

Differentiating between Spatial Ability as a Specific Rather than General Factor of Intelligence in Performance on Simple, Non-routine Problems in Mathematics

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Introduction

Spearman concluded that performance on any test of mental ability could be explained by several cognitive factors organized hierarchically as one general factor and several subordinate specific factors (Spearman, 1904, 1927). The general factor accounted for the significant amount of inter-correlation between all ability tests for any one individual while the specific factor explained the variation that was unique to each test. While there has been much debate in the literature as to what the specific factors are, with many different combinations of number and type of ability, three abilities — verbal, quantitative and spatial — consistently emerge as playing a dominant role in cognition (Kyllonen, 1996).

While Spearman was quite certain about the existence of 'g' he struggled to describe what it was in psychological terms and simply referred to it as a 'mental energy' (Spearman, 1904, 1927). Debate has since followed as to what 'g' is with one suggestion being that it may be working memory capacity since that can also explain a general aspect of performance across a range of tests (Kyllonen, 1996). Lohman (1993) favors the working memory argument but begins from the observation that tests of spatial ability serve as very good measures of general intelligence and, therefore, if working memory consists of a phonological loop and visual spatial sketchpad, with spatial ability related to the latter component, then spatial ability tests might be excellent measures of 'g'.

This leads to a conundrum: what should one conclude if a correlation is found between a test of spatial ability and, for example, a test of non-routine problem solving in mathematics? Has one observed an effect related to spatial ability as a specific, lower order factor of intelligence, or to spatial ability as a measure of 'g'? Given that spatial ability

has been defined as the ability to “generate, retain, retrieve, and transform well-structured visual images.” (Lohman, 1993, p. 3), and assuming the non-routine problems in mathematics do not contain any well-structured images, one might conclude the correlation is best explained by spatial ability having revealed ‘g’ rather than the specific factor. If so, then efforts to improve non-routine problem solving through spatial skills training may be misguided. By analyzing data collected from a sample of engineering students that was administered a test of non-routine problem solving in mathematics the purpose of this paper is to contribute to the discussion as to what spatial ability is and why measures of spatial ability can correlate with other tests that are not overtly spatial in nature.

Research Design

Two math tests were administered to a sample of 115 first year engineering students, 53 from Ohio State University and 62 from Dublin Institute of Technology. One test consisted of six simple word problems and the other of six questions to assess the core competencies needed to solve the problems, e.g. the ability to factor a quadratic equation. Thirty minutes were allowed to complete problems and questions. The Purdue Spatial Visualization Test of Rotations (PSVT:R, Guay, 1976) was administered to each group of students. College entrance test scores from the SAT and ACT tests were chosen as measures of general intelligence and these data were collected from those participants for whom they were available. Previous research suggests that both the SAT and ACT are suitable estimations of general intelligence (Coyle & Pillow, 2008). Unfortunately, the collection of this data was not included in the original research design and, therefore, SAT and ACT math data were available for 35 participants only and ACT English, Reading and Science Reasoning for 31 participants.

The first math test, problem solving, was scored in two ways based on the view that problem solving consists of two phases — representation and solution — with the representation step drawing on linguistic, semantic and schematic knowledge and the solution phase drawing on core competency knowledge (Mayer, 1992). To solve the problem a participant must first represent the problem correctly and then complete the solution phase correctly. The non-routine aspect of the problems surfaced in the representation phase only as the core competencies required for the solution phase in all problems were of a very basic standard.

Results and Discussion

A correlation matrix was created to examine the relationships between each of these variables as shown in Table 1. Correlations were calculated using the maximum number of cases available. Problem score is the combination of representation and solution.

Mathematical ability, as measured by the SAT math and ACT math tests, was found to be significantly related to performance in problem solving but not to problem representation

Table 1

Correlation matrix for all students for whom data were available. The number of cases used is shown in brackets after each correlation value.

	ACT Math	ACT English	ACT Read	ACT SCIRE	Problem score	Problem representation
PSVT:R	.249 (35)	.159 (31)	-.275 (31)	-.180 (35)	.577** (115)	.585** (115)
ACT/SAT Math		-.020 (31)	-.098 (31)	.047 (31)	.441** (35)	.289 (35)
ACT English			.345 (31)	-.006 (31)	-.065 (31)	-.120 (31)
ACT Read				.143 (31)	-.137 (31)	-.111 (31)
ACT SCIRE					-.087 (31)	-.114 (31)
Problem score						.715** (120)

and, relative to spatial ability, with smaller effect sizes. Verbal ability, as measured by the ACT English and reading tests was not found to be related to problem representation or solving. Likewise, the ACT science reasoning test was not significantly related to either aspect of problem solving. It appears that in solving the simple math word problems used in this study, both mathematical and spatial abilities are relevant and that of these, spatial ability has a slightly larger effect size. It is worth noting that the verbal ability level required for the items included in the current study would be considerably lower than the verbal ability threshold requirements for university entry. While this limits an in-depth analysis of the relationship between spatial and verbal ability, research from the last 50 years suggests that these two abilities are typically not closely related (Wai, Lubinski, & Benbow, 2009). In terms of representing the problems, however, only spatial ability was found to be relevant marking it out as separate and distinct from the other two abilities in this thought process.

A significant relationship between math ability and problem solving is to be expected as the problems are mathematical in nature. As tests of mathematical ability, the SAT and ACT have been found to have high reliability and validity and to be very good predictors of success in higher education in the US (Camara & Echternacht, 2000; Powers, Li, Suh, & Harris, 2016). One could regard them as measures of individual abilities and as metrics of general intelligence.

Conclusions

It is interesting to find relationships with problem representation being different for spatial ability on the one hand and SAT/ACT math on the other. SAT/ACT math is significantly related to problem solving but not problem representation whereas the PSVT:R measure of spatial ability is significantly related to both. Problem representation appears to draw on different aspects of cognition compared with the combination of representation and

solution. In this case, therefore, spatial ability appears to measure an aspect of thinking that is not measured by SAT/ACT math tests.

This conclusion is supported by the lack of significance in the relationship between problem solving/representation and the other ACT measures of reading, English and science reasoning. This suggests problem representation is cognitively different to Spearman's 'mental energy' or general intelligence since it is significantly related to only one of the ability tests. Problem representation, as described by the Mayer (1992) model, requires linguistic, semantic and schematic knowledge and the application of this knowledge to a non-routine scenario. The problems used in this study were linguistically and semantically very simple and required no more than common knowledge of these aspects. What these data appear to show is that development of an appropriate schema when representing a problem may draw on spatial ability as a specific cognitive factor rather than as a general factor of intelligence. If so, efforts to improve spatial ability in order to improve performance in mathematics are justified, particularly so in math courses that reward non-routine problem solving.

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