Improvement in Weighting Assignment Process in Analytic Hierarchy Process by Introducing Suggestion Matrix and Likert Scale

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***Abstract*— Analytic Hierarchy Process (AHP) has been widely used in varieties of decision making processes among several alternatives, where data on pair-wise comparisons are aggregated and the degree of importance of each alternative is quantified. The process of assigning importance or priorities against the alternatives has inherent limitations, which lead to higher possibility of inconsistency. This paper focuses on two basic limitations of the AHP, first one is its inconsistency generated from huge comparisons in judgment matrix and the second one is the use of 'ranking weightages' given by AHP. To eliminate these limitations, this research paper recommends to calculate relative importance among alternatives from the ratings assigned from "Likert scale" to form a "suggestion matrix" with zero percent CR before judgment matrix which gives privilege to decision makers to change relative importance within the range of CR. This process intensifies the effectiveness of AHP by reducing time consumption through optimizing inconsistency.**

***Keywords*—** *Suggestion matrix, Likert scale, CR, ratio, rule of transitivity, rule of reciprocity, resource allocation, times of importance etc.*

1. Introduction

The analytic hierarchy process (AHP) as a method for multi-criteria decision-making. It provides a way of breaking down the general method into a hierarchy of sub-problems, which are easier to evaluate. Through this method users can assess the relative weight of multiple criteria or multiple options against given criteria in an intuitive manner. Pairwise comparisons are more appealing to users than using quantitative ratings. Saaty established a consistent way of converting pairwise comparisons into a set of numbers. The numbers represent the relative priority of each of the criteria.

The pairwise comparison method was introduced by Fechner [1] and developed by Thurstone [2]. Based on pairwise comparison, Saaty [3], [4], [5], [6] proposes Analytical Hierarchy Process. AHP is a compensatory method. Here complete aggregation among criteria is assumed and a linear additive model is developed. The weights and scores are achieved basically by pairwise comparisons between all options with each other [7][8].

Two issues surround the use of the AHP. Firstly, according to Saaty, the allowable upper bound of consistency ratio (CR) is 10% [7][9]. Therefore pair-wise comparison matrices with CRs greater than 10% are not accepted, though it is tough to maintain required consistency in case of public opinion. The second issue surrounding the AHP involves the range of a_{ij}, the relative weight of alternative i to j. With the range of a_{ij} is changed from 1-9 to 1-15 (or 1-5), then its corresponding eigenvector and the rank of the priority of alternatives are also changed [10].

Firstly, with AHP the decision problem is decomposed into a number of subsystems, within which and between which a substantial number of pairwise comparisons need to be completed. This approach has the disadvantage that the number of pairwise comparisons may become very large (n(n−1)/2), and thus become a lengthy task [11]. Due to this lengthy task, users usually don't consider their past assigned value during giving new input value; which in turn creates inconsistency. Moreover consistent comparisons are being developed through 'trial & error' method which is troublesome in case large number of alternatives.

Secondly, the scale of relative importance plays a key role to quantify each Decision Maker's feeling. Therefore, which scale should be used in the process of a pair-wise comparison is the most controversial issue concerning the refinement of this method. Saaty's [3] 1-9 linear scale is long considered the standard of the AHP. But this scale is characterized by some deficiencies. To overcome the deficiencies of Saaty's scale, various judgment scales for a pair-wise comparison have been proposed and evaluated to date. Reducing the range of the linear scale to 1-5 was proposed by Aupetit and Genest [12]; extending the range of the linear scale to 1-13 and 1-50 was proposed by Harker and Vargas [13]. Moreover, two non-linear scales (quadratic and irrational) were also proposed by Harker and Vargas [13]. According to Lootsma [14], power scale

International Journal of Supply Chain Management
IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)

is superior to the 1-9 linear scale.

AHP has extensive application. Besides, some limitations have also been recognized. The author focuses on mainly two of them. One is huge number of pairwise comparison in judgment matrix which facilitates generating huge inconsistency and another is the rational quantification of qualitative factors.

2. Material and method

2.2 Literature review

The author of this paper has made some findings regarding AHP. In AHP, the user usually give input like whether a variable is more or less or equal important than another. Then the process assigns a quantitative value based on the qualitative factors. For pair-wise comparison, i.e. relative importance of one option over another is done using a 'scale of relative importance'. The assigned quantitative value is determined from the specified scale. The assigned value depends on the choice of scale. When a value of 3 has been assigned according to 1-9 scale, then it does not mean that the user is giving three times more importance to the variable than another. According to 9 point scale, assigning '3' means moderate importance, where to most of people three times more importance may be considered to be 'extreme'. But throughout the calculation of AHP, the values are treated as if they were multiplicative weighting of one respect to another. For example, if any user assigned a value 3 (moderate importance) to an alternative with respect to another, in the long run, the final result shows a weighting of 3 times more than another. But unfortunately three times more weighting may be considered as strong or extreme, depending on the individual user. It also differs from situation to situation.

Two different 'scale of relative importance' (1-9 and 1-5 point) is shown in table 1. Now if a user wants to give equal importance to option A & C and wants to give moderate importance on option B, then the pairwise comparison matrix may look like table 2 and table 3.

From the table 2 and table 3, it is seen that, for same typical user input, various result occurs due to scale difference. It is also noticed that assigning only moderate importance to one of the variable, generates a weighting percentage of three times (in case of 1-9 scale) [15] or two times (in case of 1-5 scale) than other alternatives. Now if a user wants to allocate resources to the alternative to which s/he wants to give moderate importance than others, how much should s/he allocate there? Whether 50% or 60% (values are particularly for this case)? Though the weighting percentage differs, but the ranking is same in both the cases.

Actually the corresponding weighting to qualitative variable varies from individual to individual. Some may consider 'five times' as extreme where some other may consider 'double or triple' as extreme. But in both the cases, same value (correspond to extreme importance) is assigned for the certain qualitative variable in AHP as per directed by the scale.

Table 1: Two different types of 'scale of relative importance'

'Scale of relative importance' (1-9 point)			**'Scale of relative importance' (1-5 point)**	
Qualitative variables	Quantitati-ve value		Qualitative variables	Quantita-tive value
Equal importance	1		Equal importance	1
Moderate importance	3		Moderate importance	2
Strong importance	5		Strong importance	3
Very Strong importance	7		Very Strong importance	4
Extreme importance	9		Extreme importance	5

Table 2: Pairwise comparison matrix formed by using traditional 1-9 point scale

	A	**B**	**C**	**Geometric Mean b_k**	**Normalized Weight, x_k**	**%**	**Rank**
A	1	1/3	1	0.69	0.2	20	2nd
B	3	1	3	2.08	0.6	60	1st
C	1	1/3	1	0.69	0.2	20	2nd

Table 3: Pairwise comparison matrix formed by using 1-5 point scale

	A	**B**	**C**	**b_k**	**x_k**	**%**	**Rank**
A	1	1/2	1	0.794	0.25	25	2nd
B	2	1	2	1.587	0.5	50	1st
C	1	1/2	1	0.794	0.25	25	2nd

So it can be concluded, that ***AHP gives weightage/rankings which can be used to prioritize the alternatives but weightage do not provide actual times of importance.***

2.3 Methodology

As the corresponding weightings to a certain qualitative input vary from individual to individual, the author of this paper recommends using Likert scale [16][17] primarily. After that, using the ratio of the data of Likert scale, a quantitative suggestion matrix has to be developed which is to be reviewed by the users. This enables the users to compare pairwise which is straight forward and convenient form of data input.

Firstly, user has to specify the 'scale of relative importance'. According to the scale, the range of scores of Likert scale would be chosen. If a user selects 1-9 or 1-5 scale of relative importance, then he has to score in 'Likert scale' out of 9 or out of 5 respectively. After scoring the alternatives users may get relative ratings of the options with respect to the one another. The process of taking relative rating from user is shown in table 4.

Table 4: Likert scale rating for n number of alternatives

Alternatives	Rating	Relative Rating (with respect to immediately previous one)
1	x_1	…
2	x_2	x_2/ x_1
3	x_3	x_3/ x_2
…	…	…
n	x_n	x_n/ x_{n-1}

Secondly, develop a quantitative suggestion matrix using relative rating as input from Likert scale. Quantitative suggestion matrix (table 5) may be formed by using rule of transitivity and rule reciprocity [18][19][20].

Here it is to be mentioned that as the span of Likert scale is equal to span of chosen 'scale of relative importance', so the derived relative importance from the ratio of inputs in Likert scale is compatible with the 'scale of relative importance'. For 1-9 scale, the maximum ratio will be 9 which correspond to maximum importance (extreme) according to 'scale of relative importance'.

Table 5: Quantitative suggestion matrix formation process for diagonal input (for n variables, n-1 input along first diagonal)

Alts	Alt1	Alt 2	Alt 3	…	Alt k	…	Alt n
Alt 1	1	$1/A_{21}$	$1/A_{31}$	…	$1/A_{k1}$	…	$1/A_{n1}$
Alt 2	A_{21}	1	$1/A_{32}$	…	$1/A_{k2}$	…	$1/A_{n2}$
Alt 3	A_{32} $*A_{21}$	A_{32}	1	…	$1/A_{k3}$	…	$1/A_{n3}$
…	…	…	…	…	…	…	…
Alt k	A_{k3} $*A_{21}$	A_{k3} $*A_{32}$	A_{k4} $*A_{43}$	…	1	…	$1/A_{nk}$
…	…	…	…	…	…	…	…
Alt n	A_{n3} $*A_{21}$	A_{n3} $*A_{32}$	A_{13} $*A_{41}$	…	$A_{n(k+1)}*$ $A_{(k+1)k}$	…	1

***Suggested values are marked with shadow*
***Alt means Alternative*

Thirdly, through reviewing the suggestion matrix, user can compare pairwise. If the suggested value seems not to be suitable, then user can change the value unless the CR remains within 10%. If the CR exceeds 10% then user has to modify the past assigned inputs.

2.4 Data collection and analysis

Let assume, a user primarily wants to give equal importance on option A & D, moderate importance on B and strong importance on C. At first the user has to give input in Likert scale and then the relative importance can be obtained (table 6).

Table 6: Getting relative rating from the input of Likert scale

	Likert scale Rating (out of 9)	Relative Rating	Likert scale Rating (out of 5)	Relative Rating
A	2	---	1	---
B	5	2.5	3	3
C	9	1.8	5	1.67
D	2	0.22	1	0.2

After that, the user has to place the relative rating along first diagonal of the pairwise comparison matrix and then using rule of reciprocity & transitivity, suggestion matrix is to be formed (table 7).

After reviewing suggestion matrix (table 7), the user has flexibility to modify the matrix in order to construct a better pairwise comparison. (Table 8 and Table 9)

Table 7: Suggestion matrix

1-9 scale						1-5 scale				
	A	B	C	D			A	B	C	D
A	1	0.4	0.22	1		A	1	0.33	0.2	1
B	2.5	1	0.56	2.5		B	3	1	0.6	3
C	4.5	1.8	1	4.5		C	5	1.67	1	5
D	1	0.4	0.22	1		D	1	0.33	0.2	1

***Suggested values are marked with shadow*

Table 8: Modified pairwise comparison matrix (using 1-9 scale)

	A	B	C	D	Geometric mean, b_i	Normalized Weight, x_i	Weightage percentage (%)
A	1	0.33	0.17	1	0.49	0.09	9
B	3	1	0.56	2.5	1.43	0.28	28
C	6	1.8	1	4.5	2.64	0.52	52
D	1	0.4	0.22	1	0.54	0.11	11
y_i	11	3.53	1.95	9			

***modified values are marked with shadow*

From table 8,
$\lambda_{max}=\sum x_i y_i=4.009$,
$CI=(\lambda_{max}-n)/(n-1)=0.003$,
$CR=CI/RI=0.0033=0.33\% \leq 10\%$

Table 9: Modified pairwise comparison matrix (using 1-5 scale)

	A	B	C	D	Geometric mean, b_i	Normalized Weight, x_i	Weightage percentage (%)
A	1	0.33	0.2	1	0.507	0.0996	9.96
B	3	1	0.5	2.5	1.392	0.2734	27.34
C	5	2	1	5	2.659	0.5225	52.25
D	1	0.4	0.2	1	0.531	0.1045	10.45
y_i	10	3.73	1.9	9.5			

***modified values are marked with shadow*

From Table 9,
$\lambda_{max}=\sum x_i y_i =4.001232$,
$CI=(\lambda_{max}-n)/(n-1)=0.00041$,
$CR=CI/RI=0.00046=0.046\% \leq 10\%$

3. Result and discussion

From the table 8 and table 9 it is noticed that, weightage percentages do not vary much with the change of scale. Here, ratio of Likert scale has been used. So weighting percentage will remain same for same user input using various scales. Use of Likert scale helps to understand the evaluation habit of individual. It indicates the higher and lower limit of user's input. As the ratio of data reflects the actual times of importance, the output weightage percentages not only give rankings but also reflect the actual times of importance. So it can be used for prioritizing as well other application like resource allocation.

Suggestion matrix gives advantage to infer pairwise comparison among alternatives before the formation of judgment matrix which limits the input of aberrant data. By this process total numbers of data input are reduced which in turns optimize inconsistency significantly.

Finally, suggested method is more user-friendly to input data and less time-consuming than traditional AHP. This method facilitates the use of AHP in critical decision making.

Conclusion

The recommended technique successfully incorporates the use of Likert scale along with pairwise comparison in Analytical Hierarchy Process. It also facilitates reduced inconsistency due to the use of suggestion matrix. It is expected global weighting found through the recommended process will represent the more acceptable decision under multi criterion environment [21][22][23].

Such method of introducing Likert scale along with suggestion matrix is equally applicable to various linear scales like 1-5, 1-9, 1-13, 1-50, 1-100 etc. But further research required regarding such application in non linear scale like quadratic, irrational etc. (Harker and Vargas, 1987).

It will be more acceptable, if the algorithm of the process of this work can executed through any software or programing language, for instance Matlab, Microsoft excel, C, C++ and so on.

More research work can be continued, about how much the value of relative importance can change from suggestion matrix to form a judgment matrix for multi-criterion environment within the range of CR and for different range of CR what could be the standard deviation.

Acknowledgments

The author would like to express gratitude towards Mr. Ainul Kabir for his unstinted help and co-operation. Finally, the author would like to convey gratitude to all of the concerned persons who helped in various ways.

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