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A Recent Study of Excited Energy Levels of Diatomics for Modified more General Exponential Screened Coulomb Potential: Extended Quantum Mechanics

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In present research paper, we investigated in detail the non-relativistic bound states of diatomics under the influence of modified more general exponential screened coulomb (MGESC) potential by means Bopp's shift method and standard perturbation theory in the noncommutative 3-dimensional space phase (NC: 3D-RSP). Through an exact analytical solution of 3-dimensional time-independent Schrödinger equation, the corrections expressions were derived for energy eigenvalues for n^{th} excited states. Furthermore, the obtained corrections of energies are depended on the discreet atomic quantum numbers ($j = |l - s|,.....(l + s)$, (n, l)) and m), in addition to the two infinitesimal parameters (Θ, χ) which are induced by position-position noncommutativity. We have also shown that, the total complete degeneracy of energy levels of modified (MGESC) potential equals the new values $2n^2$. We have also shown that, the group symmetry (NC: 3D-RSP) corresponding modified (MGESC) potential reduce to the symmetry sub-group (NC: 3D-RS).

Keywords: More general exponential screened coulomb potential, Noncommutative space and phase, Star product and Bopp's shift method.

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1. INTRODUCTION

The more general exponential screened coulomb (MGESC) potential is known to describe adequately the effective interaction in many-body environments of a variety of fields [1 – 2]. In particularity, this potential used to calculate the bounded state eigenvalues of diatomics (N_2CO, NO,…). The noncommutativity of space-time, which known firstly by Heisenberg and was formalized by Snyder at 1947, suggest by the physical recent results in string theory [3]. Very recently, several authors have attempted to obtain either the exact or approximate solutions of the non-relativistic Schrodinger equation or two relativistic (Klein-Gordon and Dirac) equations for different central and non central potentials in noncommutative space. We want to extended, this study to case of noncommutative space phase to obtaining an profound new interpretations in the sub-atomics scales on based to the previously works [3 – 5] and our previously works in this context [6 – 10]. The nonrelativistic energy levels for diatomics which interacted with (MGESC) potential in the context of noncommutative space $V_{nc\text{-}mgesci}(r)$ have not been obtained yet. The purpose of the present paper is to attempt study the non-relativistic Schrödinger equation with (MGESC) potential (see below):

$$V_{mgesci}(r) = \left(-\frac{a}{r}\right)\left(1 + (1 + b)\exp(-2b)\right) \Rightarrow$$
$$V_{nc\text{-}mgesci}(r) = V_{mgesci}(r) - \frac{a}{2r^3}\left(1 + (1 + b)\exp(-2b)\right)\vec{\mathbf{L}}\vec{\Theta} + \frac{\vec{\mathbf{L}}\vec{\vec{\theta}}}{2\mu} \quad (1)$$

in (NC: 3D-RSP) symmetries using the generalization Bopp's shift method which depend on the concepts that we present below and in the third section to discover the new symmetries and a possibility to obtain another applications to this potential in different fields. The new structure of extended quantum mechanics based to new noncommutative canonical commutations relations (NNCCRs) in both Schrödinger and Heisenberg pictures ((SP) and (HP)), respectively, as follows (Throughout this paper the natural units $c = \hbar = 1$ will be used) [4, 5]:

$$\left[x_i, p_j\right] = i\delta_{ij} \Rightarrow \left[\hat{x}_i \overset{*}{,} \hat{p}_j\right] = \left[\hat{x}_i(t) \overset{*}{,} \hat{p}_j(t)\right] = i\delta_{ij} \quad (2)$$

$$\left[x_i, x_j\right] = 0 \Rightarrow \left[\hat{x}_i \overset{*}{,} \hat{x}_j\right] = \left[\hat{x}_i(t) \overset{*}{,} \hat{x}_j(t)\right] = i\theta_{ij} \quad (3)$$

$$\left[p_i, p_j\right] = 0 \Rightarrow \left[\hat{p}_i \overset{*}{,} \hat{p}_j\right] = \left[\hat{p}_i(t) \overset{*}{,} \hat{p}_j(t)\right] = i\bar{\theta}_{ij} \quad (4)$$

However, the new operators $(\hat{x}_i(t), \hat{p}_i(t))$ in (HP) are depending to the corresponding new operators $(\hat{x}_i, \hat{p}_i)$ in (SP) from the following projections relations, respectively [6 – 9]:

$$\begin{aligned}&(x_i(t), p_i(t)) = \exp(i\hat{H}_{ni}(t - t_0))(x_i, p_i)\exp(-i\hat{H}_{ni}(t - t_0)) \Rightarrow \\ &(\hat{x}_i(t), \hat{p}_i(t)) = \exp(i\hat{H}_{nc-ni}(t - t_0)) * (\hat{x}_i, \hat{p}_i) * \exp(-i\hat{H}_{nc-ni}(t - t_0))\end{aligned} \quad (5)$$

While the dynamics of new systems ($\frac{d\hat{x}_i(t)}{dt}$, $\frac{d\hat{p}_i(t)}{dt}$) are described from the following relations:

$$\frac{dx_i(t)}{dt} = \left[\hat{x}_i(t), \hat{H}_{mgesci}\right] \Rightarrow \frac{d\hat{x}_i(t)}{dt} = \left[\hat{x}_i(t) \overset{*}{,} \hat{H}_{nc-mgesci}\right] \quad (6)$$

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$$\frac{\mathrm{dp_i}(t)}{\mathrm{dt}} = \left[\mathrm{p_i}(t), \hat{H}_{mgesci}\right] \Rightarrow \frac{\mathrm{d}\hat{p}_i(t)}{\mathrm{dt}} = \left[\hat{p}_i(t) \overset{*}{,} \hat{H}_{nc-mgesci}\right] \quad (7)$$

Here, the two operators $\hat{H}_{mgesci}$ and $\hat{H}_{nc-mgesci}$ are presents the ordinary and new quantum Hamiltonian operators in the quantum mechanics and it's extension, respectively, while $\frac{\mathrm{d}\hat{x}_i(t)}{\mathrm{dt}}$ and $\frac{\mathrm{d}\hat{p}_i(t)}{\mathrm{dt}}$ are describe the dynamics of systems in (NC: 3D-RSP). The very small two parameters $\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$ (compared to the energy) are elements of two antisymmetric real matrixes and (*) denote to the new star product, which is generalized between two arbitrary functions $(f,g)(x,p) \to (\hat{f},\hat{g})(\hat{x},\hat{p})$ to $\hat{f}(\hat{x},\hat{p})\hat{g}(\hat{x},\hat{p}) \equiv (f*g)(x,p)$ instead of the usual product $(fg)(x, p)$ in ordinary 3-dimensional spaces [4, 5]:

$$\begin{aligned}&(fg)(x,p) \Rightarrow (f*g)(x,p) = \\ &= \left(fg - \frac{i}{2}\left(\theta^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial g}{\partial x^\nu} + \bar{\theta}^{\mu\nu}\frac{\partial f}{\partial p^\mu}\frac{\partial g}{\partial p^\nu}\right)\right)(x,p)\bigg|_{(x^\mu = x^\nu,\, p^\mu = p^\nu)} + \\ &+ O\left(\theta^2, \bar{\theta}^2\right)\end{aligned} \quad (8)$$

The second and the third terms are induced by (space-space) and (phase-phase) noncommutativity properties, respectively. The organization scheme of the study is given as follows: In next section, we briefly review the Schrödinger equation with (MGESC) potential on based to ref. [2]. The Section three is devoted to studying the three modified Schrödinger equation by applying Bopp's shift method for (MGESC) potential. In the fourth section and by applying standard perturbation theory we find the quantum spectrum of n^{th} excited levels in for spin-orbital interaction in the framework of the (NC-3D: RSP) symmetries and we derive the magnetic spectrum for studied potential. In the fifth section, we resume the global spectrum and corresponding noncommutative Hamiltonian operator for (MGESC) potential and corresponding energy levels. Finally, the concluding remarks have been presented in the last section.

2. OVERVIEW OF THE EIGNENFUNCTIONS AND THE ENERGY EIGENVALUES FOR ORDINARY (MGESC) POTENTIAL

We shall recall here the time independent Schrödinger equation for a (MGESC) potential $V_{nc-mgesci}(r)$ [2]:

$$V_{mgesci}(r) = \left(-\frac{a}{r}\right)\left(1 + (1+b)\exp(-2b)\right) \quad (9)$$

where a and b are the strength coupling constant and the screened parameter, respectively. If we insert this potential into the Schrödinger equation:

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right)R_{nl}(r) + 2\mu\left[E_{nl} - \left(-\frac{a}{r}\right)\left(1+(1+b)\exp(-2b)\right)\right]R_{nl}(r) = 0 \quad (10)$$

Here μ is the reduced mass of diatomics. The electronic radial wave functions are shown as a function of the Laguerre polynomial in terms of some parameters [2]:

$$\begin{aligned}R_{nl}(r) &= \left[\frac{(n-2\alpha+1)!(2\mathrm{i}\varepsilon)^{3-2\alpha}}{(2n-2\alpha+2)(n!)^3}\right]^{1/2}(2\mathrm{i}\varepsilon)^{-\frac{3}{2}+\alpha} \times \\ &\times \exp\left(-\frac{\nu}{2}\right)\nu^{\alpha-\frac{1}{2}}L_n^{2\alpha+1}(\nu)\end{aligned} \quad (11)$$

where $\mathrm{r} = (2\mathrm{i}\varepsilon)^{-1}\nu$, therefore, the complete wave function $\Psi(r,\theta,\varphi)$ and the energy E_{nl} of the potential in eq. (9) are given by [2]:

$$\begin{aligned}\Psi(r,\theta,\phi) &= \left[\frac{(n-2\alpha+1)!(2\mathrm{i}\varepsilon)^{3-2\alpha}}{(2n-2\alpha+2)(n!)^3}\right]^{1/2}(2\mathrm{i}\varepsilon)^{-\frac{3}{2}+\alpha} \times \\ &\times \exp\left(-\frac{\nu}{2}\right)\nu^{\alpha-\frac{1}{2}}L_n^{2\alpha+1}(\nu)Y_l^m(\theta,\phi)\end{aligned} \quad (12)$$

$$E_{nl} = -abe^{-2b} + 2\mu\left(\frac{a+e^{-2b}}{1+2n+\sqrt{4l(l+1)+1}}\right)^2 \quad (13)$$

with $\alpha = \frac{1}{2}\sqrt{4l(l+1)+1}$ and $Y_l^m(\theta,\varphi)$ are the well known spherical harmonic functions.

3. OVERVIEW OF THREE DIMENSIONAL NONCOMMUTATIVE REAL SPACE-PHASE FOR MODIFIED (MGESC) POTENTIAL

In this section, we shall gives an overview or a brief preliminary for a (MGESC) potential $V_{nc-mgesci}(r)$ in (NC: 3D-RSP), to perform this task the physical form of modified Schrödinger equation (MSE), it's necessary to replace ordinary three dimensional Hamiltonian operators $\hat{H}(p_i, x_i)$, ordinary complex wave function $\Psi(\vec{r})$ and ordinary energy E_{nl} by new three Hamiltonian operators $\hat{H}_{nc-mgesci}(\hat{p}_i, \hat{x}_i)$, new complex wave function $\hat{\Psi}(\vec{\vec{r}})$ and new values $E_{nc-mgesci}$, respectively while, the last step corresponds to replace the ordinary old product by new star product (*), which allow us to constructing the modified Schrödinger equations in both (NC-3D: RSP) [9, 10]:

$$\begin{aligned}\hat{H}_{mgesci}(p_i,x_i)\Psi(\vec{r}) = E_{mgesci}\Psi(\vec{r}) \Rightarrow \hat{H}_{nc-mgesci}(\hat{p}_i,\hat{x}_i) * \hat{\Psi}(\vec{\vec{r}}) = \\ = E_{nc-mgesci}\hat{\Psi}(\vec{\vec{r}})\end{aligned} \quad (14)$$

Instead of solving any quantum systems by using directly star product procedure, a Bopp's shift method can be used [9, 10]:

$$\left[\hat{x}_i, \hat{x}_j\right] = \left[\hat{x}_i(t), \hat{x}_j(t)\right] = i\theta_{ij} \text{ and } \left[\hat{p}_i, \hat{p}_j\right] = \left[\hat{p}_i(t), \hat{p}_j(t)\right] = i\bar{\theta}_{ij} \quad (15)$$

The new generalized positions and momentum coordinates $(\hat{x}_i, \hat{p}_i)$ in (NC: 3D-RSP) are depended with corresponding usual generalized positions and momentum coordinates (x_{ii}, p_i) in ordinary quantum mechanics by the following, respectively [9, 10]:

$$x_i \Rightarrow \hat{x}_i = x_i - \frac{\theta_{ij}}{2} p_j \quad \text{and} \quad p_i \Rightarrow \hat{p}_i = p_i - \frac{\bar{\theta}_{ij}}{2} x_j \tag{16}$$

The above equation allows us to obtain the two operators $\hat{r}^2$ and $\hat{p}^2$ in (NC-3D: RSP), respectively [8 – 10]:

$$\hat{r}_i^{\,2} = r_i^{\,2} - \vec{\mathbf{L}}\vec{\Theta} \quad \text{and} \quad \hat{p}_i^{\,2} = p_i^{\,2} + \vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}} \tag{17}$$

The two couplings $\mathbf{L}\Theta$ and $\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}$ are $(L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13})$ and $(L_x\bar{\theta}_{12} + L_y\bar{\theta}_{23} + L_z\bar{\theta}_{13})$, respectively and ($L_x$, L_y and L_z) are the three components of angular momentum operator $\vec{L}$ and $\Theta_{ij} = \theta_{ij}/2$. Thus, the reduced Schrödinger equation (without star product) can be written as:

$$H(\hat{p}_i, \hat{x}_i)\psi(\vec{r}) = E_{nc-mgesci}\psi(\vec{r}) \tag{18}$$

the new operator of Hamiltonian $H(\hat{p}_i, \hat{x}_i)$ can be expressed as:

$$H_{nc-mgesci}(\hat{p}_i, \hat{x}_i) \equiv H\left(\hat{x}_i = x_i - \frac{\theta_{ij}}{2} p_j \quad \text{and} \quad \hat{p}_i = p_i - \frac{\bar{\theta}_{ij}}{2} x_j\right) \tag{19}$$

Now, we can determine the modified (MGESC) potential $V_{mgesci}(\hat{r})$ as follows:

$$V_{mgesc}(\hat{r}) = \left(-\frac{a}{\hat{r}}\right)(1 + (1+b)\exp(-2b)) \tag{20}$$

After straightforward calculations, we can obtain the important term, which will be use to determine the modified (MGESC) potential in (NC: 3D- RSP) as:

$$\left(-\frac{a}{\hat{r}}\right) = \left(-\frac{a}{r}\right) - \frac{a\vec{\mathbf{L}}\vec{\Theta}}{2r^3} \tag{21}$$

By making the substitution $\left(-\frac{a}{\hat{r}}\right)$ in eq. (20), we find the global our working potential operator $H_{\text{nc-}mgesc}(\hat{r})$ satisfies the equation in (NC: 3D-RSP):

$$\begin{aligned} H_{\text{nc-}mgesci}(\hat{r}) &= \left(-\frac{a}{r}\right)(1 + (1+b)\exp(-2b)) - \\ &- \frac{a}{2r^3}(1 + (1+b)\exp(-2b))\vec{\mathbf{L}}\vec{\Theta} + \frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2\mu} \end{aligned} \tag{22}$$

This equation is the sum of $V_{mgesc}(\hat{r})$ and $\frac{\hat{p}_i^{\,2}}{2\mu}$. It's clearly, that the first term give the ordinary (MGESC) potential in commutative space, while the rest two terms are proportional's with two infinitesimals parameters (Θ and $\bar{\theta}$) and then we can considered as a perturbations terms $H_{per\text{-}mgesci}(r)$ in (NC: 3D-RSP) as:

$$H_{\text{per-}mgesci}(r) = -\frac{a}{2r^3}(1 + (1+b)\exp(-2b))\vec{\mathbf{L}}\vec{\Theta} + \frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2\mu} \tag{23}$$

4. THE EXACT SPIN-ORBITAL SPECTRUM MODIFICATION FOR MODIFIED (WBEPM) POTENTIAL IN (NC:3D- RSP):

Again, the perturbative term $H_{per\text{-}mgesci}(r)$ can be rewritten to the equivalent physical form:

$$H_{\text{per-}mgesci}(r) = 2\left(\Theta\left(-\frac{a}{2r^3}(1 + (1+b)\exp(-2b))\right) + \frac{\bar{\theta}}{2\mu}\right)\overleftrightarrow{S}\overleftrightarrow{L} \tag{24}$$

We have chose the two vectors $\vec{\Theta}$ and $\vec{\bar{\boldsymbol{\theta}}}$ parallel to the spin $\overleftrightarrow{S}$ of diatomics (N_2CO, NO,…). Furthermore, the above perturbative terms $H_{per\text{-}mgesci}(r)$ can be rewritten to the following new form:

$$H_{\text{per-}mgesci}(r) = \left(\Theta\left(-\frac{a}{2r^3}(1 + (1+b)\exp(-2b))\right) + \frac{\bar{\theta}}{2\mu}\right)\left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right) \tag{25}$$

This operator traduces the coupling between spin $\overleftrightarrow{S}$ and orbital momentum $\overleftrightarrow{L}$. The set ($\hat{H}_{\text{so-}mgesci}(r)$, J^2, L^2, S^2 and J_z) forms a complete of conserved physics quantities and for $\vec{S} = \vec{1}/2$ the eigen-values of the spin orbital coupling operator are $k_\pm \equiv \frac{1}{2}\left\{\left(l \pm \frac{1}{2}\right)(l \pm \frac{1}{2} + 1) + l(l+1) - \frac{3}{4}\right\}$ corresponding: $j = l + \frac{1}{2}$ (spin up) and $j = l - \frac{1}{2}$ (spin down), respectively, then, one can form a diagonal (3×3) matrix, with non null elements are $(\hat{H}_{so-mgesci})_{11}$, $(\hat{H}_{so-mgesci})_{22}$ and $(\hat{H}_{so-mgesci})_{33}$ for modified (MGESC) potential in (NC: 3D-RSP) as:

$$(\hat{H}_{so-mgesci})_{11} = k_+\left(\frac{\bar{\theta}}{2\mu} - \frac{\Theta a}{2r^3}(1 + (1+b)\exp(-2b))\right) \text{if } j = l + \tfrac{1}{2} \tag{26}$$

$$(\hat{H}_{so-mgesci})_{22} = k_-\left(\frac{\bar{\theta}}{2\mu} - \frac{\Theta a}{2r^3}(1 + (1+b)\exp(-2b))\right) \text{if } j = l - \tfrac{1}{2} \tag{27}$$

$$(\hat{H}_{so-mgesci})_{33} = 0 \tag{28}$$

After profound straightforward calculation, one can show that, the radial function $R_{nl}(r)$ satisfying the following differential equation for modified (MGESC) potential:

$$\begin{aligned} &\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right)R_{nl}(r) + \\ &+2\mu\begin{bmatrix} E_{nl} + \left(\frac{a}{r}\right)(1 + (1+b)\exp(-2b)) - \frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2\mu} \\ + \frac{a}{2r^3}(1 + (1+b)\exp(-2b))\vec{\mathbf{L}}\vec{\Theta} \end{bmatrix} R_{nl}(r) = 0 \end{aligned} \tag{29}$$

The two terms which composed the expression of $H_{per\text{-}mgesci}(r)$ are proportional's with two infinitesimals parameters (Θ and $\bar{\theta}$), thus, in what follows, we proceed to solve the modified radial part of the Schrödinger equation that is, equation (29) by applying standard perturbation theory for their exact solutions at first order of two parameters Θ and $\bar{\theta}$.

4.1 4.1 The Exact Spin-orbital Spectrum Modifications for Modified (MGESC) Potential in (NC: 3D- RSP)

The purpose here is to give a complete prescription for determine the energy level of n^{th} excited levels, of diatomics, we first find the corrections $E_{u\text{-}mgesci}$ and $E_{d\text{-}mgesci}$ for diatomics which have $j = l + 1/2$ (spin up) and $j = l - 1/2$ (spin down), respectively, at first order of two parameters Θ and $\bar{\theta}$ obtained by applying the standard perturbation theory:

$$E_{\text{u-}mgesci} = \left[\frac{(n-2\alpha+1)!(2\mathrm{i}\varepsilon)^{3-2\alpha}}{(2n-2\alpha+2)(n!)^3}\right](2\mathrm{i}\varepsilon)^{2\alpha-6}k_+ \int_0^{+\infty}\exp(-\nu)\nu^{2\alpha-1}\left[L_n^{2\alpha+1}(\nu)\right]^2 \left(\frac{\bar{\theta}}{2\mu}-\frac{a\Theta}{2(2\mathrm{i}\varepsilon)^{-3}\nu^3}\left(1+(1+b)\exp(-2b)\right)\right)\nu^2 d\nu \quad (30)$$

$$E_{\text{d-}mgescii} = \left[\frac{(n-2\alpha+1)!(2\mathrm{i}\varepsilon)^{3-2\alpha}}{(2n-2\alpha+2)(n!)^3}\right](2\mathrm{i}\varepsilon)^{2\alpha-6}k_- \int_0^{+\infty}\exp(-\nu)\nu^{2\alpha-1}\left[L_n^{2\alpha+1}(\nu)\right]^2 \left(\frac{\bar{\theta}}{2\mu}-\frac{a\Theta}{2(2\mathrm{i}\varepsilon)^{-3}\nu^3}\left(1+(1+b)\exp(-2b)\right)\right)\nu^2 d\nu \quad (31)$$

The above two equations can be written in the following:

$$E_{\text{u-}mgesci} = \left[\frac{(n-2\alpha+1)!(2\mathrm{i}\varepsilon)^{3-2\alpha}}{(2n-2\alpha+2)(n!)^3}\right](2\mathrm{i}\varepsilon)^{2\alpha-6}k_+ \left\{\frac{\bar{\theta}}{2}T_2(n,\alpha)-\frac{a\Theta}{2(2\mathrm{i}\varepsilon)^{-3}}\left(1+(1+b)\exp(-2b)\right)T_1(n,\alpha)\right\} \quad (32)$$

$$E_{\text{d-}mgescii} = \left[\frac{(n-2\alpha+1)!(2\mathrm{i}\varepsilon)^{3-2\alpha}}{(2n-2\alpha+2)(n!)^3}\right](2\mathrm{i}\varepsilon)^{2\alpha-6}k_- \left\{\frac{\bar{\theta}}{2}T_2(n,\alpha)-\frac{a\Theta}{2(2\mathrm{i}\varepsilon)^{-3}}\left(1+(1+b)\exp(-2b)\right)T_1(n,\alpha)\right\} \quad (33)$$

And the expressions of the two terms $T_1(n,\alpha)$ and $T_2(n,\alpha)$ are given by:

$$T_1(n,\alpha) = \int_0^{+\infty}\exp(-\nu)\nu^{(2\alpha-1)-1}\left[L_n^{2\alpha+1}(\nu)\right]^2 d\nu \quad (34)$$

$$T_2(n,\alpha) = \int_0^{+\infty}\exp(-\nu)\nu^{(2\alpha+2)-1}\left[L_n^{2\alpha+1}(\nu)\right]^2 d\nu \quad (35)$$

We apply the following special integration [11]:

$$\int_0^{+\infty} t^{\varepsilon-1}\exp(-\omega t)L_m^{\lambda}(\omega t)L_n^{\beta}(\omega t)dt = \frac{\omega^{-\varepsilon}\Gamma(n-\varepsilon+\beta+1)\Gamma(m+\lambda+1)}{m!n!\Gamma(1-\varepsilon+\beta)\Gamma(1+\lambda)}\, {}_3F_2(-m,\varepsilon,\varepsilon-\beta;-n+\varepsilon,\lambda+1;1) \quad (36)$$

where ${}_3F_2(-m,\varepsilon,\varepsilon-\beta;-n+\varepsilon,\lambda+1;1)$ is obtained from the generalized the hypergeometric function ${}_pF_q(\alpha_1,\ldots,\alpha_p,\beta_1,\ldots,\beta_q,z)$ for $p=3$ and $q=2$ while $\Gamma(x)=\int_0^{+\infty} z^{x-1}e^{-z}dz$ denote to the usual Gamma function. After straightforward calculations, we can obtain the explicitly results:

$$T_1(n,\alpha) = \frac{\Gamma(n+3)\Gamma(n+2\alpha+2)}{(n!)^2\Gamma(1)\Gamma(2\alpha+2)}\, {}_3F_2(-n,2\alpha-1,0;-n+2\alpha-1,2\alpha+2;1) \quad (37)$$

$$T_2(n,\alpha) = \frac{\Gamma(n)\Gamma(n+2\alpha+2)}{(n!)^2\Gamma(1-2\alpha-2+2\alpha+1)\Gamma(2\alpha+2)}\, {}_3F_2(-n,2\alpha+2,1;-n+2\alpha+2,2\alpha+2;1) \quad (38)$$

Because $\Gamma(0)=(-1)!=\infty$, the factor $T_2(n,\alpha)$ reduce to zero, thus the exact modifications $E_{\text{u-}mgesci}$ and $E_{\text{d-}mgesci}$ of n^{th} excited states produced by spin-orbital effect can be expressed as:

$$E_{\text{u-}mgesci} = -\Theta\frac{a}{2}\left[\frac{(n-2\alpha+1)!}{(2n-2\alpha+2)}\right]\frac{\Gamma(n+3)\Gamma(n+2\alpha+2)}{(n!)^5\Gamma(2\alpha+2)}k_+ \left(1+(1+b)\exp(-2b)\right){}_3F_2(-n,2\alpha-1,0;-n+2\alpha-1,2\alpha+2;1) \quad (39)$$

$$E_{\text{d-}mgescii} = -\Theta\frac{a}{2}\left[\frac{(n-2\alpha+1)!}{(2n-2\alpha+2)}\right]\frac{\Gamma(n+3)\Gamma(n+2\alpha+2)}{(n!)^5\Gamma(2\alpha+2)}k_- \left(1+(1+b)\exp(-2b)\right){}_3F_2(-n,2\alpha-1,0;-n+2\alpha-1,2\alpha+2;1) \quad (40)$$

Thus, the group symmetry (NC: 3D-RSP) reduce to new sub-group symmetry (NC: 3D-RS).

4.2 The Exact Magnetic Spectrum Modifications for Modified (MGESC) Potential

Further to the important previously obtained results, now, we consider another physically meaningful phenomena produced by the effect of modified (MGESC) potential related to the influence of an external uniform magnetic field $\overleftrightarrow{B}$, to avoid the repetition in the theoretical calculations, it's sufficient to apply the following replacements:

$$\begin{cases}\vec{\Theta}\to\chi\vec{B}\\ \vec{\bar{\theta}}\to\bar{\sigma}\vec{B}\end{cases} \Rightarrow \left(\frac{\bar{\theta}}{2\mu}-\frac{a\Theta}{2r^3}\left(1+(1+b)\exp(-2b)\right)+\right) \Rightarrow \left(\frac{\bar{\sigma}}{2\varpi}-\frac{\chi a}{2r^3}\left(1+(1+b)\exp(-2b)\right)\right)\vec{B}\vec{L} \quad .(41)$$

Here χ and $\bar{\sigma}$ are two infinitesimal real proportional's constants, and we choose the arbitrary external magnetic field $\overleftrightarrow{B}$ parallel to the (Oz) axis, which allow us to introduce the new modified magnetic Hamiltonian $H_{m-mgesci}$ in (NC: 3D-RSP) as:

$$H_{m-mgesci}=\left(\frac{\bar{\sigma}}{2\mu}-\frac{a\chi}{2r^3}\left(1+(1+b)\exp(-2b)\right)\right)\aleph_{\text{mod}-z} \quad (42)$$

Here $\aleph_{\text{mod}-z}\equiv\vec{B}\vec{J}-\aleph_z$ denote to the modified Zeeman effect while $\aleph_z\equiv-\overleftrightarrow{S}\overleftrightarrow{B}$ is the ordinary Hamiltonian operator of Zeeman Effect. To obtain the exact non-commutative magnetic modifications of energy $E_{\text{mag-}mgesci}(n,m,\alpha)$, we just replace: k_+ and Θ in the eq. (27) by the following parameters: m and χ, respectively:

$$E_{\text{mag-}mgesci}(n,m,\alpha)=-\chi\frac{a}{2}\left[\frac{(n-2\alpha+1)!}{(2n-2\alpha+2)}\right]\frac{\Gamma(n+3)\Gamma(n+2\alpha+2)}{(n!)^5\Gamma(2\alpha+2)} \quad (43)$$
$$\left(1+(1+b)\exp(-2b)\right){}_3F_2(-n,2\alpha-1,0;-n+2\alpha-1,2\alpha+2;1)Bm$$

We have $-l\leq m\leq +l$, which allow us to fixing $(2l+1)$ values for discreet number m.

5. THE EXACT MODIFIED OF n^{th} EXCITES STATES FOR MODIFIED (MGESC) POTENTIAL IN (NC: 3D- RSP):

In the light of the results of the preceding sections, let us resume the modified eigenenergies ($E_{\text{nc-u}mgesci}(n,j,l,s,m,\alpha)$- $E_{\text{nc-d}mgesci}(n,j,l,s,m,\alpha)$) of a diatomics with spin $\overleftrightarrow{S}=\frac{\vec{1}}{2}$ for modified Schrödinger equation obtained in this paper, the total energies corresponding n^{th} excited states in (NC: 3D-RSP) are determined on based to our original results presented on the Eqs. (39), (40) and (43) in addition to the energy E_{nl} for commutative space which presented in the eq. (13):

$$E_{\text{nc-u}mgesci}(n,j,l,s,m,\alpha)=-abe^{-2b}+2\mu\left(\frac{a+e^{-2b}}{1+2n+\sqrt{4l(l+1)+1}}\right)^2-$$
$$-\frac{a}{2}(\Theta k_++Bm\chi)\left[\frac{(n-2\alpha+1)!}{(2n-2\alpha+2)}\right]\frac{\Gamma(n+3)\Gamma(n+2\alpha+2)}{(n!)^5\Gamma(2\alpha+2)} \quad (44)$$
$$\left(1+(1+b)\exp(-2b)\right){}_3F_2(-n,2\alpha-1,0;-n+2\alpha-1,2\alpha+2;1)$$

$$E_{\text{nc -d}mgesci}(n,j,l,s,m,\alpha)=-abe^{-2b}+2\mu\left(\frac{a+e^{-2b}}{1+2n+\sqrt{4l(l+1)+1}}\right)^2-$$
$$-\frac{a}{2}(\Theta k_-+Bm\chi)\left[\frac{(n-2\alpha+1)!}{(2n-2\alpha+2)}\right]\frac{\Gamma(n+3)\Gamma(n+2\alpha+2)}{(n!)^5\Gamma(2\alpha+2)} \quad (45)$$
$$\left(1+(1+b)\exp(-2b)\right){}_3F_2(-n,2\alpha-1,0;-n+2\alpha-1,2\alpha+2;1)$$

It's clearly, that the obtained eigenvalues of energies are real's and then the noncommutative diagonal Hamiltonian $\hat{H}_{nc-umgesci}$ is Hermitian $\left(\hat{H}_{nc-umgesci}=\left(\hat{H}_{nc-umgesci}\right)^+\right)$, furthermore it's possible to writing the three elements: $\left(H_{nc-umgesci}\right)_{11}$, $\left(\hat{H}_{nc-umgesci}\right)_{22}$ and $\left(\hat{H}_{nc-umgesci}\right)_{33}$ as follows:

$$\left(H_{nc-umgesci}\right)_{11}=-\frac{\Delta}{2\mu}-\frac{a}{r}\left(1+(1+b)\exp(-2b)\right)+$$
$$+k_+\left(\frac{\bar{\theta}}{2\mu}-\frac{\Theta a}{2r^3}\left(1+(1+b)\exp(-2b)\right)\right)+ \quad (46)$$
$$+\left(\frac{\bar{\sigma}}{2\mu}-\chi\frac{a}{2r^3}\left(1+(1+b)\exp(-2b)\right)\right)\aleph_{\text{mod}-z}$$

$$\left(H_{nc-umgesci}\right)_{22}=-\frac{\Delta}{2\mu}-\frac{a}{r}\left(1+(1+b)\exp(-2b)\right)+$$
$$+k_-\left(\frac{\bar{\theta}}{2\mu}-\frac{\Theta a}{2r^3}\left(1+(1+b)\exp(-2b)\right)\right)+ \quad (47)$$
$$+\left(\frac{\bar{\sigma}}{2\mu}-\chi\frac{a}{2r^3}\left(1+(1+b)\exp(-2b)\right)\right)\aleph_{\text{mod}-z}$$

$$\left(H_{nc-umgesci}\right)_{33}=-\frac{\Delta}{2\mu}-\frac{a}{r}\left(1+(1+b)\exp(-2b)\right) \quad (48)$$

It is pertinent to note that when the diatomics have $\overleftrightarrow{S}\neq\frac{\vec{1}}{2}$, the total operator can be obtains from the interval $|l-s|\leq j\leq|l+s|$, which allow us to obtaining the eigenvalues of the operator $\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right)$ as $k(j,l,s)\equiv j(j+1)+l(l+1)-s(s+1)$ and then the nonrelativistic energy spectrum $E_{\text{nc-}mgesci}(n,j,l,s,m,\alpha)$ reads:

$$E_{nl}\Rightarrow E_{\text{nc-}mgesci}(n,j,l,s,m,\alpha)=-abe^{-2b}+2\mu\left(\frac{a+e^{-2b}}{1+2n+\sqrt{4l(l+1)+1}}\right)^2+ \quad (49)$$
$$+\left[\frac{(n-2\alpha+1)!}{(2n-2\alpha+2)}\right]\frac{\Gamma(n+3)\Gamma(n+2\alpha+2)}{(n!)^5\Gamma(2\alpha+2)}\left(Bm\chi-\frac{a}{2}\Theta k(j,l,s)\right)$$

On the other hand, it is evident to consider the quantum number m takes $(2l+1)$ values and we have also two values for $j=l\pm 1/2$, thus every state in usually three dimensional space of energy for modified (MGESC) potential will be $2(2l+1)$ sub-states. To obtain the total complete degeneracy of energy level of the modified (MGESC) potential in noncommutative three-dimension spaces-phases, we need to sum for all allowed values of l. Total degeneracy is thus,

$$\sum_{i=0}^{n-1}(2l+1)\equiv n^2 \quad (50)$$

Note that the obtained new energy eigenvalues ($E_{\text{nc-u}mgesci}(n,j,l,s,m,\alpha)$ and $E_{\text{nc-d}mgesci}(n,j,l,s,m,\alpha)$) and $E_{\text{nc-}mgesci}(n,l,\alpha)$ now depend to new discrete atomic quantum numbers (n,j,l,s) and m in addition to the parameter α of the potential. Paying attention to the behavior of the spectrums (44), (45) and (49), it is possible to recover the results of commutative space (13) when we consider $(\Theta,\chi)\rightarrow(0,0)$.

6. CONCLUSION

In the conclusions of present work, we reviewed the exact solutions of the Schrödinger equation with a (MGESC) potential and the formalism of Bopp's shift method. Then, we have applied the Bopp's shift method to solve the modified Schrodinger equation for modified a (MGESC) potential in (NC: 3D-RSP), we obtained in present research paper:

i) The exact energy spectrum ($E_{\text{nc-umgesci}}(n, j, l, s, m, \alpha)$ - $E_{\text{nc -dmgesci}}(n, j, l, s, m, \alpha)$) and $E_{\text{nc-mgesci}}(n, j, l, s, m, \alpha)$ for n^{th} excited levels for $\overleftrightarrow{S} = \frac{\vec{1}}{2}$ and $\overleftrightarrow{S} \neq \frac{\vec{1}}{2}$, respectively.

ii) The modified Hamiltonian operator $\hat{H}_{nc-\text{umgesci}}$ for the modified (MGESC) potential,

iii) We shown that the old states are changed radically and replaced by degenerated new states, describing two new original spectrums, the first new one, produced by spin-orbital interaction while the second new spectrum produced by an external magnetic field,

iv) We have shown that, every state in usually three dimensional space of energy for modified (MGESC) potential will be $2(2l+1)$ sub-states in (NC: 3D-RSP).

v) We have shown that, the group symmetry (NC: 3D-RSP) corresponding modified (MGESC) potential reduce to the symmetry sub-group (NC: 3D-RS).

It has been shown that: the (MSE) presents useful rich spectrums for improved understanding of diatomics influenced by the modified a (MGESC) potential and we have seen also that the spin-orbital and modified Zeeman effect were appears du the presence of the two infinitesimal parameters (Θ, χ) which are induced by position-position noncommutativity property of space.

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