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Visualizing Compound Rotations with Virtual Reality

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Abstract

Mental rotations are among the most difficult of all spatial tasks to perform, and even those with high levels of spatial ability can struggle to visualize the result of compound rotations. This pilot study investigates the use of the virtual reality-based Rotation Tool, created using the Virtual Reality Modeling Language (VRML) together with MATLAB and the Simulink 3D Animation Toolbox, to assist engineering students in the visualization of compound rotations made about both fixed and mobile reference frames. This tool allows students to verify the non-commutative nature of compound rotations, as well as the relationship between fixed and mobile frame rotations. The effectiveness of the Rotation Tool is evidenced by the improved ability of students to work through questions pertaining to compound rotations, as well as their increased confidence when doing so.

Introduction

In mechanically-complex engineering courses, such as those specializing in robotics and mechatronics, it is imperative that students be able to accurately visualize three-dimensional (3D) objects, both as a sum of interconnected parts and as a whole. Moreover, students must also be able to visualize the motion of these objects in 3D space. These tasks require students to conduct complex geometric transformations consisting of translations and compound rotations. Although students typically demonstrate little difficulty visualizing the effect of linear translations, it has been observed that some have more difficulty visualizing the effect of compound rotations. In fact, of the three categories of spatial ability identified by Linn and Peterson, students have repeatedly demonstrated the largest discrepancy in their ability to perform mental rotations (Linn & Petersen, 1985), with rotations about multiple axes among the most difficult types of rotations to visualize (Onyancha, Derov, & Kinsey, 2009).

Traditionally, isometric drawings have been used as compound rotation visualization aids, depicting the starting, intermediate, and final orientations of a rotated object. They are often used in lecture notes and textbooks to demonstrate the non-commutative nature of compound rotations, as well as the difference between rotations made about a fixed frame of orthonormal axes and those made about a mobile frame that rotates with the object. Such static drawings encourage an analytic approach to breaking down compound rotations (Bruder & Wedeward, 2007), but assume that students can envision the correct motion associated with each step. In addition, examples must be

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chosen carefully to avoid the interpretation of the isometric views as two-dimensional (2D) patterns instead of the intended 3D representation (Branoff, 2000).

Animated presentations can depict the action of the rotation more clearly than static images, but still make students passive observers rather than active participants. Sketching employs a more active approach and can be beneficial in developing weak spatial abilities (Sorby, 1999), but generally involves drawing isometric views on 2D media that instructors must manually verify. Rotating physical manipulatives allows for a true visualization in 3D space, but has limited accuracy when demonstrating rotation angles that are not integer multiples of $\pi/2$. It has also been argued that students rotate physical manipulatives instinctively and too quickly to encourage proper examination of the motion (Gutiérrez, 1996). Instead, a software visualization aid that places limitations on the rotations is advocated to encourage students to predict the outcome of the motion and force a deeper analysis.

Virtual reality, with its interactive capabilities and its accurate 3D depictions, can be incorporated into a rotation visualization tool that does just that. Such a software tool will also be able to automatically verify results, providing students with immediate feedback and the ability to use the tool outside of the classroom. This paper investigates the use of such a virtual reality-based tool, the Rotation Tool, to aid students with the visualization of compound rotations in 3D, allowing them to gain deeper insight into the principles of compound rotations than is generally achieved with traditional teaching techniques alone. In the sections that follow, a review of existing rotation visualization software is provided, followed by a detailed description of the Rotation Tool. The methodology used in this study is then outlined, and the effectiveness of the tool is analyzed by examining student test results and survey feedback.

Literature Review

The importance of strong spatial skills - including mental rotation skills - in the areas of science, technology, engineering, and mathematics has long been argued (Wai, Lubinski, & Benbow, 2009), garnering the attention of engineering educators hoping to improve student performance and retention with targeted spatial training (Sorby, 2009). Sorby and her colleagues have conducted extensive research in the field (Sorby, 2009; Sorby, Casey, Veurink, & Dulaney, 2013), developing and using multimedia software and a workbook (Sorby & Wysocki, 2003) to train students in multiple aspects of spatial ability. Visualizing single and multiple rotations about a fixed frame are among the topics considered, and the non-commutative nature of compound rotations is emphasized. The modules developed by Sorby and Wysocki, along with a separate spatial assessment and training website (Blasko, Holliday-Darr, Mace, & Blasko-Drabik, 2004), have been successfully incorporated into training courses at multiple institutions (Veurink et al., 2009). Sorby (2011) cautions against using these modules with an audience of mixed spatial abilities, however, as they are intended to target those with weak spatial skills.

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Other examples of targeted mental rotation training include the exercises on single rotations developed for handheld touch screen devices by Martin-Dorta, Saorin, and Contero (2011). Rafi and Samsudin (2009) consider compound rotations with their interactive Desktop Mental Rotation Trainer (iDeMRT), prompting students to choose the set of consecutive rotations that will result in the overall change in orientation displayed. The interactive version of iDeMRT was found to be more effective than the animated version (Samsudin, Rafi, & Hanif, 2011), demonstrating the importance of active participation. Display Object (Hsi, Linn, & Bell, 1997) and the Physical Model Rotator (Kinsey, Towle, & Onyancha, 2008) pair the rotation of concrete objects with rotating computer images, allowing students to associate the motion seen on the screen with the corresponding real-world motion.

Wang, Yeh, Wang, Yang, and Rizzo (2011) use stereoscopic technology to help students visualize 3D rotations, finding that incorporating interactive control generates higher levels of student enthusiasm than more passive displays. Price and Lee (2010) also use stereoscopic technology to display rotations in 3D, finding that interpreting such displays increases the cognitive load of students compared to the visualization of rotations displayed on paper. Such findings underscore the importance of using new technology effectively, rather than relying on the technology itself to be the answer.

Desktop virtual reality tools capable of demonstrating important compound rotation principles include VRMath, a virtual learning environment that Yeh (2010) uses to demonstrate the non-commutative nature of compound rotations to disbelieving primary school students. Manseur includes two virtual reality-based tools with his textbook (Manseur, 2006) that demonstrate the motion of fixed and mobile frame rotations separately, each about a different set of non-configurable rotation axes. Bruder and Wedeward’s (2007) virtual reality rotation tool, on the other hand, compares the fixed and mobile frame rotations that result from the same set of three user-defined rotation angles and axes.

Rotation Tool

In order to break down the yaw-pitch-roll motion of a robot wrist, as shown in Figure 1a, or keep track of the effects of joint angles and link twists when considering the kinematics of a robot arm, as exemplified in Figure 1b, students must have more than strong visualization skills. They must also possess a solid understanding of the fundamental principles of compound rotations. The Rotation Tool, described below, allows students to explore these fundamental principles before having to apply them to such mechanically-complex examples.

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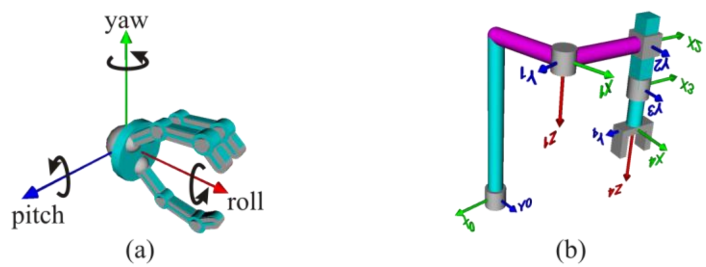

Figure 1. Examples of compound rotations in robotics. (a) The motion of a robot gripper with a spherical wrist can be broken down into compound rotations made about three orthogonal axes. (b) A link-coordinate representation of a SCARA robot arm with four degrees of freedom (DOF). Each DOF has two rotations associated with it, resulting in eight compound rotations to be considered for this relatively simple robot arm.

Tool Description

The Rotation Tool is an interactive educational tool that consists of a custom MATLAB graphical user interface (GUI) and a virtual reality window, as shown in Figure 2. Students use the GUI to enter up to three consecutive rotations about both the fixed and mobile reference frames, with complete freedom in their choice of rotation angles and axes. The GUI is designed to provide separate control of rotations made about the fixed and mobile frames, allowing students to verify the conditions that will lead to different and matching final orientations when making the two types of rotations, as exemplified in Figures 2b and 2c.

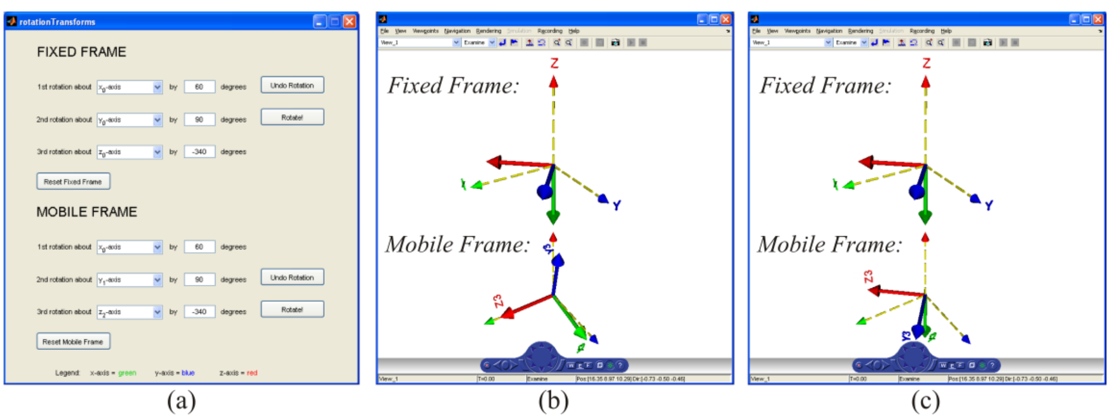

Figure 2. Rotation tool. (a) GUI showing values for three consecutive rotations around fixed and mobile frames, after the first fixed frame rotation and the first and second mobile frame rotations have been executed. (b) Virtual reality window showing final orientations based on the data entered in a). (c) Virtual reality window showing identical final orientations for both frame types when the order of the mobile rotations in a) is reversed. The comparison of the mobile frame orientations in b) and c) demonstrates the non-commutative nature of compound rotations.

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The 3D animation of each rotation in the virtual world is initiated by pressing the "Rotate!" buttons on the GUI, giving students complete, real-time control of the visualization. Undo options are also available to allow students to step back and forth between orientations, as needed, to gain a clear understanding of the rotation action. To preserve the order of the rotations, only the relevant rotation and undo buttons are enabled at any time. This built-in software constraint also serves as a way to encourage deeper analysis and understanding of the effect of each rotation on the overall orientation. Each frame can be reset at any time, and a new set of rotations can be entered and visualized.

Every effort has been made to ensure that the Rotation Tool is a user-friendly and engaging learning tool. It was designed with features intended to offset the increased cognitive load demands that virtual reality-based tools can place on students, and it incorporates feedback from students who used the tool prior to this study. For example, visual cues normally present in the virtual world's background (removed in Figure 2 for clearer printing) allow students to navigate the virtual world with ease, enabling them to change their viewpoint when occlusion occurs and gain a real sense of the 3D space being represented. The inclusion of both rotation frame types in the same virtual window guarantees that they are viewed from the same perspective, ensuring an accurate comparison of resulting orientations. Simple 3D graphics are used to represent the rotating frames, placing the focus on the rotations themselves, rather than on the visualization of geometrically-complex rotation objects. Similarly, a simple color scheme and clear axis labels ensure that minimal mental effort is required to correctly interpret the scene. Finally, smooth, animated transitions between initial and final orientations give students an unambiguous sense of the motion associated with each rotation.

Technical Considerations

The Rotation Tool's virtual world is defined by a Virtual Reality Modeling Language (VRML) (International Standard, 1997) file, and is controlled by MATLAB via the Simulink 3D Animation Toolbox. Although many programming languages can be used to interface with VRML, MATLAB was chosen because of students' existing familiarity with its environment from use in prerequisite courses and its proficiency in performing matrix multiplications. Within the MATLAB program, rotation matrices are used instead of other rotation representations, such as quaternions, to make the underlying code readable for students and to reinforce the mathematical difference between fixed and mobile rotations that is taught in the lectures. The resultant rotation matrices must be converted to their equivalent axis-angle representations in order to be displayed in VRML, and Paul's (1981) approach is used to avoid singularities when the net rotation angle is an integer multiple of π. In the case of no net rotation, the rotation axis is arbitrarily set to (1, 0, 0) to ensure that the VRML rotation axis remains well defined at all times.

Methodology

There are two robotics courses offered by the Department of Electrical and Electronic Engineering at the authors' university: "Mechatronics and Industrial Automation" offered to fourth year Electrical Engineering undergraduates, and "Mechatronics and Robotics" offered to students in the taught Masters in Mechanical Engineering program. Although these are separate courses, the classes were combined for the lectures on compound rotations.

Before the Rotation Lectures

To avoid making assumptions regarding students' spatial abilities and pre-existing knowledge of compound rotations, the students were given an Initial Rotation Knowledge Assessment (IRKA) before attending any lectures on rotations. The IRKA contained eight questions to test students' mental rotation skills, carefully chosen from the rotation section of the Purdue Spatial Visualization Test (PSVT:R) (Guay, 1976). As summarized in Table 1, these eight questions included both positive and negative rotations about single and double axes. They also encompassed all of the geometries identified by Onyancha et al. (2009), and special care was taken to correct any errors present in the original questions (Yue, 2007). Although a number of standardized mental rotation tests exist (Ekstrom, French, Harman, & Dermen, 1976; Gittler & Glück, 1998; Vandenberg & Kuse, 1978), the PSVT:R questions were chosen as the most appropriate because they do not contain impossible rotations or questions that rely on pattern recognition.

Table 1. Subset of Questions Selected from PSVT:R for IRKA

Number of Questions	Rotation Type	Rotation Angle
2	Single	$\pm \pi/2$
2	Single	π
2	Double	$\pm \pi/2$
2	Double	$\pm \pi/2, \pi$

The IRKA also contained a question to test the students' knowledge of the non-commutative nature of compound rotations, shown in Figure 3. The term "non-commutative" was strictly avoided in the question because the students may have been taught the terminology in the past without really understanding it. A cube with a regular geometry and uniquely patterned faces was used to simplify the visualization, in order to ensure that the focus remained on determining the non-commutative property of compound rotations. Rotation angles of $\pi/4$ made sketching the intermediate and final orientations difficult, forcing students to answer based on their existing knowledge or intuitive beliefs.

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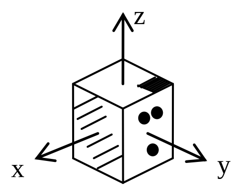
True or False?

You are given a cube (with 6 different faces) centered on the orthogonal frame {x, y, z}:



Rotating the cube 45 degrees about the z-axis, followed by a rotation of 45 degrees about the y-axis gives a **different** final orientation than rotating the cube 45 degrees about the y-axis, followed by a rotation of 45 degrees about the z-axis.

Figure 3. Non-commutative question on the IRKA.

Students were given approximately 15 minutes to complete the IRKA, after being told that participation was voluntary and would have no impact on their marks for the course. It was determined that this time interval would remove the time pressure normally associated with the PSVT:R, and allow students to solve the visualization questions either holistically or analytically, as suited their preference and visualization abilities. To avoid influencing the students' choice of strategy, coordinate axes were not added to the PSVT:R questions (Branoff, 1998); however, students were allowed to add their own axes or make sketches directly on the IRKA, if desired. Students were also encouraged to provide feedback at the end of the IRKA in the form of open comments.

Tutorial After the Rotation Lectures

After the relevant material on rotations had been covered in the traditional lecture format, the students were invited to attend a voluntary tutorial in the computer lab that would use a virtual reality-based tool to reinforce the topics covered in class. The tutorial was held during regularly scheduled class time so as not to interfere with the students' schedules. The students were again assured that participating would have no impact on their grades for the course.

The tutorial was formatted to follow a time-interrupted series method of evaluation appropriate for small populations, and consisted of a pre-test, treatment, and a post-test. In this case, the treatment was the use of the Rotation Tool to complete a worksheet. It was hypothesized that the students would have a better understanding of the fundamental principles of compound rotations after the treatment, as measured by the pre- and post-tests. It was further expected that the treatment would increase the students' confidence in their ability to understand fundamental compound rotation principles, as measured by survey feedback.

Pre-Test. The pre-test was designed to determine the students' level of understanding after being taught about compound rotations in a traditional lecture setting. Rather than

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testing students' abilities to perform mental rotations, as was already tested in the IRKA, this test was designed to focus on how well they could apply the fundamental principles of compound rotations. The first page contained detailed instructions on how to complete the test, along with two examples to ensure that there was no ambiguity or confusion over nomenclature or what was being asked. This was followed by six questions on the visualization of compound rotations and the application of their principles, as summarized in Table 2, as well as one Likert-style survey question asking students to rate their understanding of the difference between fixed and mobile frame rotations based on what they had learned in the lectures.

Table 2. Pre-Test Questions

Question	Task required
1	Visualize double rotation about fixed frame.
2	Apply knowledge of non-commutative nature of compound rotations.
3	Apply knowledge of relationship between fixed and mobile frame rotations.
4	Visualize triple rotation about mobile frame.
5	Apply knowledge of non-commutative nature of compound rotations.
6	Apply knowledge of relationship between fixed and mobile frame rotations.

The visualization questions involved the rotation of a cube by $\pm \pi/2$ or π, about either a fixed frame or a mobile frame, as illustrated in Figure 4. A cube was selected as the rotation object to simplify the visualization task, concentrating instead on testing each student's ability to make rotations about the specified frame type and to correctly apply the right-hand rule. Each pre-test question was designed to build on the previous question, thereby also minimizing the visualization effort required. For example, Questions 2 and 3 are simply permutations of the rotations done in Question 1, and do not require students to visualize the rotations if they understand the underlying compound rotation principles. Similarly, the visualization required in Question 4 is actually just one rotation added to the result of Question 1, and Questions 5 and 6 are permutations of Question 4. All of the questions that did not specifically require visualization included a choice of "Don't know" as an answer to discourage students from simply guessing.

The students were given approximately 20 minutes to complete the pre-test, an adequate amount of time for students who realized the relationship between the questions and had a good understanding of the underlying theory.

Treatment. Once the pre-tests were collected, a brief demonstration of the main features of the Rotation Tool was given. During this demonstration, two examples were completed that emphasized the main principles of compound rotations. Detailed written instructions on the Rotation Tool's use and main features, as well as the examples covered in the demonstration, were provided during the tutorial. They were also made available online to all students registered in the course, for easy reference throughout the term.

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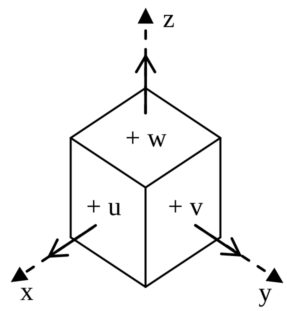
You are given a cube with an attached orthogonal *mobile* frame {u, v, w}, where each face of the cube is labelled according to its corresponding mobile axis. This cube is centered on the orthogonal *fixed* frame {x, y, z} so that the two frames coincide initially, as shown:



If you rotate the cube from its initial position by **-90°** about the fixed **x-axis**, and then **90°** about the fixed **z-axis**, what is the final orientation?

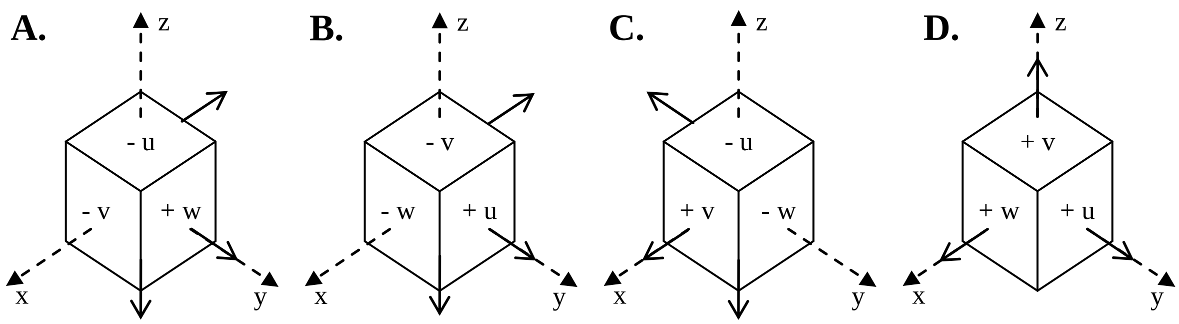


Figure 4. Question 1 from the pre-test. The incorrect solutions include the results of rotations made about the wrong rotation frame type and/or in the wrong direction.

After the demonstration, the students were asked to use the Rotation Tool to complete a worksheet, encouraging the students to explore the capabilities of the Rotation Tool with purpose. The students used the Rotation Tool to visualize different sets of compound rotations made about both fixed and mobile frames, and identified which conditions yielded the same, or different, final orientations. They were then asked to generalize their findings, thereby deriving the universal principles of compound rotations.

The students were given approximately 20 minutes to complete the worksheet using the Rotation Tool, either on their own or in pairs. They were encouraged to discuss results with each other and ask questions of the instructors as needed.

Post-Test. The general format of the post-test matched that of the pre-test, with six multiple choice questions testing the students' abilities to visualize different sets of compound rotations and their understanding of the compound rotation principles. A survey consisting of five Likert-style questions and two open comment questions was

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also included. The students were given approximately 20 minutes to complete the survey and post-test without the aid of the Rotation Tool.

Results and Discussion

Before the Rotation Lectures

A total of 28 students completed the IRKA. The percentage of students who correctly answered each question is shown in Figure 5.

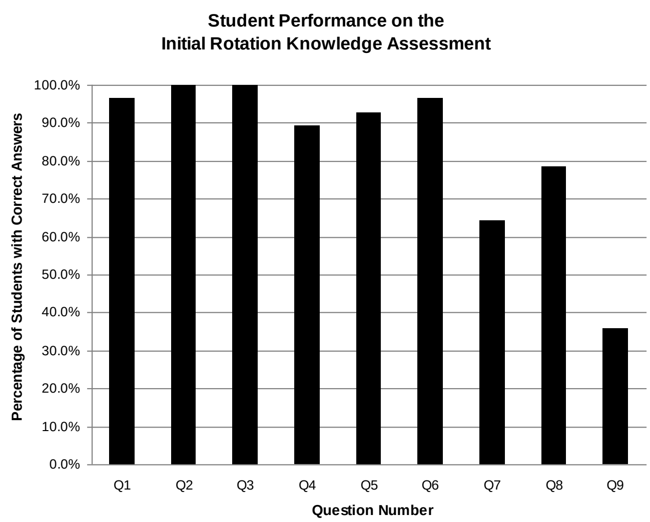

Figure 5. The breakdown of correct answers on the IRKA, where Q1-Q8 are the PSVT:R questions outlined in Table 2, and Q9 is on the non-commutative nature of compound rotations.

Mental Rotation Skills. The mean score for the subset of PSVT:R questions was 89.7%, with a standard deviation of 10.8%. Although a direct comparison with previous studies using the same subset of questions is not possible, such a high score clearly demonstrates that the students entered the course with high spatial skills and a good ability to perform mental rotations. Therefore, poor spatial abilities can be definitively ruled out in this study as a contributing factor to students' difficulties in visualizing the result of compound rotations.

As Figure 5 shows, Questions 7 and 8 each received a lower percentage of correct responses than all of the other PSVT:R questions. This indicates that double rotations with rotation angles of $\pm\ \pi/2$ and π are the most difficult rotation types on the PSVT:R for students to perform, consistent with the findings of Onyancha et al. (2009). In turn, this suggests that regardless of a student's ability to perform mental rotations, compound rotations become increasingly more difficult to visualize as the number of rotations and unique rotation angles increase. This could be because of increased spatial memory requirements. Regardless of the underlying reason, a visualization tool

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like the Rotation Tool will remove the ambiguity and difficulty in visualizing the orientation, allowing students to focus on examining the underlying compound rotation principles.

It should be noted that no distinction is made between the performance of female and male students on the PSVT:R questions, as is done in other studies. The small class size and small percentage of female students, typical of many high-level engineering courses, made such a distinction impractical. However, the large discrepancy in scores seen in other studies was not present in this study. This could be because the group of students tested here was well advanced in their engineering studies and either had naturally strong spatial abilities that enabled them to remain in engineering, or had developed them during their studies. It could also be because a subset of the PSVT:R questions was used, and measures were taken to remove biases favouring holistic strategies over analytic ones.

Non-commutative Nature of Rotations. Contrary to the high performance on the PSVT:R questions, only 35.7% of students answered the question on the non-commutative nature of compound rotations correctly. Such a low percentage indicates that the intuitive belief that rotations are commutative held by primary students (Yeh, 2010) persists into higher levels of learning. Although assumed to be a straightforward fact to teach students, this could be a significant contribution to students' inability to effectively visualize compound rotations and understand their underlying principles.

Student Feedback. Seven out of the 28 participating students chose to leave comments at the end of the IRKA. Most seemed to enjoy working through the IRKA, leaving comments along the lines of, "that was fun," and a couple of students commented on the increasing difficulty of the questions as the test went on. One student did comment specifically on having difficulty visualizing the 3D shape in Question 8, stating that the "shape can be interpreted in different ways." Although questions were carefully selected in an effort to avoid these types of problems, this does highlight the difficulty of visualizing 3D shapes from isometric drawings on a 2D medium.

One student "didn't really understand [the] last question [Q9]" and thought that a "sample would help." Although all 28 students answered the question and only one student commented on the difficulty of it, it is possible that other students also found the change in format for the last question confusing. On closer examination of the returned IRKAs, it was found that nine students made deliberate marks on their paper, indicating a good understanding of what the question asked. Of these nine, only 33.3% answered the question correctly. This corresponds closely with the overall percentage of students who answered the question correctly. Thus, despite any possible confusion with the change in question format, it can be confidently concluded that the majority of the students did not understand that rotations are non-commutative.

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Tutorial After the Rotation Lectures

During the tutorial, the students were asked to return three items: the pre-test, the worksheet, and the post-test. Of the documents that were returned, 21 complete sets could be matched up based on anonymous student identification numbers. The results for these 21 students are reported below.

Worksheet. In general, the students used the Rotation Tool to complete the worksheet without difficulty. There were very few questions asked of the instructors, and all students were observed to be eager to try the tool out for themselves, even those who were working in small groups.

Overall, students performed very well on the worksheet, further demonstrating their ability to use the Rotation Tool correctly and easily. The mean score on the worksheet was 86.2%, with a standard deviation of 19.6%. Of particular note, all 21 students correctly identified that compound rotations are non-commutative because the order of the rotations affects the final orientation. In addition, 19 of the 21 students correctly identified the relationship between fixed and mobile frame rotations.

Pre-Test vs. Post-Test. The mean pre-test score of 42.9% (standard deviation of 25.0%) was lower than expected, given that the students had already attended lectures on compound rotations, and demonstrates the need for the addition of non-traditional teaching methods. The mean post-test score was 70.6% (standard deviation of 29.3%), resulting in a mean gain of 27.8% (standard deviation of 31.8%). Despite the small sample size, the distribution of gains in test scores follows a normal distribution, as determined by visual inspection and tested using the Kolmogorov-Smirnov test ($p = 0.6512$), the Shapiro-Wilk test ($p = 0.5374$), and the Anderson-Darling test ($p = 0.4034$). The improvement in test scores after the treatment is highly significant ($t = 4.01$, $p < 0.001$), with seven students earning perfect post-test scores compared to just one earning a perfect pre-test score.

Despite the improvements in the post-test performance over that of the pre-test, it was observed that many students struggled to finish both the pre-test and post-test in the allotted time. In fact, only 11 students completed the pre-test, whereas 15 students finished the post-test. Although the Wilcoxon signed-ranks test indicates that the median increase in the number of questions answered after the treatment is statistically significant ($z = 1.91$, $p < 0.05$), it is clear that some students still had trouble finishing all six questions on the post-test. Closer examination of the returned tests reveals that a number of students made sketches for the questions that did not strictly require visualization, with a higher percentage of students making sketches for every question of the post-test than for the pre-test. Although it is encouraging to see the students take an analytical approach similar to the breakdown of rotations advocated by the Rotation Tool, it is clear that the students either did not realize the relationships between the questions or they did not trust their understanding of the relationships. Instead of applying the compound rotation principles that most were able to correctly identify when

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completing the worksheet just moments before, they relied on the concrete visualizations provided by sketches to answer all of the post-test questions.

Student Feedback. The students were asked to rate how well they understood the difference between fixed and mobile frame rotations, both before and after using the Rotation Tool. These ratings are compared in Figure 6, and clearly illustrate the students' increased confidence in their understanding after using the Rotation Tool. This corresponds with the increase in general rotation knowledge that the large majority of students felt resulted from using the Rotation Tool, as shown in Figure 7.

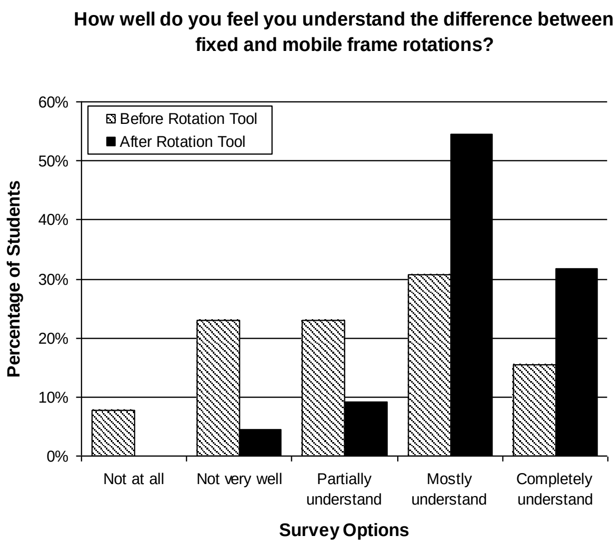

Figure 6. Comparison of students' self-rated level of understanding of the difference between fixed and mobile frame rotations, before and after using the Rotation Tool.

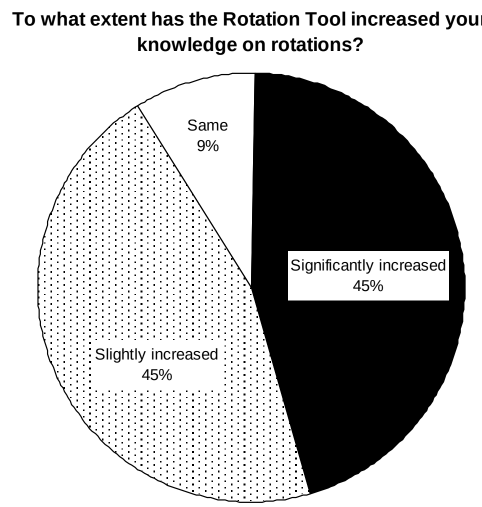

Figure 7. Student feedback after using the Rotation Tool. Note that no students felt that the Rotation Tool had significantly or slightly decreased their knowledge of rotations.

The written comments received from students at the end of the tutorial were overwhelmingly positive, with multiple comments along the lines of "this was cool" and "I'd [like] to have more classe[s] like this." The value of the Rotation Tool as a visualization aid was made particularly clear from the enthusiastic comments left by students who felt that they had struggled to understand the concepts taught in the lectures, with one student stating, "I will use it after this because I don't get this stuff at all but this will make it [a] lot easier." Even students entering the tutorial with confidence in their knowledge and ability to visualize compound rotations saw the benefit of the Rotation Tool, as evidenced by the student who commented, "Feel as if I may already have some degree of spa[t]ial awareness; however this tool definitely allows one to visualize this better."

Conclusions

Although the students demonstrated strong mental rotation abilities, many exhibited relatively low self-confidence in their understanding of fundamental compound rotation principles when taught using traditional methods. This could be because traditional visualization aids are unable to overcome incorrect intuitive beliefs held by a majority of students, such as the belief that compound rotations are commutative. Focused use of the Rotation Tool yielded significant improvements in test results and an increase in students' self-rated confidence in their understanding of compound rotation principles. This demonstrates the power of the interactive, virtual reality-based visualization aid, even when used for a very short time.

Despite the improvements seen, it is clear from the number of sketches on the returned post-tests that the students were still not completely confident in applying their knowledge of compound rotation principles, preferring to verify their answers with concrete visualizations. Although a perfectly acceptable method of solving the questions, it indicates room for improvement in how the Rotation Tool is incorporated into the curriculum. Going forward, the worksheet will be modified to encourage a deeper analysis of *why* certain combinations of rotations result in different, or matching, orientations, and will ask students to apply the compound rotation principles to predict the resulting orientations of compound rotations before checking it with the Rotation Tool. In addition, the relationship between questions on the pre- and post-tests will be made more obvious, something that may have been easily overlooked by students under pressure to complete the tests.

References

Blasko, D. G., Holliday-Darr, K., Mace, D., & Blasko-Drabik, H. (2004). VIZ: The visualization assessment and training web site. *Behavior Research Methods, Instruments, & Computers, 36*(2), 256-260.

Branoff, T. (1998). The effects of adding coordinate axes to a mental rotations task in measuring spatial visualization ability in introductory undergraduate technical graphics courses. *Engineering Design Graphics Journal, 62*(2), 16-34.

Branoff, T. J. (2000). Spatial visualization measurement: A modification of the Purdue spatial visualization test – visualization of rotations. *Engineering Design Graphics Journal, 64*(2), 14-22.

Bruder, S., & Wedeward, K. (2007). An interactive online robotics course. *Intelligent Automation & Soft Computing, 13*(1), 105-116.

Ekstrom, R. B., French, J. W., Harman, H. H., & Dermen, D. (1976). *Manual for kit of factor-referenced cognitive tests*. Princeton, NJ: Educational Testing Service.

Gittler, G., & Glück, J. (1998). Differential transfer of learning: Effects of instruction in descriptive geometry on spatial test performance. *Journal for Geometry and Graphics, 2*(1), 71-84.

Guay, R. B. (1976). Purdue spatial visualization tests: Visualization of rotations. West Lafayette, IN: Purdue Research Foundation.

Gutiérrez, A. (1996). Visualization in 3-dimensional geometry: In search of a framework. In L. Puig & A. Gutiérrez (Eds), *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education, held in Valencia, Spain, 8-12 July 1996* (Vol. 1, pp. 3-19). Valencia: Universidad de Valencia.

Hsi, S., Linn, M. C., & Bell, J. E. (1997). The role of spatial reasoning in engineering and the design of spatial instruction. *Journal of Engineering Education, 86*(2), 151-158.

International Standard (1997). ISO/IEC 14772-1:1997 Information technology -- Computer graphics and image processing -- The Virtual Reality Modeling Language (VRML) -- Part 1: Functional specification and UTF-8 encoding.

Kinsey, B. L., Towle, E., & Onyancha, R. M. (2008). Improvement of spatial ability using innovative tools: Alternative View Screen and Physical Model Rotator. *Engineering Design Graphics Journal, 72*(1), 1-8.

Linn, M. C., & Petersen, A. C. (1985). Emergence and characterization of sex differences in spatial ability: A meta-analysis. *Society for Research in Child Development, 56*(6), 1479-1498.

Manseur, R. (2006). *Robot modeling and kinematics*. Boston, MA: Charles River Media.

Martin-Dorta, N., Saorin, J. L., & Contero, M. (2011). Web-based spatial training using handheld touch screen devices. *Educational Technology & Society, 14*(3), 163-177.

Onyancha, R. M., Derov, M., & Kinsey, B. L. (2009). Improvements in spatial ability as a result of targeted training and computer-aided design software use: Analyses of object geometries and rotation types. *Journal of Engineering Education, 98*(2), 157-167.

Paul, R. P. (1981). *Robot manipulators: Mathematics, programming, and control*. Cambridge, MA: MIT press.

Price, A., & Lee, H.-S. (2010). The effect of two-dimensional and stereoscopic presentation on middle school students' performance of spatial cognition tasks. *Journal of Science Education and Technology, 19*(1), 90-103.

Rafi, A., & Samsudin, K. (2009). Practising mental rotation using interactive Desktop Mental Rotation Trainer (iDeMRT). *British Journal of Educational Technology, 40*(5), 889-900.

Samsudin, K., Rafi, A., & Hanif, A. S. (2011). Training in mental rotation and spatial visualization and its impact on orthographic drawing performance. *Educational Technology & Society, 14*(1), 179-186.

Sorby, S. A. (1999). Developing 3-D spatial visualization skills. *Engineering Design Graphics Journal, 63*(2), 21-32.

Sorby, S. A. (2009). Educational research in developing 3-D spatial skills for engineering students. *International Journal of Science Education, 31*(3), 459-480.

Sorby, S. (2011). *Improving Spatial Visualization Skills [PowerPoint slides]*. Retrieved from ENGAGE website: http://www.engageengineering.org/associations/11559/files/ENGAGE Improving Spatial Visualization Skills Webinar JAN 27 2011.pdf.

Sorby, S., Casey, B., Veurink, N., & Dulaney, A. (2013). The role of spatial training in improving spatial and calculus performance in engineering students. *Learning and Individual Differences, 26*, 20-29.

ISSN: 1949-9167

Sorby, S., & Wysocki, A. F. (2003). *Introduction to 3D spatial visualization: An active approach*. Clifton Park, NY: Thomson Delmar Learning.

Vandenberg, S. G., & Kuse, A. R. (1978). Mental rotations, a group test of three-dimensional spatial visualization. *Perceptual & Motor Skills, 47*(2), 599-604.

Veurink, N. L., Hamlin, A. J., Kampe, J. C. M., Sorby, S. A., Blasko, D. G., Holliday-Darr, K. A., . . . Knott, T. W. (2009). Enhancing Visualization Skills-Improving Options aNd Success (EnViSIONS) of engineering and technology students. *Engineering Design Graphics Journal, 73*(2), 1-17.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology, 101*(4), 817-835.

Wang, C.-Y., Yeh, S.-C., Wang, J.-L., Yang, S.-C., & Rizzo, A. (2011). A 3D motion controlled method for the training of mental rotation. In I. Aedo, N.-S. Chen, D. G. Sampson, J. M. Spector, & Kinshuk (Eds.), *Proceedings of the 11th IEEE International Conference on Advanced Learning Technologies (ICALT) held in Athens, GA, 6-8 July 2011* (pp. 1-3). Los Alamitos, CA: IEEE Computer Society.

Yeh, A. (2010). Three primary school students' cognition about 3D rotation in a virtual reality learning environment. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the Future of Mathematics Education : Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia (MERGA) held in Fremantle, Western Australia, 3-7 July 2010* (pp. 690-697). Fremantle, WA: MERGA.

Yue, J. (2007). Spatial visualization by isometric view. *Engineering Design Graphics Journal, 71*(2), 5-19.

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