Method for pulse transformations using dispersion varying optical fibre tapers

N. G. R. Broderick
Optoelectronics Research Centre, University of Southampton, Southampton SO17 1BJ, UK
ngb@orc.soton.ac.uk

Abstract: I introduce the problem of transforming one optical pulse into another through nonlinear propagation in a length of dispersion varying optical fibre. Then using a genetic algorithm to design the dispersion profiles, I show that the problem can be solved leading to high quality pulse transforms that are significantly better than what has been published previously. Finally I suggestion further work and other applications for this method.

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1. Introduction

The problem of temporal pulse shape transformation is a common one in Optics with any number of different solutions. However most such solutions rely on linear filters such as fibre Bragg gratings which restricts the range of transforms that can be applied since a linear transformation cannot create any new frequencies. In particular a linear approach cannot result in a shorter output pulse and so to create shorter pulses an element of nonlinear propagation must be used. One approach towards pulse compression that has been well studied is soliton compression in a dispersion decreasing optical fibre[1, 2]. A particularly attractive feature of soliton compression is that in the ideal lossless case all of the pulse’s energy gets transformed resulting in a pedestal free short optical pulse. More recently people have also examined the problem of generating parabolic pulses in dispersion decreasing fibre[3] and have again found that high quality pulse transformations can be achieved. In this paper I generalised this work to consider how to design bespoke optical fibre tapers that transform one pulse into another and show using several examples the success and limitations of this method.

Optical pulse propagation in an optical fibre can be well described by the Nonlinear Schrödinger equation (NLSE) given by [4]:

$$i\frac{\partial\psi}{\partial z}-\frac{\beta_2}{2}\frac{\partial^2\psi}{\partial t^2}+i\frac{\alpha}{2}\psi+\gamma|\psi|^2\psi=0 \tag{1}$$

where $\psi(z,t)$ is the slowly varying envelope of the electric field, β_2 represents the dispersion of the fibre, α is the loss and γ represents the usual Kerr nonlinearity. In a standard optical fibre the coefficients of the NLSE are all constant however it is easy to vary the the size of the dispersion through fibre tapering where the diameter of the fibre changes by a relatively small amount (typically less than 10%). Using tapered fibres can dramatically alter the optical properties of pulses propagating through them in the nonlinear regime as is well known from numerous experimental and theoretical studies.

There are in the literature several methods for making shallow tapers for controlling the dispersion of optical fibres depending on the required length of the taper. The controlled fabrication of kilometre lengths of dispersion varying fibres is possible by varying the exit speed of the fibre on the draw tower itself[5, 6] while more recently Vukovic et al. [7] described a novel taper rig capable of producing metre length fibre tapers with slowly varying diameters. Similarly, Sumetsky *et al.* [8] published a different design for making metre length fibre tapers. The experiments and modelling by Vukovic et al. [9, 10] suggest that it is possible to vary the dispersion of a highly nonlinear photonic crystal fibre between $\pm 150\,\mathrm{ps/nm/km}$ and one of the goals of this work is to consider what pulse transformations are possible within such limitations.

2. Theoretical Model

The aim of this work is to demonstrate that a carefully designed fibre taper can transform an initial pulse shape $\phi_1(t)$ into a desired pulse shape $\phi_2(t)$. In order to model propagation along an optical fibre taper the standard NLSE [Eq. (1)] needs to be modified to include a position dependent dispersion function $\beta_2(z)$. It is also worth recalling that the NLSE has two well known scaling transformations, the first for an arbitrary time T_0 is given by

$$t \to \frac{t}{T_0}, \quad \text{and} \quad \beta_2(z) \to \frac{\beta_2(z)}{T_0^2}. \tag{2}$$

While the amplitude and nonlinearity can be scaled by an arbitrary power factor P_0

$$\gamma \to \gamma P_0^2, \quad \text{and} \quad \psi \to \sqrt{P_0}\psi. \tag{3}$$

These two scalings means that I can set the initial pulse width to unity and set gamma to unity without loss of generality. Importantly the physical dimensions of dispersion are $[\text{time}^2]/[\text{distance}]$ and I discuss in the conclusion what the best possible scalings are for the various pulse transformations.

For the normalised NLSE the initial pulse shape is given by $\psi(0,t) = \phi_1(t)$ while the output from the fibre taper is $\psi(L,t)$ where L is the length of the taper. The difference between the taper output and the desired pulse shape $\phi_2(t)$ is given by the misfit parameter[11]:

$$M = \frac{\int_{-\infty}^{\infty} \left(|\psi(L,t)|^2 - |\phi_2(t)|^2\right)^2 dt}{\int_{-\infty}^{\infty} |\psi(L,t)|^4 dt}. \tag{4}$$

This is not the usual L_2 norm for functions but rather it is usual norm weighted by the square of the pulse energy at the output. The primary reason for choosing this misfit function is that it was used previously by Finot *et al.* to characterise the evolution of a pulse into a parabolic shape and so I can compare my results with the earlier published results. The main effect of the weighting is to reduce the misfit for larger pulses since they contain more energy. Note that the smaller the misfit the better the global match between the output pulse and the desired pulse shape.

For arbitrary pulse shapes $\phi_{1,2}(t)$ it is in general unknown whether there is a dispersion profile that will exactly transform ϕ_1 into ϕ_2 (clearly some transformations are impossible such as when ϕ_2 contains more energy than ϕ_1). Furthermore an analytic approach is impossible and so numerical techniques must be used. In the subsequent section I discuss the numerical method used and then proceed to discuss the results for a variety of different pulse shapes.

2.1. *Genetic Algorithms for Pulse Shape Optimisation*

The problem described above is particularly difficult since there are an infinite number of degrees of freedom as $\beta_2(z)$ is an arbitrary function. Hence I need to impose some constraints. Firstly I assume that the length of the taper is fixed and can be set to unity. Given the possible rescalings of the NLSE this restriction can be seen as fixing either the actual physical length of the fibre, or the pulse width or the value of the nonlinearity (or some combination of all three). Next I approximate the dispersion using a Taylor series approximation

$$\beta_2(z) = \sum_{n=0}^{n=N} a_n \frac{z^n}{n!} \tag{5}$$

although any other set of basis functions such as Chebyshev polynomials or trigonometric functions can be used. In this way the problem is reduced from having an infinite number of degrees of freedom to a finite number N. As long as N is sufficiently large the precise value of N is relatively unimportant as I will show later. In practice I have found that between 10 and 30 terms are sufficient (or in fact overkill).

An advantage of writing the dispersion as a sum of polynomials is that the resulting profile is relatively smooth and so can be fabricated using standard tapering techniques. An alternative approach would be to discretise the function β_2 into N discrete values along the length. This disadvantage of this is that at the interface between any two values the dispersion will change discontinuously which is hard to implement except through splicing bits of fibre together which is time-consuming and impractical for small lengths of fibre. This approach would however model the use of comb-like dispersion profiles used by some researchers[12] for parabolic pulse generation.

Using the decomposition of β_2 given in Eq. (5) I implemented a genetic algorithm to find the optimum coefficients a_i. In the algorithm each individual had a genome consisting of a list of the coefficients a_i and the initial population of size M was created using a uniform distribution of random numbers. To get each subsequent generation a mixture of asexual and sexual reproduction was used. Firstly the top N_1 individuals as ranked by the misfit function [Eq. (4)] were each cloned C times and a Gaussian random variable was added. Next two individuals where chosen at random and a new individual was created with m genes from the first parent and $N - m$ from the second with a Gaussian distributed random number being added to each coefficient. All of the new individuals were then ranked along with the old generation via their misfit function and only the top M individuals were kept. The algorithm was implemented on the IRIDIS supercomputer at the University of Southampton using the OpenMPI message passing implementation. Typically for a population size of 10000 and a maximum number of generations of 20000 the program took about 5 hours to run using 360 processors and scaled as expected with the number of processors.

3. Results

In the cases studied below I usually restricted the input pulse ϕ_1 to a Gaussian pulse given by

$$\phi_1(t) = 5e^{-t^2}. \tag{6}$$

The choice of a fixed input pulse corresponds to a common situation in many labs, namely that the number of choices of mode-locked lasers is limited and it is desirable to transform the output of the fixed laser source to an arbitrary pulse shape. I have however included a couple of different input pulse shapes in the results to show that the success of this method does not depend critically on the input pulse shape. In the subsequent sections I discuss the results of transforming the input Gaussian pulse into a Parabolic, Sech or square shaped pulse.

3.1. Generating Parabolic Pulses

The first problem I consider is the generation of parabolic pulses using a Gaussian input. The desired output pulse $\phi_2(t)$ was a parabolic pulse with the form:

$$\phi_2(t) = \begin{cases} a(1-(t/b)^2) & |t| < b \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The reason for considering this problem is that parabolic pulse generation is well studied (see the recent review by Finot *et al.* [13] for more details) and hence there are a number of different solutions available in the literature. Another useful feature of this problem is that in fibres with a constant gain, parabolic pulses are asymptotic solutions to the NLSE[14] and all pulses will evolve towards a parabolic shape. As the NLSE with decreasing dispersion and no loss is equivalent to a fibre with gain the solution to the pulse transformation problem for an infinite fibre is thus known. Hence I can compare the numerical results for a fibre of finite length to that of the known dispersion profile for an infinite fibre.

In our simulations we used a normalised fibre length of unity, $\gamma = 1$ and $\alpha = 0.2$. The input was a Gaussian pulse given by $Aexp(-t^2)$ where A was the amplitude of the pulse. As I am only interested in determining how to create the most parabolic pulse the parameters a and b in Eq. (7) were chosen by doing a least squares fit that minimised the misfit function. Using the genetic algorithm for an input power of 25 we obtained the results as shown in Fig. 1(a). Here the output pulse is shown in black while the green line shows the best parabolic fit to the pulse. The misfit parameter was 7.1052×10^{-7} which is an improvement by several orders of magnitude over previous results [15, 16]. In Fig. 1(b) I show the optimised dispersion profile in green while the red dashed line shows the evolution of the misfit parameter along the length of the taper.

The optimised dispersion profile processes some interesting features. Firstly compared to the expected monotonically decreasing profile it actually increases in the second half of the fibre before decreasing again towards the end. Surprisingly the misfit parameter also increases slightly in the second half of the taper before decreasing dramatically towards the end. In order to test the genetic algorithm I ran it several times

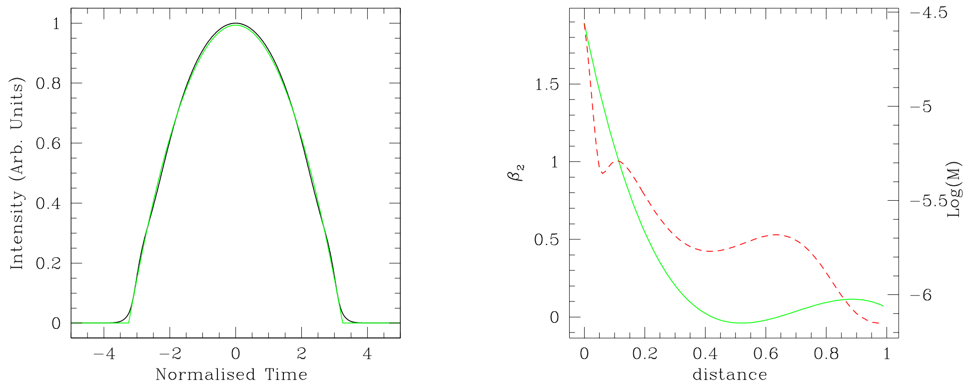


Fig. 1. (a) Intensity profile of the output pulse (black line) alone with the best parabolic fit (green line). (b) Dispersion profile (green line) and evolution of the misfit parameter (red dotted line) for the optimum fibre taper.

(with different random seeds) and with different numbers of polynomials with the results as shown in Fig. 2(a). Here three optimised dispersion profiles are shown and in all cases the resulting misfit parameter is $\approx 7.2 \times 10^{-7}$ and does not vary significantly. From Fig. 2(a) it can be seen that in the three cases shown the initial dispersion varies considerably more than the dispersion in the last half of the fibre. This suggests that in order to get the best parabolic pulse it is the dispersion profile in the later stages that is critical while the initial dispersion is not so crucial.

I also optimised $\beta_2(z)$ for two other input pulse shapes and these results are shown in Fig. 2(b). The first was for a sech shaped pulse with initial intensity of 25 and a width of 1, the optimised dispersion profile for this case is shown in black. The other case was for a Gaussian pulse with a peak intensity of 50 (rather than 25 in the previous examples) and the optimised profile is shown in green. In both cases the dispersion profile follows the same pattern as previously, i.e. there is a decreasing dispersion over the first half of the taper followed by a small hump. In these examples the misfit parameter was similar to those found earlier but was higher for the sech shaped pulse. This is not surprising as a sech shaped pulse is further from a parabolic pulse to begin with and hence it is perhaps harder to transform. Comparing the results for Gaussian pulses of different amplitudes while the global misfit parameter is very similar the local differences are more striking. For the low intensity pulse there is an excellent fit at the centre of the pulse while the error near the base is higher since the nonlinearity is not sufficient to generate enough bandwidth to copy the discontinuity of the intensity at the base of the parabolic pulse. In contrast for the high intensity pulse the error near the central part of the pulse is greater but local error near the base is lower.

Importantly in all the cases examined the resulting dispersion profile is slowly varying and does not change too much in absolute terms over the length of the taper. This

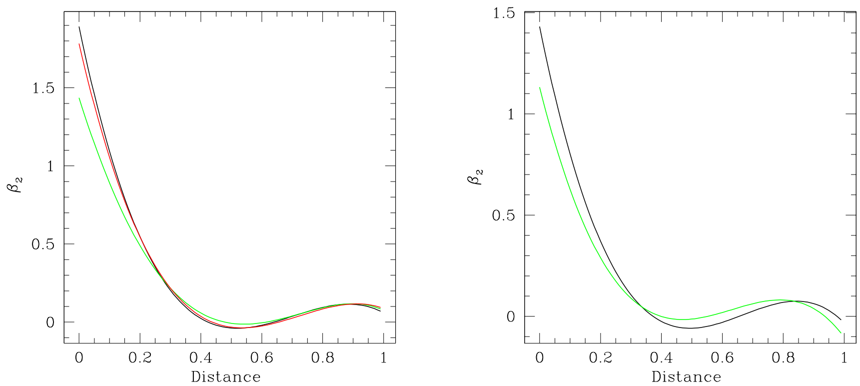


Fig. 2. (a) Differing dispersion profiles for different runs of the genetic algorithm. The green and red line are the results when there are 21 different polynomials but with different random seeds. The black line is the case for 31 polynomials. (b) The black line shows the optimised profile for converting a sech shaped pulse into a parabolic pulse while the green line shows the profile for a high intensity Gaussian pulse.

is important for a number of reason. Firstly it means that the dispersion profiles can be fabricated using existing taper rigs and so such dispersion profiles can be practically realised. Secondly from a numerical point of view it shows that the number of terms in the Taylor series expansion [Eq. (5)] can be reduced without altering the result. This would allow the program to run faster or alternatively tackle problems of greater complexity (which I will return to in the discussion).

3.2. Gaussian to Sech transformations

The next problem I consider is transforming a Gaussian shaped input pulse [Eq. (6)] to a Sech shaped output pulse $\phi_2(t)$ given by :

$$\phi_2(t) = a\text{sech}(t/b). \tag{8}$$

The reason behind looking at this transformation is that it is well known that in the lossless regime a Gaussian pulse will evolve into a soliton provided that the energy is sufficient. Hence again we can compare the optimised solution for a fibre of finite length with the constant dispersion profile expected for the lossless infinitely long case. The best results for this transformation are shown in Fig. 3.

In this case the input pulse was again a Gaussian pulse with unit width and a peak intensity of 25. After optimisation the best misfit parameter was 2.6×10^{-6} resulting in a extremely good match between desired and actual pulse shapes. It can be seen from Fig. 3(a) that the resulting pulse is well match down to the $-20\,$dB level. Fig. 3(b) shows the best dispersion profile (in green) and surprisingly the dispersion decreases monotonically from -1.8 to -8.8. This increase in the magnitude of the dispersion

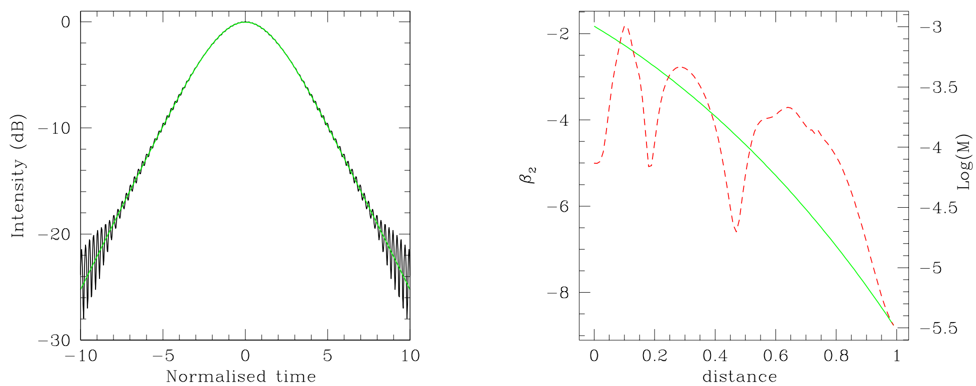


Fig. 3. (a) Intensity profile of the output pulse (black line) alone with the best sech shaped fit (green line). (b) Dispersion profile (green line) and evolution of the misfit parameter (red dotted line) for the optimum fibre taper.

during propagation is what drives the increase in the pulse width which nearly doubles during propagation. The evolution of the misfit parameter is also interesting since in undergoes three large oscillations before reaching the final value.

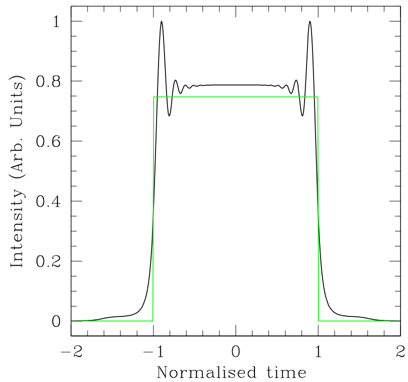
3.3. Generating Square Pulses

The last problem I consider is generating square pulses from Gaussian pulses. This problem is chosen as it illustrates some of the limitations of using tapers for pulse transformations. The desired output pulse $\phi_2(t)$ is given by:

$$\phi_2(t) = \begin{cases} a & |t| < b \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

As with the parabolic pulse a notable feature of a square pulse is that it has an extremely broad spectrum due to the discontinuities in the intensity. These discontinuities are however more significant since they occur at points of maximum amplitude rather than at points of zero intensity for the parabolic pulse. As before the input was a Gaussian pulse given by Eq. (6) and the results are shown in Fig. 4.

Compared to the previous results it is obvious that in this case the genetic algorithm has not resulted in a good match between the output pulse and the ideal square form. In particular there is noticeable ringing on the edges of the pulse and the wings of the pulse are broader than the square pulse. This however is not particularly surprising given the the desired output pulse shape is discontinuous while any output solution must be twice differentiable. In Fig. 4(b) I show the optimised dispersion profile and the evolution of the misfit parameter. The dispersion profile is interesting in that it starts off being anomalous before increasing to the normal dispersion regime and the decreases again to almost the initial value. As in the previous cases the misfit parameter



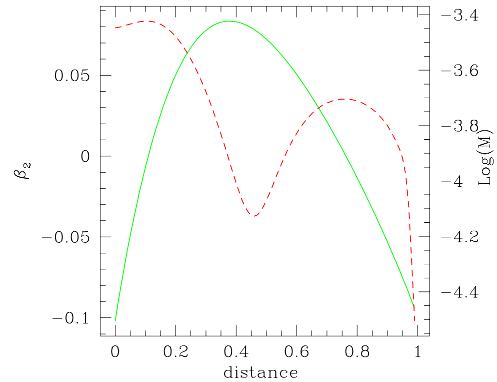


Fig. 4. (a) Intensity profile of the output pulse (black line) alone with the best square shaped fit (green line). (b) Dispersion profile (green line) and evolution of the misfit parameter (red dotted line) for the optimum fibre taper.

does not evolve monotonically but rises in the final portion of the taper before dropping very sharply towards the end.

In order to further examine the effects of the discontinuities on the performance of the pulse transformation I examined the evolution towards a super-Gaussian pulse given by $\phi_2(t) = a\exp(-(t/b)^{12})$. These results are shown in Fig. 5 and show a much better fit than the for the square pulse. Note that in the wings the resulting pulse does not decay as fast as the super-Gaussian and this is again due to the limited bandwidth that can be created during propagation. However it does suggest that the smoother the desired output pulse the smaller the optimum misfit for a given input pulse.

4. Discussion and Conclusions

In this work I have used a normalised version of the nonlinear Schrödinger equation and it is worth examining how the results scale for realistic parameters. For the parabolic pulse generation the dispersion decreases from ≈ 1.5 to zero. Assuming a one metre long fibre taper and a picosecond pulse this corresponds to a value of $\beta_2 = 1500\,\mathrm{ps}^2/\mathrm{km}$ which is significantly higher than what can be easily achieved using either conventional or even highly dispersive fibres (such as small core photonic crystal fibres). However there are two ways around this problem, firstly if we assume that instead of a metre long fibre taper we have a kilometre long fibre taper the starting dispersion becomes $\beta_2 = 1.5\,\mathrm{ps}^2/\mathrm{km}$ which is similar to standard commercial dispersion shifted fibres and can easily be fabricated. Here the loss used becomes $0.2\,\mathrm{km}^{-1}$ which is a realistic value for commercial step index fibres. In the past similar kilometre long tapers have been fabricated by changing the speed while drawing the fibres and so this is a practical method for generating parabolic pulses. Alternatively if the initial pulse width is 100 fs rather than 1 ps the dispersion scaling of the NLSE means that the required starting

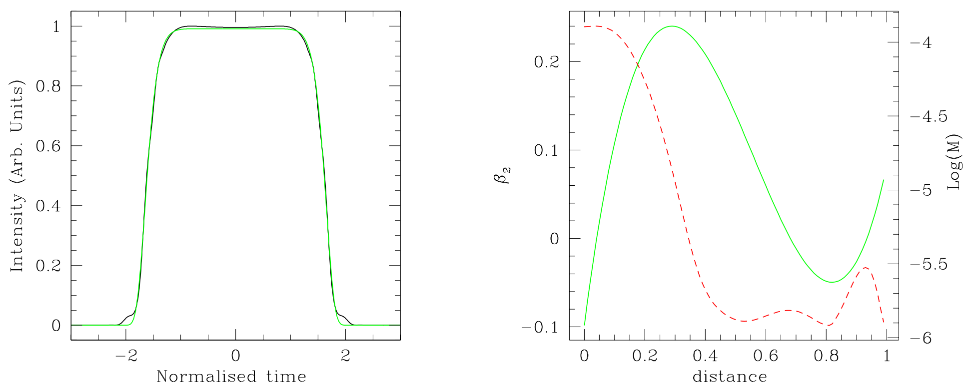


Fig. 5. (a) Intensity profile of the output pulse (black line) alone with the best super-Gaussian shaped fit (green line). (b) Dispersion profile (green line) and evolution of the misfit parameter (red dotted line) for the optimum fibre taper.

dispersion for a one metre long fibre taper becomes $\beta_2 = 15\,\mathrm{ps}^2/\mathrm{km}$ which can be easily achieved using either photonic crystal fibres or standard step index fibres. These values are typical of the dispersion profiles for the other pulse shapes and suggest that this method can be used either with picosecond pulses and kilometre long fibre tapers or with 100 femtosecond pulses and metre length fibre tapers.

In this work I have ignored any higher order corrections to the NLSE such as third order dispersion or the Raman effect. Such effects are unlikely to be important for picosecond pulses however for femtosecond pulses such effects will be more important and will be considered in future work. In addition trying to optimised tapers for effects such as super-continuum generation will require inclusion of higher order dispersion as well as the Raman effect and self-steepening. Future work will also look at modelling the geometric fibre parameters rather than the dispersion profile using the genetic algorithm. For example a photonic crystal fibre can be described by two parameters, the hole size d and the hole to hole spacing Λ, while a step index fibre is characterised by the refractive index difference Δn and the core diameter d. In a more realistic genetic algorithm both parameters would be separately optimised and the algorithm would calculate the dispersion (to all orders) which would be used in the generalised NLSE. While this doubles the number of dimensions of the problem using 15 terms in the Taylor series for each parameter rather than 30 as done in this work would keep the size of the simulation the same without reducing the accuracy of the results. Finally in this work I have only looked at the intensity of the output pulse rather than it's phase profile and clearly in the future I could look at the optimising the phase profile as well as the intensity profile.

In conclusion I shown various examples of how one pulse shape can be transformed into another during propagation down an optical fibre taper. In all cases considered the misfit parameter that described the success of the transform was extremely low showing that this method is extremely flexible. The major area where this method fails is where the desired pulse shape changes discontinuously such as in the case of the square pulse or the wings of the parabolic pulse. In all cases examined the genetic algorithm produced slowly varying tapers that can be fabricated using existing technology. Compared to linear filtering techniques for pulse shaping nonlinear pulse transformations can increase the pulse's frequency bandwidth allowing pulse compression simultaneously with pulse transformations which is an attractive feature of this method that could be exploited further in the future.

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