On functions with strongly \(\delta\)-semiclosed graphs

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Abstract. In 1997, Park et al. [5] offered a new notion called \(\delta\)-semiopen sets which are stronger than semi-open sets but weaker than \(\delta\)-open sets. It is the aim of this paper to introduce and study some properties of functions with strongly \(\delta\)-semiclosed graphs by utilizing \(\delta\)-semiopen sets and the \(\delta\)-semi-closure operator.

Keywords: \(\delta\)-semiopen set, \(\delta\)-semi\(T_1\) space, \(\delta\)-semi\(T_2\) space, strongly \(\delta\)-semiclosed graph.

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1 Introduction and preliminaries

In what follows \((X, \tau)\) and \((Y, \sigma)\) (or \(X\) and \(Y\)) denote topological spaces. Let \(A \subseteq X\). We denote the interior, the closure and the complement of a set \(A\) by \(Int(A)\), \(Cl(A)\) and \(X \setminus A\) or \(A^c\), respectively.

Levine [3] defined semiopen sets which are weaker than open sets in topological spaces. After Levine’s semiopen sets, mathematicians gave in several papers different and interesting new modifications of open sets as well as generalized open sets. In 1968, Veličko [6] introduced \(\delta\)-open sets, which are stronger than open sets, in order to investigate the characterization of \(H\)-closed spaces. In 1997, Park et al. [5] introduced the notion of \(\delta\)-semiopen sets which are stronger than semiopen sets but weaker than \(\delta\)-open sets and investigated the relationships between several types of these open sets.

A subset \(A\) of a topological space \(X\) is said to be \(\delta\)-semiopen [5] (resp.
semiopen [3]) set if there exists a \( \delta \)-open (resp. open) set \( U \) of \( X \) such that \( U \subset A \subset \text{Cl}(U) \). The complement of a \( \delta \)-semiopen (resp. semiopen) set is called a \( \delta \)-semiclosed (resp. semiclosed) set. A point \( x \in X \) is called the \( \delta \)-semicluster point of \( A \) if \( A \cap U \neq \emptyset \) for every \( \delta \)-semiopen set \( U \) of \( X \) containing \( x \). The set of all \( \delta \)-semicluster points of \( A \) is called the \( \delta \)-semiclosure of \( A \), denoted by \( \delta \text{Cl}_S(A) \). We denote the collection of all \( \delta \)-semiopen (resp. \( \delta \)-semiclosed) sets by \( \delta \text{SO}(X) \) (resp. \( \delta \text{SC}(X) \)). We set \( \delta \text{SO}(X,x) = \{ U : x \in U \in \delta \text{SO}(X) \} \) and \( \delta \text{SC}(X,x) = \{ U : x \in U \in \delta \text{SC}(X) \} \).

1 Lemma. (Park et al. [5]) The intersection (resp. union) of arbitrary collection of \( \delta \)-semiclosed (resp. \( \delta \)-semiopen) sets in \((X,\tau)\) is \( \delta \)-semiclosed (resp. \( \delta \)-semiopen).

2 Corollary. Let \( A \) be a subset of a topological space \((X,\tau)\), then \( \delta \text{Cl}_S(A) = \cap \{ F \in \delta \text{SC}(X,\tau) : A \subset F \} \).

3 Lemma. (Park et al. [5]) Let \( A, B \) and \( A_i \ (i \in I) \) be subsets of a space \((X,\tau)\), the following properties hold:

1. \( A \subset \delta \text{Cl}_S(A) \).
2. If \( A \subset B \), then \( \delta \text{Cl}_S(A) \subset \delta \text{Cl}_S(B) \).
3. \( \delta \text{Cl}_S(A) \) is \( \delta \)-semiclosed.
4. \( \delta \text{Cl}_S(\delta \text{Cl}_S(A)) = \delta \text{Cl}_S(A) \).
5. \( A \) is \( \delta \)-semiclosed if and only \( A = \delta \text{Cl}_S(A) \).

4 Corollary. (Caldas et al. [1]) Let \( A_i \ (i \in I) \) be subsets of a space \((X,\tau)\), the following properties hold:

1. \( \delta \text{Cl}_S(\cap \{ A_i : i \in I \}) \subset \cap \{ \delta \text{Cl}_S(A_i) : i \in I \} \).
2. \( \delta \text{Cl}_S(\cup \{ A_i : i \in I \}) \supset \cup \{ \delta \text{Cl}_S(A_i) : i \in I \} \).

5 Definition. A topological space \((X,\tau)\) is called:

1. \( \delta \)-semi \( T_1 \) [1] if for any distinct pair of points \( x \) and \( y \) in \( X \), there is a \( \delta \)-semiopen \( U \) in \( X \) containing \( x \) but not \( y \) and \( \delta \)-semiopen \( V \) in \( X \) containing \( y \) but not \( x \);  
2. \( \delta \)-semi \( T_2 \) [1] if for any distinct pair of points \( x \) and \( y \) in \( X \), there exist \( U \in \delta \text{SO}(X,x) \) and \( V \in \delta \text{SO}(X,y) \) such that \( U \cap V = \emptyset \).

Recall that a function \( f : X \rightarrow Y \) is said to be:
(1) \(\delta\)-semi-continuous [1] if for each \(x \in X\) and each \(V \in \delta SO(Y, f(x))\), there exists \(U \in \delta SO(X, x)\) such that \(f(U) \subset V\), equivalently if the inverse image of each \(\delta\)-semi-open set is \(\delta\)-semiopen,

(2) quasi \(\delta\)-semi-continuous [1] if for each \(x \in X\) and each \(V \in \delta SO(Y, f(x))\), there exists \(U \in \delta SO(X, x)\) such that \(f(U) \subset \delta Cl_{S}(V)\).

2 Strongly \(\delta\)-semi-closed graphs

Let \(f : (X, \tau) \to (Y, \sigma)\) be any function, then the subset \(G(f) = \{(x, f(x)) : x \in X\}\) of the product space \((X \times Y, \tau \times \sigma)\) is called the graph of \(f\) [2].

6 Definition. A function \(f : X \to Y\) has a strongly \(\delta\)-semi-closed graph if for each \((x, y) \in (X \times Y) \setminus G(f)\), there exist \(U \in \delta SO(X, x)\) and \(V \in \delta SO(Y, y)\) such that \([U \times \delta Cl_{S}(V)] \cap G(f) = \emptyset\).

7 Lemma. A function \(f : X \to Y\), has a strongly \(\delta\)-semi-closed graph if for each \((x, y) \in (X \times Y) \setminus G(f)\), there exist \(U \in \delta SO(X, x)\) and \(V \in \delta SO(Y, y)\) such that \(f(U) \cap \delta Cl_{S}(V) = \emptyset\).

8 Theorem. If \(f : X \to Y\) is a function with strongly \(\delta\)-semi-closed graph, then for each \(x \in X\), \(f(x) = \cap \{\delta Cl_{S}(f(U)) : U \in \delta SO(X, x)\}\).

Proof. Suppose the theorem is false. Then there exists a point \(y \in Y\) such that \(y \neq f(x)\) and \(y \in \cap \{\delta Cl_{S}(f(U)) : U \in \delta SO(X, x)\}\). This implies that \(y \in \delta Cl_{S}(f(U))\) for every \(U \in \delta SO(X, x)\). So \(V \cap f(U) \neq \emptyset\) for every \(V \in \delta SO(Y, y)\). This, in its turn, indicates that \(\delta Cl_{S}(V) \cap f(U) \supset V \cap f(U) \neq \emptyset\) which contradicts the hypothesis that \(f\) is a function with a strongly \(\delta\)-semi-closed graph. Hence the theorem holds.

9 Theorem. If \(f : X \to Y\) is \(\delta\)-semi-continuous and \(Y\) is \(\delta\)-semi-\(T_2\), then \(G(f)\) is strongly \(\delta\)-semi-closed.

Proof. Let \((x, y) \in (X \times Y) \setminus G(f)\). The \(\delta\)-semi \(T_2\)-ness of \(Y\) gives the existence of a set \(V \in \delta SO(Y, y)\) such that \(f(x) \not\in \delta Cl_{S}(V)\).

Now \(Y \setminus \delta Cl_{S}(V) \in \delta SO(Y, f(x))\). Therefore, by the \(\delta\)-semi-continuity of \(f\) there exists \(U \in \delta SO(X, x)\) such that \(f(U) \subset Y \setminus \delta Cl_{S}(V)\). Consequently, \(f(U) \cap \delta Cl_{S}(V) = \emptyset\) and therefore \(G(f)\) is strongly \(\delta\)-semi-closed.

10 Theorem. If \(f : X \to Y\) is surjective and has a strongly \(\delta\)-semi-closed graph \(G(f)\), then \(Y\) is both \(\delta\)-semi-\(T_2\) and \(\delta\)-semi-\(T_1\).

Proof. Let \(y_1, y_2\) \((y_1 \neq y_2)\) \(\in Y\). The surjectivity of \(f\) gives a \(x_1 \in X\) such that \(f(x_1) = y_1\). Now \((x_1, y_2) \in (X \times Y) \setminus G(f)\). The strong \(\delta\)-semi-closedness of \(G(f)\) provides \(U \in \delta SO(X, x_1)\) and \(V \in \delta SO(Y, y_2)\) such that \(f(U) \cap \delta Cl_{S}(V) = \emptyset\), whence one infers that \(y_1 \not\in \delta Cl_{S}(V)\). This means that there exists \(W \in\).
\[ \delta SO(Y, y_1) \text{ such that } W \cap V = \emptyset. \text{ So, } Y \text{ is } \delta\text{-semi } T_2 \text{ and } \delta\text{-semi } T_2\text{-ness always guarantees } \delta\text{-semi } T_1\text{-ness. Hence } Y \text{ is } \delta\text{-semi } T_1. \]

11 Theorem. A space \( X \) is \( \delta\text{-semi } T_2 \) if and only if the identity function \( \text{id} : X \to X \) has a strongly \( \delta\text{-semiclosed graph } G(\text{id}). \)

Proof. Necessity. Let \( Y \) be \( \delta\text{-semi } T_2 \). Since the identity function \( \text{id} : X \to X \) is \( \delta\text{-semicontinuous}, \) it follows from Theorem 9 that \( G(\text{id}) \) is strongly \( \delta\text{-semiclosed}. \)

Sufficiency. Let \( G(\text{id}) \) be a strongly \( \delta\text{-semiclosed graph.} \) Then the surjectivity of \( \text{id} \) and strongly \( \delta\text{-semiclosedness of } G(\text{id}) \) together imply, by Theorem 10, that \( X \) is \( \delta\text{-semi } T_2. \)

12 Theorem. If \( f : X \to Y \) is an injection and \( G(f) \) is strongly \( \delta\text{-semiclosed}, \) then \( X \) is \( \delta\text{-semi } T_1. \)

Proof. Since \( f \) is injective, for any pair of distinct points \( x_1, x_2 \in X, \) \( f(x_1) \neq f(x_2). \) Then \( (x_1, f(x_2)) \in (X \times Y) \setminus G(f). \) Since \( G(f) \) is strongly \( \delta\text{-semiclosed,} \) there exist \( U \in \delta SO(X, x_1) \) and \( V \in \delta SO(Y, f(x_2)) \) such that \( f(U) \cap \delta Cl_5(V) = \emptyset. \) Therefore \( x_2 \notin U. \) Pursuing the same reasoning as before we obtain a set \( W \in \delta SO(X, x_2) \) such that \( x_1 \notin W. \) Hence \( Y \) is \( \delta\text{-semi } T_1. \)

13 Theorem. If \( f : X \to Y \) is a bijective function with a strongly \( \delta\text{-semiclosed graph,} \) then both \( X \) and \( Y \) are \( \delta\text{-semi } T_1. \)

Proof. The proof is an immediate consequence of Theorems 10 and 12.

14 Theorem. If a quasi \( \delta\text{-semicontinuous function } f : X \to Y \) is an injection with a strongly \( \delta\text{-semiclosed graph } G(f), \) then \( X \) is \( \delta\text{-semi } T_2. \)

Proof. Since \( f \) is injective, for any pair of distinct points \( x_1, x_2 \in X, \) \( f(x_1) \neq f(x_2). \) Therefore \( (x_1, f(x_2)) \in (X \times Y) \setminus G(f). \) Since \( G(f) \) is strongly \( \delta\text{-semiclosed,} \) there exist \( U \in \delta SO(X, x_1) \) and \( V \in \delta SO(Y, f(x_2)) \) such that \( f(U) \cap \delta Cl_5(V) = \emptyset, \) hence one obtains \( U \cap f^{-1}(\delta Cl_5(V)) = \emptyset. \) Consequently, \( f^{-1}(\delta Cl_5(V)) \subset X \setminus U. \) Since \( f \) is quasi \( \delta\text{-semicontinuous,} \) it is so at \( x_2. \) Then there exists \( W \in \delta SO(X, x_2) \) such that \( f(W) \subset \delta Cl_5(V). \) From this and the foregoing follow that \( W \subset f^{-1}(\delta Cl_5(V)) \subset X \setminus U, \) hence one infers that \( W \cap U = \emptyset. \) Thus for the pair of distinct points \( x_1, x_2 \in X, \) there exist \( U \in \delta SO(X, x_1) \) and \( W \in \delta SO(X, x_2) \) such that \( W \cap U = \emptyset. \) This guarantees the \( \delta\text{-semi } T_2\text{-ness of } X. \)

15 Corollary. If a \( \delta\text{-semicontinuous function } f : X \to Y \) is an injection with a strongly \( \delta\text{-semiclosed graph,} \) then \( X \) is \( \delta\text{-semi } T_2. \)

Proof. The proof follows from Theorem 14 and the fact that every \( \delta\text{-semicontinuous is quasi } \delta\text{-semicontinuous.} \)
16 Theorem. If \( f : X \to Y \) is quasi \( \delta \)-semicontinuous bijective with strongly \( \delta \)-semiclosed graph, then both \( X \) and \( Y \) are \( \delta \)-semi \( T_2 \).

Proof. The proof follows from Theorem 14 and 10.

For the rest of this article we shall assume the Property \( \Delta: \delta SO(X) \) closed under finite intersections.

17 Definition. \( X \) is called strongly \( \Delta \)-closed (resp. A subset \( A \) of \( X \) is said to be strongly \( \Delta \)-closed relative to \( X \)), if every \( \delta \)-semiopen cover of \( X \) (resp. if every cover of \( A \) by \( \delta \)-semiopen sets) has a finite subfamily such that the union of their \( \delta \)-semiclosures covers \( X \) (resp. has a finite subfamily such that the union of their \( \delta \)-semiclosures covers \( X \)).

18 Lemma. Every \( \delta \)-semiclopen subset of a strongly \( \Delta \)-closed space \( X \) is strongly \( \Delta \)-closed relative to \( X \).

Proof. Let \( B \) be any \( \delta \)-semiclopen subset of a strongly \( \Delta \)-closed space \( X \). Let \( \{O_\lambda : \lambda \in \Omega \} \) be any cover of \( B \) by \( \delta \)-semiopen sets in \( X \). Then the family \( F = \{O_\lambda : \lambda \in \Omega \} \cup \{X \setminus B\} \) is a cover of \( X \) by \( \delta \)-semiopen sets in \( X \). Because of strongly \( \Delta \)-closedness of \( X \) there exists a finite subfamily \( F^* = \{O_{\lambda_i} : 1 \leq i \leq n\} \cup \{X \setminus B\} \) of \( F \) such that the union of their \( \delta \)-semiclosures covers \( X \). So, because of \( \delta \)-semiclopenness of \( B \) we now infer that the family \( \{\delta Cl_S(O_{\lambda_i}) : 1 \leq i \leq n\} \) covers \( B \) and hence \( B \) is strongly \( \Delta \)-closed relative to \( X \).

19 Lemma. The \( \delta \)-semiclosure of every \( \delta \)-semiopen set is \( \delta \)-semiopen.

Proof. Every regular open set is \( \delta \)-open and every \( \delta \)-open set is \( \delta \)-semiopen. Thus, every regular closed set is \( \delta \)-semiclosed. Now let \( A \) be any \( \delta \)-semiopen set. There exists a \( \delta \)-open set \( U \) such that \( U \subset A \subset Cl(U) \). Hence, we have \( U \subset \delta Cl_S(U) \subset \delta Cl_S(A) \subset Cl_S(Cl(U)) = Cl(U) \) since \( Cl(U) \) is regular closed. Therefore, \( \delta Cl_S(A) \) is \( \delta \)-semiopen.

20 Theorem. Let \( f : X \to Y \) be a function. If \( Y \) is a strongly \( \Delta \)-closed, \( \delta \)-semi-\( T_2 \) space and \( G(f) \) is strongly \( \delta \)-semiclosed, then \( f \) is quasi-\( \delta \)-semicontinuous.

Proof. Let \( x \in X \) and \( V \in \delta SO(Y,f(x)) \). Take any \( y \in Y \setminus \delta Cl_S(V) \). Then \( (x, y) \in (X \times Y) \setminus G(f) \). Now the strong \( \delta \)-semiclosedness of \( G(f) \) induces the existence of \( U_y(x) \in \delta SO(X,x) \) and \( V_y \in \delta SO(Y,y) \) such that \( f(U_y(x)) \cap \delta Cl_S(V_y) = \emptyset \). The \( \delta \)-semi-\( T_2 \)-ness of \( Y \) implies the existence of \( V_y \in \delta SO(Y,y) \) such that \( f(x) \notin \delta Cl_S(V_y) \). Now by Lemma 19 induces the \( \delta \)-semicloopenness of \( \delta Cl_S(V) \) and hence \( Y \setminus \delta Cl_S(V) \) is also \( \delta \)-semiclopen. Now \( \{V_y : y \in Y \setminus \delta Cl_S(V)\} \) is a cover of \( Y \setminus \delta Cl_S(V) \) by \( \delta \)-semiopen sets in \( Y \). By Lemma 18, there exists a finite subfamily \( \{V_{y_i} : 1 \leq i \leq n\} \) such that \( Y \setminus \delta Cl_S(V) \subset \bigcup_{i=1}^{n} \delta Cl_S(V_{y_i}) \). Let \( W = \)
Let $U_y(x)$ be the $\delta$-semiopen sets in $X$ satisfying (*). Since $X$ enjoys the Property $\Delta$, $W \in \delta SO(X, x)$. Now $f(W) \cap (Y \setminus \delta Cl_S(V)) \subset f[\bigcap_{i=1}^{n} U_y(x)] \cap (\bigcup_{i=1}^{n} \delta Cl_S(V_i)) = \emptyset$, by (*). Therefore, $f(W) \subset \delta Cl_S(V)$ and this indicates that $f$ is quasi-$\delta$-semicontinuous.

\textbf{21 Corollary.} If $Y$ is a strongly $\Delta$-closed then a surjection $f : X \to Y$ with a strongly $\delta$-semiclosed graph is quasi-$\delta$-semicontinuous.

\textbf{Proof.} The proof follows from Theorem 10 and 19.

Noiri [4] showed that if $G(f)$ is strongly closed then $f$ has the following property:

(P) For every set $B$ quasi H-closed relative to $Y$, $f^{-1}(B)$ is a closed set of $X$.

Analogously, we have

\textbf{22 Theorem.} A $f : X \to Y$ has a strongly $\delta$-semiclosed graph $G(f)$, then $f$ enjoys the following property:

(P$^*$) For every set $F$ which is strongly $\Delta$-closed relative to $Y$, $f^{-1}(F)$ is $\delta$-semiclosed in $X$.

\textbf{Proof.} If possible let $f^{-1}(F)$ be not $\delta$-semiclosed in $X$. Then there exists $x \in \delta Cl_S(f^{-1}(F)) \setminus f^{-1}(F)$. Let $y \in F$. Then $(x, y) \in (X \times Y) \setminus G(f)$. Strongly $\delta$-semiclosedness of $G(f)$ gives the existence of $U_y(x) \subset \delta SO(X, x)$ and $V_y \subset \delta SO(Y, y)$ such that $f(U_y(x)) \cap \delta Cl_S(V_y) = \emptyset$.

Clearly $\{V_y : y \in F\}$ is a cover of $F$ by $\delta$-semiopen sets in $Y$. The strong $\Delta$-closedness of $F$ relative to $Y$ guarantees the existence of $\delta$-semiopen sets $V_{y_1}, V_{y_2}, \ldots, V_{y_n}$ in $Y$ such that $F \subset \bigcup_{i=1}^{n} \delta Cl_S(V_{y_i})$.

Let $U = \bigcap_{i=1}^{n} U_{y_i}(x)$, where $U_{y_i}(x)$ are the $\delta$-semiopen sets in $X$ satisfying (*). Since $X$ enjoys the Property $\Delta$, $U \in \delta SO(X, x)$.

Now $f(U) \cap F \subset f[\bigcap_{i=1}^{n} U_{y_i}(x)] \cap (\bigcup_{i=1}^{n} \delta Cl_S(V_{y_i})) = f[U_y(x)] \cap \delta Cl_S(V_{y_i}) = \emptyset$.

But $x \in \delta Cl_S(f^{-1}(F))$. Therefore $U \cap f^{-1}(F) \neq \emptyset$ which contradicts to the above deduction. Hence the result is true.

\section*{3 Additional properties}

\textbf{23 Theorem.} A space $X$ is $\delta$-semi $T_2$ if and only if for every pair of points $x, y \in X$, $x \neq y$ there exist $U \in \delta SO(X, x)$, $V \in \delta SO(X, y)$ such that $\delta Cl_S(U) \cap \delta Cl_S(V) = \emptyset$.
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24 Theorem. If $Y$ is $\delta$-semi $T_2$ and $f : X \to Y$ is a quasi $\delta$-semicontinuous injection, then $X$ is $\delta$-semi $T_2$.

Proof. Since $f$ is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$. By Theorem 23, the $\delta$-semi $T_2$ property of $Y$ indicates that there exist $V_i \in \delta SO(Y, f(x_i))$, $i = 1, 2$ such that $\delta Cl_S(V_1) \cap \delta Cl_S(V_2) = \emptyset$. Hence $f^{-1}(\delta Cl_S(V_1)) \cap f^{-1}(\delta Cl_S(V_2)) = \emptyset$. Since $f$ is quasi $\delta$-semicontinuous, there exists $U_i \in \delta SO(X, x_i)$, $i = 1, 2$ such that $f(U_i) \subset \delta Cl_S(V_i), i = 1, 2$. It, then follows that $U_i \subset f^{-1}(\delta Cl_S(V_i), i = 1, 2$. Hence $U_1 \cap U_2 = f^{-1}(\delta Cl_S(V_1)) \cap f^{-1}(\delta Cl_S(V_2)) = \emptyset$. This guarantees the $\delta$-semi $T_2$-ness of $X$.

25 Definition. A function $f : X \to Y$ is called $\delta$-semipoint if $f(A) \in \delta SO(Y)$ for all $A \in \delta SO(X)$.

26 Theorem. If a bijection $f : X \to Y$ is $\delta$-semiopen and $X$ is $\delta$-semi $T_2$, then $G(f)$ is strongly $\delta$-semiclosed.

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since $f$ is bijective, $x \neq f^{-1}(y)$. Since $X$ is $\delta$-semi $T_2$, there exist $U_x, U_y \in \delta SO(X)$ such that $x \in U_x$, $f^{-1}(y) \in U_y$ and $U_x \cap U_y = \emptyset$. Moreover $f$ is $\delta$-semiopen and bijective, therefore $f(x) \in f(U_x) \in \delta SO(Y)$, $y \in f(U_y) \in \delta SO(Y)$ and $f(U_x) \cap f(U_y) = \emptyset$. Hence $f(U_x) \cap \delta Cl_S(f(U_y)) = \emptyset$. This shows that $G(f)$ is strongly $\delta$-semiclosed.

27 Theorem. If $f : X \to Y$ is quasi $\delta$-semicontinuous and $Y$ is $\delta$-semi $T_2$, then $G(f)$ is strongly $\delta$-semiclosed.

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since $Y$ is $\delta$-semi $T_2$, by Theorem 23 there exist $V \in \delta SO(Y, y)$ and $W \in \delta SO(Y, f(x))$ such that $\delta Cl_S(V) \cap \delta Cl_S(W) = \emptyset$. Since $f$ is quasi $\delta$-semicontinuous, there exists $U \in \delta SO(X, x)$ such that $f(U) \subset \delta Cl_S(W)$. This, therefore, implies that $f(U) \cap \delta Cl_S(V) = \emptyset$. So by Lemma 7, $G(f)$ is strongly $\delta$-semiclosed.

We close with the following questions which we were unable to answer:

Question 1. Is there any nontrivial example of a function with a strongly $\delta$-semiclosed graph?

Question 2. Are there nontrivial examples of the notions introduced in Definition 17?

Question 3. Is there a nontrivial example of a $\delta$-semipoint function?
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