Heisenberg's turbulent spectral theory determining the filtering procedure in LES models

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1. Introduction

Large-eddy simulation (LES) models represent a well-established technique to study the physical behaviour of the planetary boundary-layer (PBL). In general, the governing equations in the LES models are the incompressible Navier-Stokes equations described for a horizontally homogeneous boundary-layer. The resolved turbulent flow variables are obtained by the application of a low-pass spatial filter of characteristic width, the turbulence resolution length-scale (Pope, 2004), smaller than the scales of the resolved turbulent motions. This low-pass spatial filter width has the same order of magnitude as the numerical grid dimensions and, based on Kolmogorov spectral characteristics can be expressed in terms of the inertial subrange scales. The purpose of the present study is to show that Heisenberg´s classical theory for inertial transfer of turbulent energy (Heisenberg, 1948) can be used to select the filtering in LES models.

2. Heisenberg model for the kinematic turbulence viscosity

In his classical work, based on intuitive arguments, Heisenberg (1948) assumed that the process of the energy transfer from the small to the large wavenumbers in a Kolmogorov turbulent spectrum is similar to the conversion of mechanical energy into thermal energy through the agency of molecular viscosity. In other words, the physical picture that forms the basis of Heisenberg's theory is that, in the energy cascade process within the kinetic turbulent spectrum, the mechanism of inertial exchange of energy from large to small eddies is controlled by a kinematic turbulence viscosity (KTV). This introduces a division between scales at any arbitrary wave number in the inertial subrange. This scale separation is indeed naturally relevant in the LES frame, where a cut-off wave number is arbitrarily chosen in the inertial range, introducing then a well determinate division, to which Heisenberg's approach seems well suited to be applied. An eddy viscosity is the product of a characteristic turbulent length scale and a velocity, and thus dimensional analysis yields (Heisenberg, 1948 – see also Hinze, 1975):

$$\nu_T = \int_{k_c}^{\infty} C_H \sqrt{\frac{E(k)}{k^3}} dk \qquad (1)$$

where k is the wavenumber, C_H is Heisenberg´s dimensionless spectral transfer constant and $E(k)$ is the three-dimensional (3-D) turbulence energy spectrum in the inertial subrange, with the following form (Kolmogorov, 1941):

$$E(k) = \alpha_K \varepsilon^{2/3} k^{-5/3} \qquad (2)$$

where α_K is the Kolmogorov constant. Assuming that the small-scale turbulence (inertial subrange) should act on the large-scale turbulence like an additional eddy viscosity we substitute eq. (2) into eq. (1), where $C_H \approx$ 0.47 and $\alpha_K \approx 1.52$ (Muschinski and Roth, 1993; Corrsin, 1963), to obtain

$$\nu_T = 0.44 \varepsilon^{1/3} k_c^{-4/3} \qquad (3)$$

It is important to note that Eq. (3) was firstly derived by Muschinski and Roth (1993).

Following the philosophy of Kraichnan's eddy viscosity in spectral space, Lesieur and Metais (1996) present an eddy viscosity expressed by

$$\nu_T = \frac{2}{3} \alpha_K^{-3/2} \left[\frac{E(k_c)}{k} \right]^{1/2} \qquad (4)$$

where $E(k_c)$ is the kinetic energy spectrum at the cutoff k_c.

Now, substituting (2) with $k = k_c$ in Eq. (4) yields

$$\nu_T = 0.44 \varepsilon^{1/3} k_c^{-4/3} \qquad (5)$$

suggesting that Heisenberg's eddy viscosity is identical to the one proposed by Lesieur and Metais (1996, Eqs. (3.5) and (4.1)). A remarkable point here is the consistency of the theoretical model (4) with the experimental value for the Heisenberg's constant.

At this point it's important to note that the KTV obtained from the Heisenberg's model (eq. (3)) is expressed in terms of a cutoff wavenumber that defines the beginning of the inertial subrange and establishes the concept of sharp spectral filter. Therefore, Heisenberg's spectral transfer theory becomes the sharp spectral filter a natural filtering procedure to be applied in LES models. In fact, this type of filter has been largely employed in LES methodology.

3. Acknowledgements

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4. References

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