 9465 | . 0274 | . 9507 | . 0245 |
|  | 2 | . 9186 | . 0478 | . 9586 | . 0119 | . 9308 | . 0347 | . 9378 | . 0294 | . 9137 | . 0435 | . 9359 | . 0064 | . 9437 | . 0272 | . 9475 | . 0244 |
|  | 3 | . 9185 | . 0480 | . 9580 | . 0121 | . 9309 | . 0345 | . 9390 | . 0289 | . 9161 | . 0430 | . 9420 | . 0062 | . 9456 | . 0267 | . 9499 | . 0240 |
| 42 | 1 | . 9102 | . 0539 | . 9579 | . 0126 | . 9247 | . 0383 | . 9326 | . 0320 | . 9183 | . 0439 | . 9448 | . 0083 | . 9475 | . 0272 | . 9518 | . 0243 |
|  | 2 | . 9203 | . 0461 | . 9592 | . 0117 | . 9322 | . 0334 | . 9401 | . 0279 | . 9134 | . 0456 | . 9411 | . 0081 | . 9427 | . 0287 | . 9469 | . 0257 |
|  | 3 | . 9156 | . 0497 | . 9569 | . 0123 | . 9295 | . 0355 | . 9369 | . 0299 | . 9195 | . 0443 | . 9407 | . 0078 | . 9479 | . 0280 | . 9522 | . 0253 |

Table A. 6. CI coverage for pairwise techniques: Low skew distribution and $\rho=.8$.

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\text {u }} i_{P}$ |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\prime} i_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $K$ | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9514 | . 0255 | . 9547 | . 0172 | . 9394 | . 0308 | . 9475 | . 0269 | . 9482 | . 0250 | . 9453 | . 0151 | . 9454 | . 0266 | . 9461 | . 0258 |
|  | 2 | . 9495 | . 0268 | . 9517 | . 0208 | . 9357 | . 0352 | . 9478 | . 0278 | . 9515 | . 0250 | . 9477 | . 0158 | . 9465 | . 0273 | . 9508 | . 0257 |
|  | 3 | . 9467 | . 0297 | . 9560 | . 0193 | . 9380 | . 0327 | . 9433 | . 0303 | . 9467 | . 0263 | . 9453 | . 0137 | . 9420 | . 0283 | . 9453 | . 0260 |
| 18 | 1 | . 9495 | . 0259 | . 9528 | . 0183 | . 9359 | . 0326 | . 9472 | . 0270 | . 9509 | . 0239 | . 9462 | . 0135 | . 9476 | . 0255 | . 9485 | . 0251 |
|  | 2 | . 9503 | . 0260 | . 9523 | . 0190 | . 9373 | . 0316 | . 9485 | . 0268 | . 9505 | . 0247 | . 9471 | . 0138 | . 9472 | . 0263 | . 9486 | . 0257 |
|  | 3 | . 9488 | . 0271 | . 9523 | . 0190 | . 9357 | . 0325 | . 9469 | . 0279 | . 9497 | . 0261 | . 9469 | . 0146 | . 9458 | . 0280 | . 9475 | . 0271 |
| 30 | 1 | . 9491 | . 0272 | . 9527 | . 0194 | . 9361 | . 0335 | . 9474 | . 0277 | . 9520 | . 0237 | . 9468 | . 0135 | . 9485 | . 0252 | . 9500 | . 0244 |
|  | 2 | . 9490 | . 0257 | . 9531 | . 0180 | . 9357 | . 0320 | . 9470 | . 0265 | . 9505 | . 0257 | . 9478 | . 0146 | . 9469 | . 0271 | . 9487 | . 0265 |
|  | 3 | . 9505 | . 0250 | . 9543 | . 0172 | . 9376 | . 0316 | . 9486 | . 0258 | . 9491 | . 0256 | . 9452 | . 0149 | . 9454 | . 0271 | . 9463 | . 0270 |
| 42 | 1 | . 9503 | . 0258 | . 9537 | . 0183 | . 9373 | . 0321 | . 9483 | . 0266 | . 9506 | . 0250 | . 9470 | . 0143 | . 9470 | . 0265 | . 9486 | . 0257 |
|  | 2 | . 9505 | . 0256 | . 9541 | . 0179 | . 9382 | . 0315 | . 9489 | . 0262 | . 9496 | . 0256 | . 9462 | . 0147 | . 9455 | . 0275 | . 9477 | . 0263 |
|  | 3 | . 9491 | . 0254 | . 9524 | . 0179 | . 9363 | . 0313 | . 9471 | . 0266 | . 9489 | . 0256 | . 9442 | . 0154 | . 9451 | . 0273 | . 9465 | . 0265 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9482 | . 0259 | . 9481 | . 0198 | . 9407 | . 0301 | . 9471 | . 0269 | . 9507 | . 0250 | . 9333 | . 0086 | . 9478 | . 0265 | . 9494 | . 0261 |
|  | 2 | . 9505 | . 0252 | . 9483 | . 0188 | . 9420 | . 0287 | . 9483 | . 0268 | . 9462 | . 0258 | . 9295 | . 0082 | . 9422 | . 0290 | . 9425 | . 0280 |
|  | 3 | . 9463 | . 0240 | . 9493 | . 0163 | . 9380 | . 0290 | . 9423 | . 0260 | . 9477 | . 0267 | . 9280 | . 0097 | . 9430 | . 0290 | . 9450 | . 0263 |
| 18 | 1 | . 9487 | . 0261 | . 9499 | . 0184 | . 9396 | . 0307 | . 9467 | . 0268 | . 9493 | . 0251 | . 9318 | . 0087 | . 9466 | . 0263 | . 9468 | . 0264 |
|  | 2 | . 9509 | . 0256 | . 9514 | . 0183 | . 9418 | . 0302 | . 9490 | . 0264 | . 9503 | . 0250 | . 9326 | . 0087 | . 9476 | . 0265 | . 9484 | . 0257 |
|  | 3 | . 9486 | . 0257 | . 9509 | . 0180 | . 9416 | . 0287 | . 9477 | . 0257 | . 9489 | . 0258 | . 9302 | . 0095 | . 9448 | . 0274 | . 9456 | . 0272 |
| 30 | 1 | . 9492 | . 0267 | . 9506 | . 0189 | . 9411 | . 0304 | . 9477 | . 0272 | . 9501 | . 0264 | . 9332 | . 0093 | . 9475 | . 0274 | . 9476 | . 0277 |
|  | 2 | . 9497 | . 0263 | . 9502 | . 0187 | . 9410 | . 0303 | . 9481 | . 0271 | . 9504 | . 0242 | . 9318 | . 0083 | . 9480 | . 0252 | . 9481 | . 0252 |
|  | 3 | . 9502 | . 0266 | . 9520 | . 0185 | . 9419 | . 0300 | . 9479 | . 0278 | . 9507 | . 0252 | . 9335 | . 0091 | . 9478 | . 0266 | . 9488 | . 0261 |
| 42 | 1 | . 9508 | . 0260 | . 9524 | . 0185 | . 9427 | . 0298 | . 9490 | . 0266 | . 9505 | . 0251 | . 9322 | . 0093 | . 9481 | . 0262 | . 9482 | . 0262 |
|  | 2 | . 9505 | . 0249 | . 9514 | . 0176 | . 9425 | . 0288 | . 9487 | . 0256 | . 9499 | . 0257 | . 9326 | . 0092 | . 9470 | . 0269 | . 9476 | . 0264 |
|  | 3 | . 9484 | . 0256 | . 9502 | . 0180 | . 9403 | . 0294 | . 9466 | . 0263 | . 9483 | . 0262 | . 9301 | . 0093 | . 9453 | . 0275 | . 9453 | . 0275 |

Table A. 7. CI coverage for pairwise techniques: Normal distribution and $\rho=.2$.

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\text {un }}{ }_{P}$ |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\prime} i_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9489 | . 0290 | . 9505 | . 0140 | . 9351 | . 0356 | 退 | . 0310 | . 9510 | . 0268 | . 9309 | . 0052 | . 9460 | . 0284 | . 9479 | . 0271 |
|  | 2 | . 9505 | . 0290 | . 9550 | . 0120 | . 9410 | . 0325 | . 9488 | . 0287 | . 9500 | . 0260 | . 9275 | . 0055 | . 9462 | . 0268 | . 9467 | . 0273 |
|  | 3 | . 9417 | . 0337 | . 9510 | . 0140 | . 9340 | . 0357 | . 9410 | . 0333 | . 9457 | . 0240 | . 9183 | . 0063 | . 9423 | . 0223 | . 9450 | . 0230 |
| 18 | 1 | . 9509 | . 0259 | . 9509 | . 0120 | . 9374 | . 0319 | . 9476 | . 0273 | . 9504 | . 0253 | . 9251 | . 0062 | . 9468 | . 0265 | . 9478 | . 0261 |
|  | 2 | . 9514 | . 0257 | . 9496 | . 0122 | . 9387 | . 0311 | . 9475 | . 0271 | . 9487 | . 0254 | . 9236 | . 0056 | . 9453 | . 0264 | . 9465 | . 0258 |
|  | 3 | . 9507 | . 0269 | . 9518 | . 0130 | . 9378 | . 0328 | . 9471 | . 0284 | . 9515 | . 0241 | . 9231 | . 0062 | . 9478 | . 0255 | . 9493 | . 0242 |
| 30 | 1 | . 9494 | . 0278 | . 9522 | . 0131 | . 9361 | . 0337 | . 9459 | . 0295 | . 9493 | . 0263 | . 9258 | . 0072 | . 9456 | . 0277 | . 9468 | . 0271 |
|  | 2 | . 9505 | . 0267 | . 9510 | . 0127 | . 9371 | . 0331 | . 9469 | . 0284 | . 9486 | . 0276 | . 9255 | . 0072 | . 9449 | . 0291 | . 9460 | . 0285 |
|  | 3 | . 9482 | . 0284 | . 9481 | . 0143 | . 9340 | . 0346 | . 9442 | . 0299 | . 9499 | . 0277 | . 9288 | . 0065 | . 9466 | . 0291 | . 9476 | . 0283 |
| 42 | 1 | . 9512 | . 0270 | . 9521 | . 0133 | . 9379 | . 0332 | . 9477 | . 0285 | . 9494 | . 0229 | . 9166 | . 0111 | . 9459 | . 0241 | . 9470 | . 0235 |
|  | 2 | . 9518 | . 0258 | . 9520 | . 0123 | . 9380 | . 0320 | . 9478 | . 0275 | . 9479 | . 0280 | . 9265 | . 0092 | . 9445 | . 0291 | . 9458 | . 0285 |
|  | 3 | . 9507 | . 0268 | . 9514 | . 0128 | . 9370 | . 0332 | . 9468 | . 0286 | . 9504 | . 0267 | . 9274 | . 0089 | . 9467 | . 0282 | . 9481 | . 0273 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9480 | 289 | . 9449 | . 0131 | . 9396 | 325 | . 9445 | . 0297 | . 9528 | . 0232 | 8354 | . 0674 | . 9489 | . 0244 | . 9493 | . 0249 |
|  | 2 | . 9498 | . 0275 | . 9448 | . 0117 | . 9383 | . 0337 | . 9457 | . 0298 | . 9520 | . 0247 | . 8440 | . 0268 | . 9475 | . 0270 | . 9482 | . 0268 |
|  | 3 | . 9493 | . 0273 | . 9440 | . 0110 | . 9370 | . 0330 | . 9453 | . 0297 | . 9560 | . 0227 | . 8440 | . 1560 | . 9530 | . 0233 | . 9533 | . 0220 |
| 18 | 1 | . 9481 | . 0280 | . 9454 | . 0125 | . 9399 | . 0312 | . 9452 | . 0288 | . 9466 | . 0263 | . 8354 | . 0371 | . 9439 | . 0274 | . 9448 | . 0269 |
|  | 2 | . 9505 | . 0262 | . 9490 | . 0119 | . 9408 | . 0304 | . 9473 | . 0274 | . 9507 | . 0250 | . 8414 | . 0443 | . 9481 | . 0262 | . 9487 | . 0260 |
|  | 3 | . 9503 | . 0253 | . 9476 | . 0110 | . 9414 | . 0286 | . 9473 | . 0262 | . 9527 | . 0243 | . 8410 | . 0369 | . 9495 | . 0250 | . 9504 | . 0249 |
| 30 | 1 | . 9496 | . 0274 | . 9474 | . 0119 | . 9418 | . 0305 | . 9471 | . 0281 | . 9516 | . 0267 | . 8491 | . 0527 | . 9490 | . 0277 | . 9491 | . 0277 |
|  | 2 | . 9513 | . 0255 | . 9495 | . 0106 | . 9427 | . 0288 | . 9486 | . 0263 | . 9528 | . 0257 | . 8495 | . 0554 | . 9501 | . 0268 | . 9505 | . 0267 |
|  | 3 | . 9461 | . 0296 | . 9443 | . 0136 | . 9373 | . 0334 | . 9434 | . 0305 | . 9487 | . 0268 | . 8426 | . 0642 | . 9461 | . 0279 | . 9465 | . 0275 |
| 42 | 1 | . 9502 | . 0272 | . 9484 | . 0121 | . 9423 | . 0305 | . 9477 | . 0278 | . 9490 | . 0268 | . 8428 | . 0740 | . 9460 | . 0280 | . 9467 | . 0275 |
|  | 2 | . 9513 | . 0251 | . 9490 | . 0107 | . 9424 | . 0285 | . 9481 | . 0258 | . 9515 | . 0234 | . 8380 | . 0919 | . 9492 | . 0243 | . 9494 | . 0241 |
|  | 3 | . 9511 | . 0265 | . 9496 | . 0112 | . 9423 | . 0299 | . 9483 | . 0269 | . 9496 | . 0270 | . 8447 | . 0761 | . 9469 | . 0281 | . 9474 | . 0277 |

Table A. 8. CI coverage for pairwise techniques: Normal skew distribution and $\rho=.5$.

|  |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\prime} i_{P}$ |  | $z$ |  | Sp |  | mult $_{P}$ |  | $u^{\text {un }}{ }_{P}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  |  | $N=50$ |  |  |  |  |  |  |  | $N=500$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9501 | . 0260 | . 9445 | . 0085 | . 9359 | . 0321 | . 9411 | . 030 | . 948 | . 0276 | . 8873 | . 0105 | . 9438 | . 0285 | . 9426 | . 0287 |
|  | 2 | . 9443 | . 0332 | . 9428 | . 0125 | . 9315 | . 0405 | . 9343 | . 0390 | . 9497 | . 0255 | . 8828 | . 0227 | . 9452 | . 0272 | . 9453 | . 0272 |
|  | 3 | . 9497 | . 0260 | . 9440 | . 0070 | . 9367 | . 0310 | . 9407 | . 0310 | . 9580 | . 0217 | . 8910 | . 0750 | . 9530 | . 0213 | . 9540 | . 0217 |
| 18 | 1 | . 9529 | . 0288 | . 9500 | . 0082 | . 9395 | . 0346 | . 9449 | . 0317 | . 9519 | . 0241 | . 8849 | . 0148 | . 9476 | . 0253 | . 9487 | . 0251 |
|  | 2 | . 9509 | . 0288 | . 9476 | . 0093 | . 9382 | . 0346 | . 9425 | . 0324 | . 9528 | . 0236 | . 8889 | . 0149 | . 9491 | . 0248 | . 9500 | . 0242 |
|  | 3 | . 9505 | . 0282 | . 9465 | . 0081 | . 9364 | . 0341 | . 9429 | . 0305 | . 9500 | . 0239 | . 8853 | . 0172 | . 9470 | . 0248 | . 9473 | . 0246 |
| 30 | 1 | . 9494 | . 0269 | . 9418 | . 0082 | . 9348 | . 0327 | . 9406 | . 0301 | . 9498 | . 0282 | . 8929 | . 0152 | . 9465 | . 0288 | . 9474 | . 0284 |
|  | 2 | . 9481 | . 0315 | . 9471 | . 0093 | . 9349 | . 0369 | . 9403 | . 0345 | . 9481 | . 0273 | . 8861 | . 0185 | . 9439 | . 0285 | . 9451 | . 0279 |
|  | 3 | . 9498 | . 0300 | . 9486 | . 0095 | . 9352 | . 0359 | . 9411 | . 0336 | . 9463 | . 0273 | . 8849 | . 0266 | . 9420 | . 0283 | . 9430 | . 0278 |
| 42 | 1 | . 9540 | . 0256 | . 9491 | . 0081 | . 9401 | . 0308 | . 9457 | . 0282 | . 9507 | . 0252 | . 8901 | . 0391 | . 9467 | . 0263 | . 9475 | . 0259 |
|  | 2 | . 9464 | . 0302 | . 9435 | . 0100 | . 9326 | . 0370 | . 9377 | . 0342 | . 9503 | . 0256 | . 8872 | . 0389 | . 9461 | . 0268 | . 9466 | . 0266 |
|  | 3 | . 9524 | . 0277 | . 9469 | . 0089 | . 9386 | . 0339 | . 9444 | . 0310 | . 9500 | . 0252 | . 8869 | . 0434 | . 9465 | . 0263 | . 9469 | . 0261 |
|  |  | $N=100$ |  |  |  |  |  |  |  | $N=2000$ |  |  |  |  |  |  |  |
| 6 | 1 | . 9527 | . 0283 | . 9407 | . 0077 | . 9455 | . 0301 | . 9470 | . 2295 | . 9435 | . 0283 | . 6831 | . 2340 | . 9409 | . 0285 | . 9425 | . 0283 |
|  | 2 | . 9460 | . 0325 | . 9415 | . 0085 | . 9377 | . 0340 | . 9415 | . 0338 | . 9472 | . 0267 | . 6888 | . 2622 | . 9437 | . 0280 | . 9440 | . 0282 |
|  | 3 | . 9547 | . 0213 | . 9387 | . 0033 | . 9467 | . 0240 | . 9493 | . 0233 | . 9450 | . 0257 | . 6863 | . 3137 | . 9420 | . 0273 | . 9393 | . 0283 |
| 18 | 1 | . 9485 | . 0294 | . 9369 | . 0073 | . 9392 | . 0322 | . 9424 | . 0308 | . 9492 | . 0276 | . 6871 | . 2759 | . 9467 | . 0283 | . 9471 | . 0280 |
|  | 2 | . 9515 | . 0278 | . 9403 | . 0075 | . 9429 | . 0313 | . 9459 | . 0299 | . 9503 | . 0266 | . 6863 | . 2409 | . 9476 | . 0275 | . 9476 | . 0272 |
|  | 3 | . 9480 | . 0272 | . 9388 | . 0062 | . 9389 | . 0304 | . 9422 | . 0283 | . 9483 | . 0254 | . 6851 | . 2796 | . 9451 | . 0266 | . 9450 | . 0264 |
| 30 | 1 | . 9445 | . 0310 | . 9367 | . 0082 | . 9362 | . 0341 | . 9391 | . 0326 | . 9548 | . 0230 | . 6800 | . 2930 | . 9524 | . 0240 | . 9524 | . 0240 |
|  | 2 | . 9513 | . 0265 | . 9427 | . 0067 | . 9438 | . 0288 | . 9469 | . 0272 | . 9467 | . 0289 | . 6949 | . 2538 | . 9444 | . 0297 | . 9441 | . 0298 |
|  | 3 | . 9510 | . 0283 | . 9449 | . 0072 | . 9423 | . 0313 | . 9452 | . 0299 | . 9528 | . 0238 | . 6815 | . 2783 | . 9497 | . 0249 | . 9513 | . 0240 |
| 42 | 1 | . 9539 | . 0270 | . 9422 | . 0083 | . 9455 | . 0302 | . 9491 | . 0285 | . 9467 | . 0246 | . 6772 | . 3034 | . 9437 | . 0255 | . 9441 | . 0253 |
|  | 2 | . 9507 | . 0277 | . 9412 | . 0087 | . 9410 | . 0311 | . 9441 | . 0296 | . 9486 | . 0241 | . 6711 | . 3040 | . 9460 | . 0249 | . 9461 | . 0251 |
|  | 3 | . 9477 | . 0288 | . 9374 | . 0083 | . 9391 | . 0316 | . 9414 | . 0304 | . 9505 | . 0243 | . 6760 | . 3018 | . 9477 | . 0250 | . 9476 | . 0252 |

Table A. 9. CI coverage for pairwise techniques: Normal skew distribution and $\rho=.8$.

## APPENDIX B: CI WIDTH TABLES FOR PAIRWISE TECHNIQUES

|  |  |  | $z$ |  |  | Sp |  |  | multp |  |  | uniP |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | K | $F$ | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| 50 | 6 | 1 | . 5129 | . 5362 | . 5507 | . 5260 | . 5429 | . 5528 | . 4696 | . 5159 | . 5633 | . 5089 | . 5322 | . 5500 |
|  |  | 2 | . 5118 | . 5356 | . 5504 | . 5242 | . 5425 | . 5527 | . 4683 | . 5138 | . 5616 | . 5061 | . 5325 | . 5500 |
|  |  | 3 | . 5122 | . 5361 | . 5500 | . 5252 | . 5427 | . 5527 | . 4682 | . 5117 | . 5614 | . 5069 | . 5321 | . 5499 |
|  | 18 | 1 | . 5120 | . 5355 | . 5501 | . 5250 | . 5424 | . 5525 | . 4680 | . 5138 | . 5613 | . 5073 | . 5315 | . 5495 |
|  |  | 2 | . 5122 | . 5355 | . 5502 | . 5248 | . 5423 | . 5526 | . 4675 | . 5130 | . 5608 | . 5073 | . 5317 | . 5497 |
|  |  | 3 | . 5119 | . 5358 | . 5502 | . 5247 | . 5426 | . 5526 | . 4682 | . 5139 | . 5618 | . 5072 | . 5320 | . 5499 |
|  | 30 | 1 | . 5116 | . 5352 | . 5499 | . 5246 | . 5421 | . 5524 | . 4672 | . 5127 | . 5605 | . 5070 | . 5313 | . 5494 |
|  |  | 2 | . 5121 | . 5356 | . 5502 | . 5251 | . 5424 | . 5526 | . 4687 | . 5138 | . 5617 | . 5076 | . 5317 | . 5497 |
|  |  | 3 | . 5123 | . 5357 | . 5501 | . 5251 | . 5425 | . 5525 | . 4680 | . 5134 | . 5613 | . 5078 | . 5318 | . 5497 |
|  | 42 | 1 | . 5120 | . 5354 | . 5501 | . 5248 | . 5423 | . 5525 | . 4678 | . 5135 | . 5613 | . 5073 | . 5315 | . 5496 |
|  |  | 2 | . 5121 | . 5356 | . 5501 | . 5250 | . 5424 | . 5525 | . 4680 | . 5132 | . 5611 | . 5073 | . 5316 | . 5497 |
|  |  | 3 | . 5122 | . 5357 | . 5503 | . 5251 | . 5425 | . 5526 | . 4682 | . 5138 | . 5613 | . 5076 | . 5316 | . 5496 |
| 100 | 6 | 1 | . 3663 | . 3779 | . 3860 | . 3741 | . 3826 | . 3886 | . 3431 | . 3677 | . 3936 | . 3620 | . 3749 | . 3864 |
|  |  | 2 | . 3664 | . 3778 | . 3862 | . 3739 | . 3826 | . 3886 | . 3424 | . 3676 | . 3943 | . 3622 | . 3755 | . 3868 |
|  |  | 3 | . 3658 | . 3771 | . 3860 | . 3736 | . 3823 | . 3883 | . 3433 | . 3668 | . 3924 | . 3617 | . 3742 | . 3859 |
|  | 18 | 1 | . 3661 | . 3776 | . 3861 | . 3738 | . 3825 | . 3885 | . 3426 | . 3675 | . 3938 | . 3617 | . 3748 | . 3863 |
|  |  | 2 | . 3658 | . 3774 | . 3859 | . 3737 | . 3823 | . 3884 | . 3423 | . 3672 | . 3936 | . 3616 | . 3746 | . 3861 |
|  |  | 3 | . 3662 | . 3777 | . 3859 | . 3738 | . 3824 | . 3885 | . 3427 | . 3672 | . 3930 | . 3618 | . 3748 | . 3861 |
|  | 30 | 1 | . 3659 | . 3774 | . 3859 | . 3737 | . 3823 | . 3884 | . 3425 | . 3673 | . 3933 | . 3615 | . 3746 | . 3860 |
|  |  | 2 | . 3658 | . 3773 | . 3858 | . 3737 | . 3823 | . 3884 | . 3425 | . 3672 | . 3933 | . 3614 | . 3745 | . 3859 |
|  |  | 3 | . 3660 | . 3775 | . 3858 | . 3737 | . 3823 | . 3884 | . 3424 | . 3670 | . 3929 | . 3616 | . 3746 | . 3861 |
|  | 42 | 1 | . 3660 | . 3774 | . 3858 | . 3737 | . 3823 | . 3883 | . 3423 | . 3672 | . 3933 | . 3615 | . 3747 | . 3860 |
|  |  | 2 | . 3662 | . 3777 | . 3860 | . 3739 | . 3825 | . 3885 | . 3425 | . 3675 | . 3934 | . 3617 | . 3748 | . 3861 |
|  |  | 3 | . 3662 | . 3776 | . 3860 | . 3739 | . 3825 | . 3885 | . 3425 | . 3676 | . 3936 | . 3617 | . 3749 | . 3862 |
| 500 | 6 | 1 | . 1663 | . 1684 | . 1703 | . 1689 | . 1706 | . 1719 | . 1605 | . 1667 | . 1730 | . 1634 | . 1673 | . 1713 |
|  |  | 2 | . 1662 | . 1684 | . 1703 | . 1689 | . 1705 | . 1720 | . 1606 | . 1668 | . 1731 | . 1633 | . 1673 | . 1713 |
|  |  | 3 | . 1663 | . 1684 | . 1703 | . 1690 | . 1706 | . 1719 | . 1604 | . 1665 | . 1727 | . 1635 | . 1674 | . 1714 |
|  | 18 | 1 | . 1663 | . 1684 | . 1703 | . 1690 | . 1706 | . 1720 | . 1606 | . 1668 | . 1732 | . 1635 | . 1674 | . 1713 |
|  |  | 2 | . 1662 | . 1684 | . 1703 | . 1689 | . 1705 | . 1719 | . 1605 | . 1666 | . 1730 | . 1634 | . 1674 | . 1713 |
|  |  | 3 | . 1662 | . 1684 | . 1703 | . 1689 | . 1705 | . 1719 | . 1604 | . 1666 | . 1730 | . 1634 | . 1673 | . 1713 |
|  | 30 | 1 | . 1663 | . 1684 | . 1703 | . 1690 | . 1706 | . 1720 | . 1605 | . 1667 | . 1731 | . 1635 | . 1674 | . 1713 |
|  |  | 2 | . 1662 | . 1684 | . 1703 | . 1689 | . 1705 | . 1719 | . 1605 | . 1666 | . 1730 | . 1634 | . 1673 | . 1712 |
|  |  | 3 | . 1663 | . 1684 | . 1703 | . 1689 | . 1706 | . 1719 | . 1605 | . 1666 | . 1730 | . 1634 | . 1674 | . 1713 |
|  | 42 | 1 | . 1663 | . 1684 | . 1703 | . 1689 | . 1705 | . 1719 | . 1605 | . 1666 | . 1730 | . 1634 | . 1673 | . 1713 |
|  |  | 2 | . 1662 | . 1684 | . 1703 | . 1689 | . 1705 | . 1719 | . 1605 | . 1666 | . 1731 | . 1634 | . 1673 | . 1713 |
|  |  | 3 | . 1663 | . 1684 | . 1703 | . 1689 | . 1706 | . 1720 | . 1605 | . 1666 | . 1730 | . 1634 | . 1674 | . 1713 |
| 2000 | 6 | 1 | . 0836 | . 0842 | . 0846 | . 0848 | . 0852 | . 0856 | . 0814 | . 0836 | . 0858 | . 0820 | . 0837 | . 0855 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0848 | . 0852 | . 0856 | . 0815 | . 0835 | . 0858 | . 0819 | . 0837 | . 0855 |
|  |  | 3 | . 0836 | . 0842 | . 0847 | . 0848 | . 0852 | . 0856 | . 0815 | . 0837 | . 0858 | . 0820 | . 0837 | . 0854 |
|  | 18 | 1 | . 0836 | . 0842 | . 0846 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0819 | . 0837 | . 0855 |
|  |  | 2 | . 0836 | . 0842 | . 0846 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0819 | . 0837 | . 0855 |
|  |  | 3 | . 0836 | . 0842 | . 0847 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0819 | . 0837 | . 0855 |
|  | 30 | 1 | . 0836 | . 0841 | . 0846 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0819 | . 0837 | . 0855 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0820 | . 0837 | . 0855 |
|  |  | 3 | . 0836 | . 0842 | . 0846 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0819 | . 0837 | . 0855 |
|  | 42 | 1 | . 0836 | . 0842 | . 0846 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0820 | . 0837 | . 0855 |
|  |  | 2 | . 0836 | . 0842 | . 0846 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0820 | . 0837 | . 0855 |
|  |  | 3 | . 0836 | . 0842 | . 0847 | . 0848 | . 0852 | . 0856 | . 0815 | . 0836 | . 0858 | . 0820 | . 0837 | . 0855 |

Table B.1. CI widths: Normal distribution, $\rho=.2$. $\mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  | $z$ |  |  | Sp |  |  | multp |  |  | unip |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | K | F | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| 50 | 6 | 1 | . 3826 | . 4227 | . 4619 | . 4225 | . 4590 | . 4919 | . 3591 | . 4101 | . 4625 | . 3778 | . 4201 | . 4615 |
|  |  | 2 | . 3827 | . 4230 | . 4602 | . 4234 | . 4594 | . 4913 | . 3594 | . 4086 | . 4624 | . 3779 | . 4205 | . 4600 |
|  |  | 3 | . 3806 | . 4237 | . 4611 | . 4204 | . 4592 | . 4917 | . 3568 | . 4086 | . 4618 | . 3753 | . 4207 | . 4628 |
|  | 18 | 1 | . 3844 | . 4244 | . 4622 | . 4243 | . 4600 | . 4918 | . 3600 | . 4099 | . 4628 | . 3799 | . 4217 | . 4618 |
|  |  | 2 | . 3850 | . 4251 | . 4625 | . 4251 | . 4610 | . 4925 | . 3616 | . 4115 | . 4640 | . 3808 | . 4228 | . 4625 |
|  |  | 3 | . 3843 | . 4238 | . 4617 | . 4238 | . 4600 | . 4914 | . 3600 | . 4099 | . 4631 | . 3799 | . 4215 | . 4617 |
|  | 30 | 1 | . 3822 | . 4222 | . 4601 | . 4227 | . 4587 | . 4905 | . 3586 | . 4078 | . 4604 | . 3777 | . 4197 | . 4597 |
|  |  | 2 | . 3833 | . 4232 | . 4613 | . 4231 | . 4593 | . 4912 | . 3590 | . 4091 | . 4626 | . 3787 | . 4208 | . 4608 |
|  |  | 3 | . 3833 | . 4236 | . 4615 | . 4238 | . 4602 | . 4919 | . 3595 | . 4099 | . 4628 | . 3788 | . 4214 | . 4613 |
|  | 42 | 1 | . 3831 | . 4232 | . 4607 | . 4232 | . 4593 | . 4908 | . 3590 | . 4091 | . 4618 | . 3786 | . 4207 | . 4604 |
|  |  | 2 | . 3848 | . 4246 | . 4618 | . 4244 | . 4604 | . 4917 | . 3605 | . 4104 | . 4627 | . 3801 | . 4218 | . 4615 |
|  |  | 3 | . 3824 | . 4225 | . 4604 | . 4229 | . 4590 | . 4909 | . 3590 | . 4087 | . 4614 | . 3779 | . 4200 | . 4602 |
| 100 | 6 | 1 | . 2766 | . 2966 | . 3161 | . 3035 | . 3216 | . 3379 | . 2642 | . 2900 | . 3180 | . 2736 | . 2952 | . 3158 |
|  |  | 2 | . 2768 | . 2972 | . 3165 | . 3034 | . 3218 | . 3390 | . 2644 | . 2909 | . 3187 | . 2738 | . 2955 | . 3160 |
|  |  | 3 | . 2765 | . 2975 | . 3168 | . 3035 | . 3221 | . 3390 | . 2642 | . 2901 | . 3200 | . 2741 | . 2959 | . 3163 |
|  | 18 | 1 | . 2763 | . 2964 | . 3160 | . 3029 | . 3213 | . 3380 | . 2637 | . 2901 | . 3178 | . 2732 | . 2946 | . 3156 |
|  |  | 2 | . 2764 | . 2964 | . 3157 | . 3029 | . 3209 | . 3377 | . 2633 | . 2896 | . 3171 | . 2733 | . 2946 | . 3154 |
|  |  | 3 | . 2769 | . 2968 | . 3162 | . 3036 | . 3217 | . 3382 | . 2641 | . 2904 | . 3180 | . 2741 | . 2953 | . 3160 |
|  | 30 | 1 | . 2764 | . 2963 | . 3156 | . 3032 | . 3213 | . 3378 | . 2637 | . 2900 | . 3176 | . 2735 | . 2946 | . 3153 |
|  |  | 2 | . 2771 | . 2970 | . 3163 | . 3037 | . 3217 | . 3381 | . 2645 | . 2905 | . 3178 | . 2741 | . 2953 | . 3159 |
|  |  | 3 | . 2761 | . 2964 | . 3160 | . 3028 | . 3212 | . 3380 | . 2634 | . 2901 | . 3180 | . 2732 | . 2949 | . 3158 |
|  | 42 | 1 | . 2763 | . 2964 | . 3158 | . 3031 | . 3212 | . 3378 | . 2638 | . 2900 | . 3177 | . 2733 | . 2947 | . 3152 |
|  |  | 2 | . 2769 | . 2968 | . 3160 | . 3036 | . 3215 | . 3381 | . 2643 | . 2903 | . 3179 | . 2738 | . 2951 | . 3157 |
|  |  | 3 | . 2768 | . 2969 | . 3160 | . 3034 | . 3214 | . 3379 | . 2638 | . 2899 | . 3175 | . 2738 | . 2951 | . 3156 |
| 500 | 6 | 1 | . 1278 | . 1317 | . 1356 | . 1388 | . 1423 | . 1458 | . 1244 | . 1304 | . 1368 | . 1261 | . 1309 | . 1358 |
|  |  | 2 | . 1277 | . 1317 | . 1356 | . 1387 | . 1424 | . 1458 | . 1246 | . 1305 | . 1368 | . 1261 | . 1310 | . 1357 |
|  |  | 3 | . 1276 | . 1316 | . 1358 | . 1389 | . 1424 | . 1458 | . 1245 | . 1305 | . 1367 | . 1262 | . 1310 | . 1359 |
|  | 18 | 1 | . 1277 | . 1316 | . 1356 | . 1388 | . 1424 | . 1459 | . 1245 | . 1305 | . 1367 | . 1261 | . 1309 | . 1358 |
|  |  | 2 | . 1278 | . 1318 | . 1357 | . 1389 | . 1425 | . 1460 | . 1247 | . 1307 | . 1369 | . 1262 | . 1311 | . 1359 |
|  |  | 3 | . 1279 | . 1318 | . 1358 | . 1389 | . 1425 | . 1460 | . 1246 | . 1306 | . 1369 | . 1263 | . 1312 | . 1360 |
|  | 30 | 1 | . 1277 | . 1317 | . 1356 | . 1388 | . 1424 | . 1459 | . 1244 | . 1304 | . 1367 | . 1261 | . 1309 | . 1358 |
|  |  | 2 | . 1276 | . 1316 | . 1356 | . 1387 | . 1424 | . 1458 | . 1244 | . 1304 | . 1366 | . 1261 | . 1309 | . 1358 |
|  |  | 3 | . 1276 | . 1315 | . 1355 | . 1387 | . 1423 | . 1457 | . 1243 | . 1303 | . 1365 | . 1260 | . 1308 | . 1356 |
|  | 42 | 1 | . 1280 | . 1320 | . 1359 | . 1391 | . 1427 | . 1462 | . 1249 | . 1309 | . 1372 | . 1264 | . 1313 | . 1362 |
|  |  | 2 | . 1276 | . 1316 | . 1356 | . 1387 | . 1423 | . 1458 | . 1244 | . 1304 | . 1366 | . 1260 | . 1309 | . 1357 |
|  |  | 3 | . 1276 | . 1316 | . 1355 | . 1387 | . 1423 | . 1458 | . 1243 | . 1303 | . 1365 | . 1260 | . 1309 | . 1357 |
| 2000 | 6 | 1 | . 0648 | . 0658 | . 0668 | . 0702 | . 0711 | . 0720 | . 0635 | . 0654 | . 0674 | . 0638 | . 0655 | . 0672 |
|  |  | 2 | . 0648 | . 0658 | . 0668 | . 0702 | . 0711 | . 0720 | . 0635 | . 0653 | . 0673 | . 0637 | . 0654 | . 0672 |
|  |  | 3 | . 0648 | . 0658 | . 0667 | . 0702 | . 0711 | . 0720 | . 0634 | . 0654 | . 0674 | . 0638 | . 0654 | . 0672 |
|  | 18 | 1 | . 0648 | . 0658 | . 0668 | . 0702 | . 0711 | . 0720 | . 0635 | . 0654 | . 0673 | . 0638 | . 0654 | . 0671 |
|  |  | 2 | . 0648 | . 0658 | . 0667 | . 0702 | . 0711 | . 0720 | . 0635 | . 0653 | . 0673 | . 0638 | . 0654 | . 0671 |
|  |  | 3 | . 0648 | . 0658 | . 0668 | . 0702 | . 0711 | . 0720 | . 0634 | . 0654 | . 0673 | . 0638 | . 0654 | . 0671 |
|  | 30 | 1 | . 0647 | . 0657 | . 0667 | . 0702 | . 0711 | . 0719 | . 0634 | . 0653 | . 0673 | . 0637 | . 0654 | . 0671 |
|  |  | 2 | . 0647 | . 0657 | . 0667 | . 0702 | . 0710 | . 0719 | . 0634 | . 0653 | . 0673 | . 0637 | . 0654 | . 0671 |
|  |  | 3 | . 0648 | . 0658 | . 0668 | . 0702 | . 0711 | . 0720 | . 0635 | . 0654 | . 0673 | . 0638 | . 0654 | . 0671 |
|  | 42 | 1 | . 0647 | . 0657 | . 0667 | . 0702 | . 0711 | . 0719 | . 0634 | . 0653 | . 0673 | . 0637 | . 0654 | . 0671 |
|  |  | 2 | . 0648 | . 0658 | . 0668 | . 0702 | . 0711 | . 0720 | . 0635 | . 0654 | . 0673 | . 0638 | . 0654 | . 0671 |
|  |  | 3 | . 0647 | . 0658 | . 0667 | . 0702 | . 0711 | . 0719 | . 0635 | . 0653 | . 0673 | . 0637 | . 0654 | . 0671 |

Table B.2. CI widths: Normal distribution, $\rho=.5 . \mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  | $z$ |  |  | Sp |  |  | multp |  |  | unip |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | K | $F$ | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| 50 | 6 | 1 | . 1786 | . 2088 | . 2429 | . 2251 | . 2626 | . 3024 | . 1692 | . 2034 | . 2423 | . 1735 | . 2058 | . 2411 |
|  |  | 2 | . 1770 | . 2075 | . 2409 | . 2228 | . 2603 | . 3012 | . 1676 | . 2004 | . 2410 | . 1716 | . 2039 | . 2385 |
|  |  | 3 | . 1796 | . 2093 | . 2421 | . 2286 | . 2629 | . 3013 | . 1710 | . 2028 | . 2400 | . 1749 | . 2054 | . 2407 |
|  | 18 | 1 | . 1776 | . 2072 | . 2401 | . 2243 | . 2610 | . 2999 | . 1684 | . 2013 | . 2395 | . 1728 | . 2041 | . 2388 |
|  |  | 2 | . 1778 | . 2076 | . 2409 | . 2236 | . 2612 | . 3009 | . 1683 | . 2018 | . 2398 | . 1726 | . 2042 | . 2390 |
|  |  | 3 | . 1779 | . 2076 | . 2411 | . 2243 | . 2612 | . 3008 | . 1689 | . 2022 | . 2410 | . 1728 | . 2041 | . 2401 |
|  | 30 | 1 | . 1794 | . 2099 | . 2435 | . 2254 | . 2632 | . 3032 | . 1700 | . 2037 | . 2429 | . 1740 | . 2063 | . 2421 |
|  |  | 2 | . 1769 | . 2069 | . 2404 | . 2227 | . 2602 | . 2998 | . 1675 | . 2007 | . 2393 | . 1716 | . 2033 | . 2383 |
|  |  | 3 | . 1777 | . 2077 | . 2413 | . 2237 | . 2609 | . 3007 | . 1683 | . 2017 | . 2398 | . 1727 | . 2046 | . 2397 |
|  | 42 | 1 | . 1793 | . 2093 | . 2423 | . 2268 | . 2635 | . 3023 | . 1708 | . 2039 | . 2420 | . 1750 | . 2063 | . 2408 |
|  |  | 2 | . 1774 | . 2078 | . 2418 | . 2226 | . 2603 | . 3007 | . 1674 | . 2010 | . 2403 | . 1716 | . 2038 | . 2396 |
|  |  | 3 | . 1781 | . 2080 | . 2411 | . 2247 | . 2616 | . 3013 | . 1686 | . 2020 | . 2406 | . 1727 | . 2044 | . 2398 |
| 100 | 6 | 1 | . 1293 | . 1442 | . 1598 | . 1605 | . 1793 | . 1981 | . 1250 | . 1421 | . 1607 | . 1271 | . 1430 | . 1600 |
|  |  | 2 | . 1285 | . 1433 | . 1601 | . 1591 | . 1778 | . 1968 | . 1233 | . 1409 | . 1593 | . 1256 | . 1427 | . 1598 |
|  |  | 3 | . 1292 | . 1443 | . 1597 | . 1595 | . 1782 | . 1976 | . 1242 | . 1411 | . 1609 | . 1262 | . 1421 | . 1599 |
|  | 18 | 1 | . 1292 | . 1445 | . 1609 | . 1597 | . 1786 | . 1984 | . 1245 | . 1418 | . 1610 | . 1264 | . 1427 | . 1604 |
|  |  | 2 | . 1289 | . 1438 | . 1596 | . 1591 | . 1776 | . 1971 | . 1239 | . 1409 | . 1601 | . 1258 | . 1417 | . 1591 |
|  |  | 3 | . 1290 | . 1438 | . 1601 | . 1595 | . 1778 | . 1976 | . 1243 | . 1412 | . 1604 | . 1261 | . 1422 | . 1599 |
|  | 30 | 1 | . 1284 | . 1435 | . 1597 | . 1589 | . 1774 | . 1971 | . 1234 | . 1406 | . 1597 | . 1254 | . 1416 | . 1592 |
|  |  | 2 | . 1295 | . 1444 | . 1601 | . 1604 | . 1786 | . 1975 | . 1249 | . 1418 | . 1604 | . 1270 | . 1428 | . 1597 |
|  |  | 3 | . 1289 | . 1438 | . 1597 | . 1592 | . 1776 | . 1966 | . 1241 | . 1408 | . 1596 | . 1262 | . 1421 | . 1591 |
|  | 42 | 1 | . 1286 | . 1433 | . 1592 | . 1595 | . 1780 | . 1973 | . 1243 | . 1411 | . 1598 | . 1262 | . 1421 | . 1592 |
|  |  | 2 | . 1291 | . 1440 | . 1599 | . 1598 | . 1781 | . 1974 | . 1243 | . 1413 | . 1600 | . 1263 | . 1423 | . 1595 |
|  |  | 3 | . 1290 | . 1440 | . 1603 | . 1598 | . 1784 | . 1981 | . 1246 | . 1416 | . 1608 | . 1266 | . 1428 | . 1603 |
| 500 | 6 | 1 | . 0603 | . 0634 | . 0665 | . 0735 | . 0772 | . 0810 | . 0592 | . 0628 | . 0668 | . 0594 | . 0629 | . 0665 |
|  |  | 2 | . 0603 | . 0634 | . 0666 | . 0736 | . 0774 | . 0812 | . 0592 | . 0630 | . 0669 | . 0596 | . 0631 | . 0666 |
|  |  | 3 | . 0605 | . 0634 | . 0665 | . 0737 | . 0774 | . 0811 | . 0592 | . 0630 | . 0667 | . 0596 | . 0631 | . 0668 |
|  | 18 | 1 | . 0605 | . 0635 | . 0666 | . 0738 | . 0774 | . 0812 | . 0593 | . 0630 | . 0668 | . 0596 | . 0630 | . 0666 |
|  |  | 2 | . 0605 | . 0634 | . 0665 | . 0737 | . 0773 | . 0810 | . 0592 | . 0629 | . 0667 | . 0595 | . 0630 | . 0665 |
|  |  | 3 | . 0604 | . 0634 | . 0666 | . 0737 | . 0773 | . 0811 | . 0592 | . 0629 | . 0668 | . 0596 | . 0630 | . 0666 |
|  | 30 | 1 | . 0602 | . 0632 | . 0663 | . 0735 | . 0772 | . 0809 | . 0591 | . 0628 | . 0667 | . 0595 | . 0629 | . 0665 |
|  |  | 2 | . 0603 | . 0634 | . 0665 | . 0735 | . 0772 | . 0811 | . 0591 | . 0628 | . 0667 | . 0594 | . 0629 | . 0666 |
|  |  | 3 | . 0604 | . 0634 | . 0666 | . 0735 | . 0773 | . 0811 | . 0591 | . 0629 | . 0668 | . 0595 | . 0630 | . 0666 |
|  | 42 | 1 | . 0603 | . 0633 | . 0664 | . 0735 | . 0772 | . 0810 | . 0591 | . 0628 | . 0667 | . 0595 | . 0629 | . 0665 |
|  |  | 2 | . 0604 | . 0634 | . 0665 | . 0735 | . 0773 | . 0811 | . 0591 | . 0628 | . 0668 | . 0594 | . 0630 | . 0666 |
|  |  | 3 | . 0603 | . 0633 | . 0664 | . 0735 | . 0772 | . 0811 | . 0591 | . 0628 | . 0667 | . 0595 | . 0629 | . 0666 |
| 2000 | 6 | 1 | . 0308 | . 0316 | . 0324 | . 0375 | . 0384 | . 0394 | . 0303 | . 0314 | . 0325 | . 0304 | . 0314 | . 0325 |
|  |  | 2 | . 0308 | . 0316 | . 0323 | . 0375 | . 0384 | . 0393 | . 0303 | . 0314 | . 0326 | . 0304 | . 0314 | . 0325 |
|  |  | 3 | . 0309 | . 0316 | . 0324 | . 0375 | . 0384 | . 0393 | . 0303 | . 0314 | . 0325 | . 0304 | . 0314 | . 0325 |
|  | 18 | 1 | . 0308 | . 0316 | . 0323 | . 0375 | . 0384 | . 0393 | . 0303 | . 0314 | . 0325 | . 0304 | . 0314 | . 0325 |
|  |  | 2 | . 0308 | . 0316 | . 0323 | . 0375 | . 0384 | . 0393 | . 0303 | . 0314 | . 0325 | . 0304 | . 0314 | . 0325 |
|  |  | 3 | . 0308 | . 0316 | . 0324 | . 0375 | . 0384 | . 0394 | . 0303 | . 0314 | . 0325 | . 0304 | . 0314 | . 0325 |
|  | 30 | 1 | . 0309 | . 0316 | . 0324 | . 0375 | . 0384 | . 0394 | . 0303 | . 0314 | . 0325 | . 0304 | . 0314 | . 0325 |
|  |  | 2 | . 0308 | . 0316 | . 0323 | . 0374 | . 0384 | . 0393 | . 0303 | . 0314 | . 0325 | . 0303 | . 0314 | . 0324 |
|  |  | 3 | . 0308 | . 0316 | . 0324 | . 0375 | . 0384 | . 0394 | . 0303 | . 0314 | . 0325 | . 0304 | . 0314 | . 0325 |
|  | 42 | 1 | . 0308 | . 0316 | . 0324 | . 0375 | . 0384 | . 0394 | . 0303 | . 0314 | . 0325 | . 0304 | . 0314 | . 0325 |
|  |  | 2 | . 0309 | . 0316 | . 0324 | . 0375 | . 0385 | . 0394 | . 0304 | . 0314 | . 0326 | . 0304 | . 0315 | . 0325 |
|  |  | 3 | . 0308 | . 0316 | . 0324 | . 0375 | . 0384 | . 0394 | . 0303 | . 0314 | . 0326 | . 0304 | . 0314 | . 0325 |

Table B.3. CI widths: Normal distribution, $\rho=.8$. $\mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  | $z$ |  |  | Sp |  |  | multp |  |  | unip |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | K | $F$ | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| 50 | 6 | 1 | . 5100 | . 5349 | . 5499 | . 5215 | . 5400 | . 5515 | . 4746 | . 5193 | . 5697 | . 5103 | . 5334 | . 5507 |
|  |  | 2 | . 5099 | . 5352 | . 5501 | . 5216 | . 5401 | . 5514 | . 4752 | . 5205 | . 5712 | . 5103 | . 5332 | . 5511 |
|  |  | 3 | . 5083 | . 5353 | . 5502 | . 5214 | . 5404 | . 5517 | . 4736 | . 5201 | . 5727 | . 5104 | . 5331 | . 5504 |
|  | 18 | 1 | . 5109 | . 5359 | . 5507 | . 5227 | . 5410 | . 5519 | . 4752 | . 5213 | . 5721 | . 5109 | . 5339 | . 5512 |
|  |  | 2 | . 5108 | . 5358 | . 5505 | . 5226 | . 5408 | . 5518 | . 4747 | . 5216 | . 5720 | . 5108 | . 5338 | . 5513 |
|  |  | 3 | . 5108 | . 5356 | . 5506 | . 5223 | . 5407 | . 5518 | . 4736 | . 5210 | . 5725 | . 5107 | . 5337 | . 5512 |
|  | 30 | 1 | . 5105 | . 5357 | . 5505 | . 5226 | . 5409 | . 5519 | . 4748 | . 5215 | . 5720 | . 5107 | . 5338 | . 5512 |
|  |  | 2 | . 5110 | . 5358 | . 5506 | . 5225 | . 5407 | . 5518 | . 4743 | . 5208 | . 5716 | . 5109 | . 5339 | . 5511 |
|  |  | 3 | . 5107 | . 5358 | . 5505 | . 5223 | . 5408 | . 5518 | . 4744 | . 5209 | . 5709 | . 5103 | . 5335 | . 5510 |
|  | 42 | 1 | . 5113 | . 5360 | . 5506 | . 5229 | . 5411 | . 5520 | . 4752 | . 5218 | . 5726 | . 5111 | . 5339 | . 5512 |
|  |  | 2 | . 5111 | . 5360 | . 5506 | . 5229 | . 5410 | . 5519 | . 4750 | . 5213 | . 5717 | . 5109 | . 5338 | . 5512 |
|  |  | 3 | . 5111 | . 5362 | . 5507 | . 5228 | . 5411 | . 5519 | . 4752 | . 5215 | . 5722 | . 5110 | . 5341 | . 5513 |
| 100 | 6 | 1 | . 3657 | . 3776 | . 3861 | . 3722 | . 3810 | . 3875 | . 3486 | . 3750 | . 4045 | . 3647 | . 3769 | . 3879 |
|  |  | 2 | . 3653 | . 3773 | . 3861 | . 3720 | . 3812 | . 3875 | . 3488 | . 3759 | . 4051 | . 3647 | . 3766 | . 3879 |
|  |  | 3 | . 3658 | . 3778 | . 3868 | . 3729 | . 3814 | . 3877 | . 3472 | . 3744 | . 4038 | . 3651 | . 3769 | . 3883 |
|  | 18 | 1 | . 3656 | . 3777 | . 3862 | . 3723 | . 3812 | . 3876 | . 3488 | . 3756 | . 4053 | . 3649 | . 3772 | . 3881 |
|  |  | 2 | . 3658 | . 3778 | . 3864 | . 3724 | . 3813 | . 3877 | . 3489 | . 3757 | . 4056 | . 3650 | . 3773 | . 3881 |
|  |  | 3 | . 3659 | . 3781 | . 3865 | . 3724 | . 3813 | . 3877 | . 3485 | . 3757 | . 4054 | . 3653 | . 3775 | . 3881 |
|  | 30 | 1 | . 3655 | . 3777 | . 3863 | . 3723 | . 3813 | . 3877 | . 3487 | . 3757 | . 4055 | . 3649 | . 3773 | . 3881 |
|  |  | 2 | . 3657 | . 3778 | . 3864 | . 3723 | . 3813 | . 3877 | . 3488 | . 3756 | . 4056 | . 3650 | . 3773 | . 3881 |
|  |  | 3 | . 3658 | . 3778 | . 3864 | . 3724 | . 3813 | . 3877 | . 3487 | . 3756 | . 4053 | . 3651 | . 3774 | . 3882 |
|  | 42 | 1 | . 3651 | . 3773 | . 3860 | . 3719 | . 3810 | . 3875 | . 3484 | . 3752 | . 4051 | . 3646 | . 3769 | . 3878 |
|  |  | 2 | . 3655 | . 3776 | . 3862 | . 3721 | . 3811 | . 3875 | . 3483 | . 3751 | . 4049 | . 3649 | . 3772 | . 3880 |
|  |  | 3 | . 3655 | . 3777 | . 3863 | . 3722 | . 3812 | . 3876 | . 3482 | . 3751 | . 4048 | . 3649 | . 3773 | . 3880 |
| 500 | 6 | 1 | . 1662 | . 1685 | . 1704 | . 1683 | . 1700 | . 1715 | . 1656 | . 1728 | . 1807 | . 1651 | . 1689 | . 1727 |
|  |  | 2 | . 1662 | . 1684 | . 1703 | . 1683 | . 1700 | . 1714 | . 1656 | . 1731 | . 1812 | . 1652 | . 1690 | . 1728 |
|  |  | 3 | . 1662 | . 1684 | . 1704 | . 1683 | . 1700 | . 1714 | . 1654 | . 1727 | . 1806 | . 1651 | . 1689 | . 1728 |
|  | 18 | 1 | . 1661 | . 1684 | . 1704 | . 1683 | . 1700 | . 1714 | . 1654 | . 1727 | . 1807 | . 1651 | . 1689 | . 1727 |
|  |  | 2 | . 1661 | . 1684 | . 1704 | . 1683 | . 1700 | . 1715 | . 1655 | . 1727 | . 1807 | . 1651 | . 1689 | . 1727 |
|  |  | 3 | . 1661 | . 1684 | . 1703 | . 1683 | . 1700 | . 1714 | . 1654 | . 1727 | . 1807 | . 1650 | . 1689 | . 1727 |
|  | 30 | 1 | . 1662 | . 1685 | . 1704 | . 1683 | . 1700 | . 1715 | . 1655 | . 1728 | . 1808 | . 1651 | . 1689 | . 1728 |
|  |  | 2 | . 1661 | . 1684 | . 1704 | . 1683 | . 1700 | . 1715 | . 1655 | . 1728 | . 1808 | . 1651 | . 1689 | . 1728 |
|  |  | 3 | . 1661 | . 1684 | . 1704 | . 1683 | . 1700 | . 1715 | . 1655 | . 1728 | . 1808 | . 1651 | . 1689 | . 1727 |
|  | 42 | 1 | . 1661 | . 1684 | . 1704 | . 1683 | . 1700 | . 1714 | . 1654 | . 1727 | . 1807 | . 1651 | . 1689 | . 1727 |
|  |  | 2 | . 1661 | . 1684 | . 1704 | . 1683 | . 1700 | . 1715 | . 1654 | . 1728 | . 1807 | . 1651 | . 1689 | . 1727 |
|  |  | 3 | . 1662 | . 1684 | . 1704 | . 1683 | . 1700 | . 1715 | . 1655 | . 1728 | . 1808 | . 1651 | . 1689 | . 1728 |
| 2000 | 6 | 1 | . 0836 | . 0842 | . 0847 | . 0845 | . 0849 | . 0853 | . 0847 | . 0872 | . 0899 | . 0828 | . 0846 | . 0863 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0845 | . 0850 | . 0853 | . 0846 | . 0872 | . 0898 | . 0827 | . 0845 | . 0863 |
|  |  | 3 | . 0836 | . 0842 | . 0847 | . 0845 | . 0849 | . 0853 | . 0846 | . 0872 | . 0898 | . 0827 | . 0845 | . 0863 |
|  | 18 | 1 | . 0836 | . 0842 | . 0847 | . 0845 | . 0850 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0845 | . 0850 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |
|  |  | 3 | . 0836 | . 0842 | . 0847 | . 0845 | . 0850 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |
|  | 30 | 1 | . 0836 | . 0842 | . 0847 | . 0845 | . 0849 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0845 | . 0850 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |
|  |  | 3 | . 0836 | . 0842 | . 0847 | . 0845 | . 0850 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |
|  | 42 | 1 | . 0836 | . 0842 | . 0847 | . 0845 | . 0850 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0845 | . 0849 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |
|  |  | 3 | . 0836 | . 0842 | . 0847 | . 0845 | . 0850 | . 0853 | . 0847 | . 0872 | . 0898 | . 0828 | . 0845 | . 0863 |

Table B.4. CI widths: Low skew distribution, $\rho=.2$. $\mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  | $z$ |  |  | Sp |  |  | multp |  |  | unip |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | K | $F$ | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| 50 | 6 | 1 | . 3814 | . 4258 | . 4657 | . 4173 | . 4537 | . 4862 | . 3809 | . 4318 | . 4864 | . 3944 | . 4379 | . 4771 |
|  |  | 2 | . 3772 | . 4226 | . 4639 | . 4134 | . 4517 | . 4851 | . 3765 | . 4290 | . 4820 | . 3914 | . 4361 | . 4757 |
|  |  | 3 | . 3770 | . 4225 | . 4659 | . 4164 | . 4522 | . 4841 | . 3818 | . 4300 | . 4831 | . 3890 | . 4354 | . 4767 |
|  | 18 | 1 | . 3797 | . 4239 | . 4641 | . 4175 | . 4538 | . 4856 | . 3789 | . 4307 | . 4849 | . 3928 | . 4363 | . 4765 |
|  |  | 2 | . 3792 | . 4226 | . 4636 | . 4151 | . 4523 | . 4851 | . 3783 | . 4306 | . 4848 | . 3915 | . 4352 | . 4751 |
|  |  | 3 | . 3800 | . 4240 | . 4648 | . 4161 | . 4528 | . 4852 | . 3778 | . 4306 | . 4851 | . 3921 | . 4361 | . 4771 |
|  | 30 | 1 | . 3806 | . 4246 | . 4648 | . 4164 | . 4531 | . 4854 | . 3783 | . 4305 | . 4848 | . 3929 | . 4367 | . 4768 |
|  |  | 2 | . 3794 | . 4238 | . 4645 | . 4156 | . 4529 | . 4855 | . 3777 | . 4301 | . 4848 | . 3911 | . 4355 | . 4763 |
|  |  | 3 | . 3780 | . 4233 | . 4641 | . 4153 | . 4525 | . 4848 | . 3783 | . 4302 | . 4850 | . 3914 | . 4359 | . 4760 |
|  | 42 | 1 | . 3803 | . 4245 | . 4649 | . 4165 | . 4531 | . 4856 | . 3783 | . 4301 | . 4841 | . 3924 | . 4362 | . 4765 |
|  |  | 2 | . 3797 | . 4239 | . 4645 | . 4151 | . 4520 | . 4849 | . 3773 | . 4294 | . 4836 | . 3917 | . 4357 | . 4762 |
|  |  | 3 | . 3804 | . 4250 | . 4659 | . 4162 | . 4531 | . 4858 | . 3788 | . 4309 | . 4852 | . 3928 | . 4373 | . 4775 |
| 100 | 6 | 1 | . 2752 | . 2970 | . 3177 | . 2976 | . 3168 | . 3335 | . 2801 | . 3091 | . 3391 | . 2860 | . 3083 | . 3290 |
|  |  | 2 | . 2750 | . 2972 | . 3185 | . 2982 | . 3169 | . 3344 | . 2807 | . 3100 | . 3405 | . 2861 | . 3087 | . 3299 |
|  |  | 3 | . 2734 | . 2967 | . 3167 | . 2975 | . 3158 | . 3323 | . 2777 | . 3078 | . 3385 | . 2854 | . 3078 | . 3288 |
|  | 18 | 1 | . 2746 | . 2966 | . 3174 | . 2976 | . 3160 | . 3330 | . 2802 | . 3087 | . 3392 | . 2856 | . 3078 | . 3289 |
|  |  | 2 | . 2739 | . 2964 | . 3175 | . 2972 | . 3157 | . 3330 | . 2792 | . 3078 | . 3381 | . 2852 | . 3076 | . 3286 |
|  |  | 3 | . 2744 | . 2967 | . 3177 | . 2980 | . 3164 | . 3332 | . 2799 | . 3087 | . 3391 | . 2860 | . 3082 | . 3288 |
|  | 30 | 1 | . 2738 | . 2961 | . 3173 | . 2974 | . 3159 | . 3330 | . 2795 | . 3081 | . 3384 | . 2852 | . 3075 | . 3288 |
|  |  | 2 | . 2754 | . 2976 | . 3184 | . 2983 | . 3168 | . 3339 | . 2801 | . 3089 | . 3393 | . 2863 | . 3085 | . 3296 |
|  |  | 3 | . 2746 | . 2968 | . 3179 | . 2978 | . 3163 | . 3335 | . 2801 | . 3087 | . 3393 | . 2859 | . 3081 | . 3294 |
|  | 42 | 1 | . 2744 | . 2966 | . 3177 | . 2978 | . 3165 | . 3337 | . 2796 | . 3086 | . 3393 | . 2858 | . 3082 | . 3294 |
|  |  | 2 | . 2741 | . 2963 | . 3171 | . 2977 | . 3160 | . 3328 | . 2798 | . 3082 | . 3390 | . 2857 | . 3079 | . 3289 |
|  |  | 3 | . 2740 | . 2961 | . 3170 | . 2974 | . 3158 | . 3328 | . 2796 | . 3083 | . 3389 | . 2853 | . 3077 | . 3288 |
| 500 | 6 | 1 | . 1272 | . 1315 | . 1359 | . 1362 | . 1399 | . 1435 | . 1343 | . 1413 | . 1488 | . 1330 | . 1382 | . 1434 |
|  |  | 2 | . 1273 | . 1316 | . 1359 | . 1362 | . 1400 | . 1436 | . 1343 | . 1415 | . 1492 | . 1332 | . 1384 | . 1434 |
|  |  | 3 | . 1274 | . 1319 | . 1360 | . 1365 | . 1402 | . 1436 | . 1345 | . 1417 | . 1490 | . 1332 | . 1386 | . 1435 |
|  | 18 | 1 | . 1273 | . 1317 | . 1360 | . 1364 | . 1400 | . 1436 | . 1343 | . 1414 | . 1490 | . 1332 | . 1383 | . 1435 |
|  |  | 2 | . 1274 | . 1318 | . 1360 | . 1364 | . 1400 | . 1436 | . 1343 | . 1414 | . 1490 | . 1332 | . 1383 | . 1434 |
|  |  | 3 | . 1273 | . 1317 | . 1360 | . 1364 | . 1400 | . 1436 | . 1343 | . 1414 | . 1489 | . 1332 | . 1384 | . 1434 |
|  | 30 | 1 | . 1273 | . 1317 | . 1360 | . 1365 | . 1402 | . 1437 | . 1345 | . 1416 | . 1492 | . 1333 | . 1384 | . 1435 |
|  |  | 2 | . 1274 | . 1318 | . 1361 | . 1365 | . 1402 | . 1437 | . 1345 | . 1416 | . 1492 | . 1334 | . 1385 | . 1436 |
|  |  | 3 | . 1274 | . 1318 | . 1361 | . 1364 | . 1401 | . 1437 | . 1343 | . 1414 | . 1490 | . 1333 | . 1383 | . 1435 |
|  | 42 | 1 | . 1273 | . 1316 | . 1359 | . 1362 | . 1399 | . 1434 | . 1341 | . 1412 | . 1487 | . 1331 | . 1382 | . 1433 |
|  |  | 2 | . 1275 | . 1319 | . 1361 | . 1366 | . 1402 | . 1437 | . 1346 | . 1418 | . 1493 | . 1335 | . 1385 | . 1436 |
|  |  | 3 | . 1273 | . 1316 | . 1359 | . 1363 | . 1400 | . 1435 | . 1343 | . 1415 | . 1491 | . 1332 | . 1383 | . 1434 |
| 2000 | 6 | 1 | . 0647 | . 0658 | . 0669 | . 0690 | . 0699 | . 0708 | . 0691 | . 0713 | . 0737 | . 0675 | . 0693 | . 0711 |
|  |  | 2 | . 0647 | . 0658 | . 0669 | . 0690 | . 0699 | . 0708 | . 0691 | . 0713 | . 0737 | . 0675 | . 0693 | . 0712 |
|  |  | 3 | . 0647 | . 0658 | . 0669 | . 0690 | . 0699 | . 0708 | . 0692 | . 0714 | . 0737 | . 0676 | . 0693 | . 0711 |
|  | 18 | 1 | . 0647 | . 0658 | . 0669 | . 0690 | . 0699 | . 0708 | . 0690 | . 0713 | . 0736 | . 0675 | . 0693 | . 0711 |
|  |  | 2 | . 0647 | . 0658 | . 0669 | . 0690 | . 0699 | . 0708 | . 0690 | . 0713 | . 0736 | . 0675 | . 0693 | . 0711 |
|  |  | 3 | . 0647 | . 0658 | . 0669 | . 0690 | . 0699 | . 0708 | . 0691 | . 0714 | . 0737 | . 0676 | . 0693 | . 0711 |
|  | 30 | 1 | . 0647 | . 0658 | . 0668 | . 0690 | . 0699 | . 0708 | . 0691 | . 0713 | . 0737 | . 0676 | . 0693 | . 0711 |
|  |  | 2 | . 0646 | . 0658 | . 0668 | . 0690 | . 0699 | . 0708 | . 0690 | . 0713 | . 0736 | . 0675 | . 0693 | . 0711 |
|  |  | 3 | . 0647 | . 0658 | . 0669 | . 0690 | . 0699 | . 0708 | . 0690 | . 0713 | . 0736 | . 0676 | . 0693 | . 0711 |
|  | 42 | 1 | . 0648 | . 0658 | . 0669 | . 0690 | . 0699 | . 0708 | . 0691 | . 0713 | . 0737 | . 0676 | . 0693 | . 0711 |
|  |  | 2 | . 0647 | . 0658 | . 0668 | . 0690 | . 0699 | . 0708 | . 0691 | . 0714 | . 0737 | . 0676 | . 0693 | . 0711 |
|  |  | 3 | . 0647 | . 0658 | . 0668 | . 0690 | . 0699 | . 0708 | . 0691 | . 0714 | . 0737 | . 0676 | . 0693 | . 0711 |

Table B.5. CI widths: Low skew distribution, $\rho=.5 . \mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  | $z$ |  |  | Sp |  |  | multp |  |  | unip |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | K | $F$ | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| 50 | 6 | 1 | . 1734 | . 2069 | . 2431 | . 2149 | . 2514 | . 2903 | . 1812 | . 2180 | . 2601 | . 1880 | . 2233 | . 2629 |
|  |  | 2 | . 1749 | . 2087 | . 2451 | . 2173 | . 2523 | . 2896 | . 1814 | . 2194 | . 2613 | . 1880 | . 2232 | . 2639 |
|  |  | 3 | . 1759 | . 2092 | . 2463 | . 2158 | . 2511 | . 2909 | . 1816 | . 2188 | . 2620 | . 1892 | . 2253 | . 2645 |
|  | 18 | 1 | . 1738 | . 2080 | . 2451 | . 2168 | . 2531 | . 2926 | . 1802 | . 2183 | . 2616 | . 1878 | . 2246 | . 2650 |
|  |  | 2 | . 1735 | . 2070 | . 2444 | . 2159 | . 2513 | . 2900 | . 1802 | . 2171 | . 2596 | . 1866 | . 2227 | . 2624 |
|  |  | 3 | . 1753 | . 2088 | . 2461 | . 2165 | . 2523 | . 2911 | . 1811 | . 2187 | . 2613 | . 1891 | . 2249 | . 2645 |
|  | 30 | 1 | . 1727 | . 2056 | . 2421 | . 2152 | . 2513 | . 2898 | . 1796 | . 2167 | . 2593 | . 1867 | . 2226 | . 2623 |
|  |  | 2 | . 1725 | . 2064 | . 2441 | . 2153 | . 2514 | . 2908 | . 1804 | . 2178 | . 2608 | . 1872 | . 2234 | . 2636 |
|  |  | 3 | . 1730 | . 2070 | . 2442 | . 2153 | . 2511 | . 2898 | . 1799 | . 2172 | . 2599 | . 1868 | . 2225 | . 2626 |
|  | 42 | 1 | . 1736 | . 2069 | . 2431 | . 2145 | . 2501 | . 2880 | . 1790 | . 2160 | . 2582 | . 1857 | . 2217 | . 2614 |
|  |  | 2 | . 1752 | . 2086 | . 2458 | . 2170 | . 2530 | . 2914 | . 1816 | . 2189 | . 2622 | . 1890 | . 2249 | . 2651 |
|  |  | 3 | . 1750 | . 2091 | . 2459 | . 2177 | . 2535 | . 2924 | . 1821 | . 2193 | . 2619 | . 1889 | . 2251 | . 2648 |
| 100 | 6 | 1 | . 1271 | . 1436 | . 1611 | . 1537 | . 1711 | . 1895 | . 1353 | . 1548 | . 1755 | . 1397 | . 1581 | . 1783 |
|  |  | 2 | . 1267 | . 1439 | . 1617 | . 1536 | . 1717 | . 1902 | . 1351 | . 1549 | . 1776 | . 1394 | . 1584 | . 1789 |
|  |  | 3 | . 1262 | . 1432 | . 1630 | . 1529 | . 1714 | . 1898 | . 1337 | . 1543 | . 1766 | . 1385 | . 1578 | . 1787 |
|  | 18 | 1 | . 1270 | . 1437 | . 1616 | . 1534 | . 1710 | . 1897 | . 1349 | . 1545 | . 1766 | . 1397 | . 1581 | . 1784 |
|  |  | 2 | . 1266 | . 1437 | . 1618 | . 1540 | . 1715 | . 1902 | . 1352 | . 1551 | . 1769 | . 1400 | . 1586 | . 1789 |
|  |  | 3 | . 1271 | . 1441 | . 1618 | . 1536 | . 1714 | . 1902 | . 1349 | . 1545 | . 1764 | . 1396 | . 1582 | . 1783 |
|  | 30 | 1 | . 1276 | . 1442 | . 1621 | . 1537 | . 1713 | . 1899 | . 1351 | . 1547 | . 1767 | . 1396 | . 1583 | . 1785 |
|  |  | 2 | . 1267 | . 1436 | . 1618 | . 1532 | . 1710 | . 1900 | . 1347 | . 1544 | . 1766 | . 1392 | . 1580 | . 1785 |
|  |  | 3 | . 1266 | . 1435 | . 1616 | . 1531 | . 1712 | . 1901 | . 1347 | . 1543 | . 1764 | . 1391 | . 1576 | . 1781 |
|  | 42 | 1 | . 1257 | . 1429 | . 1615 | . 1530 | . 1708 | . 1898 | . 1342 | . 1541 | . 1761 | . 1388 | . 1576 | . 1777 |
|  |  | 2 | . 1270 | . 1439 | . 1618 | . 1536 | . 1712 | . 1899 | . 1350 | . 1548 | . 1766 | . 1396 | . 1584 | . 1786 |
|  |  | 3 | . 1264 | . 1432 | . 1613 | . 1535 | . 1714 | . 1905 | . 1351 | . 1550 | . 1772 | . 1396 | . 1586 | . 1792 |
| 500 | 6 | 1 | . 0599 | . 0633 | . 0669 | . 0707 | . 0742 | . 0780 | . 0658 | . 0704 | . 0753 | . 0675 | . 0717 | . 0763 |
|  |  | 2 | . 0599 | . 0633 | . 0669 | . 0706 | . 0743 | . 0780 | . 0656 | . 0703 | . 0752 | . 0673 | . 0716 | . 0763 |
|  |  | 3 | . 0600 | . 0633 | . 0668 | . 0706 | . 0742 | . 0780 | . 0657 | . 0702 | . 0753 | . 0674 | . 0717 | . 0762 |
|  | 18 | 1 | . 0600 | . 0633 | . 0668 | . 0706 | . 0742 | . 0778 | . 0658 | . 0704 | . 0752 | . 0674 | . 0717 | . 0763 |
|  |  | 2 | . 0599 | . 0633 | . 0668 | . 0707 | . 0743 | . 0779 | . 0657 | . 0703 | . 0752 | . 0673 | . 0716 | . 0761 |
|  |  | 3 | . 0598 | . 0632 | . 0667 | . 0705 | . 0741 | . 0777 | . 0656 | . 0702 | . 0751 | . 0672 | . 0714 | . 0760 |
|  | 30 | 1 | . 0599 | . 0633 | . 0668 | . 0706 | . 0742 | . 0779 | . 0656 | . 0702 | . 0751 | . 0672 | . 0715 | . 0761 |
|  |  | 2 | . 0600 | . 0634 | . 0669 | . 0706 | . 0742 | . 0780 | . 0658 | . 0703 | . 0752 | . 0673 | . 0716 | . 0761 |
|  |  | 3 | . 0599 | . 0633 | . 0668 | . 0706 | . 0742 | . 0779 | . 0657 | . 0702 | . 0750 | . 0672 | . 0715 | . 0759 |
|  | 42 | 1 | . 0600 | . 0635 | . 0671 | . 0707 | . 0744 | . 0783 | . 0658 | . 0704 | . 0753 | . 0673 | . 0717 | . 0763 |
|  |  | 2 | . 0599 | . 0633 | . 0667 | . 0706 | . 0741 | . 0778 | . 0656 | . 0702 | . 0750 | . 0672 | . 0715 | . 0761 |
|  |  | 3 | . 0600 | . 0634 | . 0669 | . 0706 | . 0742 | . 0778 | . 0658 | . 0703 | . 0751 | . 0674 | . 0716 | . 0760 |
| 2000 | 6 | 1 | . 0308 | . 0316 | . 0325 | . 0360 | . 0369 | . 0378 | . 0341 | . 0354 | . 0368 | . 0347 | . 0360 | . 0373 |
|  |  | 2 | . 0307 | . 0316 | . 0325 | . 0360 | . 0369 | . 0378 | . 0341 | . 0354 | . 0368 | . 0347 | . 0360 | . 0373 |
|  |  | 3 | . 0307 | . 0315 | . 0324 | . 0360 | . 0369 | . 0378 | . 0340 | . 0353 | . 0367 | . 0347 | . 0360 | . 0373 |
|  | 18 | 1 | . 0307 | . 0316 | . 0325 | . 0360 | . 0369 | . 0378 | . 0340 | . 0354 | . 0368 | . 0347 | . 0360 | . 0373 |
|  |  | 2 | . 0307 | . 0316 | . 0325 | . 0360 | . 0369 | . 0378 | . 0340 | . 0354 | . 0368 | . 0347 | . 0360 | . 0374 |
|  |  | 3 | . 0307 | . 0316 | . 0325 | . 0360 | . 0369 | . 0378 | . 0340 | . 0354 | . 0367 | . 0347 | . 0360 | . 0373 |
|  | 30 | 1 | . 0307 | . 0316 | . 0324 | . 0360 | . 0369 | . 0378 | . 0340 | . 0354 | . 0368 | . 0347 | . 0360 | . 0373 |
|  |  | 2 | . 0307 | . 0316 | . 0325 | . 0360 | . 0369 | . 0378 | . 0340 | . 0354 | . 0368 | . 0347 | . 0360 | . 0373 |
|  |  | 3 | . 0307 | . 0316 | . 0325 | . 0360 | . 0369 | . 0378 | . 0340 | . 0354 | . 0367 | . 0347 | . 0360 | . 0373 |
|  | 42 | 1 | . 0307 | . 0316 | . 0324 | . 0360 | . 0369 | . 0378 | . 0340 | . 0353 | . 0367 | . 0347 | . 0359 | . 0373 |
|  |  | 2 | . 0307 | . 0315 | . 0324 | . 0359 | . 0368 | . 0377 | . 0339 | . 0353 | . 0367 | . 0346 | . 0359 | . 0372 |
|  |  | 3 | . 0307 | . 0315 | . 0324 | . 0360 | . 0369 | . 0378 | . 0340 | . 0353 | . 0367 | . 0347 | . 0360 | . 0373 |

Table B.6. CI widths: Low skew distribution, $\rho=.8 \mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  | $z$ |  |  | Sp |  |  | multp |  |  | UniP |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | K | $F$ | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| 50 | 6 | 1 | . 5095 | . 5369 | . 5516 | . 5194 | . 5386 | . 5505 | . 4675 | . 5285 | . 6017 | . 5279 | . 5459 | . 5634 |
|  |  | 2 | . 5084 | . 5362 | . 5514 | . 5178 | . 5377 | . 5505 | . 4657 | . 5279 | . 6000 | . 5270 | . 5460 | . 5636 |
|  |  | 3 | . 5108 | . 5379 | . 5517 | . 5206 | . 5387 | . 5508 | . 4634 | . 5227 | . 5974 | . 5278 | . 5457 | . 5628 |
|  | 18 | 1 | . 5093 | . 5370 | . 5516 | . 5193 | . 5385 | . 5507 | . 4656 | . 5278 | . 6014 | . 5282 | . 5466 | . 5639 |
|  |  | 2 | . 5084 | . 5366 | . 5514 | . 5187 | . 5382 | . 5506 | . 4662 | . 5282 | . 6008 | . 5276 | . 5460 | . 5633 |
|  |  | 3 | . 5084 | . 5367 | . 5514 | . 5188 | . 5384 | . 5506 | . 4652 | . 5279 | . 6003 | . 5279 | . 5467 | . 5636 |
|  | 30 | 1 | . 5078 | . 5363 | . 5514 | . 5184 | . 5379 | . 5504 | . 4658 | . 5279 | . 6003 | . 5275 | . 5462 | . 5637 |
|  |  | 2 | . 5090 | . 5370 | . 5515 | . 5194 | . 5386 | . 5507 | . 4663 | . 5283 | . 6012 | . 5281 | . 5465 | . 5639 |
|  |  | 3 | . 5094 | . 5372 | . 5517 | . 5195 | . 5388 | . 5508 | . 4669 | . 5288 | . 6016 | . 5280 | . 5466 | . 5638 |
|  | 42 | 1 | . 5090 | . 5370 | . 5516 | . 5189 | . 5384 | . 5507 | . 4658 | . 5277 | . 6005 | . 5280 | . 5466 | . 5640 |
|  |  | 2 | . 5081 | . 5364 | . 5513 | . 5184 | . 5380 | . 5504 | . 4664 | . 5282 | . 6011 | . 5275 | . 5463 | . 5639 |
|  |  | 3 | . 5083 | . 5367 | . 5515 | . 5188 | . 5382 | . 5506 | . 4662 | . 5281 | . 6010 | . 5276 | . 5462 | . 5637 |
| 100 | 6 | 1 | . 3642 | . 3778 | . 3868 | . 3692 | . 3790 | . 3861 | . 3462 | . 3860 | . 4344 | . 3789 | . 3898 | . 4016 |
|  |  | 2 | . 3653 | . 3784 | . 3872 | . 3700 | . 3797 | . 3866 | . 3458 | . 3874 | . 4353 | . 3788 | . 3899 | . 4019 |
|  |  | 3 | . 3644 | . 3778 | . 3869 | . 3694 | . 3790 | . 3865 | . 3462 | . 3874 | . 4370 | . 3793 | . 3906 | . 4019 |
|  | 18 | 1 | . 3645 | . 3782 | . 3871 | . 3698 | . 3792 | . 3863 | . 3459 | . 3861 | . 4356 | . 3789 | . 3900 | . 4017 |
|  |  | 2 | . 3644 | . 3781 | . 3870 | . 3698 | . 3793 | . 3864 | . 3463 | . 3864 | . 4348 | . 3787 | . 3897 | . 4016 |
|  |  | 3 | . 3645 | . 3782 | . 3872 | . 3700 | . 3794 | . 3864 | . 3465 | . 3863 | . 4356 | . 3787 | . 3896 | . 4014 |
|  | 30 | 1 | . 3647 | . 3782 | . 3871 | . 3697 | . 3793 | . 3864 | . 3460 | . 3862 | . 4347 | . 3787 | . 3897 | . 4014 |
|  |  | 2 | . 3640 | . 3779 | . 3869 | . 3695 | . 3791 | . 3862 | . 3461 | . 3868 | . 4363 | . 3788 | . 3899 | . 4017 |
|  |  | 3 | . 3644 | . 3781 | . 3870 | . 3696 | . 3792 | . 3863 | . 3460 | . 3864 | . 4353 | . 3787 | . 3898 | . 4016 |
|  | 42 | 1 | . 3642 | . 3779 | . 3869 | . 3696 | . 3792 | . 3862 | . 3463 | . 3866 | . 4360 | . 3789 | . 3899 | . 4018 |
|  |  | 2 | . 3644 | . 3780 | . 3869 | . 3697 | . 3792 | . 3863 | . 3464 | . 3867 | . 4360 | . 3790 | . 3899 | . 4016 |
|  |  | 3 | . 3638 | . 3777 | . 3868 | . 3693 | . 3789 | . 3861 | . 3466 | . 3869 | . 4356 | . 3786 | . 3896 | . 4014 |
| 500 | 6 | 1 | . 1658 | . 1684 | . 1705 | . 1673 | . 1692 | . 1707 | . 1726 | . 1853 | . 2015 | . 1725 | . 1766 | . 1810 |
|  |  | 2 | . 1659 | . 1684 | . 1705 | . 1672 | . 1690 | . 1707 | . 1722 | . 1852 | . 2015 | . 1725 | . 1766 | . 1811 |
|  |  | 3 | . 1659 | . 1685 | . 1704 | . 1673 | . 1690 | . 1706 | . 1725 | . 1857 | . 2017 | . 1724 | . 1765 | . 1811 |
|  | 18 | 1 | . 1659 | . 1684 | . 1705 | . 1673 | . 1691 | . 1707 | . 1720 | . 1850 | . 2012 | . 1723 | . 1765 | . 1810 |
|  |  | 2 | . 1659 | . 1685 | . 1706 | . 1673 | . 1691 | . 1707 | . 1718 | . 1848 | . 2009 | . 1724 | . 1766 | . 1811 |
|  |  | 3 | . 1659 | . 1685 | . 1706 | . 1673 | . 1691 | . 1707 | . 1718 | . 1848 | . 2012 | . 1723 | . 1765 | . 1811 |
|  | 30 | 1 | . 1658 | . 1684 | . 1705 | . 1673 | . 1691 | . 1707 | . 1720 | . 1851 | . 2013 | . 1724 | . 1765 | . 1811 |
|  |  | 2 | . 1660 | . 1685 | . 1706 | . 1673 | . 1691 | . 1707 | . 1719 | . 1850 | . 2010 | . 1724 | . 1765 | . 1810 |
|  |  | 3 | . 1660 | . 1685 | . 1706 | . 1673 | . 1691 | . 1707 | . 1719 | . 1849 | . 2011 | . 1724 | . 1766 | . 1811 |
|  | 42 | 1 | . 1659 | . 1684 | . 1706 | . 1673 | . 1691 | . 1707 | . 1718 | . 1849 | . 2010 | . 1724 | . 1765 | . 1810 |
|  |  | 2 | . 1659 | . 1685 | . 1706 | . 1673 | . 1691 | . 1707 | . 1719 | . 1850 | . 2010 | . 1724 | . 1765 | . 1810 |
|  |  | 3 | . 1659 | . 1685 | . 1706 | . 1673 | . 1691 | . 1707 | . 1718 | . 1846 | . 2006 | . 1724 | . 1765 | . 1811 |
| 2000 | 6 | 1 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0909 | . 0954 | . 1007 | . 0867 | . 0886 | . 0906 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0907 | . 0952 | . 1005 | . 0867 | . 0886 | . 0906 |
|  |  | 3 | . 0835 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0907 | . 0953 | . 1007 | . 0868 | . 0886 | . 0906 |
|  | 18 | 1 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0908 | . 0952 | . 1005 | . 0866 | . 0885 | . 0905 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0909 | . 0954 | . 1006 | . 0866 | . 0885 | . 0905 |
|  |  | 3 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0908 | . 0952 | . 1006 | . 0866 | . 0885 | . 0905 |
|  | 30 | 1 | . 0835 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0908 | . 0953 | . 1006 | . 0866 | . 0886 | . 0905 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0908 | . 0953 | . 1006 | . 0866 | . 0886 | . 0905 |
|  |  | 3 | . 0835 | . 0842 | . 0847 | . 0840 | . 0845 | . 0849 | . 0908 | . 0953 | . 1006 | . 0866 | . 0885 | . 0905 |
|  | 42 | 1 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0908 | . 0953 | . 1006 | . 0866 | . 0885 | . 0905 |
|  |  | 2 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0908 | . 0953 | . 1006 | . 0866 | . 0885 | . 0905 |
|  |  | 3 | . 0836 | . 0842 | . 0847 | . 0841 | . 0845 | . 0849 | . 0908 | . 0952 | . 1005 | . 0866 | . 0885 | . 0905 |

Table B.7. CI widths: High skew distribution, $\rho=.2$. $\mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  | $z$ |  |  | Sp |  |  | multp |  |  | unip |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | K | $F$ | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| 50 | 6 | 1 | . 3717 | . 4259 | . 4714 | . 4062 | . 4433 | . 4769 | . 4017 | . 4623 | . 5274 | . 4395 | . 4852 | . 5245 |
|  |  | 2 | . 3737 | . 4247 | . 4721 | . 4072 | . 4450 | . 4778 | . 4051 | . 4639 | . 5283 | . 4396 | . 4857 | . 5257 |
|  |  | 3 | . 3702 | . 4219 | . 4676 | . 4060 | . 4439 | . 4767 | . 4026 | . 4631 | . 5250 | . 4364 | . 4833 | . 5239 |
|  | 18 | 1 | . 3704 | . 4245 | . 4708 | . 4065 | . 4438 | . 4770 | . 4014 | . 4602 | . 5251 | . 4379 | . 4847 | . 5249 |
|  |  | 2 | . 3683 | . 4225 | . 4695 | . 4063 | . 4434 | . 4770 | . 4024 | . 4615 | . 5261 | . 4368 | . 4844 | . 5248 |
|  |  | 3 | . 3689 | . 4234 | . 4693 | . 4075 | . 4444 | . 4775 | . 4040 | . 4627 | . 5287 | . 4380 | . 4848 | . 5251 |
|  | 30 | 1 | . 3700 | . 4234 | . 4700 | . 4066 | . 4444 | . 4779 | . 4028 | . 4625 | . 5278 | . 4384 | . 4852 | . 5253 |
|  |  | 2 | . 3695 | . 4236 | . 4703 | . 4068 | . 4444 | . 4776 | . 4025 | . 4619 | . 5271 | . 4385 | . 4857 | . 5256 |
|  |  | 3 | . 3713 | . 4245 | . 4706 | . 4069 | . 4443 | . 4775 | . 4032 | . 4628 | . 5280 | . 4392 | . 4857 | . 5263 |
|  | 42 | 1 | . 3709 | . 4247 | . 4712 | . 4075 | . 4444 | . 4776 | . 4029 | . 4617 | . 5261 | . 4391 | . 4857 | . 5259 |
|  |  | 2 | . 3693 | . 4234 | . 4702 | . 4067 | . 4440 | . 4775 | . 4029 | . 4621 | . 5275 | . 4382 | . 4850 | . 5258 |
|  |  | 3 | . 3699 | . 4230 | . 4695 | . 4066 | . 4438 | . 4771 | . 4026 | . 4624 | . 5271 | . 4376 | . 4842 | . 5242 |
| 100 | 6 | 1 | . 2709 | . 2975 | . 3221 | . 2918 | . 3105 | . 3275 | . 3045 | . 3412 | . 3832 | . 3291 | . 3531 | . 3778 |
|  |  | 2 | . 2703 | . 2973 | . 3211 | . 2911 | . 3098 | . 3270 | . 3036 | . 3402 | . 3824 | . 3276 | . 3534 | . 3787 |
|  |  | 3 | . 2698 | . 2968 | . 3214 | . 2910 | . 3103 | . 3276 | . 3049 | . 3395 | . 3798 | . 3284 | . 3526 | . 3781 |
|  | 18 | 1 | . 2698 | . 2966 | . 3206 | . 2913 | . 3101 | . 3272 | . 3033 | . 3404 | . 3825 | . 3278 | . 3527 | . 3780 |
|  |  | 2 | . 2693 | . 2966 | . 3209 | . 2912 | . 3098 | . 3274 | . 3039 | . 3412 | . 3833 | . 3281 | . 3531 | . 3777 |
|  |  | 3 | . 2689 | . 2963 | . 3209 | . 2908 | . 3094 | . 3267 | . 3041 | . 3407 | . 3830 | . 3281 | . 3534 | . 3781 |
|  | 30 | 1 | . 2697 | . 2965 | . 3204 | . 2909 | . 3095 | . 3269 | . 3032 | . 3405 | . 3826 | . 3277 | . 3529 | . 3780 |
|  |  | 2 | . 2698 | . 2967 | . 3209 | . 2908 | . 3098 | . 3271 | . 3033 | . 3402 | . 3827 | . 3276 | . 3530 | . 3781 |
|  |  | 3 | . 2691 | . 2963 | . 3208 | . 2910 | . 3097 | . 3271 | . 3037 | . 3407 | . 3831 | . 3284 | . 3535 | . 3788 |
|  | 42 | 1 | . 2694 | . 2963 | . 3205 | . 2901 | . 3089 | . 3264 | . 3021 | . 3387 | . 3804 | . 3267 | . 3520 | . 3769 |
|  |  | 2 | . 2695 | . 2964 | . 3208 | . 2907 | . 3092 | . 3266 | . 3028 | . 3396 | . 3815 | . 3272 | . 3525 | . 3777 |
|  |  | 3 | . 2708 | . 2977 | . 3219 | . 2912 | . 3098 | . 3272 | . 3030 | . 3395 | . 3810 | . 3276 | . 3527 | . 3776 |
| 500 | 6 |  | . 1262 | . 1315 | . 1366 | . 1334 | . 1370 | . 1406 | . 1531 | . 1647 | . 1788 | . 1592 | . 1669 | . 1755 |
|  |  | 2 | . 1263 | . 1317 | . 1367 | . 1331 | . 1370 | . 1407 | . 1530 | . 1648 | . 1783 | . 1588 | . 1666 | . 1754 |
|  |  | 3 | . 1261 | . 1318 | . 1371 | . 1333 | . 1369 | . 1405 | . 1524 | . 1648 | . 1796 | . 1592 | . 1666 | . 1756 |
|  | 18 | 1 | . 1263 | . 1317 | . 1368 | . 1335 | . 1372 | . 1408 | . 1531 | . 1649 | . 1791 | . 1593 | . 1670 | . 1761 |
|  |  | 2 | . 1265 | . 1318 | . 1369 | . 1334 | . 1371 | . 1407 | . 1530 | . 1649 | . 1788 | . 1592 | . 1667 | . 1756 |
|  |  | 3 | . 1265 | . 1319 | . 1369 | . 1334 | . 1371 | . 1407 | . 1529 | . 1646 | . 1787 | . 1591 | . 1666 | . 1755 |
|  | 30 | 1 | . 1263 | . 1316 | . 1366 | . 1333 | . 1370 | . 1405 | . 1529 | . 1647 | . 1789 | . 1591 | . 1666 | . 1757 |
|  |  | 2 | . 1263 | . 1316 | . 1368 | . 1332 | . 1369 | . 1406 | . 1529 | . 1646 | . 1787 | . 1590 | . 1666 | . 1755 |
|  |  | 3 | . 1266 | . 1319 | . 1370 | . 1335 | . 1372 | . 1408 | . 1529 | . 1645 | . 1784 | . 1590 | . 1666 | . 1754 |
|  | 42 | 1 | . 1266 | . 1320 | . 1371 | . 1333 | . 1371 | . 1407 | . 1530 | . 1646 | . 1785 | . 1590 | . 1665 | . 1753 |
|  |  | 2 | . 1262 | . 1315 | . 1366 | . 1332 | . 1370 | . 1406 | . 1528 | . 1646 | . 1785 | . 1590 | . 1666 | . 1755 |
|  |  | 3 | . 1266 | . 1320 | . 1371 | . 1335 | . 1372 | . 1408 | . 1530 | . 1647 | . 1786 | . 1591 | . 1667 | . 1756 |
| 2000 | 6 | 1 | . 0645 | . 0658 | . 0671 | . 0675 | . 0684 | . 0693 | . 0811 | . 0854 | . 0900 | . 0825 | . 0853 | . 0886 |
|  |  | 2 | . 0644 | . 0658 | . 0670 | . 0675 | . 0684 | . 0693 | . 0813 | . 0851 | . 0897 | . 0825 | . 0851 | . 0884 |
|  |  | 3 | . 0644 | . 0658 | . 0671 | . 0675 | . 0684 | . 0693 | . 0814 | . 0854 | . 0899 | . 0825 | . 0854 | . 0886 |
|  | 18 | 1 | . 0644 | . 0658 | . 0671 | . 0674 | . 0684 | . 0693 | . 0812 | . 0853 | . 0900 | . 0826 | . 0854 | . 0887 |
|  |  | 2 | . 0644 | . 0658 | . 0671 | . 0675 | . 0684 | . 0693 | . 0811 | . 0852 | . 0899 | . 0825 | . 0854 | . 0886 |
|  |  | 3 | . 0644 | . 0658 | . 0670 | . 0674 | . 0684 | . 0693 | . 0812 | . 0852 | . 0899 | . 0825 | . 0854 | . 0886 |
|  | 30 | 1 | . 0645 | . 0658 | . 0671 | . 0675 | . 0684 | . 0693 | . 0812 | . 0852 | . 0899 | . 0825 | . 0853 | . 0886 |
|  |  | 2 | . 0645 | . 0658 | . 0671 | . 0675 | . 0684 | . 0694 | . 0813 | . 0853 | . 0901 | . 0825 | . 0854 | . 0886 |
|  |  | 3 | . 0644 | . 0658 | . 0671 | . 0675 | . 0684 | . 0693 | . 0812 | . 0853 | . 0901 | . 0825 | . 0854 | . 0887 |
|  | 42 | 1 | . 0644 | . 0658 | . 0670 | . 0674 | . 0684 | . 0693 | . 0811 | . 0851 | . 0898 | . 0824 | . 0853 | . 0884 |
|  |  | 2 | . 0644 | . 0658 | . 0671 | . 0674 | . 0684 | . 0693 | . 0812 | . 0853 | . 0900 | . 0825 | . 0853 | . 0886 |
|  |  | 3 | . 0644 | . 0657 | . 0670 | . 0674 | . 0683 | . 0693 | . 0812 | . 0853 | . 0900 | . 0825 | . 0854 | . 0886 |

Table B.8. CI widths: High skew distribution, $\rho=.5 . \mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  | $z$ |  |  | Sp |  |  | multp |  |  | unip |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | K | $F$ | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 | Q1 | M | Q3 |
| 50 | 6 | 1 | . 1639 | . 2060 | . 2518 | . 2097 | . 2434 | . 2808 | . 2021 | . 2457 | . 2969 | . 2343 | . 2826 | . 3359 |
|  |  | 2 | . 1646 | . 2060 | . 2520 | . 2091 | . 2422 | . 2795 | . 2016 | . 2466 | . 2985 | . 2345 | . 2813 | . 3379 |
|  |  | 3 | . 1647 | . 2072 | . 2525 | . 2080 | . 2434 | . 2824 | . 2008 | . 2459 | . 2965 | . 2325 | . 2813 | . 3376 |
|  | 18 | 1 | . 1656 | . 2069 | . 2522 | . 2090 | . 2438 | . 2809 | . 2014 | . 2467 | . 2985 | . 2338 | . 2830 | . 3376 |
|  |  | 2 | . 1642 | . 2061 | . 2516 | . 2085 | . 2424 | . 2793 | . 2000 | . 2452 | . 2971 | . 2315 | . 2802 | . 3349 |
|  |  | 3 | . 1646 | . 2055 | . 2510 | . 2094 | . 2431 | . 2795 | . 2002 | . 2450 | . 2960 | . 2313 | . 2795 | . 3350 |
|  | 30 | 1 | . 1619 | . 2039 | . 2498 | . 2071 | . 2417 | . 2790 | . 1991 | . 2441 | . 2957 | . 2309 | . 2790 | . 3338 |
|  |  | 2 | . 1647 | . 2058 | . 2511 | . 2088 | . 2429 | . 2798 | . 2005 | . 2457 | . 2975 | . 2328 | . 2812 | . 3361 |
|  |  | 3 | . 1645 | . 2056 | . 2512 | . 2093 | . 2438 | . 2809 | . 2011 | . 2465 | . 2988 | . 2331 | . 2823 | . 3379 |
|  | 42 | 1 | . 1636 | . 2045 | . 2490 | . 2078 | . 2418 | . 2786 | . 1991 | . 2441 | . 2957 | . 2311 | . 2796 | . 3344 |
|  |  | 2 | . 1640 | . 2061 | . 2517 | . 2098 | . 2438 | . 2805 | . 2004 | . 2448 | . 2956 | . 2324 | . 2804 | . 3348 |
|  |  | 3 | . 1643 | . 2057 | . 2510 | . 2085 | . 2431 | . 2796 | . 2006 | . 2455 | . 2967 | . 2328 | . 2813 | . 3358 |
| 100 | 6 | 1 | . 1212 | . 1429 | . 1657 | . 1484 | . 1656 | . 1836 | . 1529 | . 1792 | . 2089 | . 1805 | . 2084 | . 2400 |
|  |  | 2 | . 1223 | . 1429 | . 1651 | . 1469 | . 1652 | . 1830 | . 1538 | . 1790 | . 2084 | . 1798 | . 2070 | . 2370 |
|  |  | 3 | . 1226 | . 1438 | . 1662 | . 1478 | . 1650 | . 1827 | . 1539 | . 1790 | . 2097 | . 1792 | . 2084 | . 2400 |
|  | 18 | 1 | . 1225 | . 1438 | . 1663 | . 1483 | . 1649 | . 1827 | . 1545 | . 1803 | . 2097 | . 1812 | . 2086 | . 2392 |
|  |  | 2 | . 1227 | . 1438 | . 1665 | . 1485 | . 1654 | . 1835 | . 1544 | . 1802 | . 2097 | . 1810 | . 2085 | . 2395 |
|  |  | 3 | . 1219 | . 1431 | . 1658 | . 1478 | . 1650 | . 1832 | . 1534 | . 1791 | . 2084 | . 1799 | . 2075 | . 2384 |
|  | 30 | 1 | . 1222 | . 1433 | . 1657 | . 1486 | . 1654 | . 1833 | . 1537 | . 1791 | . 2086 | . 1799 | . 2070 | . 2378 |
|  |  | 2 | . 1221 | . 1432 | . 1658 | . 1480 | . 1649 | . 1830 | . 1535 | . 1792 | . 2083 | . 1796 | . 2068 | . 2374 |
|  |  | 3 | . 1217 | . 1426 | . 1651 | . 1484 | . 1654 | . 1833 | . 1541 | . 1798 | . 2091 | . 1804 | . 2080 | . 2389 |
|  | 42 | 1 | . 1235 | . 1444 | . 1665 | . 1469 | . 1637 | . 1815 | . 1519 | . 1771 | . 2062 | . 1776 | . 2045 | . 2347 |
|  |  | 2 | . 1212 | . 1423 | . 1648 | . 1476 | . 1649 | . 1833 | . 1527 | . 1789 | . 2087 | . 1795 | . 2070 | . 2381 |
|  |  | 3 | . 1219 | . 1429 | . 1652 | . 1486 | . 1657 | . 1837 | . 1539 | . 1799 | . 2092 | . 1809 | . 2081 | . 2388 |
| 500 | 6 | 1 | . 0589 | . 0633 | . 0677 | . 0679 | . 0712 | . 0747 | . 0790 | . 0864 | . 0947 | . 0931 | . 1010 | . 1104 |
|  |  | 2 | . 0593 | . 0636 | . 0680 | . 0680 | . 0713 | . 0748 | . 0790 | . 0865 | . 0950 | . 0934 | . 1009 | . 1101 |
|  |  | 3 | . 0589 | . 0632 | . 0678 | . 0681 | . 0713 | . 0747 | . 0786 | . 0860 | . 0949 | . 0930 | . 1004 | . 1096 |
|  | 18 | 1 | . 0589 | . 0632 | . 0677 | . 0681 | . 0715 | . 0750 | . 0792 | . 0867 | . 0953 | . 0935 | . 1015 | . 1109 |
|  |  | 2 | . 0591 | . 0634 | . 0678 | . 0680 | . 0714 | . 0748 | . 0791 | . 0862 | . 0945 | . 0932 | . 1009 | . 1097 |
|  |  | 3 | . 0588 | . 0632 | . 0676 | . 0680 | . 0713 | . 0748 | . 0790 | . 0864 | . 0951 | . 0933 | . 1012 | . 1106 |
|  | 30 | 1 | . 0588 | . 0631 | . 0676 | . 0680 | . 0714 | . 0748 | . 0789 | . 0862 | . 0947 | . 0933 | . 1010 | . 1100 |
|  |  | 2 | . 0589 | . 0632 | . 0676 | . 0680 | . 0714 | . 0748 | . 0790 | . 0864 | . 0949 | . 0932 | . 1011 | . 1103 |
|  |  | 3 | . 0589 | . 0633 | . 0678 | . 0680 | . 0714 | . 0749 | . 0791 | . 0864 | . 0949 | . 0934 | . 1012 | . 1103 |
|  | 42 | 1 | . 0594 | . 0637 | . 0682 | . 0682 | . 0716 | . 0750 | . 0794 | . 0867 | . 0952 | . 0934 | . 1011 | . 1102 |
|  |  | 2 | . 0589 | . 0634 | . 0679 | . 0680 | . 0714 | . 0748 | . 0792 | . 0864 | . 0948 | . 0933 | . 1011 | . 1101 |
|  |  | 3 | . 0591 | . 0635 | . 0680 | . 0680 | . 0714 | . 0749 | . 0791 | . 0865 | . 0950 | . 0932 | . 1010 | . 1102 |
| 2000 | 6 | 1 | . 0306 | . 0316 | . 0328 | . 0346 | . 0354 | . 0363 | . 0422 | . 0446 | . 0472 | . 0497 | . 0523 | . 0553 |
|  |  | 2 | . 0305 | . 0316 | . 0327 | . 0346 | . 0354 | . 0363 | . 0421 | . 0445 | . 0473 | . 0495 | . 0522 | . 0553 |
|  |  | 3 | . 0305 | . 0316 | . 0327 | . 0346 | . 0355 | . 0363 | . 0422 | . 0446 | . 0473 | . 0498 | . 0523 | . 0553 |
|  | 18 | 1 | . 0305 | . 0316 | . 0327 | . 0346 | . 0355 | . 0363 | . 0422 | . 0446 | . 0473 | . 0497 | . 0524 | . 0554 |
|  |  | 2 | . 0305 | . 0316 | . 0327 | . 0346 | . 0354 | . 0363 | . 0422 | . 0446 | . 0473 | . 0497 | . 0523 | . 0552 |
|  |  | 3 | . 0305 | . 0316 | . 0327 | . 0346 | . 0355 | . 0363 | . 0423 | . 0447 | . 0474 | . 0498 | . 0525 | . 0555 |
|  | 30 | 1 | . 0305 | . 0316 | . 0327 | . 0347 | . 0355 | . 0364 | . 0423 | . 0446 | . 0473 | . 0498 | . 0523 | . 0552 |
|  |  | 2 | . 0305 | . 0316 | . 0327 | . 0346 | . 0354 | . 0363 | . 0423 | . 0446 | . 0473 | . 0498 | . 0523 | . 0553 |
|  |  | 3 | . 0305 | . 0316 | . 0327 | . 0346 | . 0355 | . 0363 | . 0423 | . 0447 | . 0474 | . 0498 | . 0524 | . 0554 |
|  | 42 | 1 | . 0304 | . 0315 | . 0327 | . 0346 | . 0355 | . 0363 | . 0423 | . 0447 | . 0474 | . 0498 | . 0525 | . 0555 |
|  |  | 2 | . 0305 | . 0316 | . 0328 | . 0346 | . 0355 | . 0363 | . 0422 | . 0446 | . 0473 | . 0497 | . 0523 | . 0553 |
|  |  | 3 | . 0305 | . 0316 | . 0327 | . 0346 | . 0355 | . 0364 | . 0423 | . 0447 | . 0474 | . 0498 | . 0524 | . 0555 |

Table B.9. CI widths: High skew distribution, $\rho=.8$. $\mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

APPENDIX C: CI COVERAGE TABLES FOR MATRIXWISE TECHNIQUES

|  |  |  | $\rho=.2$ |  |  |  | $\rho=.5$ |  |  |  | $\rho=.8$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | mult $_{M}$ | ипі $_{\text {м }}$ |  |  | mult $_{M}$ | uni $_{M}$ |  |  | mult $_{M}$ | uni $_{\text {M }}$ |  |  |
|  | K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
| $\begin{aligned} & \overline{\tilde{n}} \\ & \text { Z } \\ & \text { Z } \end{aligned}$ | 6 | 1 | . 9387 | . 0313 | . 9373 | . 0379 | . 9354 | . 0353 | . 9141 | . 0601 | . 9369 | . 0324 | . 8357 | . 1093 |
|  |  | 2 | . 9377 | . 0350 | . 9307 | . 0432 | . 9405 | . 0325 | . 9040 | . 0705 | . 9322 | . 0398 | . 8003 | . 1475 |
|  |  | 3 | . 9380 | . 0330 | . 9260 | . 0463 | . 9317 | . 0380 | . 8917 | . 0793 | . 9370 | . 0300 | . 7993 | . 1430 |
|  | 18 | 1 | . 9365 | . 0324 | . 9391 | . 0339 | . 9372 | . 0321 | . 9274 | . 0429 | . 9399 | . 0340 | . 8691 | . 0862 |
|  |  | 2 | . 9377 | . 0316 | . 9391 | . 0345 | . 9380 | . 0316 | . 9278 | . 0440 | . 9375 | . 0347 | . 8744 | . 0828 |
|  |  | 3 | . 9356 | . 0328 | . 9361 | . 0378 | . 9377 | . 0329 | . 9263 | . 0468 | . 9364 | . 0338 | . 8787 | . 0782 |
|  | 30 | 1 | . 9361 | . 0332 | . 9401 | . 0332 | . 9358 | . 0339 | . 9286 | . 0423 | . 9351 | . 0325 | . 8767 | . 0717 |
|  |  | 2 | . 9354 | . 0322 | . 9389 | . 0329 | . 9371 | . 0333 | . 9321 | . 0398 | . 9352 | . 0369 | . 8886 | . 0719 |
|  |  | 3 | . 9375 | . 0316 | . 9408 | . 0323 | . 9346 | . 0343 | . 9290 | . 0422 | . 9352 | . 0360 | . 8952 | . 0665 |
|  | 42 | 1 | . 9373 | . 0320 | . 9425 | . 0305 | . 9381 | . 0332 | . 9331 | . 0384 | . 9405 | . 0305 | . 8886 | . 0649 |
|  |  | 2 | . 9382 | . 0315 | . 9422 | . 0309 | . 9382 | . 0317 | . 9358 | . 0361 | . 9322 | . 0371 | . 8946 | . 0635 |
|  |  | 3 | . 9366 | . 0312 | . 9404 | . 0310 | . 9372 | . 0331 | . 9344 | . 0381 | . 9386 | . 0342 | . 9074 | . 0565 |
|  | 6 | 1 | . 9319 | . 0328 | . 9143 | . 0611 | . 9297 | . 0345 | . 8803 | . 0873 | . 9275 | . 0380 | . 7828 | . 1541 |
|  |  | 2 | . 9345 | . 0328 | . 9178 | . 0615 | . 9275 | . 0357 | . 8605 | . 1058 | . 9208 | . 0372 | . 7480 | . 1793 |
|  |  | 3 | . 9317 | . 0327 | . 9143 | . 0620 | . 9273 | . 0347 | . 8567 | . 1110 | . 9280 | . 0350 | . 7417 | . 1887 |
|  | 18 | 1 | . 9298 | . 0325 | . 9219 | . 0524 | . 9298 | . 0337 | . 8953 | . 0696 | . 9215 | . 0391 | . 8137 | . 1195 |
|  |  | 2 | . 9298 | . 0332 | . 9201 | . 0545 | . 9272 | . 0357 | . 8920 | . 0738 | . 9225 | . 0387 | . 8180 | . 1193 |
|  |  | 3 | . 9313 | . 0317 | . 9224 | . 0524 | . 9248 | . 0363 | . 8903 | . 0750 | . 9242 | . 0362 | . 8289 | . 1104 |
|  | 30 | 1 | . 9313 | . 0326 | . 9248 | . 0492 | . 9268 | . 0348 | . 8988 | . 0628 | . 9248 | . 0405 | . 8268 | . 1147 |
|  |  | 2 | . 9318 | . 0313 | . 9258 | . 0479 | . 9260 | . 0363 | . 8962 | . 0670 | . 9203 | . 0412 | . 8313 | . 1098 |
|  |  | 3 | . 9298 | . 0332 | . 9226 | . 0508 | . 9265 | . 0373 | . 8966 | . 0697 | . 9203 | . 0410 | . 8408 | . 1028 |
|  | 42 | 1 | . 9310 | . 0313 | . 9277 | . 0445 | . 9273 | . 0341 | . 9020 | . 0588 | . 9265 | . 0398 | . 8401 | 1026 |
|  |  | 2 | . 9299 | . 0324 | . 9256 | . 0467 | . 9263 | . 0356 | . 9021 | . 0604 | . 9251 | . 0356 | . 8543 | . 0877 |
|  |  | 3 | . 9307 | . 0318 | . 9261 | . 0467 | . 9265 | . 0340 | . 9035 | . 0585 | . 9250 | . 0361 | . 8599 | . 0852 |
| $\begin{aligned} & 3 \\ & 0 \\ & 0 \\ & \text { N } \\ & \text { E0.0. } \\ & \text { in } \end{aligned}$ | 6 | 1 | . 9218 | . 0285 | . 8830 | . 1000 | . 9057 | . 0365 | . 8049 | . 1479 | . 9029 | . 0404 | . 6735 | . 2304 |
|  |  | 2 | . 9132 | . 0323 | . 8777 | . 1062 | . 9080 | . 0355 | . 7973 | . 1590 | . 8967 | . 0470 | . 6502 | . 2552 |
|  |  | 3 | . 9070 | . 0303 | . 8773 | . 1023 | . 9177 | . 0323 | . 7930 | . 1697 | . 9043 | . 0410 | . 6457 | . 2657 |
|  | 18 | 1 | . 9146 | . 0316 | . 8882 | . 0883 | . 9036 | . 0403 | . 8163 | . 1306 | . 9011 | . 0440 | . 7144 | . 1885 |
|  |  | 2 | . 9152 | . 0324 | . 8865 | . 0920 | . 9036 | . 0408 | . 8083 | . 1407 | . 8982 | . 0455 | . 7133 | . 1934 |
|  |  | 3 | . 9146 | . 0326 | . 8852 | . 0938 | . 9080 | . 0378 | . 8144 | . 1378 | . 8998 | . 0444 | . 7252 | . 1874 |
|  | 30 | 1 | . 9139 | . 0335 | . 8894 | . 0850 | . 9055 | . 0389 | . 8219 | . 1208 | . 8963 | . 0482 | . 7081 | . 1961 |
|  |  | 2 | . 9150 | . 0311 | . 8888 | . 0851 | . 9045 | . 0398 | . 8213 | . 1250 | . 9010 | . 0448 | . 7389 | . 1733 |
|  |  | 3 | . 9152 | . 0301 | . 8910 | . 0834 | . 9063 | . 0376 | . 8265 | . 1200 | . 8994 | . 0447 | . 7439 | . 1703 |
|  | 42 | 1 | . 9149 | . 0314 | . 8922 | . 0783 | . 9044 | . 0385 | . 8225 | . 1171 | . 9038 | . 0463 | . 7318 | . 1793 |
|  |  | 2 | . 9147 | . 0325 | . 8902 | . 0820 | . 9043 | . 0395 | . 8265 | . 1172 | . 8984 | . 0440 | . 7437 | . 1665 |
|  |  | 3 | . 9162 | . 0315 | . 8916 | . 0813 | . 9075 | . 0395 | . 8297 | . 1171 | . 9021 | . 0430 | . 7555 | . 1605 |

Table C. 1. CI coverage for matrixwise techniques: $N=50$ conditions.

|  |  |  | $\rho=.2$ |  |  |  | $\rho=.5$ |  |  |  | $\rho=.8$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | mult $_{M}$ | uni $_{\text {M }}$ |  |  | mult $_{\text {M }}$ | ипі $_{\text {M }}$ |  |  | mult $_{M}$ | ипі $_{\text {M }}$ |  |  |
|  | K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
| $\begin{aligned} & \text { न् } \\ & \text { हुँ } \\ & \text { Z } \end{aligned}$ | 6 | 1 | . 9410 | . 0301 | . 9385 | . 0350 | . 9399 | . 0323 | . 9168 | . 0528 | . 9458 | . 0299 | . 8452 | . 1005 |
|  |  | 2 | . 9410 | . 0285 | . 9377 | . 0358 | . 9400 | . 0328 | . 9072 | . 0623 | . 9368 | . 0362 | . 8032 | . 1365 |
|  |  | 3 | . 9347 | . 0287 | . 9337 | . 0343 | . 9417 | . 0300 | . 9013 | . 0643 | . 9497 | . 0223 | . 8000 | . 1353 |
|  | 18 | 1 | . 9399 | . 0302 | . 9407 | . 0323 | . 9397 | . 0312 | . 9235 | . 0448 | . 9383 | . 0327 | . 8580 | . 0855 |
|  |  | 2 | . 9421 | . 0297 | . 9416 | . 0331 | . 9405 | . 0302 | . 9248 | . 0452 | . 9430 | . 0311 | . 8672 | . 0838 |
|  |  | 3 | . 9407 | . 0288 | . 9396 | . 0335 | . 9419 | . 0285 | . 9265 | . 0434 | . 9392 | . 0301 | . 8681 | . 0789 |
|  | 30 | 1 | . 9410 | . 0306 | . 9421 | . 0318 | . 9416 | . 0309 | . 9280 | . 0421 | . 9366 | . 0339 | . 8625 | . 0823 |
|  |  | 2 | . 9415 | . 0302 | . 9423 | . 0323 | . 9424 | . 0288 | . 9304 | . 0394 | . 9433 | . 0292 | . 8833 | . 0686 |
|  |  | 3 | . 9422 | . 0301 | . 9421 | . 0329 | . 9369 | . 0338 | . 9253 | . 0449 | . 9415 | . 0317 | . 8863 | . 0690 |
|  | 42 | 1 | . 9424 | . 0300 | . 9440 | . 0306 | . 9419 | . 0308 | . 9302 | . 0401 | . 9460 | . 0298 | . 8801 | . 0733 |
|  |  | 2 | . 9420 | . 0289 | . 9436 | . 0298 | . 9424 | . 0286 | . 9321 | . 0374 | . 9407 | . 0310 | . 8871 | . 0665 |
|  |  | 3 | . 9401 | . 0294 | . 9415 | . 0307 | . 9415 | . 0301 | . 9334 | . 0382 | . 9391 | . 0312 | . 8906 | . 0634 |
|  | 6 | 1 | . 9367 | . 0288 | . 9254 | . 0493 | . 9317 | . 0323 | . 8855 | . 0779 | . 9341 | . 0338 | . 7873 | . 1396 |
|  |  | 2 | . 9390 | . 0312 | . 9222 | . 0572 | . 9355 | . 0315 | . 8783 | . 0842 | . 9270 | . 0397 | . 7458 | . 1735 |
|  |  | 3 | . 9373 | . 0273 | . 9217 | . 0520 | . 9297 | . 0393 | . 8663 | . 1007 | . 9257 | . 0417 | . 7233 | . 1927 |
|  | 18 | 1 | . 9370 | . 0296 | . 9282 | . 0466 | . 9333 | . 0330 | . 8920 | . 0698 | . 9325 | . 0332 | . 8137 | . 1148 |
|  |  | 2 | . 9364 | . 0294 | . 9253 | . 0479 | . 9299 | . 0348 | . 8889 | . 0728 | . 9290 | . 0366 | . 8046 | . 1244 |
|  |  | 3 | . 9359 | . 0291 | . 9251 | . 0496 | . 9329 | . 0335 | . 8915 | . 0727 | . 9322 | . 0354 | . 8197 | . 1147 |
|  | 30 | 1 | . 9359 | . 0304 | . 9281 | . 0459 | . 9309 | . 0340 | . 8923 | . 0682 | . 9333 | . 0316 | . 8252 | . 1029 |
|  |  | 2 | . 9362 | . 0295 | . 9280 | . 0462 | . 9321 | . 0312 | . 8967 | . 0630 | . 9308 | . 0348 | . 8263 | . 1076 |
|  |  | 3 | . 9376 | . 0288 | . 9282 | . 0465 | . 9321 | . 0325 | . 8974 | . 0641 | . 9312 | . 0347 | . 8344 | . 1032 |
|  | 42 | 1 | . 9363 | . 0316 | . 9285 | . 0461 | . 9299 | . 0355 | . 8941 | . 0661 | . 9245 | . 0383 | . 8096 | . 1188 |
|  |  | 2 | . 9369 | . 0300 | . 9299 | . 0443 | . 9338 | . 0334 | . 9002 | . 0635 | . 9325 | . 0333 | . 8382 | . 0981 |
|  |  | 3 | . 9359 | . 0305 | . 9297 | . 0444 | . 9332 | . 0340 | . 8984 | . 0658 | . 9291 | . 0356 | . 8399 | . 0986 |
|  | 6 | 1 | . 9218 | . 0321 | . 8848 | . 0955 | . 9127 | . 0353 | . 8072 | . 1393 | . 9073 | . 0434 | . 6703 | . 2247 |
|  |  | 2 | . 9205 | . 0303 | . 8890 | . 0925 | . 9135 | . 0355 | . 7932 | . 1573 | . 9163 | . 0403 | . 6537 | . 2442 |
|  |  | 3 | . 9203 | . 0320 | . 8793 | . 1003 | . 9147 | . 0330 | . 7873 | . 1623 | . 9080 | . 0407 | . 6330 | . 2583 |
|  | 18 | 1 | . 9202 | . 0313 | . 8897 | . 0861 | . 9158 | . 0368 | . 8190 | . 1276 | . 9085 | . 0395 | . 6999 | . 1913 |
|  |  | 2 | . 9188 | . 0333 | . 8876 | . 0899 | . 9100 | . 0384 | . 8096 | . 1352 | . 9098 | . 0385 | . 7019 | . 1911 |
|  |  | 3 | . 9201 | . 0311 | . 8892 | . 0884 | . 9118 | . 0391 | . 8102 | . 1370 | . 9044 | . 0430 | . 7039 | . 1940 |
|  | 30 | 1 | . 9204 | . 0304 | . 8936 | . 0793 | . 9166 | . 0371 | . 8219 | . 1226 | . 9094 | . 0408 | . 7108 | . 1839 |
|  |  | 2 | . 9205 | . 0324 | . 8888 | . 0863 | . 9133 | . 0367 | . 8212 | . 1222 | . 9078 | . 0416 | . 7167 | . 1817 |
|  |  | 3 | . 9190 | . 0324 | . 88897 | . 0850 | . 9125 | . 0377 | . 8161 | . 1287 | . 9129 | . 0401 | . 7284 | . 1800 |
|  | 42 | 1 | . 9205 | . 0324 | . 8920 | . 0807 | . 9125 | . 0384 | . 8218 | . 1194 | . 9083 | . 0408 | . 7291 | . 1625 |
|  |  | 2 | . 9219 | . 0320 | . 8933 | . 0807 | . 9133 | . 0377 | . 8235 | . 1193 | . 9077 | . 0449 | . 7231 | . 1812 |
|  |  | 3 | . 9209 | . 0330 | . 8897 | . 0846 | . 9105 | . 0352 | . 8253 | . 1138 | . 9126 | . 0402 | . 7385 | . 1694 |

Table C. 2. CI coverage for matrixwise techniques: $N=100$ conditions.

|  |  |  | $\rho=.2$ |  |  |  | $\rho=.5$ |  |  |  | $\rho=.8$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | mult $_{M}$ | ипім |  |  | mult $_{M}$ | ипі $_{\text {M }}$ |  |  | mult $_{M}$ | ипі $_{\text {M }}$ |  |  |
|  | K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
|  | 6 | 1 | . 9457 | . 0262 | . 9409 | . 0304 | . 9470 | . 0275 | . 9219 | . 0452 | . 9425 | . 0291 | . 8359 | . 0920 |
|  |  | 2 | . 9467 | . 0268 | . 9428 | . 0315 | . 9442 | . 0282 | . 9123 | . 0503 | . 9445 | . 0285 | . 8050 | . 1120 |
|  |  | 3 | . 9417 | . 0287 | . 9400 | . 0330 | . 9397 | . 0240 | . 9020 | . 0487 | . 9517 | . 0213 | . 7987 | . 1153 |
|  | 18 | 1 | . 9470 | . 0258 | . 9453 | . 0278 | . 9466 | . 0267 | . 9268 | . 0393 | . 9473 | . 0255 | . 8619 | . 0723 |
|  |  | 2 | . 9471 | . 0265 | . 9444 | . 0292 | . 9453 | . 0264 | . 9233 | . 0411 | . 9490 | . 0249 | . 8610 | . 0740 |
|  |  | 3 | . 9459 | . 0274 | . 9431 | . 0308 | . 9476 | . 0251 | . 9277 | . 0384 | . 9459 | . 0255 | . 8622 | . 0731 |
|  | 30 | 1 | . 9484 | . 0252 | . 9466 | . 0273 | . 9457 | . 0276 | . 9255 | . 0401 | . 9464 | . 0290 | . 8592 | . 0806 |
|  |  | 2 | . 9471 | . 0273 | . 9451 | . 0295 | . 9452 | . 0286 | . 9245 | . 0427 | . 9441 | . 0286 | . 8572 | . 0774 |
|  |  | 3 | . 9453 | . 0274 | . 9428 | . 0299 | . 9464 | . 0290 | . 9284 | . 0414 | . 9422 | . 0282 | . 8656 | . 0720 |
|  | 42 | 1 | . 9468 | . 0266 | . 9452 | . 0282 | . 9462 | . 0238 | . 9260 | . 0353 | . 9470 | . 0261 | . 8610 | . 0754 |
|  |  | 2 | . 9455 | . 0273 | . 9440 | . 0295 | . 9444 | . 0292 | . 9253 | . 0417 | . 9461 | . 0267 | . 8655 | . 0719 |
|  |  | 3 | . 9453 | . 0275 | . 9430 | . 0297 | . 9469 | . 0281 | . 9297 | . 0394 | . 9466 | . 0262 | . 8756 | . 0664 |
|  | 6 | 1 | . 9435 | . 0271 | . 9325 | . 0395 | . 9434 | . 0285 | . 8903 | . 0662 | . 9394 | . 0305 | . 7825 | . 1283 |
|  |  | 2 | . 9468 | . 0253 | . 9327 | . 0402 | . 9455 | . 0288 | . 8835 | . 0740 | . 9425 | . 0283 | . 7478 | . 1490 |
|  |  | 3 | . 9423 | . 0280 | . 9290 | . 0433 | . 9463 | . 0237 | . 8840 | . 0673 | . 9453 | . 0263 | . 7433 | . 1533 |
|  | 18 | 1 | . 9436 | . 0278 | . 9314 | . 0403 | . 9451 | . 0276 | . 8988 | . 0593 | . 9444 | . 0281 | . 8077 | . 1090 |
|  |  | 2 | . 9433 | . 0282 | . 9296 | . 0425 | . 9423 | . 0278 | . 8936 | . 0616 | . 9402 | . 0299 | . 7979 | . 1162 |
|  |  | 3 | . 9447 | . 0273 | . 9324 | . 0404 | . 9434 | . 0292 | . 8951 | . 0628 | . 9432 | . 0303 | . 8086 | . 1148 |
|  | 30 | 1 | . 9427 | . 0271 | . 9310 | . 0390 | . 9413 | . 0294 | . 8944 | . 0605 | . 9413 | . 0299 | . 8077 | . 1081 |
|  |  | 2 | . 9441 | . 0270 | . 9315 | . 0400 | . 9435 | . 0271 | . 8974 | . 0583 | . 9430 | . 0269 | . 8110 | . 1032 |
|  |  | 3 | . 9444 | . 0281 | . 9318 | . 0410 | . 9431 | . 0269 | . 8987 | . 0571 | . 9410 | . 0286 | . 8192 | . 1017 |
|  | 42 | 1 | . 9447 | . 0275 | . 9324 | . 0397 | . 9432 | . 0290 | . 8981 | . 0593 | . 9429 | . 0263 | . 8067 | . 1028 |
|  |  | 2 | . 9447 | . 0274 | . 9323 | . 0398 | . 9447 | . 0258 | . 9001 | . 0550 | . 9427 | . 0299 | . 8128 | . 1083 |
|  |  | 3 | . 9442 | . 0275 | . 9318 | . 0396 | . 9429 | . 0284 | . 8995 | . 0585 | . 9436 | . 0264 | . 8295 | . 0946 |
| $\begin{aligned} & \frac{3}{0} \\ & \text { y } \\ & \text { 菏 } \\ & \text { in } \end{aligned}$ | 6 | 1 | . 9320 | . 0302 | . 8900 | . 0785 | . 9351 | . 0307 | . 8120 | . 1235 | . 9299 | . 0339 | . 6693 | . 2005 |
|  |  | 2 | . 9372 | . 0313 | . 8933 | . 0807 | . 9330 | . 0333 | . 8043 | . 1323 | . 9337 | . 0275 | . 6445 | . 2065 |
|  |  | 3 | . 9320 | . 0330 | . 8887 | . 0827 | . 9330 | . 0327 | . 7880 | . 1450 | . 9237 | . 0357 | . 6130 | . 2397 |
|  | 18 | 1 | . 9375 | . 0287 | . 8982 | . 0722 | . 9331 | . 0313 | . 8170 | . 1159 | . 9303 | . 0319 | . 6904 | . 1831 |
|  |  | 2 | . 9357 | . 0292 | . 8960 | . 0739 | . 9333 | . 0305 | . 8184 | . 1149 | . 9295 | . 0315 | . 6914 | . 1784 |
|  |  | 3 | . 9344 | . 0291 | . 8946 | . 0748 | . 9313 | . 0311 | . 8163 | . 1154 | . 9301 | . 0342 | . 6923 | . 1870 |
|  | 30 | 1 | . 9363 | . 0305 | . 8970 | . 0730 | . 9350 | . 0312 | . 8209 | . 1150 | . 9300 | . 0355 | . 6906 | . 1880 |
|  |  | 2 | . 9356 | . 0291 | . 8977 | . 0708 | . 9309 | . 0320 | . 8162 | . 1162 | . 9326 | . 0331 | . 6967 | . 1820 |
|  |  | 3 | . 9339 | . 0301 | . 8944 | . 0736 | . 9318 | . 0305 | . 8243 | . 1085 | . 9280 | . 0329 | . 7027 | . 1745 |
|  | 42 | 1 | . 9350 | . 0302 | . 8963 | . 0718 | . 9300 | . 0304 | . 8175 | . 1081 | . 9301 | . 0287 | . 6996 | . 1621 |
|  |  | 2 | . 9342 | . 0297 | . 8965 | . 0713 | . 9330 | . 0341 | . 8192 | . 1183 | . 9262 | . 0352 | . 6917 | . 1802 |
|  |  | 3 | . 9336 | . 0303 | . 8964 | . 0713 | . 9295 | . 0301 | . 8208 | . 1074 | . 9276 | . 0316 | . 7123 | . 1623 |

Table C. 3. CI coverage for matrixwise techniques: $N=500$ conditions.

|  |  |  | $\rho=.2$ |  |  |  | $\rho=.5$ |  |  |  | $\rho=.8$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | mult $_{M}$ | ипі $_{\text {м }}$ |  |  | mult $_{M}$ | ипім |  |  | mult $_{\text {M }}$ | ипi $_{\text {M }}$ |  |  |
|  | K | $F$ | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left | Cov | Left |
| $\begin{aligned} & \bar{\Pi} \\ & \tilde{0} \\ & \text { Zn } \end{aligned}$ | 6 | 1 | . 9488 | . 0259 | . 9460 | . 0278 | . 9500 | . 0237 | . 9249 | . 0383 | . 9423 | . 0277 | . 8337 | . 0848 |
|  |  | 2 | . 9418 | . 0282 | . 9405 | . 0288 | . 9463 | . 0265 | . 9155 | . 0448 | . 9438 | . 0275 | . 8108 | . 1048 |
|  |  | 3 | . 9457 | . 0267 | . 9403 | . 0313 | . 9513 | . 0237 | . 9187 | . 0423 | . 9437 | . 0273 | . 7877 | . 1180 |
|  | 18 | 1 | . 9466 | . 0265 | . 9441 | . 0283 | . 9438 | . 0272 | . 9219 | . 0399 | . 9471 | . 0278 | . 8566 | . 0780 |
|  |  | 2 | . 9487 | . 0258 | . 9447 | . 0288 | . 9481 | . 0262 | . 9262 | . 0388 | . 9478 | . 0276 | . 8505 | . 0795 |
|  |  | 3 | . 9461 | . 0265 | . 9427 | . 0297 | . 9497 | . 0252 | . 9279 | . 0378 | . 9458 | . 0261 | . 8569 | . 0746 |
|  | 30 | 1 | . 9473 | . 0276 | . 9445 | . 0297 | . 9486 | . 0279 | . 9281 | . 0403 | . 9520 | . 0242 | . 8650 | . 0693 |
|  |  | 2 | . 9480 | . 0253 | . 9453 | . 0271 | . 9497 | . 0270 | . 9295 | . 0390 | . 9437 | . 0303 | . 8557 | . 0796 |
|  |  | 3 | . 9480 | . 0265 | . 9457 | . 0285 | . 9463 | . 0278 | . 9264 | . 0391 | . 9508 | . 0242 | . 8716 | . 0666 |
|  | 42 | 1 | . 9482 | . 0262 | . 9454 | . 0281 | . 9463 | . 0276 | . 9244 | . 0407 | . 9439 | . 0254 | . 8509 | . 0728 |
|  |  | 2 | . 9475 | . 0267 | . 9446 | . 0287 | . 9492 | . 0244 | . 9290 | . 0351 | . 9456 | . 0252 | . 8572 | . 0705 |
|  |  | 3 | . 9453 | . 0276 | . 9430 | . 0294 | . 9465 | . 0285 | . 9278 | . 0393 | . 9480 | . 0251 | . 8704 | . 0649 |
|  | 6 | 1 | . 9461 | . 0270 | . 9313 | . 0387 | . 9461 | . 0255 | . 8918 | . 0579 | . 9486 | . 0266 | . 7944 | . 1079 |
|  |  | 2 | . 9462 | . 0273 | . 9300 | . 0392 | . 9485 | . 0245 | . 8898 | . 0583 | . 9473 | . 0262 | . 7470 | . 1350 |
|  |  | 3 | . 9467 | . 0283 | . 9313 | . 0370 | . 9507 | . 0257 | . 8887 | . 0650 | . 9473 | . 0270 | . 7313 | . 1513 |
|  | 18 | 1 | . 9465 | . 0268 | . 9325 | . 0370 | . 9486 | . 0254 | . 9001 | . 0534 | . 9459 | . 0262 | . 8053 | . 1026 |
|  |  | 2 | . 9467 | . 0266 | . 9335 | . 0372 | . 9456 | . 0271 | . 8934 | . 0576 | . 9428 | . 0268 | . 7972 | . 1063 |
|  |  | 3 | . 9477 | . 0246 | . 9348 | . 0343 | . 9501 | . 0244 | . 9024 | . 0529 | . 9439 | . 0282 | . 8040 | . 1068 |
|  | 30 | 1 | . 9470 | . 0260 | . 9332 | . 0360 | . 9477 | . 0259 | . 8997 | . 0533 | . 9466 | . 0275 | . 8065 | . 1045 |
|  |  | 2 | . 9464 | . 0261 | . 9324 | . 0365 | . 9431 | . 0283 | . 8922 | . 0589 | . 9436 | . 0270 | . 8005 | . 1035 |
|  |  | 3 | . 9472 | . 0260 | . 9335 | . 0359 | . 9460 | . 0266 | . 9008 | . 0530 | . 9456 | . 0270 | . 8178 | . 0971 |
|  | 42 | 1 | . 9476 | . 0259 | . 9339 | . 0356 | . 9472 | . 0241 | . 9000 | . 0493 | . 9474 | . 0271 | . 8062 | . 1050 |
|  |  | 2 | . 9468 | . 0267 | . 9334 | . 0366 | . 9463 | . 0264 | . 8973 | . 0552 | . 9428 | . 0284 | . 8017 | . 1082 |
|  |  | 3 | . 9460 | . 0267 | . 9330 | . 0359 | . 9478 | . 0273 | . 9022 | . 0546 | . 9477 | . 0283 | . 8269 | . 0978 |
| $\begin{aligned} & \frac{3}{0} \\ & \frac{2}{n} \\ & \text { B } \\ & \text { in } \end{aligned}$ | 6 | 1 | . 9435 | . 0267 | . 9003 | . 0627 | . 9411 | . 0279 | . 8197 | . 1051 | . 9391 | . 0253 | . 6770 | . 1658 |
|  |  | 2 | . 9437 | . 0278 | . 9008 | . 0600 | . 9465 | . 0265 | . 8038 | . 1192 | . 9373 | . 0312 | . 6287 | . 2108 |
|  |  | 3 | . 9443 | . 0257 | . 8987 | . 0663 | . 9353 | . 0293 | . 8020 | . 1180 | . 9443 | . 0233 | . 6310 | . 2067 |
|  | 18 | 1 | . 9429 | . 0263 | . 9013 | . 0592 | . 9413 | . 0292 | . 8195 | . 1054 | . 9390 | . 0312 | . 6850 | . 1777 |
|  |  | 2 | . 9413 | . 0288 | . 8979 | . 0643 | . 9392 | . 0298 | . 8148 | . 1080 | . 9390 | . 0297 | . 6820 | . 1705 |
|  |  | 3 | . 9430 | . 0276 | . 8997 | . 0626 | . 9435 | . 0278 | . 8211 | . 1054 | . 9410 | . 0288 | . 6948 | . 1710 |
|  | 30 | 1 | . 9421 | . 0284 | . 8985 | . 0633 | . 9403 | . 0284 | . 8182 | . 1023 | . 9380 | . 0288 | . 6863 | . 1650 |
|  |  | 2 | . 9426 | . 0274 | . 8998 | . 0618 | . 9395 | . 0280 | . 8161 | . 1028 | . 9410 | . 0278 | . 6912 | . 1698 |
|  |  | 3 | . 9422 | . 0287 | . 8978 | . 0653 | . 9400 | . 0301 | . 8208 | . 1057 | . 9426 | . 0276 | . 7056 | . 1591 |
|  | 42 | 1 | . 9412 | . 0287 | . 8988 | . 0622 | . 9419 | . 0293 | . 8197 | . 1052 | . 9395 | . 0315 | . 6868 | . 1788 |
|  |  | 2 | . 9425 | . 0273 | . 9006 | . 0607 | . 9413 | . 0283 | . 8186 | . 1037 | . 9364 | . 0280 | . 6857 | . 1656 |
|  |  | 3 | . 9421 | . 0281 | . 8998 | . 0619 | . 9387 | . 0315 | . 8181 | . 1093 | . 9413 | . 0260 | . 7069 | . 1549 |

Table C. 4. CI coverage for matrixwise techniques: $N=2000$ conditions.

APPENDIX D: CI WIDTH TABLES FOR MATRIXWISE TECHNIQUES


Table D.1. CI widths for matrixwise techniques: $N=50$. Q1 $=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  |  | mult |  |  |  |  |  |  | mult |  |  |  |  |  |  | mult |  |  | uni |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | K | $F$ | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 |
| $\begin{aligned} & \text { ت̈ } \\ & \text { Z } \\ & \text { Z } \end{aligned}$ | 6 | 1 | . 2 | . 3428 | . 3674 | . 3936 | . 3530 | . 3695 | . 3823 | . 5 | . 2638 | . 2899 | . 3184 | . 2449 | . 2689 | . 2921 | . 8 | . 1247 | . 1422 | . 1607 | . 0897 | . 1042 | . 1193 |
|  |  | 2 |  | . 3426 | . 3674 | . 3931 | . 3511 | . 3680 | . 3822 |  | . 2645 | . 2904 | . 3185 | . 2377 | . 2634 | . 2889 |  | . 1240 | . 1408 | . 1597 | . 0805 | . 0959 | . 1124 |
|  |  | 3 |  | . 3423 | . 3670 | . 3936 | . 3505 | . 3679 | . 3821 |  | . 2642 | . 2900 | . 3192 | . 2345 | . 2613 | . 2868 |  | . 1241 | . 1413 | . 1602 | . 0771 | . 0926 | . 1096 |
|  | 18 | 1 |  | . 3424 | . 3674 | . 3937 | . 3550 | . 3699 | . 3822 |  | . 2636 | . 2899 | . 3177 | . 2507 | . 2730 | . 2951 |  | . 1244 | . 1418 | . 1611 | . 0958 | . 1097 | . 1249 |
|  |  | 2 |  | . 3423 | . 3671 | . 3936 | . 3539 | . 3694 | . 3823 |  | . 2634 | . 2898 | . 3175 | . 2498 | . 2728 | . 2955 |  | . 1238 | . 1409 | . 1599 | . 0960 | . 1099 | . 1250 |
|  |  | 3 |  | . 3422 | . 3672 | . 3933 | . 3539 | . 3695 | . 3824 |  | . 2640 | . 2903 | . 3182 | . 2503 | . 2742 | . 2973 |  | . 1242 | . 1411 | . 1603 | . 0971 | . 1123 | . 1291 |
|  | 30 | 1 |  | . 3426 | . 3673 | . 3935 | . 3555 | . 3698 | . 3818 |  | . 2636 | . 2900 | . 3175 | . 2528 | . 2743 | . 2955 |  | . 1235 | . 1407 | . 1599 | . 0972 | . 1105 | . 1253 |
|  |  | 2 |  | . 3426 | . 3674 | . 3935 | . 3550 | . 3698 | . 3823 |  | . 2645 | . 2904 | . 3177 | . 2542 | . 2766 | . 2982 |  | . 1249 | . 1418 | . 1604 | . 1003 | . 1143 | . 1295 |
|  |  | 3 |  | . 3425 | . 3673 | . 3931 | . 3550 | . 3699 | . 3824 |  | . 2633 | . 2902 | . 3179 | . 2539 | . 2768 | . 2993 |  | . 1240 | . 1408 | . 1595 | . 1015 | . 1169 | . 1332 |
|  | 42 | 1 |  | . 3424 | . 3672 | . 3933 | . 3561 | . 3699 | . 3818 |  | . 2637 | . 2901 | . 3177 | . 2542 | . 2756 | . 2965 |  | . 1244 | . 1412 | . 1600 | . 0989 | . 1118 | . 1262 |
|  |  | 2 |  | . 3425 | . 3674 | . 3933 | . 3560 | . 3703 | . 3823 |  | . 2643 | . 2904 | . 3180 | . 2562 | . 2779 | . 2992 |  | . 1243 | . 1413 | . 1601 | . 1021 | . 1167 | . 1327 |
|  |  | 3 |  | . 3424 | . 3676 | . 3934 | . 3561 | . 3704 | . 3826 |  | . 2636 | . 2896 | . 3171 | . 2570 | . 2790 | . 3007 |  | . 1246 | . 1417 | . 1608 | . 1042 | . 1196 | . 1365 |
| $\begin{aligned} & 3 \\ & 0 \\ & 0 \\ & 0 \\ & 3 \\ & 0 \end{aligned}$ | 6 | 1 | . 2 | . 3478 | . 3752 | . 4047 | . 3517 | . 3689 | . 3821 | . 5 | . 2804 | . 3091 | . 3397 | . 2422 | . 2685 | . 2937 | . 8 | . 1354 | . 1548 | . 1758 | . 0878 | . 1036 | . 1203 |
|  |  | 2 |  | . 3494 | . 3754 | . 4054 | . 3495 | . 3677 | . 3819 |  | . 2807 | . 3094 | . 3389 | . 2340 | . 2629 | . 2904 |  | . 1356 | . 1544 | . 1771 | . 0797 | . 0962 | . 1136 |
|  |  | 3 |  | . 3474 | . 3753 | . 4049 | . 3500 | . 3681 | . 3830 |  | . 2797 | . 3082 | . 3386 | . 2299 | . 2596 | . 2865 |  | . 1335 | . 1545 | . 1755 | . 0755 | . 0923 | . 1115 |
|  | 18 | 1 |  | . 3488 | . 3758 | . 4054 | . 3541 | . 3697 | . 3824 |  | . 2802 | . 3087 | . 3391 | . 2486 | . 2729 | . 2963 |  | . 1349 | . 1546 | . 1767 | . 0946 | . 1094 | . 1257 |
|  |  | 2 |  | . 3488 | . 3756 | . 4051 | . 3534 | . 3696 | . 3828 |  | . 2793 | . 3079 | . 3381 | . 2465 | . 2721 | . 2969 |  | . 1352 | . 1550 | . 1771 | . 0939 | . 1098 | . 1271 |
|  |  | 3 |  | . 3488 | . 3760 | . 4055 | . 3532 | . 3697 | . 3829 |  | . 2799 | . 3088 | . 3388 | . 2468 | . 2730 | . 2984 |  | . 1348 | . 1543 | . 1764 | . 0956 | . 1123 | . 1308 |
|  | 30 | 1 |  | . 3487 | . 3757 | . 4056 | . 3549 | . 3699 | . 3823 |  | . 2795 | . 3080 | . 3382 | . 2498 | . 2738 | . 2974 |  | . 1350 | . 1546 | . 1766 | . 0973 | . 1119 | . 1281 |
|  |  | 2 |  | . 3486 | . 3757 | . 4054 | . 3546 | . 3701 | . 3828 |  | . 2802 | . 3087 | . 3393 | . 2521 | . 2766 | . 3000 |  | . 1346 | . 1544 | . 1765 | . 0980 | . 1138 | . 1314 |
|  |  | 3 |  | . 3487 | . 3755 | . 4053 | . 3545 | . 3701 | . 3830 |  | . 2802 | . 3088 | . 3393 | . 2521 | . 2768 | . 3012 |  | . 1346 | . 1543 | . 1764 | . 0998 | . 1165 | . 1348 |
|  | 42 | 1 |  | . 3483 | . 3751 | . 4051 | . 3549 | . 3696 | . 3818 |  | . 2797 | . 3086 | . 3392 | . 2524 | . 2760 | . 2987 |  | . 1343 | . 1542 | . 1763 | . 0970 | . 1124 | . 1294 |
|  |  | 2 |  | . 3484 | . 3752 | . 4049 | . 3551 | . 3700 | . 3824 |  | . 2798 | . 3083 | . 3388 | . 2530 | . 2771 | . 3001 |  | . 1350 | . 1549 | . 1767 | . 1005 | . 1167 | . 1344 |
|  |  | 3 |  | . 3483 | . 3752 | . 4048 | . 3550 | . 3703 | . 3827 |  | . 2795 | . 3085 | . 3391 | . 2538 | . 2784 | . 3019 |  | . 1351 | . 1550 | . 1772 | . 1021 | . 1191 | . 1376 |
| $\begin{aligned} & 3 \\ & \frac{3}{0} \\ & \text { un } \\ & \frac{\pi}{60} \\ & i=1 \end{aligned}$ | 6 | 1 | . 2 | . 3466 | . 3863 | . 4358 | . 3526 | . 3711 | . 3861 | . 5 | . 3045 | . 3413 | . 3831 | . 2378 | . 2696 | . 2991 | . 8 | . 1532 | . 1789 | . 2085 | . 0839 | . 1033 | . 1241 |
|  |  | 2 |  | . 3461 | . 3867 | . 4352 | . 3519 | . 3713 | . 3867 |  | . 3034 | . 3404 | . 3816 | . 2306 | . 2626 | . 2937 |  | . 1538 | . 1790 | . 2085 | . 0771 | . 0956 | . 1169 |
|  |  | 3 |  | . 3464 | . 3872 | . 4368 | . 3504 | . 3705 | . 3861 |  | . 3051 | . 3398 | . 3814 | . 2281 | . 2605 | . 2932 |  | . 1538 | . 1793 | . 2097 | . 0737 | . 0935 | . 1157 |
|  | 18 | 1 |  | . 3459 | . 3861 | . 4354 | . 3546 | . 3717 | . 3853 |  | . 3035 | . 3404 | . 3826 | . 2442 | . 2732 | . 3002 |  | . 1545 | . 1804 | . 2098 | . 0918 | . 1105 | . 1311 |
|  |  | 2 |  | . 3463 | . 3863 | . 4349 | . 3538 | . 3717 | . 3855 |  | . 3038 | . 3410 | . 3833 | . 2416 | . 2725 | . 3010 |  | . 1543 | . 1802 | . 2096 | . 0912 | . 1104 | . 1316 |
|  |  | 3 |  | . 3461 | . 3861 | . 4355 | . 3536 | . 3717 | . 3857 |  | . 3037 | . 3404 | . 3826 | . 2419 | . 2728 | . 3018 |  | . 1535 | . 1791 | . 2083 | . 0915 | . 1119 | . 1343 |
|  | 30 | 1 |  | . 3459 | . 3863 | . 4348 | . 3552 | . 3716 | . 3846 |  | . 3033 | . 3406 | . 3827 | . 2463 | . 2749 | . 3011 |  | . 1535 | . 1790 | . 2084 | . 0938 | . 1125 | . 1329 |
|  |  | 2 |  | . 3463 | . 3868 | . 4364 | . 3543 | . 3713 | . 3848 |  | . 3034 | . 3402 | . 3827 | . 2464 | . 2757 | . 3028 |  | . 1535 | . 1791 | . 2081 | . 0947 | . 1140 | . 1354 |
|  |  | 3 |  | . 3460 | . 3866 | . 4352 | . 3544 | . 3716 | . 3852 |  | . 3035 | . 3405 | . 3830 | . 2462 | . 2765 | . 3044 |  | . 1539 | . 1798 | . 2091 | . 0959 | . 1160 | . 1383 |
|  | 42 | 1 |  | . 3462 | . 3867 | . 4361 | . 3551 | . 3711 | . 3840 |  | . 3020 | . 3388 | . 3803 | . 2476 | . 2761 | . 3027 |  | . 1517 | . 1768 | . 2058 | . 0968 | . 1154 | . 1356 |
|  |  | 2 |  | . 3464 | . 3868 | . 4361 | . 3550 | . 3715 | . 3845 |  | . 3027 | . 3396 | . 3814 | . 2484 | . 2773 | . 3042 |  | . 1529 | . 1790 | . 2086 | . 0965 | . 1161 | . 1378 |
|  |  | 3 |  | . 3466 | . 3870 | . 4357 | . 3543 | . 3712 | . 3845 |  | . 3030 | . 3394 | . 3810 | . 2504 | . 2801 | . 3073 |  | . 1540 | . 1798 | . 2093 | . 0987 | . 1192 | . 1419 |

Table D.2. CI widths for matrixwise techniques: $N=100$. $\mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  |  | mult |  |  |  |  |  |  | mult |  |  |  |  |  |  | mult |  |  | uni |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | K | $F$ | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 |
| $\begin{aligned} & \text { ت̈ } \\ & \text { Z } \\ & \text { Z } \end{aligned}$ | 6 | 1 | . 2 | . 1606 | . 1667 | . 1730 | . 1605 | . 1648 | . 1691 | . 5 | . 1244 | . 1304 | . 1366 | . 1138 | . 1195 | . 1248 | . 8 | . 0591 | . 0628 | . 0667 | . 0429 | . 0463 | . 0496 |
|  |  | 2 |  | . 1604 | . 1665 | . 1728 | . 1598 | . 1643 | . 1687 |  | . 1246 | . 1305 | . 1367 | . 1104 | . 1163 | . 1221 |  | . 0593 | . 0631 | . 0669 | . 0386 | . 0424 | . 0464 |
|  |  | 3 |  | . 1606 | . 1668 | . 1728 | . 1598 | . 1644 | . 1686 |  | . 1244 | . 1303 | . 1366 | . 1093 | . 1155 | . 1213 |  | . 0591 | . 0630 | . 0669 | . 0366 | . 0402 | . 0451 |
|  | 18 | 1 |  | . 1605 | . 1667 | . 1731 | . 1612 | . 1654 | . 1695 |  | . 1245 | . 1304 | . 1367 | . 1158 | . 1207 | . 1258 |  | . 0593 | . 0630 | . 0669 | . 0449 | . 0478 | . 0508 |
|  |  | 2 |  | . 1604 | . 1666 | . 1730 | . 1608 | . 1651 | . 1693 |  | . 1246 | . 1306 | . 1369 | . 1153 | . 1206 | . 1258 |  | . 0592 | . 0629 | . 0667 | . 0445 | . 0475 | . 0506 |
|  |  | 3 |  | . 1604 | . 1666 | . 1730 | . 1606 | . 1650 | . 1692 |  | . 1247 | . 1307 | . 1368 | . 1155 | . 1210 | . 1264 |  | . 0592 | . 0629 | . 0668 | . 0448 | . 0486 | . 0525 |
|  | 30 | 1 |  | . 1606 | . 1667 | . 1731 | . 1613 | . 1654 | . 1695 |  | . 1245 | . 1305 | . 1367 | . 1160 | . 1209 | . 1258 |  | . 0591 | . 0628 | . 0667 | . 0449 | . 0476 | . 0506 |
|  |  | 2 |  | . 1605 | . 1666 | . 1730 | . 1611 | . 1653 | . 1694 |  | . 1245 | . 1304 | . 1367 | . 1159 | . 1209 | . 1260 |  | . 0591 | . 0628 | . 0668 | . 0451 | . 0481 | . 0512 |
|  |  | 3 |  | . 1604 | . 1666 | . 1730 | . 1611 | . 1654 | . 1696 |  | . 1243 | . 1303 | . 1365 | . 1164 | . 1216 | . 1268 |  | . 0592 | . 0629 | . 0668 | . 0460 | . 0496 | . 0535 |
|  | 42 | 1 |  | . 1605 | . 1667 | . 1730 | . 1613 | . 1654 | . 1694 |  | . 1249 | . 1310 | . 1372 | . 1163 | . 1212 | . 1261 |  | . 0591 | . 0628 | . 0667 | . 0450 | . 0478 | . 0507 |
|  |  | 2 |  | . 1605 | . 1667 | . 1730 | . 1613 | . 1654 | . 1695 |  | . 1244 | . 1304 | . 1366 | . 1162 | . 1212 | . 1262 |  | . 0592 | . 0629 | . 0668 | . 0456 | . 0485 | . 0515 |
|  |  | 3 |  | . 1605 | . 1667 | . 1731 | . 1613 | . 1655 | . 1696 |  | . 1243 | . 1303 | . 1365 | . 1169 | . 1220 | . 1272 |  | . 0592 | . 0629 | . 0668 | . 0465 | . 0499 | . 0537 |
| $\begin{aligned} & 3 \\ & 0 \\ & 0 \\ & 0 \\ & 3 \\ & 0 \end{aligned}$ | 6 | 1 | . 2 | . 1655 | . 1727 | . 1806 | . 1605 | . 1649 | . 1692 | . 5 | . 1344 | . 1415 | . 1490 | . 1132 | . 1190 | . 1248 | . 8 | . 0657 | . 0704 | . 0752 | . 0424 | . 0462 | . 0498 |
|  |  | 2 |  | . 1655 | . 1727 | . 1808 | . 1597 | . 1642 | . 1687 |  | . 1344 | . 1413 | . 1489 | . 1100 | . 1161 | . 1222 |  | . 0657 | . 0703 | . 0754 | . 0382 | . 0422 | . 0465 |
|  |  | 3 |  | . 1653 | . 1727 | . 1810 | . 1596 | . 1643 | . 1688 |  | . 1347 | . 1416 | . 1492 | . 1088 | . 1154 | . 1216 |  | . 0659 | . 0701 | . 0750 | . 0363 | . 0404 | . 0454 |
|  | 18 | 1 |  | . 1654 | . 1727 | . 1808 | . 1610 | . 1653 | . 1695 |  | . 1343 | . 1414 | . 1490 | . 1154 | . 1207 | . 1260 |  | . 0659 | . 0704 | . 0752 | . 0445 | . 0476 | . 0509 |
|  |  | 2 |  | . 1655 | . 1727 | . 1807 | . 1607 | . 1651 | . 1693 |  | . 1343 | . 1414 | . 1490 | . 1148 | . 1204 | . 1260 |  | . 0657 | . 0703 | . 0752 | . 0439 | . 0473 | . 0508 |
|  |  | 3 |  | . 1654 | . 1727 | . 1807 | . 1606 | . 1650 | . 1692 |  | . 1343 | . 1414 | . 1490 | . 1148 | . 1207 | . 1265 |  | . 0656 | . 0702 | . 0750 | . 0442 | . 0483 | . 0525 |
|  | 30 | 1 |  | . 1655 | . 1728 | . 1808 | . 1612 | . 1654 | . 1695 |  | . 1345 | . 1417 | . 1492 | . 1156 | . 1208 | . 1261 |  | . 0656 | . 0702 | . 0751 | . 0448 | . 0479 | . 0512 |
|  |  | 2 |  | . 1655 | . 1728 | . 1808 | . 1610 | . 1653 | . 1695 |  | . 1345 | . 1416 | . 1493 | . 1157 | . 1212 | . 1265 |  | . 0658 | . 0703 | . 0751 | . 0449 | . 0482 | . 0516 |
|  |  | 3 |  | . 1654 | . 1728 | . 1808 | . 1610 | . 1653 | . 1696 |  | . 1344 | . 1414 | . 1489 | . 1162 | . 1218 | . 1273 |  | . 0656 | . 0701 | . 0750 | . 0456 | . 0495 | . 0536 |
|  | 42 | 1 |  | . 1654 | . 1727 | . 1808 | . 1612 | . 1654 | . 1695 |  | . 1341 | . 1412 | . 1487 | . 1156 | . 1208 | . 1259 |  | . 0658 | . 0704 | . 0753 | . 0450 | . 0481 | . 0515 |
|  |  | 2 |  | . 1655 | . 1728 | . 1807 | . 1612 | . 1654 | . 1695 |  | . 1346 | . 1418 | . 1493 | . 1161 | . 1214 | . 1267 |  | . 0656 | . 0702 | . 0751 | . 0452 | . 0484 | . 0517 |
|  |  | 3 |  | . 1655 | . 1728 | . 1808 | . 1612 | . 1654 | . 1696 |  | . 1344 | . 1415 | . 1491 | . 1166 | . 1220 | . 1275 |  | . 0658 | . 0703 | . 0751 | . 0462 | . 0501 | . 0541 |
| $\begin{aligned} & 3 \\ & \frac{3}{0} \\ & \sqrt{n} \\ & \frac{\pi}{60} \\ & \sqrt[31]{2} \end{aligned}$ | 6 | 1 | . 2 | . 1725 | . 1853 | . 2016 | . 1607 | . 1656 | . 1703 | . 5 | . 1529 | . 1647 | . 1787 | . 1126 | . 1193 | . 1260 | . 8 | . 0792 | . 0865 | . 0948 | . 0418 | . 0462 | . 0505 |
|  |  | 2 |  | . 1720 | . 1850 | . 2014 | . 1601 | . 1649 | . 1697 |  | . 1532 | . 1648 | . 1783 | . 1096 | . 1167 | . 1234 |  | . 0791 | . 0863 | . 0951 | . 0381 | . 0428 | . 0475 |
|  |  | 3 |  | . 1722 | . 1851 | . 2013 | . 1598 | . 1651 | . 1701 |  | . 1526 | . 1641 | . 1792 | . 1083 | . 1158 | . 1227 |  | . 0789 | . 0861 | . 0945 | . 0359 | . 0407 | . 0459 |
|  | 18 | 1 |  | . 1720 | . 1851 | . 2012 | . 1615 | . 1660 | . 1705 |  | . 1532 | . 1650 | . 1790 | . 1148 | . 1210 | . 1271 |  | . 0793 | . 0867 | . 0953 | . 0439 | . 0478 | . 0519 |
|  |  | 2 |  | . 1717 | . 1849 | . 2010 | . 1613 | . 1659 | . 1705 |  | . 1530 | . 1648 | . 1787 | . 1142 | . 1207 | . 1270 |  | . 0791 | . 0862 | . 0945 | . 0435 | . 0476 | . 0518 |
|  |  | 3 |  | . 1718 | . 1849 | . 2013 | . 1612 | . 1659 | . 1704 |  | . 1528 | . 1645 | . 1787 | . 1144 | . 1211 | . 1276 |  | . 0790 | . 0865 | . 0951 | . 0436 | . 0484 | . 0534 |
|  | 30 | 1 |  | . 1720 | . 1850 | . 2013 | . 1616 | . 1660 | . 1704 |  | . 1529 | . 1646 | . 1788 | . 1150 | . 1210 | . 1269 |  | . 0790 | . 0862 | . 0947 | . 0442 | . 0480 | . 0520 |
|  |  | 2 |  | . 1720 | . 1849 | . 2010 | . 1616 | . 1661 | . 1705 |  | . 1529 | . 1647 | . 1786 | . 1149 | . 1212 | . 1273 |  | . 0790 | . 0863 | . 0949 | . 0442 | . 0482 | . 0523 |
|  |  | 3 |  | . 1719 | . 1849 | . 2011 | . 1615 | . 1661 | . 1706 |  | . 1530 | . 1646 | . 1785 | . 1157 | . 1221 | . 1284 |  | . 0790 | . 0864 | . 0948 | . 0451 | . 0497 | . 0545 |
|  | 42 | 1 |  | . 1719 | . 1848 | . 2009 | . 1616 | . 1660 | . 1703 |  | . 1530 | . 1646 | . 1785 | . 1155 | . 1215 | . 1275 |  | . 0794 | . 0867 | . 0951 | . 0450 | . 0489 | . 0528 |
|  |  | 2 |  | . 1719 | . 1849 | . 2011 | . 1617 | . 1662 | . 1705 |  | . 1528 | . 1645 | . 1785 | . 1151 | . 1212 | . 1273 |  | . 0791 | . 0864 | . 0947 | . 0447 | . 0488 | . 0530 |
|  |  | 3 |  | . 1717 | . 1846 | . 2006 | . 1617 | . 1662 | . 1706 |  | . 1530 | . 1647 | . 1785 | . 1163 | . 1226 | . 1289 |  | . 0791 | . 0865 | . 0950 | . 0459 | . 0504 | . 0552 |

Table D.3. CI widths for matrixwise techniques: $N=500$. $\mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

|  |  |  |  | mult |  |  |  |  |  |  | mult |  |  |  |  |  |  | mult |  |  | uni |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | K | $F$ | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 | $\rho$ | Q1 | M | Q3 | Q1 | M | Q3 |
|  | 6 | 1 | . 2 | . 0815 | . 0837 | . 0858 | . 0806 | . 0824 | . 0843 | . 5 | . 0635 | . 0654 | . 0674 | . 0578 | . 0597 | . 0616 | . 8 | . 0303 | . 0314 | . 0325 | . 0220 | . 0232 | . 0242 |
|  |  | 2 |  | . 0815 | . 0836 | . 0859 | . 0803 | . 0821 | . 0840 |  | . 0635 | . 0653 | . 0674 | . 0562 | . 0581 | . 0601 |  | . 0303 | . 0314 | . 0325 | . 0195 | . 0212 | . 0227 |
|  |  | 3 |  | . 0814 | . 0835 | . 0857 | . 0804 | . 0820 | . 0839 |  | . 0635 | . 0654 | . 0674 | . 0557 | . 0575 | . 0595 |  | . 0304 | . 0314 | . 0325 | . 0185 | . 0196 | . 0223 |
|  |  | 1 |  | . 0815 | . 0836 | . 0858 | . 0809 | . 0827 | . 0845 |  | . 0635 | . 0654 | . 0673 | . 0586 | . 0603 | . 0620 |  | . 0303 | . 0314 | . 0325 | . 0228 | . 0237 | . 0246 |
|  |  | 2 |  | . 0815 | . 0836 | . 0858 | . 0808 | . 0826 | . 0844 |  | . 0635 | . 0654 | . 0673 | . 0583 | . 0601 | . 0618 |  | . 0303 | . 0314 | . 0325 | . 0226 | . 0236 | . 0245 |
|  |  | 3 |  | . 0815 | . 0836 | . 0858 | . 0808 | . 0825 | . 0844 |  | . 0634 | . 0654 | . 0673 | . 0583 | . 0603 | . 0622 |  | . 0303 | . 0314 | . 0325 | . 0226 | . 0240 | . 0256 |
|  | 30 | 1 |  | . 0815 | . 0836 | . 0858 | . 0809 | . 0827 | . 0845 |  | . 0634 | . 0653 | . 0672 | . 0586 | . 0602 | . 0618 |  | . 0303 | . 0314 | . 0325 | . 0229 | . 0237 | . 0246 |
|  |  | 2 |  | . 0815 | . 0836 | . 0858 | . 0809 | . 0827 | . 0845 |  | . 0634 | . 0653 | . 0673 | . 0586 | . 0603 | . 0620 |  | . 0303 | . 0314 | . 0325 | . 0229 | . 0238 | . 0247 |
|  |  | 3 |  | . 0815 | . 0836 | . 0858 | . 0809 | . 0827 | . 0845 |  | . 0634 | . 0653 | . 0673 | . 0589 | . 0607 | . 0625 |  | . 0303 | . 0314 | . 0325 | . 0232 | . 0245 | . 0260 |
|  | 42 | 1 |  | . 0815 | . 0836 | . 0858 | . 0810 | . 0827 | . 0845 |  | . 0634 | . 0653 | . 0673 | . 0585 | . 0601 | . 0618 |  | . 0304 | . 0314 | . 0326 | . 0229 | . 0237 | . 0246 |
|  |  | 2 |  | . 0815 | . 0836 | . 0858 | . 0810 | . 0827 | . 0845 |  | . 0635 | . 0654 | . 0674 | . 0587 | . 0604 | . 0621 |  | . 0303 | . 0314 | . 0326 | . 0230 | . 0238 | . 0248 |
|  |  | 3 |  | . 0815 | . 0836 | . 0858 | . 0810 | . 0828 | . 0846 |  | . 0634 | . 0653 | . 0673 | . 0590 | . 0608 | . 0625 |  | . 0303 | . 0314 | . 0326 | . 0233 | . 0246 | . 0261 |
| $\begin{gathered} 3 \\ \frac{3}{0} \\ \text { v } \\ 3 \\ 0 \\ 0 \end{gathered}$ | 6 | 1 | . 2 | . 0846 | . 0872 | . 0899 | . 0806 | . 0824 | . 0842 | . 5 | . 0691 | . 0713 | . 0737 | . 0577 | . 0597 | . 0616 | . 8 | . 0341 | . 0354 | . 0368 | . 0220 | . 0232 | . 0243 |
|  |  | 2 |  | . 0848 | . 0872 | . 0899 | . 0804 | . 0822 | . 0840 |  | . 0691 | . 0713 | . 0737 | . 0560 | . 0581 | . 0602 |  | . 0341 | . 0354 | . 0367 | . 0194 | . 0212 | . 0227 |
|  |  | 3 |  | . 0847 | . 0869 | . 0896 | . 0803 | . 0820 | . 0840 |  | . 0690 | . 0713 | . 0736 | . 0556 | . 0575 | . 0595 |  | . 0340 | . 0354 | . 0367 | . 0183 | . 0196 | . 0224 |
|  | 18 | 1 |  | . 0847 | . 0872 | . 0898 | . 0809 | . 0827 | . 0845 |  | . 0691 | . 0713 | . 0736 | . 0585 | . 0603 | . 0620 |  | . 0340 | . 0354 | . 0368 | . 0228 | . 0237 | . 0247 |
|  |  | 2 |  | . 0847 | . 0872 | . 0898 | . 0808 | . 0826 | . 0844 |  | . 0690 | . 0712 | . 0736 | . 0582 | . 0601 | . 0619 |  | . 0340 | . 0354 | . 0368 | . 0225 | . 0236 | . 0246 |
|  | 30 | 3 |  | . 0847 | . 0872 | . 0898 | . 0808 | . 0826 | . 0844 |  | . 0691 | . 0714 | . 0737 | . 0583 | . 0603 | . 0622 |  | . 0340 | . 0354 | . 0368 | . 0226 | . 0240 | . 0256 |
|  |  | 1 |  | . 0847 | . 0872 | . 0898 | . 0809 | . 0827 | . 0845 |  | . 0691 | . 0713 | . 0737 | . 0585 | . 0602 | . 0620 |  | . 0340 | . 0354 | . 0368 | . 0228 | . 0237 | . 0247 |
|  |  | 2 |  | . 0847 | . 0872 | . 0898 | . 0809 | . 0827 | . 0845 |  | . 0690 | . 0713 | . 0736 | . 0585 | . 0603 | . 0621 |  | . 0340 | . 0354 | . 0368 | . 0228 | . 0238 | . 0248 |
|  |  | 3 |  | . 0847 | . 0872 | . 0898 | . 0809 | . 0827 | . 0845 |  | . 0691 | . 0713 | . 0736 | . 0588 | . 0607 | . 0626 |  | . 0340 | . 0354 | . 0367 | . 0231 | . 0245 | . 0260 |
|  |  | 1 |  | . 0847 | . 0872 | . 0898 | . 0809 | . 0827 | . 0845 |  | . 0691 | . 0713 | . 0736 | . 0586 | . 0603 | . 0620 |  | . 0340 | . 0353 | . 0367 | . 0228 | . 0237 | . 0246 |
|  |  | 2 |  | . 0847 | . 0872 | . 0898 | . 0810 | . 0827 | . 0845 |  | . 0691 | . 0714 | . 0738 | . 0586 | . 0603 | . 0621 |  | . 0340 | . 0353 | . 0367 | . 0229 | . 0238 | . 0248 |
|  |  | 3 |  | . 0847 | . 0872 | . 0898 | . 0810 | . 0828 | . 0846 |  | . 0691 | . 0713 | . 0737 | . 0590 | . 0608 | . 0626 |  | . 0340 | . 0353 | . 0367 | . 0232 | . 0245 | . 0261 |
|  | 6 | 1 | . 2 | . 0908 | . 0954 | . 1007 | . 0807 | . 0826 | . 0845 | . 5 | . 0811 | . 0853 | . 0901 | . 0575 | . 0597 | . 0618 | . 8 | . 0422 | . 0446 | . 0472 | . 0219 | . 0232 | . 0245 |
|  |  | 2 |  | . 0907 | . 0952 | . 1006 | . 0805 | . 0824 | . 0843 |  | . 0812 | . 0852 | . 0900 | . 0559 | . 0581 | . 0603 |  | . 0421 | . 0445 | . 0473 | . 0194 | . 0212 | . 0228 |
|  |  | 3 |  | . 0909 | . 0953 | . 1006 | . 0804 | . 0823 | . 0843 |  | . 0813 | . 0854 | . 0899 | . 0554 | . 0576 | . 0599 |  | . 0423 | . 0448 | . 0474 | . 0184 | . 0199 | . 0224 |
|  | 18 | 1 |  | . 0908 | . 0952 | . 1005 | . 0810 | . 0829 | . 0847 |  | . 0812 | . 0852 | . 0900 | . 0584 | . 0603 | . 0623 |  | . 0423 | . 0446 | . 0473 | . 0226 | . 0237 | . 0249 |
|  |  | 2 |  | . 0909 | . 0953 | . 1006 | . 0808 | . 0827 | . 0846 |  | . 0811 | . 0852 | . 0899 | . 0581 | . 0601 | . 0622 |  | . 0422 | . 0446 | . 0473 | . 0224 | . 0236 | . 0248 |
|  |  | 3 |  | . 0907 | . 0953 | . 1006 | . 0808 | . 0827 | . 0846 |  | . 0812 | . 0852 | . 0899 | . 0581 | . 0603 | . 0624 |  | . 0423 | . 0447 | . 0474 | . 0225 | . 0241 | . 0257 |
|  | 30 | 1 |  | . 0908 | . 0953 | . 1006 | . 0810 | . 0828 | . 0847 |  | . 0812 | . 0852 | . 0899 | . 0584 | . 0603 | . 0622 |  | . 0423 | . 0446 | . 0473 | . 0227 | . 0238 | . 0250 |
|  |  | 2 |  | . 0908 | . 0953 | . 1006 | . 0810 | . 0828 | . 0847 |  | . 0812 | . 0853 | . 0900 | . 0584 | . 0604 | . 0623 |  | . 0423 | . 0446 | . 0473 | . 0227 | . 0238 | . 0250 |
|  |  | 3 |  | . 0908 | . 0953 | . 1006 | . 0810 | . 0828 | . 0847 |  | . 0812 | . 0853 | . 0901 | . 0587 | . 0607 | . 0628 |  | . 0423 | . 0447 | . 0474 | . 0231 | . 0245 | . 0261 |
|  | 42 | 1 |  | . 0908 | . 0953 | . 1006 | . 0810 | . 0828 | . 0847 |  | . 0812 | . 0852 | . 0899 | . 0584 | . 0602 | . 0621 |  | . 0423 | . 0447 | . 0474 | . 0227 | . 0237 | . 0248 |
|  |  | 2 |  | . 0908 | . 0953 | . 1006 | . 0810 | . 0828 | . 0847 |  | . 0812 | . 0853 | . 0900 | . 0585 | . 0604 | . 0623 |  | . 0423 | . 0446 | . 0474 | . 0228 | . 0239 | . 0251 |
|  |  | 3 |  | . 0908 | . 0952 | . 1005 | . 0811 | . 0829 | . 0848 |  | . 0812 | . 0853 | . 0900 | . 0588 | . 0608 | . 0628 |  | . 0423 | . 0447 | . 0474 | . 0232 | . 0247 | . 0263 |

Table D.4. CI widths for matrixwise techniques: $N=2000$. $\mathrm{Q} 1=$ first quartile, $\mathrm{M}=$ median, $\mathrm{Q} 3=$ third quartile.

